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Invited Review

Retail shelf space planning problems: A comprehensive review and classification framework

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A R T I C L E I N F O

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A B S T R A C T

The retail shelf space planning problem has long been addressed by Marketing and Operations Research (OR) professionals and researchers, with the first empirical studies tracing back to the 1960s and the first modelling approaches back to the 1970s. Due to this long history, this field presents a wide range of different mathematical modelling approaches that deal with the decisions surrounding a set of products and not only define their space assignment and related quantity, but also their vertical and horizontal positioning within a retail shelf. These decisions affect customer demand, namely in the form of space- and position-dependent demand and replenishment requirements. Current literature provides either more comprehensive decision models with a wide range of demand effects but limited practical applicability, or more simplistic model formulations with greater practical application but limited consideration of the associated demand.

Despite the recent progress seen in this research area, no work has yet systematised published research with a clear focus on shelf space planning. As a result, there is neither any up-to-date structured literature nor a unique model approach, and no benchmark sets are available. This paper provides a description and a state-of-the-art literature review of this problem, focusing on optimisation models. Based on this review, a classification framework is proposed to systematise the research into a set of sub-problems, followed by a unified approach with a univocal notation of model classes. Future lines of research point to the most promising open questions in this field.

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1. Introduction

The ultimate objective in shelf space planning is to maximise profit by making product assignments to shelves while taking into account limited shelf space and allocation constraints. This planning problem consists of distributing the scarce shelf space of a retail store among the different products to be displayed. Specifically, this embraces the questions of how much shelf space to assign to each product, which ultimately equates to the quantity of each product (=space assignment), at which vertical shelf level to position each product (=vertical allocation), and at which horizontal position to put each product and what products to put next to each other (=horizontal location). While shopping, customer demand is highly influenced by in-store factors that positively influence the visibility and awareness of the products. Therefore, these three questions do not just depend on the product margins, but also on customer preferences and the associated options for influencing customer choice.

The shelf space planning problem is of great importance to retailers because the increasing number of products conflicts with limited shelf space. The number of products grew significantly during the last years (EHI Retail Institute, 2014). Availability of the right products is one of the main drivers behind customer satisfaction. Retailers have recognised this fact and identified category management as a key task for achieving superior performance (Eltze, Goergens, & Loury, 2013) and improving space productivity (Gutgeld, Sauer, & Wachinger, 2009). Shelf space has in fact been referred to as the retailer's scarcest resource (e.g. Brown & Tucker, 1961; Geismar, Dawande, Murthi, & Sriskandarajah, 2015; Irion, Lu, Al-Khayyal, & Tsao, 2012; Lim, Rodrigues, & Zhang, 2004). The increasing number of products to allocate, the shortage of shelf space, the narrow margins in retail, and the intensity of compe-

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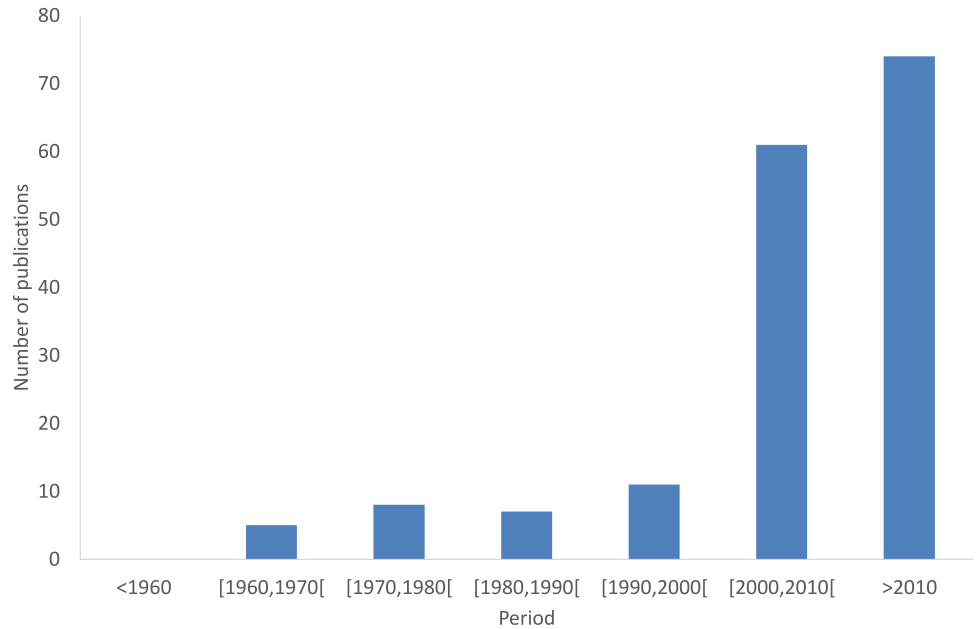

Fig. 1. The evolution of the publication activity on shelf space planning.

tition have greatly magnified the importance of retail shelf space planning (Hübner, Kuhn, & Sternbeck, 2013b). These developments have turned this into an increasingly challenging and active field of research.

Despite the practical relevance that this area of research has been experiencing, Bai (2005) and Hübner and Kuhn (2012) have established a misalignment in shelf space planning between existing commercial software applications and state-of-the art research. Software vendors focus mainly on the development of applications with large-scale data processing technologies with limited or no use of mathematical optimisation and disregard the main effects of space on demand (Hübner & Kuhn, 2012). Consequently, these applications require significant human interaction and are essentially used for visual and handling purposes. State-of-the-art optimisation methods, on the other hand, have practical limitations mainly due to their computational complexity and expensive parameter estimation requirements. Most of the complexity comes from the integration of demand effects related to the space assignment and the vertical and horizontal position of products. Considering these demand effects results in non-linear mixed-integer models. Shelf space planning problems may also differ between retail formats (e.g. hypermarket vs. discounter), product categories (e.g. fresh vs. ambient assortments) and retailer–supplier relationships (e.g. category captain roles), among others. Additionally, the planning problem is also related to other areas such as assortment, pricing, promotion or logistics and replenishment.

The retail shelf space planning problem has long been addressed by marketing professionals and researchers, with the first empirical studies tracing back to the 1960s. The main objective of these studies was to assess the impact that changes in shelf space have on customer demand. These empirical findings were first applied in decision models in the early 1970s. The number of publications with optimisation approaches related to this topic grew sharply after 2000, when the number of published works doubled, and projections for the current decade anticipate sustained growth. Fig. 1 shows the increasing number of publications between 1960 and 2019, with a total of 182 publications. These publications focused not only on building decision models and optimisation algorithms but also on more empirical analysis concerning shelf space. Conference proceedings and working papers have been excluded, but all other shelf space-related and published papers were taken into consideration. The European Journal of Operational Research has been the major journal for the presentation of new developments in the modelling field (with 25 published articles). Other journals such as the Journal of Retailing (13 articles), International Journal of Production Economics (12 articles), Production and Operations Management (8 articles), Operations Research (7 articles) and Omega (7 articles) have also published significant contributions until 2019.

Despite the recent progress seen in this research area, no work has yet been done to systematise published research with a clear focus on shelf space planning. The literature presents a great variety of models that incorporate different estimates of some of those effects. Models also differ in the level of detail of the decisions, ranging from rough space assignments to almost complete shelf space descriptions. This also leads to the use of different terms such as shelf space planning, shelf space allocation, allocation planning, etc. As a consequence, there is no unique definition of various terms, no unique shelf space planning model and no benchmark instances. Only the reviews of Hübner and Kuhn (2012) and Kök, Fisher, and Vaidyanathan (2015) address these areas partially, as they review papers on assortment planning. Based on our knowledge, our paper is the first comprehensive literature review that centres on shelf space planning. Kök et al. (2015) summarise different modelling techniques for assortment planning and related problems. They develop a unified modelling approach for assortment and inventory planning but do not integrate for example limited shelf space or space elasticity, which are at the heart of shelf space planning. The discussion on shelf space planning is based on only five different sources that were all published (at least as working papers) before 2004. Hübner and Kuhn (2012) focus on the integration of shelf space planning and assortment planning. Their work does not derive the different demand sources in a structured manner, does not include vertical allocation and horizontal location, does not provide a comprehensive literature classification and does not develop a unified mathematical model and notation. Because their focus is not just shelf space planning, they do not provide unified models and do not investigate all planning aspects related to shelf space planning decisions.

To summarise, there is neither any up-to-date structured literature nor any historically grown modelling approach available. The contribution of this paper is an up-to-date literature review that focuses fully on shelf space planning. To comprehensively structure the emerging literature on shelf space planning while taking into account the retailers' actual planning problem and possible impacts of decisions on consumer demand, associated costs and revenues, it is required to take a broader view than the scattered literature only. The starting point needs to be informed by the literature and the insights gathered from actual retail practice. This gives a much broader picture and allows analysing literature gaps from a theoretical standpoint and from the industry perspective. We follow this approach as no work has yet systematised published research.

For that purpose, the remainder of the review is organised as follows. We first develop the structure of the shelf space planning problem in Section 2 by defining the problem, relate it to other planning areas and identify the basic features that models must capture to support decision-making. Based on the definition of the planning problem, we review quantitative decision models in Section 3 and propose a classification framework to systematise the research into a set of sub-problems, followed by a unified approach with a univocal notation of model classes in Section 4. The unified modelling approach is not intended to be a new formulation to the shelf space planning problem but to enable us to classify the different contributions, to take into consideration key features, and to comprehensively discuss open areas of research in shelf space planning in Section 5.

