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Retail Shelf Allocation: A Comparative Analysis of Heuristic and Meta-Heuristic Approaches

Jared M. Hansen[a,*], Sumit Raut[b,1], Sanjeev Swami[c,2]

[a] *University of North Carolina at Charlotte, 9201 University City Blvd, Charlotte, NC 28262, United States*
[b] *Tata Consultancy Services, Akruti Trade Center, MDIC, Andheri East, Mumbai 400093, India*
[c] *Dayalbagh Educational Institute, Dayalbagh, Agra, UP 282110, India*

Abstract

This research presents a retail shelf-space decision model that incorporates a nonlinear profit function, vertical and horizontal location effects, and product cross-elasticity. We propose a linear programming formulation of the nonlinear profit function that can solve the shelf-space problem optimally. We describe potential advances in heuristic and meta-heuristic algorithms and compare the approaches through simulations and a field experiment. We discuss the impact of the number of item facings, vertical location, and horizontal location (e.g., we find the vertical location effect is approximately double the size of the horizontal location effect on profit performance).

Keywords: Retail shelf space; Meta heuristic; Nonlinear profit function

Retail shelf-space management is one of the most difficult aspects of retailing. A significant reason is that while retail shelf-space is fixed, the numbers of new potential products (Dreze, Hoch & Purk 1994), customer wants (Corstjens & Doyle 1983), and competitors (Grewal et al. 1999; Hansen 2009) are constantly growing and evolving. At the same time, customers are consolidating shopping trips toward multi-purpose shopping (e.g., Popkowski et al. 2004). Thus, the success of any retailer depends on its ability to match its changing environment by continually deciding between *how much* of *which* products to shelve *where* and *when*. Indeed, the shelf *location* of products can significantly affect the products, and thus merchandise category, performance (Dreze, Hoch & Purk 1994). Thus, retailers benefit by expanding their focus from product-level performance to the total shelf-space configuration.

The practice of analyzing shelf-space costs and product performance has been standard for several decades in retail practice and literature (e.g., Wickern 1966); many retailers have now adopted software programs such as Spaceman or Prospace for creating planograms. These programs can display historical product sales, profits, or inventory turnover information. In past years, the actual decisions of items to shelf location were usually made through human judgment because of the near infinitesimal combinations of shelf-space arrangements. As a result, shelf-space software programs are often only used as a visual template, and not to perform analysis. However, advancements in computing resources have permitted the development of more complex shelf-space models that are more consistent with consumer decision making (e.g., Borin & Farris 1995; Urban 1998).[3] Retail corporate buyers, category managers (who are employed by manufacturers), and retailer consultants can use these shelf-space models to improve their decision making, resulting in better financial performance.

In this research, we first integrate the following three important elements into the basic shelf-space decision model: (1) a nonlinear profit function, (2) location effects, and (3) product cross-elasticity. In contrast to much of the research that has used *nonlinear programming* to model the nonlinear profit function, we propose a novel *linear programming* formulation of the nonlinear profit function that can solve the shelf-space problem

* Corresponding author. Tel.: +1 704 687 7303.
E-mail addresses: mktg.theory@gmail.com (J.M. Hansen), sumit.raut@tcs.com (S. Raut), sswami1853@gmail.com (S. Swami).
[1] Tel.: +91 9004767498.
[2] Tel.: +91 562 2801545.

[3] We recommend readers interested in early shelf space model evolution literature to Anderson and Amato (1974), Hansen and Heinsbroek (1979), Corstjens and Doyle (1981), Gochet and Smeers (1979), and Bultez and Naert (1988).

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optimally. We then extend the retail shelf-space literature by comparing potential advances in heuristic and meta-heuristic algorithms of the shelf-space model. We compare the different approaches/extensions through simulations and a field experiment. Unlike prior experiments that look at one store using a before-after scenario (i.e., lacking a control group), we investigate differences *between* three different configurations (i.e., including a control group) using a before-after comparison. Thus, we can control for both natural growth in consumer populations (in the control group) as well as change due to the novelty of making any changes at all (by having two different change groups of stores). The results indicate that the meta-heuristic approach outperforms the heuristic approach (and is closest to the linear programming formulation) in the simulations and the heuristic and control group in the natural field experiment. In summary, we find that the number of facings, vertical location, and horizontal location each have a significant, near-equivalent impact. We conclude with a discussion of limitations and implications.

The shelf-space allocation model

To develop a basic understanding of how heuristic and meta-heuristic models can aid retailers, we first describe the basic shelf-space management problem as follows:

$$\text{Maximize} \sum_{j=1}^{N} \sum_{k=1}^{S} P_{jk} * x_{jk} \tag{1}$$

In essence, the formula multiplies the profit of an item (P_{jk}) by the number of facings of that item (x_{jk}). We use the term "item" throughout this paper to refer to an individual product stock keeping unit (SKU). Although many products may have one facing on a shelf, at times other products have multiple facings. It is assumed that increasing the facings of an item normally has a positive effect on the item's sales, as found in the literature (e.g., Baker and Urban 1988; Urban 1998, 2002, 2005). Otherwise, the number of facings would be kept to one per item, thus allowing room for additional items on the shelf.[4] The formula repeats the process for each product (*j*) on each shelf (*k*) to calculate the total profit across all products (*N*) on all shelves (*S*). The manager's goal becomes finding the right combinations of items and facing across the shelf-space to maximize total profit. Following Yang and Chen (1999), the problem is presented here as an integer programming formulation because retailers are interested in solutions that give integers, or whole numbers, of suggested product facings.

[4] Retail practice provides prima facie evidence of this assumption about the incremental value of multiple facings. We urge the reader to observe any aisle of a retailer. The reader will witness several products with multiple facings. We ask: As retailers have more products available to them from manufacturers than actual shelf space permit, *why* would retailers choose for any product to ever receive more than one facing? The only logical reason is if, on average, the *additional* space allocated to the product results in more sales or profit than if the next best non-displayed product was included.

Accounting for the nonlinear profit function

A nonlinear profit function is one approach that retailers can use to make better shelf-space arrangement decisions (see, e.g., Lim, Rodrigues & Zhang 2004). To demonstrate this function, we examine the basic retail shelf model shown in Eq. (1). The total profit generated by a particular product in that equation is equal to the per unit product profit dollars multiplied by the number of shelf facings. However, while a retailer might double the sales of an item by giving it two facings on the shelf, the sales or profit increase from two to three facings is usually less than the increase from one to two facings. Likewise, the increase from three to four facings will be even less, and so forth. That is, there is a nonlinear relationship between the number of facings and the profit dollars produced—referred to as a nonlinear profit function.

We emphasize that the term "nonlinear profit function" is not equivalent to—nor does it necessarily imply—"nonlinear programming." Indeed, nonlinear formulations of shelf-space problems with nonlinear profits in the literature are unable to find optimal solutions for the majority of shelf-space problems (e.g., Lim, Rodrigues & Zhang 2004). Consistent with Hansen, Raut and Swami (2006), we propose that retailers can use a "linear integer programming formulation" of the "nonlinear profit function" in the shelf-space model.[5] As such, the linear model retains the advantages of the diminishing returns that accrue with additional space for any given product. In summary, as shown in Appendix A, a nonlinear profit function can be modeled using linear programming.

