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CSE 546: Homework #0

**A.1**

According to the problem we have:

where is the event the test is positive and is the event you have the disease and is the event you do not have the disease. Thus, using Bayes rule, the probability of having the disease when you test positive is:

**A.2**

**a)**

If , then using the law of total expectation:

**b)**

From (a):

If and are independent, . Thus,

**A.3**

**a)** To calculate the PDF you can first calculate the CDF and take the derivative.

where is probability. Using the rule of total probability:

But using the definition of CDF we can now see:

Thus:

Now we can take the derivative to find the PDF:

**b)** The PDF of the sum of two random variables is a convolution as shown in part (a). Thus, we simply need to convolve the two uniform distributions together:

**A.4**

indicates a mean of 0 and variance of 1. With linear transformations of random variables, we know that (from lecture and Murphy chapter 2):

Since only affects the variance, we can first find a such that :

Next, must offset the newly scaled mean, and thus:

**A.5**

If and are independent and identically distributed, then we know (from lecture and Murphy chapter 2):

Using these in conjunction with how the mean and variance change due to the linear transformation (as in A.4) we know:

Applying another linear transformation again changes the mean and variance:

**A.6**

**a)** The expected value of a sum of independent random variables is the sum of the expected values of each random variable:

Thus,

**b)** To find the variance of , we can first find the variance of :

due to how is defined to be 1 when less than and 0 when greater than, the square of the function is equal to itself. Thus:

Now, using the answer from A.5, we can show:

**c)** To find the maximum, we must find:

We find the derivative and set it equal to zero to find the maximum:

Plugging that into the variance we found in part (b):

Thus:

**B.1**

To calculate the expected value, we want to find the PDF which we obtain through the CDF:

Since the maximum means everything is less than or equal to, we can simply convert that to:

Since all are independent, we can further convert that to:

Since all are identically distributed, we have:

The PDF is then the derivative of the CDF:

is the CDF of . Since they are a uniform distribution from 0 to 1:

Now we can calculate the expected value using the definition, from 0 to 1 as that is the limit of the distribution:

**A.7**

**a)** The rank of matrix is 2. The third column is three times the first column minus the second column, but columns 1 and 2 are linearly independent. The rank of matrix is also 2. The third column is the sum of the first two columns, but the first two columns are linearly independent.

**b)** The basis for both matrices is the first two columns:

**A.8**

**a)** Basic matrix multiplication. The row/column of the result is the dot product of the row from the first matrix and the column from the second.

b) To solve, we need to find the inverse of matrix :

First, make sure the determinant is not zero to ensure the matrix can be inverted:

Next, transpose the matrix and find the cofactors (determinants of each minor matrix with correct signs applied)

And finally multiply by one over the determinant:

**A.9**

**a)** We set the equation:

**b)** We set the equation:

c) Since

**A.10**

**B.2**

The definition of matrix multiplication provides:

Therefore, for the diagonals, we have:

So then we can rewrite trace:

Now, simply rearranging:

**B.3**