2. Shelf space planning problem

This section describes the shelf space planning issues from the retailers' actual planning problem, modelling literature and empirical insights into consumer behaviour. This starting point gives a more comprehensive perspective on the decision problem and coherent modelling requirements. This section first explains the scope of shelf space planning and describes associated problems that are inputs for shelf space planning (Section 2.1). The decisions to be made are defined in Section 2.2, the associated effects of space on demand are analysed in Section 2.3, the related profit and cost functions are identified in Section 2.4, and the constraints are presented in Section 2.5.

2.1. Scope of shelf space planning and related planning problems

The ultimate objective of shelf space planning is to maximise the retailers' profit that stems from realised customer demand, which in turn depends on the vertical and horizontal positioning and space allocated to the products on the shelf, the product margins and inventory and operational costs. Shelf management comprises two hierarchical levels. The first (macro) level is strategic and involves deciding the space for product categories (e.g. beverages, chocolate) and shelf types. The second (micro) level is tactical and involves allocating individual products within each category. Due to the large number of products within categories, it may also be practical to manage shelves on a disaggregated level (e.g. subcategory or brand level). Shelf space planning is usually connected with the micro level and considers the allocation of a category of products to a set of defined shelves. Hence, shelf space planning is part of the tactical category planning process (Hübner & Kuhn, 2012). In category planning, Hübner and Kuhn (2012), Kök et al. (2015) and Hübner (2017) differentiate among a series of hierarchical planning steps:

- *Assortment planning* involves listing and delisting products and hence the selection of products within a category. The demand model needs to take into account substitution effects for delisted products.
- *Shelf space planning* assigns space and defines the horizontal and vertical location of listed products on the shelves, taking into account the constraints of limited capacity. The product demand may depend on the available quantity and position on the shelves.
- *In-store replenishment planning* determines refill policies. It includes areas such as in-store logistics processes and inventory review procedures. Its purpose is to achieve the required on-shelf service levels based on the shelf space plans determined.

The three tactical planning areas are interdependent if shelf space and replenishment capacity are limited. For example, a broader assortment with more products requires fewer units per product on the shelf or more frequent restocking. Therefore, retailers have to carefully consider the effect of carrying large assortments due to space limitations. Increasing the assortment reduces the visibility of the products on the shelves and drives lower inventory levels, which increases the risk of out-of-stocks and imposes frequent replenishment operations. On the other hand, not including some products in the assortment may generate lost demand from loyal customers. Moreover, retailers only display a limited amount of the inventory on the shelves, keeping the additional store inventory in the backroom. Alignment between shelf replenishment operations and shelf space planning is thus also vital to avoid out-of-stocks with the in-store inventory. This means that a careful alignment of assortment, shelf and replenishment planning is needed.

Retailers typically solve problems hierarchically: first they determine their assortment, next they allocate it to the shelf, and finally, they manage in-store replenishment (see Desmet & Renaudin, 1998; Hübner, Kühn, & Kuhn, 2016; Kuhn & Sternbeck, 2013). The main reasons for this sequential approach are that most retail category planning steps take place from a functional perspective and have multiple decision owners and different planning horizons. Assortment planning is traditionally the domain of a central marketing planning unit, whereas mid-term shelf space planning typically belongs to the sales organisation team. Store managers are responsible for in-store logistics planning. The breakdown into these three planning steps, as applied in practice, helps to overcome the analytical complexity and supports the development of analytical models that can capture and solve the problem. The planning modules must relate to the organisational hierarchies and responsibilities as well as to the planning horizon (Hübner, Kuhn, & Sternbeck, 2013a; Schneeweiss, 2003). The interconnection between planning modules can be incorporated as constraints. The most important constraint is the limit on the total *shelf space* that is determined by the macro space planning. Furthermore, *control* and *availability* constraints for lower and upper bounds may also be set for the products to integrate the assortment and replenishment planning. Lower bounds are defined when the retailer wants to maintain a minimum shelf representation (e.g. for new products) or a minimum service level. Upper bounds can be set for fast-moving products when the retailer wants to leave space for other products.

Finally, there are further related decisions that also impact shelf space planning. For example, pricing decisions impact demand and ultimately shelf availability and replenishment requirements. The role of category captains, general promotion policy (e.g. everyday low price vs. high-low prices), options for secondary placements (e.g. at the end of the gondola) or sales during seasonal weeks also impact demand, volatility and seasonality, and available shelf capacity. This requires dedicated approaches to estimate demand and calibrate it to a normal week. However, this goes beyond the scope of this paper.

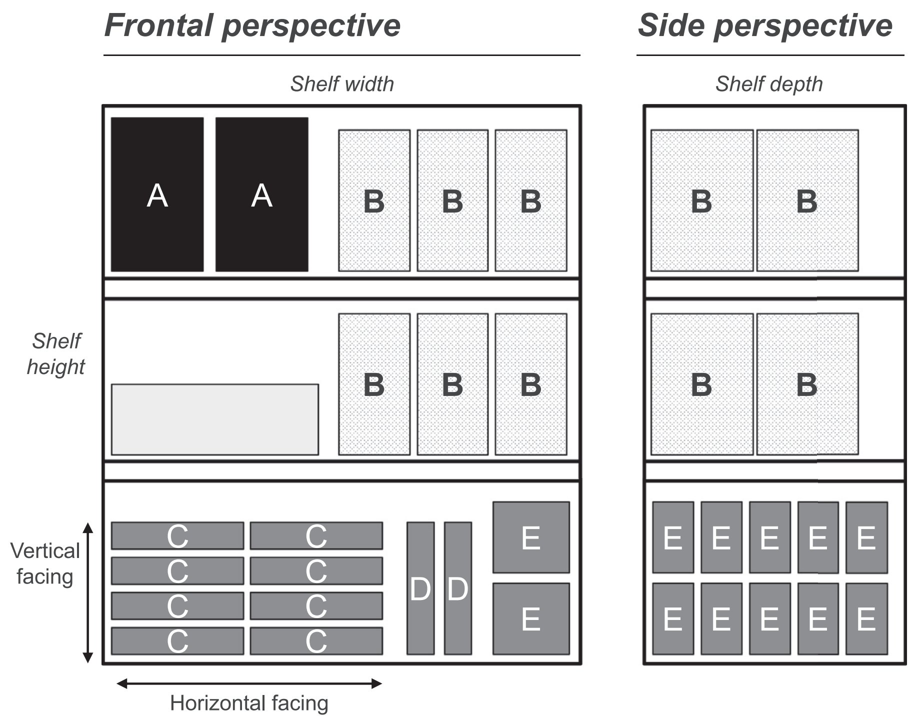

Fig. 2. Example of a planogram.

This paper focuses only on the shelf space planning problem. It is assumed that – according to the aforementioned hierarchical planning concept usually applied by retailers – assortment, pricing and promotion decisions have been made in the previous step by a different decision owner and replenishment is operated efficiently at the store level. All of the above decisions are product-related. The shelf-related decisions (e.g. shelf height and depth, number of levels per shelf) are usually given as inputs to the models after macro space planning because retailers are not likely to change the layout of the shelves during product reallocation.

2.2. Decisions to be made in shelf space planning

The traditional shelf space planning tool for retailers is a planogram, representing an illustration of a shelf space plan of a specific category, showing exactly where each product should physically be displayed at the different shelf levels (vertical allocation), how it will be positioned horizontally (horizontal location), and how much space that product should have (space assignment). Fig. 2 presents an example of a planogram. From the frontal view of the shelf, product A gets 2 facings (=number of visible units of a product in the front row) on the top shelf level and is positioned horizontally on the left side of the shelf, while product B gets 3 facings on 2 shelf levels and is positioned in the middle. Product C is stacked, has 2 horizontal facings and 4 vertical facings and 8 facings in total. It is located at the beginning of the first shelf level. Product D is positioned upright, whereas product E is positioned crosswise. Fig. 2 also shows that retailers typically choose a rectangular block formation (see product B for example), i.e. they place the same number of facings on several shelf levels exactly below one another such that the product presentation is rectangular.

The right part of Fig. 2 illustrates the difference between a *facing*, as the first visible unit of a product on the shelf, and the *shelf quantity* of a product. It shows that behind each facing there is a certain stock of additional units of a respective product such that the total shelf quantity is determined by the number of facings times the quantity per facing. For example, product E has 2 facings and 5 units behind each facing and therefore a shelf quantity of 10 units. If the number of units behind each facing is fixed (as usually the shelf is filled to the maximum), then the shelf quantity is no longer a primary decision and just an auxiliary variable.