Accounting for the horizontal directional effect

Including directional effects in the shelf-space model is another approach that retailers can use to make better shelf-space arrangement decisions. The general concept of "location" affects many aspects of retailing, from the store location to departmental adjacencies to category and subcategory shelf-space arrangements. Research on the general topic of location goes back to concepts such as Reilly's (1929) gravitational models. Specifically, in reference to the shelf-space configurations, the location of a product can significantly affect its profit generation (see, e.g., Dreze, Hoch & Purk 1994). For example, most retailers intuitively expect cold cereal products placed at or above the shopping cart level generate more sales and profit than if the same products were placed on the bottom shelf. Additionally, people might read shelf tags (habitually) from left to right within a given section of shelf (i.e., a horizontal effect). Products closer to the end of the aisle might experience more passing traffic than items located toward the center of the shelf, and thus have increased sales due to their proximity toward the end of the aisle.

[5] Nonlinear models can be transformed to linear models by a conversion. For example, consider the nonlinear objective function: max ($x_1 * x_2$), where x_1 and x_2 are binary variables (e.g., Corstjens & Doyle 1983). This nonlinear function can be transformed into a linear function as follows: max z, where $z \le x_1$; $z \le x_2$ (so that all the terms are linear).

Thus, the product's location, whether vertical or horizontal, can affect the product's, and thus (as a group) the category's performance. Referring to such affect as space elasticity, Curhan (1972) took a large sample from store experiments, and found the average value of 0.212 for space elasticity. Corstjens and Doyle (1981) observed the average space-elasticity at the item level is 0.086. Dreze, Hoch and Purk (1994) found that by moving from the worst to the best horizontal position, a brand sale can increase by 15 percent on average, while, the difference on brand sales between the worst and the best vertical position was 39 percent. The above findings motivated several researchers to develop efficient shelf-space allocation model to maximize the total sales volume.

However, the total profit remains unchanged in Eq. (1) if the product is moved to a different shelf position. Likewise, according to interviews we conducted with several retail corporate buyers/purchasing managers, such effects are usually not explicitly considered when buyers actually make shelf-space changes. This article advances the literature on shelf-space location effects by accounting for the horizontal location effect *within* a particular shelf section.

Accounting for the cross-elasticity effect

Products within a section of retail shelf that are either complements or substitutes exhibit a cross-elasticity effect. That is, they have an impact on the demand of the other products. In reference to the basic shelf model in Eq. (1), the total profit remains unchanged if a different product receives more shelf-space that competes with the original product. In improving the basic shelf model in Eq. (1), we note that cross-elasticity values can significantly vary, as shown in Hwang et al's (2005, p. 186) review of cross-elasticity values. Additionally, they can be represented in different ways—each approach having its advantages and disadvantages (Bultez & Naert 1988).

One approach to modeling the cross-elasticity is to use a function of the shelf length or number of facings that the product occupies. If (1) changes in the number of facings has an impact on the quantity sold for a given product, and (2) cross-elasticity exists between two products, then (3) an increase or decrease in the allocation of one product (i.e., the number of facings or the package size) should affect the sales units, and thus profit dollars, of the other product. We emphasize that the cross-elasticity effects are not necessarily symmetric or monotonic. For example, increased display of cake decorating sprinkles might affect the sales of cake frosting *more* than increased display of frosting might affect the sales of sprinkles.[6] See Appendix A for details of the linear integer programming model that combines the nonlinear profit function, directional effect, and cross-elasticity effect.

[6] Cake frosting and sprinkles are both normally located in the cake decorating subcategory. The nature of the relationship between them can vary depending on several factors, including which is promoted or more prominently displayed. Prior research has used examples such as cake mix and cake frosting, which could be considered a between category effect. We propose that within category effects can also exist.

Heuristic and meta-heuristic models

While retail mangers follow the industry mantra "retail is detail," most retail managers have little time to consider the details of different shelf-space arrangements. Consequently, the benefits of using shelf-space models to supplement human decision making depend on how quickly the shelf-space models can run on computers. Researchers have attempted to reduce the necessary solution time by applying heuristic or meta-heuristic search algorithms to the basic shelf-space allocation parameters. These methods are appropriate and can benefit retailers because the retail shelf-space allocation problem is usually "NP-hard" (i.e., nondeterministic polynomial-time hard). NP-hardness implies that no efficient algorithm exists to solve large instances of this problem in a reasonable time (in terms of the problem parameters). For example, as shown by Urban (1998), a forty product decision results in more than a trillion possible shelf-space choices (i.e., 2^{40}). In this section, we briefly discuss differences in how heuristic and meta-heuristic approaches attempt to reduce the search time to find a near optimal shelf-space configuration.

Applying the heuristic method

Since retail shelf-space problems remain complex in nature and managers lack error-free estimates of the parameters that affect product performance (Borin & Farris 1995), some researchers have proposed using heuristic modeling to obtain near-optimal solutions to the shelf-space design (e.g., Yang 2001). A heuristic is an algorithmic approach designed to quickly find a close to optimal solution (here, in terms of profit), though the approach may not provide *the* optimal solution. The proposed heuristic approach is a modification of Yang (2001) and consists of two major phases. First, the algorithm rank orders all products by average profit without shelf consideration. Second, starting with the most profitable product, it allocates the minimum number of facings for each product to get an initial solution. Third, using a "neighborhood search," the algorithm moves the configuration (e.g., number of facings) of a product around on the same shelf, examining the increase or decrease in profit. Finally, it swaps pairs of products around on the same shelf and between shelves to determine if the new configurations increase the total generated shelf-space profit. Appendix B provides the details of the proposed heuristic.

Applying the meta-heuristic method

In contrast to the heuristic approach, other researchers have focused on developing meta-heuristic models for the retail shelf-space decision (e.g., Esparcia-Alcázar et al. 2006; Hansen, Raut & Swami 2006; Hwang, Choi & Lee 2005; Lim, Rodrigues & Zhang 2004; Moholkar & Swami 2001; Raut & Swami 2002; Urban 1998). One advantage of the meta-heuristic approach (over the heuristic approach) is the avoidance of getting stuck in a "local solution." That is, meta-heuristic models are more flexible in covering a wider range of possible solutions in a shorter amount of time. Meta-heuristics have increased in popularity and nomenclature (e.g., genetic algorithms, tabu search, etc.) across the physical and social sciences over the last twenty years. See Osman and Laporte (1996) for a good review of meta-heuristic approaches.

