

# Challenge 07: Extremal Higher-Dimensional CFTs with Stress Tensor

Pure Thought AI Challenge 07

Pure Thought AI Challenges Project

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## Abstract

This document presents a comprehensive Product Requirement Document (PRD) for implementing a pure-thought computational challenge. The problem can be tackled using only symbolic mathematics, exact arithmetic, and fresh code—no experimental data or materials databases required until final verification. All results must be accompanied by machine-checkable certificates.

## Contents

**Domain:** Quantum Gravity **Difficulty:** Medium-High **Timeline:** 6-12 months

## 0.1 Problem Statement

Determine whether "nearly extremal" unitary CFTs exist in  $d=3,4$  with large gaps to higher-spin currents (pure gravity holographic duals).

## 0.2 Core Question

What is the maximum gap to spin- $J$  currents in a  $d$ -dimensional CFT?

## 0.3 Mathematical Formulation

**Setup:** CFT $_d$  with stress tensor  $T$ , central charge  $c_T$

**Gap assumption:** No spin- $J$  conserved current for  $J > J_{max}$

**Bootstrap equations:**

$$\langle T(u) T(v) T(u) T(v) \rangle = \sum_{J=0}^{\infty} \frac{C_{T,T,0} G_{-J}(u,v)}{C_{T,T,0}}$$

**Mixed correlator:** Also include  $\langle T T \rangle$  for scalar

**Constraints:**

- Crossing symmetry
- Unitarity:  $C_{T,T,0}^2$
- Ward identities for  $T$
- Assuming no spin-4, 6, 8, ... currents

## 0.4 Implementation

```

1 def conformal_block_4pt_Tmunu(Delta, J, d=3):
2     # Solve Casimir differential equation
3     pass
4
5 def setup_crossing_T4(J_max_current):
6     # Include identity, T, and operators
7     # Exclude spin-4, 6, ... if assuming their absence
8     pass
9
10 def extremal_functional_method():
11     # Find such that (crossing eq) < 0
12     # with 0 for excluded region
13     pass

```

## 0.5 Example Prompt

```

1 Bootstrap the stress tensor 4-point function in d=3.
2 Assume no spin-4 current exists. Derive lower bound on the gap
   _gap to
3 first non-conserved operator. Use extremal functional method.
4 Compare to known CFTs (Ising, O(N), etc.).

```

## 0.6 Success Criteria

**MVR:** Reproduce known bounds for  $d=3$  Ising-like CFTs

**Strong:** New universal bound on spin-4 gap in holographic window

**Publication:** Proof that pure AdS gravity requires large higher-spin gaps

## 0.7 Verification

```
1 def verify_cft_bound(Delta_gap_min, extremal_func):  
2     # Check extremal functional is positive on allowed region  
3     # Check it proves _gap < _gap_min is impossible  
4     assert functional_certifies_bound(extremal_func, Delta_gap_min)
```