Retailers use different software providers to create the planograms (see Hübner & Kuhn, 2012). Shelf space planning applications such as "Spaceman Suite", "Apollo Professional" and "Space Planning" visualise shelf plans. The main purpose of these tools is to simulate alternative product placements on-screen and the related scenario analyses. To create a planogram, a retail shelf space planner (usually referred to as a merchandiser) needs to make decisions for each category related to the following three areas:

- *(I) Space assignment*: Determines the *number of horizontal facings*, *number of vertical facings*, *quantity per facing* and ultimately also the *total shelf quantity* for each product $i \in \mathbb{N}$. The facings can be arranged horizontally next to each other, stacked within one shelf level or spread over multiple levels. This is expressed by the integer variable x^h_{ik} for the number of horizontal facings for a product $i \in \mathbb{N}$ of a shelf level $k \in \mathbb{K}$. The associated integer variable for stacking x^v_{ik} expresses the number of vertical facings of a product $i \in \mathbb{N}$ of a shelf level $k \in \mathbb{K}$. The total number of facings x_{ik} per shelf level results from the multiplication of all vertical and horizontal facings ($x_{ik} = x^h_{ik} \cdot x^v_{ik}$).
  The shelf quantity can be denoted by the integer variable q_i and can be computed by $q_i = \sum_{k \in K} x^d_{ik} \cdot x_{ik}$. The variable x^d_{ik} in this equation denotes the quantity per facing that can be positioned in the depth of the shelf, i.e. the number of units per product that should be placed in one shelf slot (=facing itself plus the number of units behind the facing).
  A further problem related to the spacing decision is the display orientation. This specifies the way products are displayed on shelves, as products can be displayed length- or crosswise, upright or across (see Fig. 2). This can be expressed by an integer variable w_i that characterises how a product is displayed and its associated visible width. The retailer must also decide on the display orientation of the carton when single units of a product are stored in cartons.

Table 1
Overview of decisions in shelf space planning and associated OR problems.

Decision area	(I) Space assignment	(II) Vertical allocation	(III) Horizontal location
Decision variables	Facing (x_i), quantity (q_i), display orientation (w_i)	Vertical allocation (z_{ik})	Horizontal location (y_{il})
Related OR problems	Knapsack	Assignment, layout	Scheduling

- *(II) Vertical allocation*: Determines to which shelf level a product is assigned, i.e. the vertical position within the shelf. This can be expressed by the binary variable z_{ik}, which is equal to 1 if product $i \in \mathbb{N}$ is allocated to shelf $k \in \mathbb{K}$, and otherwise 0. It should be noted that the allocation decision can also be expressed with x_{ik}, as $z_{ik} = 1$, if $x_{ik} \geq 1$, and otherwise 0.
- *(III) Horizontal location*: Determines how products are arranged next to each other and how far a product is positioned from the aisle, i.e. the horizontal location. The binary variable y_{il} expresses whether the location $l \in \mathbb{L}$ is chosen for product $i \in \mathbb{N}$ (=1), or otherwise (=0). It should be noted that the vertical allocation variable z_{ik} becomes redundant if the decisions are taken simultaneously for vertical and horizontal position by y_{ikl}, which also includes assignment to a shelf level $k \in \mathbb{K}$. Alternatively, it is also possible to identify how far the product is from the beginning of the shelf (y_i) or the product adjacencies on each shelf (y_{ijk} (=1) if product i is immediately before product $j \in \mathbb{N} : j \neq i$ on shelf k). The horizontal location may also change between shelves (although this is not commonly seen).

We use the term “vertical allocation” to describe the allocation of a specific product to a certain vertical shelf (see e.g. also in Düsterhöft, Hübner, & Schaal, 2020; Hübner & Schaal, 2017b; Yang, 2001) and the term “horizontal location” for the sequence and horizontal position. Both definitions will later help to univocally define model classes and decision problems. It is also worth noting that some literature uses alternative terms such as vertical/horizontal “position”, “assignment” or “location”.

Association of these decisions with other OR problems. Shelf space planning decisions can be ascribed to classical OR problems as depicted in Table 1. (I) Space assignment is a knapsack problem as different products need to be assigned to a capacitated shelf (=knapsack). (II) Vertical allocation is an assignment and layout problem, as items need to be assigned to different levels. (III) Horizontal location can be ascribed to a scheduling problem, as sequences of products need to be defined.

Interdependency of decisions. All three decisions are interrelated. For example, the quantity of a product may be spread across multiple shelf levels, and if a product is positioned at eye-level or in an attractive horizontal position, it requires more space to fulfil demand. It needs to be noted that other decisions also impact the demand for a product and ultimately impact shelf space planning, e.g. price changes or temporary promotions result in higher demand, and these products will require more shelf space. The investigation of these related problems is considered to be beyond the scope of this paper. The following section will focus on the impact that shelf space planning decisions have on customer demand.

2.3. Effects of shelf space planning on demand

Empirical studies have proven the positive influence of shelf decisions (I) to (III) on stimulating consumer demand, and have identified different demand types: (1) space-dependent demand and (2) position-dependent demand.

(1) Space-dependent demand impacted by (I) space assignment. Space assignment impacts the visibility and availability of a product. A differentiation needs to be made between (a) space-elastic demand, (b) cross-space elastic demand, and (c) out-of-stock substitution demand.

(a) Space-elastic demand was defined by Curhan (1972) as “the ratio of relative change in unit sales to relative change in shelf space”. It expresses that the higher the visibility of a product, the higher its demand. This phenomenon is well known in research and has been investigated in various studies over the last 50 years. Cox (1964) tests the impact that variations in shelf space have on sales of staple and impulse purchase products. He finds that the extent to which staples react to space variations is not very different from the extent to which impulse purchase products react. Frank and Massy (1970) use a cross-sectional experiment to test the influence that shelf space has on sales of grocery products and conclude that the effect of shelf space is directly linked to store size. Curhan (1972) applies multiple regressions to investigate the results of shelf space variations and finds a positive correlation between shelf space and units sales, although not all products are found to be space elastic. Furthermore, the authors point out the difficulty of carrying out tests because of unrecognised or uncontrolled relationships between items and the impact of other variables, such as prices or promotions. In their experiment, Drèze, Hoch, and Purk (1994) also investigate the impact of space on sales and conclude that it is one of the least important factors as most of the products have a number of facings that put them on the flat portion of the S-curve. Desmet and Renaudin (1998) find that store and category characteristics can explain why different products react differently to space variations. Although opinions about the magnitude of space elasticity impacts differ, all researchers conclude that product sales and product shelf space are positively correlated. Recently, Eisend (2014) conducted a meta-analysis comprising 1,268 estimates of space elasticity and concluded that the average space elasticity amounts to 17%.

In summary, demand for a product grows with the number of facings assigned to it. The magnitude of the demand increase depends on the space elasticity β_i, the number of facings x_i ($= \Sigma_k x_{ik}$) and the unmodified base demand α_i. The space-elastic demand is denoted by $d_i^{sp}(x_i)$ and calculated according to Eq. (1), as first proposed by Corstjens and Doyle (1981). We omit the shelf level and the different facing arrangements in the vertical, horizontal and depth dimensions to simplify the presentation of the notation, but the same relation holds true when the related positioning indices are attached.

$$d_i^{sp}(x_i) = \alpha_i \cdot x_i^{\beta_i} \qquad i \in \mathbb{N} \tag{1}$$

The unmodified base demand α_i is equal to the minimum demand for a product if it were represented with one facing ($x_i = 1$), i.e. $\alpha_i = d_i^{sp}(x_i = 1)$. It should be noted that the space-elastic demand for a product i with $x_i = 0$ results in no demand, as $d_i^{sp}(x_i = 0) = \alpha_i \cdot 0^{\beta_i} = 0$, and grows at a diminishing rate with $x_i^{\beta_i}$ (for $x_i > 1$) and $0 \leq \beta_i \leq 1$. However, as no assortment decisions are taken in this phase, the number of facings needs to be greater than or equal to one ($x_i \geq 1, i \in \mathbb{N}$).

To incorporate different display orientations, the demand function can be extended to express the associated width w_i (e.g. measured in centimetres) of a display orientation. This allows the demand function above to be formulated as $d_i(x_i, w_i) = \alpha_i' \cdot (x_i \cdot w_i)^{\beta_i}$ (Hübner & Schaal, 2017a). The minimum base demand α_i' is calibrated additionally on product width. The vertical number of facings x_i^v (i.e. if products are stacked) can also be similarly incorporated as a multiple: $d_i(x_i^h, x_i^v) = \alpha_i \cdot (x_i^h \cdot x_i^v)^{\beta_i}$.

(b) Cross-space elastic demand was introduced by Corstjens and Doyle (1981) to evaluate the interdependency between two different products, i.e. when the demand for a product depends on the space allocation of other products. Cross-space elasticities are considered to be positive for complementary products and negative for substitution products. Corstjens and Doyle (1981) use regression analyses and identify an average cross-space elasticity of -0.028 for five products across 140 stores. Drèze et al. (1994) experience a boost in sales of more than 5% in complementary merchandising. However, the discussion of cross-space elasticity effects is ambiguous in the pertinent literature. Brown and Lee (1996) and Kök et al. (2015) state that there is no empirical evidence that product-level demand can be modelled with cross-space elasticity. Zufryden (1986) argues that considering cross-space elasticity at an individual level would be impossible in practice due to the overwhelming number of cross-elasticity terms that would need to be estimated. In his meta-analysis, Eisend (2014) finds only five studies over the past 40 years that have been able to identify cross-space elasticity. He computes an average cross-space elasticity of −0.016, and Schaal and Hübner (2018) find that facing decisions are only significantly affected if cross-space elasticity differs substantially from this average value.