Most meta-heuristics are classified as either (1) single-point solutions (e.g., simulated annealing, tabu search) that produce a single proposed solution at each iteration or (2) population-based solutions (e.g., genetic algorithms) that generates a population (or group) of proposed solutions at each iteration (Blum & Roli 2003). While the literature continues to advance both single-point solutions (e.g., Lim, Rodrigues & Zhang 2004) and population-based solutions (e.g., Hwang, Choi & Lee 2005), there can be a substantial gain in search quality for retail managers when exploiting the information present in a population of solutions (Thierens 2004). Using a genetic algorithm, or population-based solution, Urban (1998) finds that a meta-heuristic performs better than a heuristic approach. However, the simulation assumes that the product profit is not dependent upon the horizontal shelf location within a section of the shelf. This study builds on Urban's (1998) work by considering this effect. More recent research, such as Lim, Rodrigues and Zhang (2004), using a single point method, and Hwang, Choi and Lee (2005), using a population-based method, incorporate vertical location effects for *across* shelf-space sections. As mentioned earlier, this article helps retailers by extending the horizontal effect to *within* a shelf section.

Building upon the genetic algorithm shelf-space research stream of Urban (1998), Moholkar and Swami (2001), Raut and Swami (2002), Hwang, Choi and Lee (2005), Esparcia-Alcázar et al. (2006), and Hansen et al. (2006), we propose using a genetic algorithm as a method of improving the retail shelf-space configuration (over using a heuristic approach). The advantages of the proposed meta-heuristic approach include that (1) it requires less computational effort and (2) the mangers can directly apply the solution to the retail shelf-space.

The meta-heuristic uses a different approach relative to the heuristic of comparing shelf-space configurations. First, it generates several different shelf-space arrangements which, taken together, are called a population. Second, it uses a selection procedure called "binary tournament selection," which is similar to the construction of a sports bracket, in which it randomly selects two different shelf-space arrangements and compares them. The more profitable arrangement is selected and the process repeats to generate a new population of shelf-space arrangements. Third, the meta-heuristic uses a "cross over process" where it randomly selects two of the tournament winners—called "parents"—and generates two new arrangements—called "children"—by taking different parts from the two parents—in the sports bracket analogy, swapping the offensive or defensive lines. Fourth, it uses a "mutation" operation in which it randomly selects a configuration and makes structural changes to it. These stages are repeated several times until the meta-heuristic converges to a close to optimal solution. Through this meta-heuristic process, it is proposed that the shelf-space model covers a wider range of possible shelf-space configurations and in a shorter amount of time as compare to the heuristic process. See Appendix C for details.

Table 1
Simulation solution time by analytic approach.

Problem size	Linear programming	Heuristic or meta-heuristic
Smaller problems	3 min	Less than 1 min
Larger problems	NA after 4 days	Couple of hours
Extremely large problems	Insufficient memory at 24 h	About 1 day

Read: As described in detail in the text, the heuristic and meta-heuristic approaches took significantly less time than the linear programming and are capable of solving larger problems. For example, a 10 shelf × 100 product problem contains roughly 2^{100}—or more than a nonillion (10^{30}) different configurations—which the linear programming cannot currently compute on a regular desktop machine.

Simulation studies

Consistent with prior shelf-space research, we conducted simulation studies to compare the sensitivity of model parameters of the two different algorithmic approaches to the linear programming model. We designed linear integer formulation in the simulation studies using the nonlinear profit functions and parameters of Dreze, Hoch and Purk (1994) (see Appendix D). Following Dreze, Hoch and Purk (1994), we simulated the profits of different products on different location of shelf-space. We generated the shelf-space and product parameters consistent with the guidelines of Lim, Rodrigues and Zhang (2004) (see Appendix D). We categorized the problems for the simulation study on the basis of problem sizes (i.e., large and small size problems, see Tables 1–3). For each of the small size problems, we randomly generated 25 datasets and compare the heuristic, the modified heuristic, and the meta-heuristic solutions (from Appendixes B and C) to the optimal linear integer programming formulation (shown in Appendix A) obtained by using CPLEX 7.0 (shown in Appendix D). For each of the larger size problems, we randomly generated 100 datasets. The two versions of the Yang's (2001) heuristic and genetic algorithm programs were written in the C language and ran in Linux on a P4 machine.

Results

The CPLEX 7.0 software took approximately three minutes to generate the optimal linear integer programming solutions for the smaller problems. Both the heuristic and meta-heuristic programs solved the same smaller problems in less than a minute. For the large problems, the CPLEX software was unable to produce an optimal solution for the large problems after running the program for four days using a branch and bound method. In contrast, the heuristics and meta-heuristic approaches took a couple of hours to solve the same large problems. For extremely large problems (e.g., 10 shelves and 100 products), the CPLEX software showed insufficient memory after 24 h and, hence, failed to find an optimal solution using the linear programming. Again in contrast, the heuristics and meta-heuristic took approximately 24 h for the same extremely large problems. See Table 1 for a summary. Thus, it appears that not only are the heuristic and meta-heuristic capable of solving retail shelf-space problems in much less time than linear integration modeling, but

Table 2
Simulated profit performance gaps between modeling approaches are significant.

(#Shelves, #Products)	With horizontal location effect					
	Average performance gap (percent)			Maximum performance gap (percent)		
	Heuristic	Modified heuristic	Meta heuristic	Heuristic	Modified heuristic	Meta heuristic
(2, 10)	9.8	4.2	1.3	23.4	10.5	6.9
(2, 20)	12.2	4.0	1.6	47.9	17.8	7.6
(5, 10)	21.9	7.5	0.25	61.3	32.9	4.7
(5, 30)	20.1	5.4	0.33	65.1	39.8	4.5
(5, 50)	21.5	6.0	0.42	71.0	32.1	7.9
(10, 30)	28.8	8.4	0.18	78.2	35.6	2.1
(10, 50)	28.8	8.6	0.72	80.8	31.2	7.4
(10, 100)	9.9	7.0	1.9	58.0	37.9	8.8
(# Shelves, # Products)	Without horizontal location effect					
	Average performance gap (percent)			Maximum performance gap (percent)		
	Heuristic	Modified heuristic	Meta heuristic	Heuristic	Modified heuristic	Meta heuristic
(2, 10)	8.1	4.0	2.2	21.0	11.9	7.6
(2, 20)	12.8	3.8	2.2	29.5	9.1	7.6
(5, 10)	21.8	9.8	2.0	64.7	41.3	4.8
(5, 30)	19.5	7.0	3.1	67.1	36.5	8.0
(5, 50)	24.2	10.7	1.1	74.6	36.5	7.1
(10, 30)	29.3	7.3	0.98	78.4	58.2	2.5
(10, 50)	25.9	6.4	1.5	76.9	31.7	7.1
(10, 100)	26.4	6.6	2.8	79.7	30.4	8.9

Read: The performance gap is the difference between the solution generated by the approach and the best solution of all the approaches, calculated for each simulation run and then average across all the runs for each configuration. Consideration with and without the horizontal effect leads to the same conclusion: the modified heuristic outperforms the heuristic and, in turn, the meta-heuristic (i.e., genetic algorithm) outperforms the modified heuristic.

also they can solve larger size problems that the linear integer programming cannot solve. Discussions with corporate buyers indicate that they often have to wait several days for planograms to be drawn in Spaceman or Prospace, etc. and that the planogram analysis process normally spans at least a few weeks. Thus, the computational time does not appear to be a problem.