Cross-space elastic demand can be expressed as formulated in Eq. (2), which is an extension of Eq. (1) (Corstjens & Doyle, 1981). The total demand (accounting for space- and cross-space elasticities) for a product i, $d_i^{csp}(\bar{x})$, no longer depends exclusively on the number of facings of the product i (x_i), but also on the number of facings of all other products, with $j \neq i$ ($\bar{x}$, where $\bar{x}$ is a vector of shelf quantities for all products). The cross-space elasticity δ_{ij} between products i and j implies that every time the number of facings of product j is changed, the demand for product i changes by the factor δ_{ij}. Depending on the relationship between two products, δ_{ij} can be positive or negative, $-1 \leq \delta_{ij} \leq 1$. If two products are substitutes (complements), their cross-space elasticity is negative (positive). It must be noted that δ_{ij} is not necessarily equal to δ_{ji} as the two products may have a distinct impact on each other.

$$d_i^{csp}(\bar{x}) = \alpha_i \cdot x_i^{\beta_i} \cdot \prod_{j \in \mathbb{N}, j \neq i} x_j^{\delta_{ij}} \qquad (2)$$

(c) Out-of-stock substitution demand. The spacing assignment (x_i^v, x_i^h, x_i^d) and the associated shelf quantity (q_i) impact the availability of a product and may consequently lead to out-of-stock substitution demand. If demand d_j exceeds the available quantity q_j of a product $j \in \mathbb{N}$, the resulting shortages may lead to demand gains of other products $i \in \mathbb{N}$ by substitution. This means that, in the event of shortages, customers may tend to substitute products that are temporarily out-of-stock (OOS). The OOS risk can be minimised if shelves are replenished frequently and backroom inventory is available (see e.g. Holzapfel, Hübner, Kuhn, & Sternbeck, 2016; Hübner & Schaal, 2017a; Kök & Fisher, 2007). Empirical studies show that between 45% and 84% of demand can be substituted (Ge, Messinger, & Li, 2009; Gruen, Corsten, & Bharadwaj, 2002; van Woensel, van Donselaar, Broekmeulen, & Fransoo, 2007). The average potential for substitution depends on product-, situation- and consumer-specific characteristics (Fitzsimons, 2000; Ge et al., 2009), and on the sequence of customer arrivals (Gilland & Heese, 2012). This effect is intensified, as demand for retail products is usually stochastic.

OOS demand for a product i occurs if another substitute product j ($i, j \in \mathbb{N}$) is OOS, i.e. if demand d_j for product j exceeds the available shelf quantity of product j. The OOS demand depends on the available shelf quantity and not directly on the number of facings. In this case, customers substitute a certain share of the shortfall of product j with product i. The shortfall of product j is calculated by $(d_j - q_j | d_j > q_j)$ and the substitution share is denoted by δ_{ji}^{OOS}. Eq. (3) shows the OOS demand calculation.

$$d_i^{OOS}(\bar{q}) = \sum_{j \in \mathbb{N}, j \neq i} [(d_j - q_j) | d_j > q_j] \cdot \delta_{ji}^{OOS} \qquad i \in \mathbb{N} \qquad (3)$$

(2) Position-dependent demand impacted by (II) vertical allocation and (III) horizontal location. Position-dependent demand can be further differentiated between (d) vertical position-, (e) horizontal position- and (f) arrangement-dependent demand. The vertical and horizontal position effects are often studied jointly, which is why we also analyse them together (see e.g. Drèze et al., 1994).

(d)(e) Vertical and horizontal position-dependent demand measures the impact that the position of a product within a shelf has on demand. As various studies have found, the position of a product impacts the likelihood of it getting noticed and purchased. In general, studies show a higher impact of products located on the top- and middle-shelf positions (at eye and hand level). For example, Drèze et al. (1994) report an average 39% sales increase from the worst to the best vertical allocation. Underhill (1999) identifies a “reliable zone” ranging roughly from eye- to knee-level. Products positioned within this zone are likely to be seen; products outside this zone are not. Chandon, Hutchinson, Bradlow, and Young (2009) found that products positioned on the top-shelf level are 17% more likely to be noticed and 20% more likely to be chosen than products on the bottom-shelf level. The horizontal position effects have a less significant impact than the vertical. For example, Drèze et al. (1994) report an average 15% sales increase from the worst to the best horizontal location. They also found an average 59% increase if both vertical and horizontal effects are combined. Chandon et al. (2009) found a significant drop in the likelihood of noticing products that are not centrally positioned versus products that are positioned in the centre of a shelf.

The demand arising from the horizontal location and vertical allocation, denoted as d_i^{VH}, can have two different sources. The first source is the vertical allocation ϕ_{ik}, which depends on z_{ik}, and the second source is the horizontal location ρ_{il}, which depends on y_{il}. These two factors are aggregated in the function $\gamma(\ldots)$, which expresses the demand effect from vertical and horizontal positioning effects. Multiplying this function by the basic demand α_i, as denoted in Eq. (4), results in the demand d_i^{VH}.

$$d_i^{VH}(y_{il}, z_{ik}) = \alpha_i \cdot \gamma(\phi_{ik}(z_{ik}), \rho_{il}(y_{il})) \qquad (4)$$

(f) Arrangement-dependent demand describes the way product facings are arranged on the shelves. This can also have an important role in gaining consumer attention. Alternative terms for this effect are family grouping or product grouping. Pieters, Wedel, and Batra (2010) show that carefully organising a display in families increases viewer attention, but excessive complexity (i.e. variations in the basic visual content) can indeed decrease their interest. As a result, products are organised in families and rectangular shapes.

In summary, the total positioning-dependent demand, denoted as d_i^{pos}, can have three different sources. The first two sources arise from d_i^{VH}, as denoted in Eq. (4), and are extended by the arrangement effect of the facings θ_i, which depends on x_{ik}, y_{il} and z_{ik}. These three factors are summarised in the aggregated demand factor for positioning decisions, which is denoted as γ. Eq. (5) summarises this demand effect:

$$d_i^{pos}(x_{ik}, y_{il}, z_{ik}) = \alpha_i \cdot \gamma(\phi_i(z_{ik}), \rho_i(y_{il}), \theta_i(x_{ik}, y_{il}, z_{ik})) \qquad (5)$$

Summary of related demand effects. Total demand can be expressed in the function of components (a)–(f) as formulated in Eq. (6).

$$d_i^{total}(\bar{x}, y_{il}, z_{ik}, \bar{q}) = d_i^{csp}(\bar{x}) \cdot \gamma(\phi_i(z_{ik}), \rho_i(y_{il}), \theta_i(x_i, y_{il}, z_{ik})) + d_i^{OOS}(\bar{q}) \qquad i, j \in \mathbb{N} \qquad (6)$$

Total demand d_i^{total} depends on the decision about the facings of all products ($\bar{x}$), the vertical allocation (y_{il}), the horizontal location (z_{ik}) and an additive part of the substitution from other products based on the shelf quantity $\bar{q}$. To streamline the notation, we have provided the demand components as deterministic formulations. Nevertheless, our formulation is generic enough to cope with stochastic demand. In such cases, each demand component would need to be represented with a density function and a convolution can be used to calculate the distribution of the sum of the demand of the items. For example, the probability density function $f_{d_i^{sp}}$ to account for space-elastic demand is calculated by Eq. (1), and the corresponding density function is denoted by $f_{d_i^{sp}}(x_i)$ and calculated with $\circledast_{\mathrm{i} \in \mathbb{N}} f_{d_i^{sp}} = \int \cdots \int_{\mathbb{R}_0^{+,n}, i \in \mathbb{N}} f_{d_i^{sp}} d\tau \ldots d\upsilon$. The convolution – represented by the operator $\circ$*ledast* – can be used to calculate the distribution of the sum of the demand of the two items. To include the other demand effects (b)–(f), one need to adjust the density functions accordingly (see also Hübner & Schaal, 2017b; Hübner & Schaal, 2017c).

However, it needs to be noted that not all components are relevant to each category and problem setting. For example, in some categories, space-elastic demand may be low or zero (e.g. commodities like toilet paper), other may not benefit from vertical positioning effects (e.g. a particular type of pet food that is presented on a very granular shelf type) or horizontal location effects can be ignored (e.g. very small shelves). Furthermore, due to the high testing costs, experiments have not been sufficiently extensive, with some of them dating back to the 1960s and 1970s. For example, to complete a full factorial design, it would be required to test across different category types (e.g. commodities vs. impulse buying), store types (e.g. small store vs. hypermarket), shopping purpose (e.g. impulse vs. planned weekend shopping), customer segments and different levels (e.g. increase vs. decrease shelf space) and variation across the variables (e.g. increase of shelf space together with different vertical level). This results in an arbitrarily large number of scenarios to be tested for obtaining general results. This explains why the current empirical studies provide mostly indications, with few conclusions that are common across different empirical studies. A more comprehensive synthesis of general empirical findings on consumer behaviour will be beneficial. In addition, some results are contradictory. As an example, while Chandon et al. (2009) reveal that the variation in the number of facings is the most significant in-store factor, Drèze et al. (1994) state that position has a larger impact as long as a minimum inventory is maintained to avoid out-of-stock.

2.4. Related profit and cost functions in shelf space planning

Shelf space planning and the associated demand effects impact the profit of retailers in different ways. First of all, the *expected revenues* represent the realised demand multiplied by a unit sales price. We use the term “expected revenues” as retail demand is usually stochastic (see e.g. Agrawal & Smith, 1996), and although most models take a more simplistic approach by assuming a known demand (Hübner et al., 2016).

This is offset by multiple cost parameters, namely *purchasing costs* for each product, *inventory holding costs*, *salvage costs* (if products remain in stock at the end of a given period, they must be disposed of at a salvage value and the retailer incurs a loss on each product, i.e. the delta between purchase costs and salvage value), and *shortage costs* (if the expected demand is greater than the available shelf quantity for the product). In this case, the excess demand is lost and the retailer suffers the shortage costs per unsold unit. Furthermore, there are *replenishment and operational costs* for shelf refills that are impacted by the decision about the shelf quantity. If the shelf space is limited, the shelf and replenishment decisions are interdependent because a higher number of facings (and consequently a higher shelf quantity) for a product also implies that the product must be replenished less frequently, and vice versa. Finally, there may be further *backroom costs* for inventory holding of products in the backroom and shelf replenishment from there.

2.5. Constraints to be considered in shelf space planning

Shelf space planning constraints entail business and modelling requirements, and range from hard to soft, depending on the need to entirely satisfy these requirements.

(1) The term *integer facings* means that only positive integer values for the number of facings can be applied; (2) *lower and upper bounds* on facings limit the solution space. Lower bounds are defined when the retailer wants to maintain a minimum number of facings of all products or because of new products that need a chance to make an impact. Upper bounds are set for products at a later stage of the life cycle, or for sales champions, when the retailer wants to leave space to refresh the product assortment of the stores; (3) *shelf capacity* limits the available shelf space; and (4) *availability* limits product quantity (despite of the demand) because of production capacity or availability limits.

Category-level shelf space planning may contain hundreds of different products that, if placed considering only space-, vertical- and horizontal-elasticity effects, may confuse buyers during their search for products. (5) *Merchandising rules* introduce constraints for product arrangements and define families of products that should be placed together on the shelves, preferably in rectangular shapes (e.g. all products of the same brand should be placed together and, within each brand, all products of the same flavour should also be placed together). These merchandising rules may also specify other requirements such as family sequences, shape orientation (either columns or lines), and special locations for special products, among others. Merchandising rules try to reflect consumer buying behaviour and the strategy of the company (and suppliers) for the different categories (Bianchi-Aguiar et al., 2016).

2.6. Summary

Fig. 3 summarises the related planning problems, the decisions to be taken in shelf space planning, the impact on customer demand, and the relevant costs. Shelf space planning defines the space assignment, as well as the vertical and horizontal position of all products for a given assortment. The space assignment may also include other decisions such as the determination of shelf quantity, display orientation and stacking possibilities. Each of these has different demand effects and a different impact on margins and costs.

3. Classification of shelf space planning literature

This literature review follows the guidelines for systematic review (Kotzab, Seuring, Müller, & Reiner, 2006). First, we defined the scope (as in Section 2). Then, we identified the related literature. The identification step included the collection/selection of material and the selection of categories. Finally, we completed a content analysis. Our systematic literature review covers two lines. First, we searched peer-reviewed, international research journals. Because the number of publications in this area is growing, we conducted an Internet search in this field to also find up-to-date working papers and dissertations. The first search, completed simultaneously as an iterative review, included all major international journals that publish research in Operations Research, Retailing and Operations Management. Furthermore, we utilised initial search results from open databases (Google Scholar, SSRN), library service databases (EBSCO, Scopus, Metapress) and major pub-

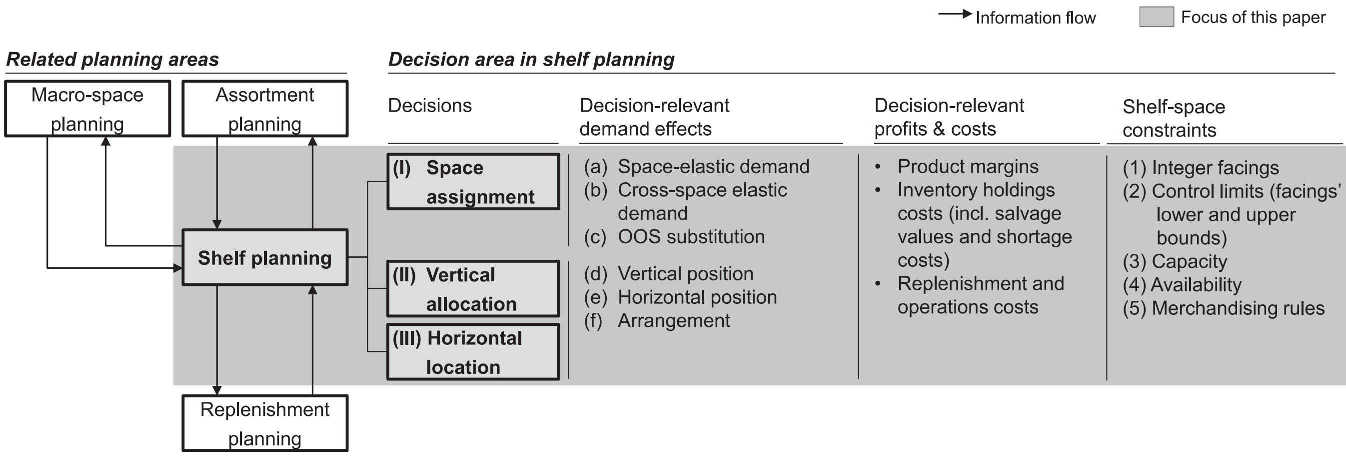

Fig. 3. Overview of related planning problems, decisions and demand effects in shelf space planning.

Table 2
Notation for different components of shelf space planning models.

Decisions		Related problems		Demand effects		Demand types		Constraints	
S	Space assignment	A-	Assortment	-spe	Space elasticity	-sto	Stochastic	-int	Integer facings
A	Vertical allocation	P-	Pricing	-csp	Cross-space elasticity	-det	Deterministic	-avai	Availability
L	Horizontal location	M-	Macro-shelf space	-oos	OOS substitution	-stat	Stationary	-cap	Capacity
		R-	Replenishment	-ver	Vertical demand effect	-seas	Seasonal	-merch	Merchandising rules
				-hor	Horizontal demand effect				
				-arr	Arrangement effect				

lishers (Emerald Insight, INFORMS, ScienceDirect, Springer Link, Taylor & Francis, Wiley), and checked literature cited in the papers identified. Related keywords in the full-text searches included all forms of shelf space management, shelf allocation, shelf location, space elasticity, shelf positioning, including plural forms (e.g. "shelves"), delimiters (e.g. "space-elasticity"), prefixes, and suffixes. The related keywords were combined for the search. However, it is important in a literature review to define distinct boundaries (Seuring, Müller, Westhaus, & Morana, 2005). Because we focus on quantitative decision models, we excluded literature that strictly covers general management, marketing and service management issues, and that does not discuss Operations Research aspects at all. This literature mainly addresses customer behaviour and empirical studies. We used content analysis to identify the conceptual and modelling content that is related to the shelf space planning field. Papers were assessed based on their decision modelling, demand models and solution approaches. To increase the reliability of the research, databases, journals and individual papers were checked by a second researcher and updated iteratively during the revision cycle of the paper by the entire team of authors.

This section starts by proposing a classification framework and notation in Section 3.1. Based on this classification, the literature is then reviewed in Section 3.2. The review mainly tackles the type of decisions, demand models and constraints. The last part also provides an overview of the solution approaches and data instances used in the publications during numerical analysis.

3.1. Classification framework for shelf space planning problems

To foster a comparison among models and their further development, we propose a unified notation for the models, as seen, for example, in the literature of vehicle routing or lot sizing. The notation is based on acronyms for the decisions to be taken, the type of demand considered within the model, and further extensions of the model to integrate other planning problems. We structure the notation according to the dimension decisions, related problems,

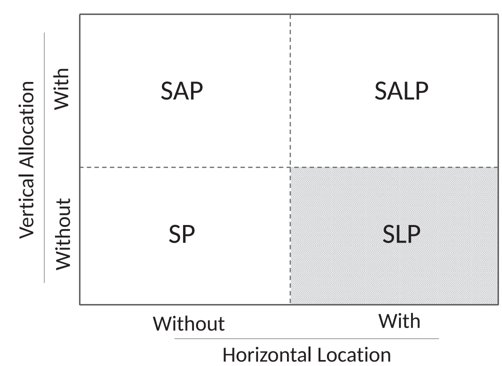

Fig. 4. Classification and notation framework for the shelf space planning problem.

demand effects, demand types and constraints, as summarised in Table 2.

In shelf space planning, the *decisions* to be taken and the demand function are at the centre of the problem, and different approaches aggregate different types of decision and demand functions. Fig. 4 gives an overview of the different decisions that can be modelled. A model denoted with *SP*- makes decisions for the *S*pace assignment *P*roblem, whereas the model type *SAP*- considers the joint *S*pace assignment and vertical *A*llocation *P*roblem. Extending the *SAP*- to horizontal *L*ocation *P*roblems will be denoted as *SALP*. It should be noted that all shelf space planning models have the space assignment decision in common. Therefore, all model notations include an *S* for space assignment.

Further extensions of the problem can be denoted with an *acronym* before the shelf space notation. The literature extends shelf space planning into four major *related planning problems*. These are assortment planning (i.e. the product selection), pricing (i.e. determining the shelf price), macro-shelf space planning (i.e. determining the overall shelf space of a category and the location of a category within a store), and replenishment planning (i.e. order quantity and refill frequency). We denote the related planning problems accordingly using *A*-, *P*-, *M*- and *R*-.