Up to this point, we have focused the analysis on the computational performance of the different approaches. We now turn our attention to the effect size, or financial performance, of the different approaches. One method of assessing performance commonly found in shelf-space literature is to compute a "performance gap," defined as the difference between the method being assessed and the best result of all the methods being compared, first calculated per simulation, and second averaged across all the simulations runs for a scenario. In the case of the small size problems, this includes the linear programming optimal solution. As the CPLEX software was unable to compute the linear programming optimal solution for the larger sized problems, we compute the difference between each approach (heuristic, modified heuristic, meta-heuristic) and the best solution of the three approaches, consistent with the literature. We do this for each simulation run, and then we calculate the average gap across the hundred simulations, as reported in Table 2. The results indicate an average performance gap of 9.5–28.8 percent for the heuristic. The modified heuristic (i.e., that includes neighborhood algorithm) has a gap of

Table 3
Simulated profit dollars between shelf-space modeling approaches are significant.

# Shelves, # Products	*N*	Heuristic		Modified heuristic		Meta-heuristic	
		μ	SD	μ	SD	μ	SD
Smaller	50	19.8	7.7	21.1	8.0	21.6	8.0
5, 10	151	63.1	22.7	75.2	20.2	80.8	19.9
5, 30	119	194.5	75.9	228.9	61.3	239.4	57.1
5, 50	120	312.5	121.0	372.1	90.9	393.0	87.6
10, 30	121	549.0	283.8	704.7	242.0	761.2	242.8
10, 50	118	938.6	490.0	1207.9	406.3	1300.0	393.3
10, 100	119	1894.8	966.3	2434.4	770.1	2543.3	720.6

Read: Smaller refers to the 2–10 and 2–20 shelf-product configurations. Across all simulation studies, the modified heuristic outperformed the heuristic. Likewise, the meta-heuristic (i.e., genetic algorithm) outperformed the modified heuristic. All results are statistical significant at the .01 level or better using *t*-tests, nonparametric Wilcoxon sign rank tests, and repeated measure ANOVA.

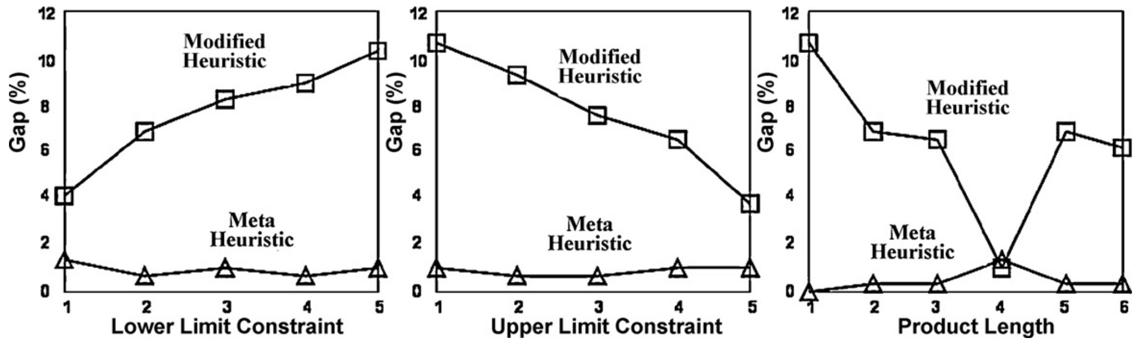

Fig. 1. The effects of shelf-space parameters on heuristic and meta-heuristic performance.
Read: The performance gap (%) is the difference between the solution generated by the approach and the best solution of all the approaches, calculated for each simulation run and then average across all the runs for each configuration. The constraint values refer to the constraint "levels," not the absolute values of the variables. As explained in text, the meta-heuristic outperforms the heuristic even when varying the model constraints.

3.9–8.6 percent on average—a significant improvement over the heuristic. The genetic algorithm performs the best, with an average gap of 0.25–1.88 percent, meaning it was the best solution most of the time when the linear programming was unable to produce a solution. The genetic algorithm was also the closest to the linear programming solution when one was computed: repeated measure ANOVA (($F(147,3)$, 12.64, $p<.001$), $M_{\text{linear program}} = 21.99$, $M_{\text{heuristic}} = 19.78$, $M_{\text{mod heuristic}} = 21.13$, $M_{\text{genetic algorithm}} = 21.65$). Interestingly, the results also show that the performance advantage of the genetic algorithm decreases as the ratio between number of products and number of shelves increases.

As shown in Fig. 1, the performance gap of the modified heuristic is significantly affected by all three of the shown parameters. In contrast, the meta-heuristic performance gap is only affected marginally by the product length parameter. The lower limit plot reveals that the heuristic's performance decreases as the lower limit constraint value increases. Similar effects are observed for other combinations of shelves and products not shown in Fig. 1. Further, these same patterns are still observed when the horizontal directional effect is removed from the model. Thus, the overall simulation results indicate that, consistent with prior research on retail shelf allocation (e.g., Hwang, Choi & Lee 2005; Urban 1998) and other applications (e.g., Golden, Raghavan and Stanojevic 2005), the genetic algorithm performs better than the heuristic.

Another method of assessing performance is to use a repeated measure ANOVA of the simulated total profit dollar differences. As shown in Table 3, the modified (i.e., two phase) heuristic outperforms the heuristic in the simulation studies across all different combinations of products and shelves. In turn, the meta-heuristic outperforms the modified heuristic. See Table 3 for detailed comparisons.

Field experiment

Much of the advancements in the literature on shelf-space optimization remain grounded in simulation studies (e.g., Borin, Farris & Freeland 1994; Borin & Farris 1995; Hansen, Raut & Swami 2006; Hwang, Choi & Lee 2005; Lim, Rodrigues & Zhang 2004; Urban 1998). However, little research has been performed to examine whether similar results may be found if retailers actually make such changes to store shelf-space configurations. This section provides a corresponding in-store experiment of the heuristic and meta-heuristic models proposed in this article.

To investigate the effects of the two algorithm approaches, we analyzed data from a natural experiment. We approached a retail category manager of a large retail chain (hereafter, Priceko), and asked if they would be willing to participate. In exchange for a copy of the results and confidentiality, we were provided with a sample of twelve stores. The experimental setting consists of 67 consumable products representing more than half a dozen brands in category of health and beauty products. The products are spread across seven shelves of a four foot gondola section. The setting is appropriate given the large number of merchandise subcategories that are restricted to small shelf runs (e.g., four or eight feet) within larger runs in which the subcategories need to be kept distinct from each other. The Priceko stores were located in the Eastern United States, and consumer demographics are consistent across the dozen stores.[7]

Four weeks of item-level performance for each of the 67 products is averaged across the Priceko stores to provide a baseline for the Yang heuristic and the genetic algorithm. Stores were divided into three groups: control, Yang heuristic, and genetic algorithm. We computed the Yang heuristic and genetic algorithms shelf-space configurations using the baseline profit data. Profit was modeled using item gross margin (i.e., handling costs are the same across the merchandise category for the retailer). The algorithms used a cross-elasticity value of .1, per the retailer's data. We note that the retailer's cross-elasticity value is between the 0.086 found by Corstjens and Doyle (1981) and the 0.212 found by Curhan (1972).