The third part of the model notation will consist of the demand components. The effect that space has on demand can be organised according to the structure above into space elasticity (*-spe*), cross-space elasticity (*-csp*), OOS substitution (*-oos*), vertical demand effect (*-ver*), horizontal demand effect (*-hor*) and arrangement effect (*-arr*). The fourth part denotes the demand types. These can be differentiated into stochastic (*-sto*) vs. deterministic (*-det*) and stationary (*-stat*) vs. seasonal (*-seas*) demand. Demand-related terms are denoted with a suffix in small letters. Finally, the major constraints are described using *-int* for integer facings, *-ctr* for control, *-avai* for product availability, *-cap* for capacitated shelf space, and *-merch* for merchandising rules. Constraints are also denoted with a suffix in small letters.

As an example, a problem that is called SALP-spe-csp-ver-hor-sto-int-cap-merch considers the Space, Allocation and Location decisions with space (-spe), cross-space (-csp), vertical (-ver)- and horizontal (-hor) demand effects and stochastic demand (-sto). It also considers integer facings (-int), capacity (-cap) and merchandising rules (-merch) as constraints. If the assortment problem is included, the model name can be preceded by the acronym “A-” for example as A-SP, that stands for a joint *A*ssortment and *S*helf assignment *P*roblem.

3.2. Literature review

This section summarises and structures the approaches to modelling the shelf space planning problem. It first gives a general overview in Section 3.2.1 about the most common shelf space planning problems, demand models, solution approaches and instances. Section 3.2.2 analyses the different streams in detail.

3.2.1. Overview

Table 3 presents the characteristics of modelling papers related to shelf space planning (only peer-reviewed papers were considered). The first two columns summarise the decision problems and related problems, demand effects and demand types are represented in the middle, and the last column describes the constraints.

Decisions and related problems. Existing mathematical models focus particularly on determining space for the products (SP and SAP are the most common problems). The reason behind this emphasis lies in the fact that most authors consider space elasticity as the most important effect. However, it must be noted that if vertical allocation and horizontal location are disregarded, solutions do not translate into a complete description of a planogram. Moreover, in terms of the objective function, the literature has focused more on demand effects and less on the cost side. Most publications only subtract the corresponding purchasing costs from the sales price of each unit sold, resulting in its gross margin. Corstjens and Doyle (1981) and Reyes and Frazier (2007) are some of the exceptions. A limited number of papers have expanded the scope of the decision problem. The dominant direction so far has been to integrate inventory replenishment decisions into shelf space planning (Hwang, Choi, & Lee, 2005; Urban, 1998; 2002). Some deal with assortment selection (Borin, Farris, & Freeland, 1994; Hübner & Schaal, 2017b; Kök & Fisher, 2007). Few align shelf space planning with macro-shelf space planning (Ghoniem, Flamand, & Haouari, 2016a; 2016b; Irion, Lu, Al-Khayyal, & Tsao, 2011).

Space effect on demand and types. The current literature provides a great variety of demand models that incorporate different demand effects as well as different methods of aggregating these effects. Moreover, many of the proposed functions tend to focus on particular effects while disregarding others. The consideration of space elasticity is common to all approaches and this effect is frequently aggregated with other elasticities using a multiplicative form. Cross-space elasticity and vertical position effects are also frequently considered, but only a very limited number of approaches aggregate both effects (Hwang et al., 2005; Lim et al., 2004; Yang & Chen, 1999). The OOS and horizontal dimensions are often disregarded, whereas the arrangement effect is only factored in one approach (Lim et al., 2004).

Constraints. Integer facings, capacitated shelf space, and lower and upper bounds on the number of facings are the most common constraints of the problem. Availability considerations were introduced by Corstjens and Doyle (1981). However, in most models, it is assumed that retailers can ensure availability and hence prevent out-of-stock occurrences by building effective logistics systems (e.g. Yang & Chen, 1999). Merchandising rules have been formulated as constraints in a very limited number of publications so far (Bianchi-Aguiar, Silva, Guimarães, Carravilla, & Oliveira, 2018; Russell & Urban, 2010).

Solution approaches and data instances. Beyond the criteria listed in Table 3, it is important to assess the solution approaches and problem scope. Most models were solved using meta-heuristics or specialised heuristics. Exact solutions (either generated by full enumerations or solver applications) are only possible for simplified demand models or very restricted data instances. Scope and data sets studied vary widely. Many real-world instances are coming from a wide range of categories, such as quality candies, bottled juices, canned dog food and distilled spirits. The size of the instances can be measured by the number of products considered. We observe that the first instances of retail applications usually have small problem sizes as they often aggregate products into brands or subcategories. Indeed, a large share of the instances approach the problem from an aggregated perspective or consider a very small set of products, leading to instances with less than 10 products. Other instances present a more detailed perspective by approaching the problem at the product level, with more than 100 products. Nevertheless, these bigger instances are tackled in less constrained and simplistic problem descriptions.

Similarly to the variety of planning models, there is not a unique set of benchmark instances, and authors have mainly generated and used their own data sets, with few comparisons between approaches. Moreover, few instances are published online and made available to the research community (Bianchi-Aguiar et al., 2018; Bianchi-Aguiar et al., 2016).

3.2.2. Detailed analysis

As the method of modelling demand determines the mathematical complexity and solvability, we combine the review of demand models with the associated solution approaches. The following analysis is therefore organised in six subsections by the demand models that have been identified in Section 2.3. Despite the existence of many different demand estimation approaches, this field presents some key demand functions that are almost consensual and used across multiple publications. The literature will be structured in narrative order, showing how publications have developed over time. The extensions of the demand models are the drivers of this development. Each literature stream will be introduced and structured similarly by first providing an overview of the demand considered, then describing the fundamental decision model and its solution approach, and finally giving an overview of further developments in terms of decision models and solution approaches.

Models with space-elastic demand. This stream is based on the demand estimate with space-elastic demand, i.e. the product's demand is a polynomial function of the space allocated to a product (see Eq. (1) in Section 2.3). A major study was published by Hansen and Heinsbroek (1979) for this first stream of space plan-

Table 3
Overview of planning problems, demand sources and constraints considered (please see Table 2 for the notation).

Reference	Decisions	Related areas				Demand effects						Dem. type	Constraints			
		A-	P-	M-	R-	-spe	-csp	-oos	-ver	-hor	-arr	-det/-sto	-int	-avail	-cap	-merch
Anderson and Amato (1974)	S	•										det	•		•	
Anderson (1979)	S	•			•							det			•	
Hansen and Heinsbroek (1979)	S	•				•						det	•		•	
Corstjens and Doyle (1981)	S					•	•					det		•	•	
Corstjens and Doyle (1983)	S			•		•	•					det			•	
Zufryden (1986)	S	•				•						det	•	•	•	
Bultez and Naert (1988)	S	•				•	•					det				
Bultez et al. (1989)	S	•				•	•					det				
Preston and Mercer (1990)	S					•						det	•		•	
Borin et al. (1994)	S	•				•	•	•				det	•		•	
Brown and Lee (1996)	S					•	•	•				det			•	
Urban (1998)	S				•	•						det			•	
Yang and Chen (1999)	SA					•	•		•			det		•	•	
Bookbinder and Zarour (2001)	S					•	•					det		•	•	
Yang (2001)	SA					•			•			det	•		•	
Urban (2002)	S				•	•						det			•	
Lim et al. (2004)	SA					•	•		•		•	det	•	•	•	
Hwang et al. (2005)	SA				•	•	•		•			det			•	
Maiti and Maiti (2006)	S				•	•	•					det			•	
Hariga et al. (2007)	SA	•			•	•	•	•	•			det			•	
Kök and Fisher (2007)	S	•						•				sto	•		•	
Reyes and Frazier (2007)	S					•						det			•	
Bai and Kendall (2008)	S				•	•						det	•		•	
Ramaseshan, Achuthan, and Collinson (2008)	S	•			•	•	•					det	•		•	
van Nierop et al. (2008)	SAL					•			•	•		det			•	
Hwang et al. (2009)	SAL					•	•		•	•		det			•	
Ramaseshan, Achuthan, and Collinson (2009)	S	•			•	•	•					det	•		•	
Raut et al. (2009)	SA				•	•	•		•			det	•		•	
Gajjar and Adil (2010)	S					•						det	•		•	
Hansen et al. (2010)	SAL					•	•		•	•		det	•		•	
Murray et al. (2010)	SA		•			•			•			det	•		•	
Russell and Urban (2010)	SAL					•			•	•		det	•		•	•
Baron et al. (2011)	S				•	•	•					det			•	
Gajjar and Adil (2011b)	S					•						det	•		•	
Irion et al. (2011)	S	•		•		•	•					det	•		•	
Lotfi, Rabbani, and Ghaderi (2011)	S				•	•						det	•		•	
Lotfi and Torabi (2011)	S	•			•	•						det	•		•	
Irion et al. (2012)	S				•	•	•	•				det	•		•	
Bai, van Woensel, Kendall, and Burke (2013)	SA					•	•		•			det	•		•	
Piramuthu and Zhou (2013)	S				•	•						det	•		•	
Gajjar and Adil (2015)	S				•	•						det	•		•	
Geismar et al. (2015)	SAL					•			•	•		det	•		•	
Bianchi-Aguiar et al. (2016)	SAL					•			•	•		det	•		•	•
Ghoniem et al. (2016a)	SL			•		•			•	•		det			•	
Ghoniem, Flamand, and Haouari (2016b)	SL			•		•			•	•		det			•	
Zhao, Zhou, and Wahab (2016)	SAL				•	•	•		•	•		det	•		•	
Bianchi-Aguiar et al. (2018)	SAL					•			•	•		det	•		•	•
Hübner and Schaal (2017a)	S				•	•						det	•		•	
Hübner (2017)	SA					•	•		•			sto	•		•	
Hübner and Schaal (2017b)	S	•				•		•				sto	•	•	•	
Xiao and Yang (2017)	S				•	•						det			•	
Flamand, Ghoniem, Haouari, and Maddah (2017)	SL	•		•						•		det			•	
Schaal and Hübner (2018)	S					•	•					sto	•		•	
Yu, Maglasang, and Tsao (2018)	S					•	•					det	•		•	
Rabbani, Salmanzadeh-Meydani, Farshbaf-Geranmayeh, and Fadakar-Gabalou (2018)	SA					•	•		•			det	•		•	