The retailer made planogram changes to the Yang heuristic store group and genetic algorithm store group that closely approximated the proposed models from the algorithms. Stores received the new shelf-space configurations from the retailer's

[7] For these stores, the average customer income is \$48,000; half of the customers are between 30 and 65 years old; and, as to customer ethnicity, 72 are percent Caucasian, 14 percent Hispanic, and eight percent African American, with six percent reporting other ethnicities.

Table 4
Field experiment shelf variations.

Baselines 7 shelf, 4 foot section	1a 1b 1c 1d 1e 1f 1g 1h 1i 1i 1j 1k 1l 1m 1n 2a 2a 2b 2c 2d 2e 2f 2g 2h 2i 2j 2k 2l 2m 2m 2n 2n 2o 2o 3a 3b 3c 3d 3e 3f 3g 3h 3h 3h 3i 3i 3j 3j 3j 4a 4b 4c 4d 4e 4e 4e 4e 4f 4f 4f 4g 4g 4g 4h 4h 4h 4i 4i 4i 5a 5b 5c 5d 5e 5f 5f 5f 5f 5g 5g 5g 5h 5h 5h 5i 5i 5i 5j 5j 5j 6a 6a 6b 6b 6c 6c 6d 6e 6f 6g 6h 6i 6i 6i 6j 6j 6j 6k 6k 6k 7a 7a 7a 7b 7b 7b 7c 7c 7c 7d 7d 7d 7e 7e 7e 7f 7f 7f
Improved heuristic 7 shelf, 4 foot section	1f 1g 1h 1i 1i 1j 1k 1l 1e1 d 1d 6f 6g 6h 1m 1n 2n 2n 2o 2o 2l 2k 6d 2f 2e 2g 2d 2b 2a 2a 2i 1b 1a 3a 3b 3c 3d 3e 3g 3g 3g 3f 3f 3h 3h 3j 3j 3j 4g 4g 4g 4a 4a 5b 4c 5e 4f 4f 4f 4e 4e 4e 3i 3i 3i 4i 4i 4i 2h 2h 2h 5a 5a 5b 4d 5d 5g 5g 5g 5f 5f 5f 4h 4h 4h 5i 5i 5i 5j 5j 5j 6a 6a 6b 6b 6c 6c 2m 2m 5h 5h 5h 6i 6i 6i 6j 6j 6j 6k 6k 6k 7a 7a 7a 7b 7b 7b 7e 7e 7e 7f 7f 7f 7c 7c 7c 7d 7d 7d
Genetic algorithm 7 shelf, 4 foot section	6f 6g 2f 2e 2g 2k 1d 1d 1f 1g 1i 1j 1k 1m 1n 3b 3c 3d 3e 6d 2c 2d 2b 2a 2a 1c 1c 2i 1a 1a 3a 4a 4a 4b 4c 3g 3g 3g 3f 3f 3h 3h 3h 3j 3j 3j 4i 4i 4i 5a 5a 5b 4d 4f 4f 4f 4e 4e 4e 3i 3i 3i 5i 5i 5i 2h 2h 2h 6b 6b 6c 6c 5g 5g 5g 5f 5f 5f 4h 4h 4h 4g 4g 4g 5j 5j 5j 5d 5e 2m 2m 2m 5h 5h 5h 6i 6i 6i 6j 6j 6j 6k 6k 6k 2j 2j 2j 7a 7a 7a 7b 7b 7b 7e 7e 7e 7f 7f 7f 7c 7c 7c 7d 7d 7d

Note: Each cell contains the products (from left to right) actually present on the four foot section of retail shelf. Thus, in the baseline stores, the highest shelf from the ground contains 15 products, starting on the left with product 1a and ending with product 1*n* at the left, etc. Some products appear more than once because they were allocated more than one facing for the given configuration.

corporate office as part of the department's annual planogram reevaluation. Department managers reset the shelves and changed the products' locations to match the new configurations; the reset took a few hours. Item-level performance data were queried from the retailer's sales database for both the four week periods before and after the planogram resets. See Table 4 for an aggregated comparison.

Store level results

We first model the impact using the *percentage change in profit dollars* for the total shelf-space as the dependant measure. This controls for natural initial variations in profit dollar performance due to exogenous store-level factors. The stores assigned to the control group experienced an overall 1.7 percent increase in profit dollars between the two time periods. While the confidentiality agreement of the field experiment precludes description of product specific performance changes, 47 percent of the number of products experienced *some* level of profit dollar increase. In contrast, the Modified Yang (2001) heuristic stores experienced an overall 6.0 percent increase in profit dollars. In particular, 57 percent of the number of items experienced a profit increase. Interestingly, the genetic algorithm model stores experienced an overall 11.7 percent increase in profit dollars. In particular, 55 percent of the number of products experienced some level of profit dollar increase. Thus, while there was an increase in the *number* of items that had a sales increase for both the heuristic and meta-heuristic, the *amount* of increase was almost double in stores assigned to the meta-heuristic group (versus stores assigned to the heuristic group). In summary, consistent with the simulation results and prior research (e.g., Hwang, Choi & Lee 2005; Urban 1998), stores assigned to either the genetic algorithm and heuristic models experienced an increase in profitability beyond the control group stores. Consistent with the simulation conclusions, the genetic algorithm store group profitability increased at a rate almost double to that of the Modified Yang heuristic store group.

Item level effect decomposition

Having investigated the overall performance between the three different shelf-space configurations, we next evaluate what aspects might have improved the performance. We continue to use *percentage change in profit dollars* as the dependent measure (rather than actual profit dollars) to control for the varying sales and profit dollars that different stores naturally experienced in the pre-planogram change period. As we have the physical dimensions of all the products, we are able to calculate the exact distance changes in vertical and horizontal location on the four foot shelf-space. Vertical position is calculated as the distance from the bottom of the lowest shelf to the bottom of the product (i.e., toward the ceiling) in feet; items that were lowered had negative coefficients while raised items had positive coefficients. Horizontal position is calculated as the distance from the left edge of the shelf section to the left edge of the product (i.e., toward the right edge of the gondola) in feet.[8]

Decomposing the differences between the three configurations, regression analysis indicates significant product level effects for changes in the number of facings, vertical posi-

[8] Using bottom of shelf, bottom of product and left shelf edge, left product edge is the same as using center of product from the shelf space across the products and shelves (i.e., simply transposed). If an item has multiple facings, we use the leftmost edge of the item group in making the calculation for simplicity.

Table 5
OLS regression coefficients indicate facing and location effects are significant.

Coefficient	Unstandardized Coefficient (B)	Standardized Coefficient (β)	p-Value (sig.)
Constant	3.4	n.a.	.02
Horizontal position (toward right)	1.6	.25	.02
Vertical position (toward top)	2.8	.59	.01
Number of facings	14.5	1.55	.03
Number of facings^2	−7.1	−1.48	.04
Adjusted R^2	.33		

Read: Horizontal and vertical positions are in feet from the floor and left hand side, respectively. The number of facings is an integer value. The number of facings squared accounts for the theorized diminishing effect (i.e., nonlinear profit function).

tion, and horizontal position for the model ($F(66,3) = 5.850$, $p < .001$). Reviewing Table 5, we find support for a nonlinear profit function, given a positive, significant facing coefficient and a negative, significant squared facing coefficient. The standardized effects show that the vertical location effect size is approximately double the size of the horizontal location effect (see Table 5). We note that these results are consistent with the findings of Dreze, Hoch and Purk (1994). Reviewing the standardized coefficients, we also note that the facing effect size is less than the locational effects. The results indicate that the genetic algorithm does not just produce a more efficient model in terms of product facings, but it also produces a better horizontal and vertical arrangement. In short, retailers have multiple methods of affecting sales in shelf-space design.

At the same time, there were a few items deleted in the stores assigned to modified heuristic and meta-heuristic store groups that continued to be present in the baseline stores. Analysis of their deletion is difficult, as they cannot be included in the regression analysis. That is, they continue to sell, and thus, produce some profit in the baseline stores. However, they are not included in the other two store groups to give space to other items to have more facings. Unfortunately, we have no way of assessing *which* deletions are associated with *which* particular facing increases. Indeed, they may be associated with increases in multiple products. The finding and limitation of the current investigation introduce an interesting, important inquiry for future research to explore.[9]

Discussion and implications

The results of the simulation studies and the exploratory natural field experiment provide initial support to the assertion that the meta-heuristic approach outperforms the modified Yang (2001) heuristic for a shelf-space model that accounts for a nonlinear profit function, location effects, and product cross-elasticity. The (1) number of facings, (2) vertical position, and (3) horizontal position all have a significant impact on the item level profitability for as small as a four foot section of retail shelf-space as seen in the different across the three configurations in the natural experiment, as well as in the simulation studies.

Still, refinements are needed on many fronts. The experiment uses a four foot section of retail shelf-space, and as such should be treated as exploratory. Further, while we use total planogram profits as the objective, other performance criteria exist that need be evaluated such as market basket or cross-category metrics (Kamakura & Kang 2007). Since the space arrangement will often interact with other marketing mix activities, future research is needed that investigates the effect of price promotions and advertising on market basket performance as they affect shelf-space models. Likewise, it is assumed here that product out-of-stocks are not a concern (i.e., that one facing of the product on the shelf is sufficient for less profitable products) or else the buyer adjusts either the vendor pack order size or change the shipping method to reduce lead time given that these are existing products. These variables could be included in future shelf-space models for retailer scenarios where the retailer is not capable or able to make such changes, or taking into account back room storage (see Hariga, Al-Ahmari & Mohamed 2007). Further, promotional allowances are not included in the model because these allowances are normally not accounted for in a retail buyer's performance evaluation or scorecard—who is the individual actually making the shelf allocation decision. These allowances are, instead, calculated by accounting personnel at the end of the business period and rolled up to the company.

Additionally, future research should model the effect of branding on cross-elasticity estimation. To do so would require a multiple configurations of a larger span of shelf-space than was available here. We do not know which side of the aisle the shelf-space configurations were located on (given the natural field occurred in stores across multiple states). Also, our model is only applied here to a brick and mortar retail scenario. While we do not address virtual store layouts here due to space constraints (see, e.g., Laroche et al. 2005; Vrechopoulos et al. 2004), and while online retailers currently hold a minority of retail market share, we propose that an extension of our space allocation model could be applied to virtual store layouts. In particular, while online retailers can include an infinite number of product photos and prices on a Web page, the Web browser can only shown a certain number either horizontally or vertically before customer have to scroll down or to the side. The differential costs of the first, second, third, etc., sponsored listing positions on major search engines indicates that the location of a product on a given Web page configuration has a financial

[9] While the research design prevents analyzing direct combinations of deletions and expansions, we can assess that because (1) these few items did contribute profit to the baseline stores overall profit percent increase, and (2) the other two store groups had a better profit increase than the baseline stores, then (3) the gain in the increase in facings of other items and changing of locations more than offset the loss of the half dozen items contributions.

impact. Further, we only had access to four feet of shelf-space in the exploratory field study. Research is needed to establish the generalizability of these results. Indeed, several other variants of meta-heuristics exist that could also be tested. Last, we echo the call for research that moves beyond within category analysis to consider cross category space effects; such research could assist retailers in determining optimal category sizes versus simply focusing on the spacing of products within a category.[10] Many questions in this area have been unaddressed, such as the effect of seasonality on the cross category effect.

Advances in optimization programming can assist retailers in increasingly diversified markets in which "store of the community" efforts multiply the number of planograms (e.g., Grewal et al. 1999). It also can be a greater supplement to buyer decision making given that the alternative of employing category captains is being publicly scrutinized and legally debated. We caution, however, that allocation modeling requires correct *presentation* dimensions, not *product* dimensions. For example, a buyer at one retailer reported to the authors an occasion where a twelve foot notebook planogram was built using the (notebook) product dimensions. However, when the stores set the dividers and attempted to fill in the products, they found that there was not room for the notebook wire bindings to hang over the notebook underneath. The products bowed in the center and the nonoverlapped, alternating bindings took on greater height, resulting in a fewer products being able to fit vertically in the space. According to the buyer, it was a disaster. The planogram had to be redrawn; retail store employees in every store had to reset the planogram—increasing the payroll expense across the several hundred stores. The implication for practitioners is that planograms built in optimization software should be physically built before being sent to retail stores. This does not infer, however, that package dimensions should be adjusted to better fit shelf-spaces without accounting for the impact of package shape on consumer purchasing—see Yang and Raghubir (2005).

We additionally note that these shelf-space models can be applied to most retailing contexts in which products are allocated along "lines," and not just actual shelves. For instance, some department store merchandise categories may be on walls that are cubed (e.g., jeans, shirts) or pegged (e.g., hosiery, socks, stationery, fishing lures, tools). As long as the products are horizontally level (e.g., pegs placed at the same height), retailers can use the existing shelf-space models to analyze the business and supplement the merchandise managers' decision making.