ning models. They formulate the total category profit as a maximisation problem, with the number of facings as a decision variable and the space-elastic demand in a polynomial form. The proposed algorithm is based on a generalised Lagrange multiplier technique, which is generally only guaranteed to find local solutions of nonconvex programs. The authors do not consider cross-product relations, as they predict problems in generating data for cross-product relations. This work built the foundation for further developments in this stream, which have been published by the following researchers, amongst others: Zufryden (1986) developed a dynamic programming formulation for a problem with space elasticity and marketing variables. Murray, Talukdar, and Gosavi (2010) developed a model that jointly optimises shelf selection, facings, prices, and also the display orientation in a category. A branch-and-bound-based MINLP solver solves the model. Hübner and Kuhn (2011) developed a mixed-integer model for space assignment planning that also takes assortment decisions into account.

An interesting paper by Yang and Chen (1999) assumes a linear space-elastic demand function within a constrained number of facings and translates the problem into a multi-knapsack formulation. The authors state that in practice it is difficult to obtain an estimate for space elasticity and assume that the profit of any product is linear with regard to a range of facings if it is kept in a controlled range defined by proper upper and lower bounds. They propose a formulation that maximises the benefit of including products (additional facings) in a set of knapsacks (each shelf is a knapsack), while not exceeding the knapsack capacity. They propose a multi-knapsack heuristic for the resulting model and find an optimal solution for simplified versions only. Lim et al. (2004) build on Yang's linear model and use meta-heuristics.

Several authors have criticised this simplified approximation of the demand because it contradicted previous experiments revealing the diminishing growth rate of space elasticity (see e.g. Eisend, 2014; Irion et al., 2012). Despite the models having been extended step by step, major criticism for this stream of papers still arose from the fact that heuristics are applied to find near-optimal solutions and the different solution approaches have not been compared so far, that cross-space effects are not reflected, and that model extensions have not been structured comprehensively. The following subsection identifies shelf space models that take into account cross-space elasticity relations.

Models with cross-space elastic demand. One of the first important extensions related to the demand functions was introduced by Corstjens and Doyle (1981). They formulate the problem in a non-linear multiplicative form and include space- and cross-space elasticities. The demand is formulated as in Eq. (2). This demand formulation uses polynomial terms to model the decreasing demand rate as the space increases. The profit is maximised by applying a signomial geometric programming method, but Borin et al. (1994) show that the solutions reported violate the constraints in seven out of ten cases. A limitation lies in the small number of products, as the model is supposed to optimise product groups rather than individual products. However, the polynomial demand model was widely used by other authors, who extended this formulation in many different ways. Corstjens and Doyle (1983) and Raut, Swami, and Moholkar (2009) include the time dimension and assume that past demand influences demand in the current period. The model of Raut et al. (2009) is solved through two greedy and one genetic algorithm. Further applications of demand models with cross-space elastic demand can be found, e.g. in Bultez, Gijsbrechts, Vanden Abeele, and Naert (1989) and Irion et al. (2012). Bultez et al. (1989) apply Corstjens' model at a brand level, assuming identical cross-elasticity within product groups. Irion et al. (2012) further extend Corstjens' model to a product level instead of a category level. Using a linearisation framework, they transform the model into a mixed-integer problem with linear constraints. Their approach provides only near-optimal solutions with an a posteriori error bound. Gajjar and Adil (2010) and Gajjar and Adil (2011b) build on Irion's study and develop a local search heuristic. An issue with these multiplicative models with cross-space elasticity is that they result in no demand for a given category if the space of any other product is set to zero (which means it is delisted from the assortment). Furthermore, the papers above simplify the inventory control and replenishment system by assuming an efficient in-store logistics system that always ensures availability and avoids any OOS. Hence, these papers do not factor in the effects of substitution in the event of out-of-stocks, nor the costs of replenishment.

Models with inventory control and OOS substitution. There is a rich literature on *inventory control* models that also relate to shelf space planning. Abbott and Palekar (2008) integrate inventory aspects into their deterministic shelf space problem to obtain optimal replenishment quantities and frequencies. They formulate an economic order quantity problem and assume a linear relationship between product shelf space and product sales. They determine – exactly for a single-product case and approximately for a multi-product case – the optimal replenishment cycles for products, given the cost of restocking and the sales effects of inventory-elastic demand. Baron, Berman, and Perry (2011) investigate the joint shelf space allocation and inventory control problem with demand that depends on both the inventory level seen by customers and the shelf space allocated. Based on a numerical study with two products, they analyse the sensitivity of the decision variables and the effect of integrating inventory control and space assignment. These stylised models related to shelf space and replenishment planning have limitations in their applicability to practical problem sizes, as they are mainly developed to derive analytical insights, without focusing on efficient solution approaches for practice-relevant problems. Furthermore, some apply non-integer facings to obtain continuous decision variables. Additionally, all models are restricted to individual product replenishment via immediate refill by the retailer whenever a product is out-of-stock, i.e. ad hoc restocking from backroom inventory. This represents an additional limitation, as retailers normally replenish products jointly because of joint delivery cycles from the central warehouses. Bianchi-Aguiar, Carravilla, and Oliveira (2015) overcome this problem by proposing a model that minimises the days of supply differences among products (days of supply refers to how many days it will take for the stock of a product to run out if sales continue at the same rate as recent sales).

The other sub-area deals with demand gains for OOS substitutions. Borin et al. (1994) and Urban (1998) extend the demand function of Hansen and Heinsbroek (1979) and Corstjens and Doyle (1981) by considering the demand coming from the consumers who are willing to purchase a replacement product if their preferred product is not available. Borin et al. (1994) propose a model that additionally considers substitution effects and solve it with a simulated annealing heuristic. Urban (1998) integrates inventory aspects into shelf space management by considering demand as a function of the shelf inventory displayed. The deterministic, continuous-review model takes into account inventory-elastic demand, since sales before replenishment reduce the number of products displayed. Consequently, the effective shelf space assigned to products diminishes until replenishment takes place. The problem is solved by a greedy heuristic and a genetic algorithm but violates integer constraints for facings and order quantities. The costs for replenishment and the availability of a backroom are investigated by an extended model of Hübner and Schaal (2017a). The model integrates fixed and variable replenishment costs for direct refills from the backroom.

Models with vertical position effects on demand. Demand models with vertical position effects factor in additional demand gains/losses depending on the shelf height the product has been assigned to (see the first part of Eq. (4)). Yang and Chen (1999) integrate the vertical position effect of a product and consider different demand values depending on the height of the shelves. The model is formulated as a multi-knapsack problem and solved using an associated heuristic. Hwang et al. (2005) assume an average vertical position effect in case the same product is displayed on different shelves at the same time. A genetic algorithm and a gradient search heuristic are applied to solve the problem. Hariga, Al-Ahmari, and Mohamed (2007) propose an optimisation model to determine assortment, replenishment, positioning and space assignment taking into account shelf and storage constraints. The decision variables are the display locations, order quantities, and the number of facings in each display area. The deterministic, nonlinear problem could be solved exactly by using a solver only for a four-product case. In addition, they omit integer facing values. Hwang, Choi, and Lee (2009) develop an optimisation model, also considering vertical positioning effects, and solve it via a genetic algorithm. However, the model is only tested on a problem instance with 4 products. Building on Yang (2001), Gajjar and Adil (2011a) develop a shelf space model that accounts for linear space elasticity and vertical positioning effects. The model is solved via specialised heuristics. Afterwards, Hübner and Schaal (2017c) formulate the first stochastic shelf space model that efficiently generates near-optimal results for stochastic problems. They include space and cross-space elasticity as well as vertical positioning effects and salvage and shortage costs based on a Newsvendor formulation.

Demand models with horizontal position effects. The aforementioned models disregard the exact position of the products and assign the same effect regardless of the product's position on a particular shelf. This has been extended by also taking into account position effects in the demand calculation (see Eq. (4)). van Nierop, Fok, and Franses (2008) propose a hierarchical Bayesian model to estimate the impact of shelf layouts on retail sales. Furthermore, they solve an optimisation model that includes space elasticities as well as vertical and horizontal positioning effects by utilising simulated annealing. The model is tested on a real, large-scale data set for an ambient category. Hansen, Raut, and Swami (2010) and Russell and Urban (2010) also consider decisions and propose (quasi) horizontal effects on the demand function. Hansen et al. (2010) extend the simplified version of Yang and Chen (1999) and present a formulation where the variables were discretised to consider the horizontal position of the products on the shelves. The authors divide the shelf into multiple horizontal segments and the model decides on the allocation of each product on a certain shelf and a horizontal segment by taking into account the product-specific face-length. The model also differentiates between horizontal and vertical positions. They compare the performance of various heuristic and meta-heuristic algorithms.