Conclusions

Retail shelf-space optimization may be complex, but it is not a conundrum. The model put forth in this paper (1) assists practitioners in resolving the decision on *how much* of *which* products to shelf *when* and *where*, and (2) moves the literature towards a resolution on the debate concerning whether a heuristic or meta-heuristic approach provides more robust and faster estimation. Tests of the model show that the horizontal position, vertical position, and number of facings each affect the performance of the items within the shelf-space. Nonetheless, we caution that this model, or any other model, needs to be balanced with an appropriate perspective on store atmospherics and "retailtainment." We echo the caution of Wickern (1966, p. 41), that "the success of retailing consists not only of selling merchandise, but also of the nature and completeness of the assortment." For a growing number of customers, shopping is a hedonic event (e.g., Arnold & Reynolds 2003), affecting store patronage intentions (Grewal et al. 2003). Retailers often place product SKUS together according to size or brand, creating a category image. Shelf-space profit maximization *could* work against such imagery. Research is missing that balances shelf-space optimization and atmospherics. Until it is addressed, practitioners would be wise to also keep a "human touch" in the planogram design process.

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Appendix A. Improving the linear integer programming shelf-space model: considering nonlinear profit function, directional effects, and cross-elasticity effects

$$\text{Maximize} \quad Z = \sum_{j=1}^{N}\sum_{k=1}^{S}\sum_{h=1}^{T_k}\sum_{f=1}^{T_k} P_{jkhf} * x_{jkhf} + \sum_{i=1}^{N}\sum_{j=1}^{N}\frac{V_{ij}}{2} * e_{ij} \tag{A1}$$

$$\text{Subject to} \quad \sum_{h=1}^{T_k}\sum_{f=1}^{T_k} x_{jkhf} \le 1 \quad \forall j = 1, \ldots, N;\ k = 1, \ldots, S \tag{A2}$$

$$\sum_{j=1}^{N}\sum_{f=1}^{h}\sum_{q=h-f*a_j}^{h} x_{jkqf} \le 1 \quad \forall h = 1, 2, \ldots, T_k; \quad k = 1, \ldots, S \tag{A3}$$

$$L_j \le \sum_{k=1}^{S}\sum_{h=1}^{T_k}\sum_{f=1}^{T_k} f * x_{jkhf} \le U_j \quad \forall j = 1, 2, \ldots, N \tag{A4}$$

$$b_j = \sum_{k=1}^{S}\sum_{h=1}^{T_k}\sum_{f=1}^{T_k} f * a_j * x_{jkhf} \quad \forall j = 1, \ldots, N \tag{A5}$$

$$V_{ij} \le b_i, V_{ij} \le b_j, V_{ij} \ge 0, b_j \ge 0 \quad \forall j = 1, 2, \ldots, N \tag{A6}$$

$$x_{jkhf} = 0 \quad \forall j = 1, \ldots N; \quad k = 1, \ldots, S; \quad h = 1, \ldots, T_k; \quad f > (T_k - h)/a_j \tag{A7}$$

$$x_{jkhf} \in \{0, 1\} \quad \forall j = 1, \ldots, N; \quad k = 1, \ldots S; \quad h = 1, \ldots, T_k; \quad f = 1, \ldots, T_k \tag{A8}$$

Read: (A1)–(A8) provide a linear integer programming formulation of the shelf-space model. P_{jkhf} is the profit of product j on shelf k of length T at horizontal level h for face-length f and x_{jkhf} represents the allocation decision for product j on shelf k starting at horizontal level h for face-length f (i.e., 1 = yes,

[10] We thank an anonymous reviewer for bringing up this important topic.

$0 = \text{no}$). The lower and upper bounds of x_{jk} are L_j and U_j, respectively. Z is the objective function (i.e., cumulative profit). Constraint (A2) ensures that product j is not allocated more than once for its f number of facing on a particular shelf. Constraint (A3) ensures that the products do not overlap. Constraints (A4) and (A7) ensure that product j is allocated between its limits.

Constraints (A5) and (A6) and the second part of the objective function (A1) account the cross-elasticity effects. For N products and S shelves, we define the cross-elasticity effect as an $N \times N$ matrix, E, where each value of $e_{ij} \in E$ represents the incremental profit or loss due to the cross-product effects. The cross-product elasticity is represented as min $(b_i, b_j) * e_{ij}$ where b_i and b_j are the total length of products i and j, respectively, on the shelf (A5). To avoid a nonlinear objective function, we use $V_{ij} = \min(b_i, b_j)$ and constraint (A6)—resulting in a linear objective function, and, thereby, a linear integer model. Note that the cross-elasticity effects are not symmetric (i.e., $e_{ij} \neq e_{ji}$). Eq. (A5) could easily be modified to use the number of facings or other variables, if preferred.

Appendix B. Applying the modified heuristic method

Phase 1

Step 1: Arrange the products $B = \{(j,k) \mid j = 1, 2, \ldots, n;\ k = 1, 2, \ldots, s\}$ according to the order of $r_{jk} = E(P_{jkh})/a_j$ where $E\,(P_{jkh})$ is the average profit of product j without a shelf k.

Step 2: For successive $(j, k) \in B$, allocate the available space of shelf k to product j to meet the minimal display requirement of product j. If the procedure is unable to allocate all the products within the minimum display requirement, produce an infeasible result and terminate. Otherwise, go to Step 3.

Step 3: Select $(j, k) \in B$ on the basis of maximum profit improvement per unit length. If $T_k > 0$ for some k, then allocate the next selected $(j, k) \in B$ according to the maximum profit improvement per unit length. This step repeats as long as allocation is possible and maintains the constraint's feasibility (e.g., upper bound constraint U_j of product j). Otherwise, stop the process.

Step 4: Group the same product within the same shelf. Arrange the product sequence in a shelf k in the horizontal direction by the following steps.

(i) Let the total length of product j be q_j and the initial horizontal position $h = 0$.

(ii) Select the product $j \in N$ to allocate in position h, such that, $\max_{j \in N} \{P_{jkh}/q_j\}$.

(iii) $h = h + q_j$, $N = N - j$. If $h > T_k$ or $N = \{\emptyset\}$, stop the process. The same procedure is carried out for all the shelves.

Phase 2

Step 1: Swap product facings on a particular shelf until feasibility is maintained and the shelf profit is optimized.

Step 2: Apply adjacent pairwise interchange on the products' group sequence within a shelf to improve profit.

Step 3: Swap product facings between shelves until the joint profit is optimized.

Step 4: Compute the total profit.

Read: The proposed, modified heuristic approach consists of two major phases. The first phase generates the feasible, best-possible allocation following Yang (2001). In the second phase, the heuristic attempts to improve the solution by a neighborhood search method (i.e., using adjoining solutions). Phase 1, Step 3, modifies the Yang's (2001) heuristic to help find a better solution for the nonlinear profit function). The modifications in Phase I, Step 4, and Phase 2, Steps 1 and 3, address the horizontal directional effect.

Appendix C. Applying the meta-heuristic method

Step 1: We define a two-dimensional array, consisting of five rows and $S \times N$ columns, to represent a chromosome for a feasible solution of the proposed model. An example is included below.

(i) The first two bottom rows represent the products (N) and shelves (S), respectively.

(ii) The third and fourth rows from the bottom represent the face-length of a product in each shelf, filled with random integer numbers generated such that the total face-length of a product j in row three is equal to the lower bound (L_j) and the fourth row values are between 0 and $U_j - L_j$.

(iii) The fifth row from the bottom represents the relative horizontal position of a product on a shelf using random keys or a uniform random number in the range of (0, 1).

Example:

Parent 1						Parent 2					
0.12	0.41	0.21	0.80	0.65	0.19	0.21	0.11	0.61	0.28	0.52	0.11
1	0	0	0	0	2	1	2	1	0	0	2
2	0	1	1	3	1	1	1	1	1	2	2
S_1	S_2	S_1	S_2	S_1	S_2	S_1	S_2	S_1	S_2	S_1	S_2
J_1		J_2		J_3		J_1		J_2		J_3	

Child 1						Child 2					
0.12	0.41	0.21	0.80	0.65	0.19	0.21	0.11	0.61	0.28	0.65	0.19
1	0	0	0	0	2	1	2	1	0	0	2
2	0	1	1	2	2	1	1	1	1	3	1
S_1	S_2	S_1	S_2	S_1	S_2	S_1	S_2	S_1	S_2	S_1	S_2
J_1		J_2		J_3		J_1		J_2		J_3	

Step 2. The allocation of products to different shelves is performed first using the minimal display requirement in row three; then, it is repeated using row four. The results are evaluated using the total profit generated for the allocation of products on different shelves.

(i) First product j is chosen for allocation on shelf s on the basis of increasing order of random keys or uniform random numbers. The selected product j is allocated on shelf s for integer number of facing, that is, the integer number on third row corresponding to the product j and shelf s. The allocation is continued till feasibility is maintained (i.e., shelf capacity limit), otherwise, stop the allocation.

(ii) After the allocation of minimal display requirement, we select product j for allocation on shelf s according to the increasing order of random keys. The selected product i is allocated on shelf s for integer number of facing, that is, the integer number on fourth row corresponding to the product j and shelf s. The allocation is continued till feasibility is maintained (i.e., upper and lower bound constraints), otherwise, stop the allocation. The fitness of a chromosome is evaluated on the basis of total profit generated for the allocation of products on different shelves.

Step 3. Next, randomly generated chromosomes are subjected to binary tournament selection. That is, two chromosomes are randomly chosen from the population and the most profitable one is selected for the next generation.

Step 4. A cross over process is applied to the binary tournament winners in which we consider the single point crossover approach for mating with mating probability p_c. The single point crossover chooses a random cutoff point in each of the two strings to form two sub strings one to the left of the point, and one to the right. We splice together the left part of the string of one parent with the right part of the string of the other parent. The splicing point only occurs between products to maintain the feasibility of the product's lower boundary constraint.

Step 5. The chromosomes undergo mutation process to avoid being stuck in local optimal solutions. We consider two different types of mutation procedures: uniform and swapping.

(i) For row three, we apply the swapping technique. We randomly choose two cells with mutation rate (p_m) and swap the face-lengths. This swapping mutation maintains the feasibility of the lower bound constraints.

(ii) The cells in the fourth and fifth rows are subjected to uniform mutation. The main idea of uniform mutation is that each cell has an equal chance (p_m) of undergoing the mutative process. The selected cell value is replaced by the random value generated in the feasible solution domain. We consider the random value generation domains as $(0,\ldots,U_j - L_j)$ and (0, 1) for fourth and fifth rows, respectively.

Read: We set the population size = $S*N$, number of generations = $5*S*N$, crossover rate = 0.99 and mutation rate = 0.03, for the genetic algorithm, where S and N are the shelves and products, respectively.

Appendix D. Nonlinear profit function and shelf-space parameters for shelf-space model

Step 1. Following Dreze, Hoch and Purk (1994), model profit as:

$$\log(U_{jkh}) = C_0 + C_1 * X_{jkh} + C_2 * X_{jkh}^2 + C_3 * Y_{jkh} + C_4 * Y_{jkh}^2 + C_5 * Y_{jkh}^3 + C_6 * e^{-C_7 * A_{jkh}} \quad \text{(A9)}$$

Step 2: Simulate the profits of different products on different location of shelf-space. For that purpose, we generate the coefficients of the model randomly in a specified range (i.e., C_0 = .5, 3.5; C_1 = 0.9991, 0.0006; C_2 = −0.000041, 0.000015; C_3 = .001, 3; C_4 = −0.0091, 0.0002; C_5 = 0.01, 0.1; C_6 = −400, 0; C_7 = 0,1).

Step 3: Generate the shelf-space and product parameters consistent with the guidelines of Lim, Rodrigues and Zhang (2004):

Parameter	Range for small problem	Range for large problem
(N, S)	(2, 10), (2, 20)	(5, 10), (5, 30), (5, 50), (10, 30), (10, 50), (10, 100)
$A_i \in (1, A)$	$A = 10$	A = 5, 10, 30, 50, 100, 300
$L_i \in (0, L)$	$L = 0$	L = 0, 10, 30, 50, 100
$\delta_i \in (1, \delta)$	$\delta = 10$	δ = 10, 30, 50, 100, 300
$T_k \in (T_l/4, T_u)$	$T_l = \sum_{i=1}^{N} L_i * a_i/m$, $T_u = \sum_{i=1}^{N} (L_i + \delta_i) * a_i/m$ and $U_i = \delta_i + L_i$	

Step 4. Measure the performance of the heuristics by the optimal gap, defined as:

$$OG_h = (\hat{Z}_o - Z_h) * 100/\hat{Z}_o \quad \text{(A10)}$$

Read: In Step 1, U_{jkh} is the profit function, X_{jkh} is the horizontal axis position, Y_{jkh} is the axis position, and A_{jkh} is the area consumed by each product. In Step 3, the values of T_l and T_u are chosen so that the total shelf-space is generally sufficient to allocate all the products, satisfying the lower-bound constraint, but not large enough to reach the upper bounds. We categorize the problems for the simulation study on the basis of problem sizes (i.e., large and small size problems. In Step 4, $\hat{Z}_o$ is the best solution among the heuristic, modified heuristic, genetic algorithm, or linear programming solution when available, and Z_h is the solution of the heuristic being considered.

We note that the ranges in Step 2 permit a U-shaped location effect. We chose the ranges following the findings of Dreze, Hoch and Purk (1994) who had negative coefficients in some of their parameters (e.g., their Table 3); we use a Gompertz specification to model the U-shaped profit function. We do this because higher shelves usually perform better than lower shelves, with the exception that the top-most shelves perform less than eye level shelves. Further, the highest shelves are at times more difficult for many customers to reach. Because of these about eye level maximizations, the U-shape seems reasonable. Thus, one of the coefficients (either X or $X\hat{2}$) should be negative. Because of the function form, the negative value does not imply that log (u) would be negative; the parameters have been scaled so that the right hand side is positive.

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