Demand models with arrangement effects. One important practical limitation in the previous references from the literature is that merchandising rules are neglected during the allocation of products. To include this effect, the demand model needs to be extended as denoted in Eq. (5). Lim et al. (2004) introduce an additional element on the objective function and attribute additional benefits if two products with affinity are placed on the same shelf. Russell and Urban (2010) introduce the first formulation with continuous horizontal locations for the products and include product groupings in their shelf space problem. They explicitly consider the products as part of a family, which can be based on a variety of characteristics, such as brand, flavour and price set, among others. Products of these families should be kept together and, for aesthetic reasons, in uniform and rectangular shapes. The mixed-integer quadratic program is solved using a solver in combination with an improvement heuristic. Bianchi-Aguiar et al. (2018) use a mixed-integer programming approach to formulate a model that considers product-grouping and display-direction constraints and incorporates merchandising rules. Space elasticity is modelled as a linear function, as in Yang (2001). Geismar et al. (2015) propose a two-dimensional shelf space optimisation model that allows displays to extend across multiple shelf levels. The model is solved using a decomposition approach. Hübner, Schäfer, and Schaal (2020) extend this by including substitutions and stochastic and space elastic-demand.

4. General and unified decision model for the shelf space planning problem

In this section, we propose a unified modelling approach to the shelf space planning problem based on the literature reviewed in the previous section. This is not intended to be a new formulation to the problem but a generalisation to clearly differentiate the various model scopes and extensions and to provide a common terminology. It is also a starting point for future research and may serve to develop benchmark models, data sets and solution approaches.

The retailers' objective in shelf space planning is usually to maximise the category profit Π across all products $i \in \mathbb{N}$ as denoted in Eq. (7), although other objectives may be considered as well. The decision variables are x^h_{ik}, representing the number of horizontal facings for product i on shelf level $k \in \mathbb{K}$, x^v_{ik}, the number of vertical facings for product i on shelf level $k \in \mathbb{K}$ (for simplicity purposes, we consider that $x^v_{ik} = 1$), x^d_{ik}, the stock per facing for product i on shelf level $k \in \mathbb{K}$, z_{ik}, the vertical allocation of product i to a shelf level k, and y_{ik}, the horizontal location of product i in shelf level k. The auxiliary variables x_i, the total number of facings for product i, are obtained by the sum of horizontal facings on all shelves, and the auxiliary variables q_i, shelf quantity of product i, are obtained by the multiplication of x_i with the stock per facing x^d_{ik}. The objective function is represented by the sum of the profit per product π_i that depends on demand d_i, where π_i represents all product-specific revenues and costs offset by general costs F that depend on demand $\overline{d}$ and on the shelf quantity $\overline{q}$, where $\overline{d}$ is the vector of all d_i and $\overline{q}$ is the vector of all q_i for all products i.

$$Max \quad \Pi(x_i, z_{ik}, y_i) = \sum_{i \in \mathbb{N}} \pi_i(d_i) - F(\overline{d}, \overline{q}) \tag{7}$$

The *demand* can include all different forms, from space-elastic demand to arrangement effects, as identified in Fig. 3. The various demand functions of Eqs. (1)–(6) hence serve as input to the objective function. The *product profit* π_i represents the expected revenues p_i offset by the purchasing costs c_i and further variable operations costs o_i, with $\pi_i = p_i - c_i - o_i$. The operations costs o_i consist of four parts. The first part represents variable ordering and replenishment costs that consider a constant cost rate that is multiplied by the number of replenishment activities per period, where r_i is the cost rate for each replenishment of product i and (d_i/q_i) represents the replenishment frequency. These costs can be further differentiated if the replenishment is conducted with inventory from the backroom (as this causes additional sorting) or if it is performed directly after delivery without intermediate backroom storing. The second part corresponds to inventory holding costs represented by an interest rate int.rate multiplied by the purchasing cost of the product. The third and fourth parts represent the overage and underage costs. These mainly need to be applied in the event of perishable products where products cannot be stored for an unlimited period. In cases where the shelf quantity exceeds the demand ($q_i > d_i$), the remaining quantity needs to be disposed of with a salvage value ν_i. In the other case ($d_i > q_i$), out-of-stock costs s_i can be modelled as penalty costs when products are depleted. Hence, the generic formulation of the operations costs o_i is depicted in Eq. (8).

$$\begin{aligned} o_i = r_i \cdot \frac{d_i}{q_i} + \text{int.rate} \cdot c_i \cdot \frac{q_i}{2} + \nu_i \cdot [(q_i - d_i)|q_i > d_i] \\ + s_i \cdot [(d_i - q_i)|d_i > q_i] \quad \forall i \in \mathbb{N} \end{aligned} \tag{8}$$

Further *fixed replenishment costs* occur per replenishment activity (e.g. transport of all products to the shelf), whereas r_i are product-specific costs (e.g. order processing costs). The total replenishment costs can be modelled by the factor F that depends on the *fixed replenishment costs* and the replenishment frequency of all products (represented by the ratio $\overline{d}$ to $\overline{q}$).

The most prominent *constraints* can be modelled in the following manner. The constraint for *integer facings*, Eq. (9), expresses that the number of facings x^h_{ik}, x^d_{ik} can only have positive integer values $\mathbb{Z}^+$. The other variables need to be defined either as binary variables, $z_{ik} \in \{0, 1\}$, the vertical allocation of product i to a shelf level k, or as continuous variables, $y_{ik} \in \mathbb{R}^+_0$, the horizontal location of product i on shelf level k. The relation between x_i and q_i needs to be defined as $x_i = \sum_{k \in K} x_{ik}^h$ and $q_i = \sum_{k \in K} x_{ik}^h \cdot x_{ik}^d$ to compute the total quantity. The relation between x_{ik}^h, x_{ik}^d and z_{ik} is given by Eq. (10), where "M" is a sufficiently large number.

$$x_{ik}^h, x_{ik}^d \in \mathbb{Z}^+, \forall i \in \mathbb{N}, \quad \forall k \in \mathbb{K} \tag{9}$$

$$x_{ik}^h + x_{ik}^d \le M \cdot z_{ik}, \quad \forall i \in \mathbb{N}, \forall k \in \mathbb{K} \tag{10}$$

The auxiliary variables x_i, total number of facings for product i, are calculated in Eq. (11), and q_i, shelf quantity of product i, is calculated in Eq. (12).

$$x_i = \sum_{k \in \mathbb{K}} x_{ik}^h, \quad \forall i \in \mathbb{N} \tag{11}$$

$$q_i = \sum_{k \in \mathbb{K}} x_{ik}^h \cdot x_{ik}^d, \quad \forall i \in \mathbb{N} \tag{12}$$

Lower and upper bounds, denoted by L_i and U_i, limit the solution space for the number of facings.

$$L_i \le x_i \le U_i, \quad \forall i \in \mathbb{N} \tag{13}$$

The assignment of products is limited to the *capacitated shelf space*, C^h (width of the shelves) and C_k^d (depth of shelf k) as denoted in Constraints (14) and (15), whereas size_i^h and size_i^d are the width and depth of product i. The assignment of products is also limited based on the available shelf height C_k^v, which cannot be exceeded by the allocated products with height size_i^v, as denoted in Constraint (16).

$$\sum_{i \in \mathbb{N}} \text{size}_i^h \cdot x_{ik}^h \le C^h, \quad \forall k \in \mathbb{K} \tag{14}$$

$$\sum_{i \in \mathbb{N}} \text{size}_i^d \cdot x_{ik}^d \le C_k^d, \quad \forall k \in \mathbb{K} \tag{15}$$

$$\text{size}_i^v \cdot z_{ik} \le C_k^v, \quad \forall i \in \mathbb{N}, \forall k \in \mathbb{K} \tag{16}$$

The *availability constraints* relate the shelf quantity q_i of product i to A_i, the maximum quantity available of product i.

$$q_i \le A_i, \forall i \in \mathbb{N} \tag{17}$$

Merchandising rules ensure that products are arranged as required for the category. These rules may have different forms, e.g. products need to be ordered according to their attributes, represented in columns. A formulation for all these rules is beyond the scope of this paper, and we will present only the constraints (18)–(27) that ensure that the shelf layout for each product i forms a rectangular shape (within a two-dimensional shelf layout), as these are the most popular *merchandising rules*. As an extension to this unified modelling approach, similar restrictions can be built to ensure that product families form rectangular shapes.

New auxiliary decision variables must be added to build these constraints: $H_i, V_i \in \mathbb{R}_0^+$, representing respectively the total horizontal/vertical space occupied by product i, $h_i \in \mathbb{R}_0^+$, representing the initial vertical position of product i, and the binary variables l_{ij} and b_{ij} that determine whether product i is respectively on the left or below product j. As the location of product i will be the same on all the shelves, the decision variable y_{ik} is independent of the shelf ($y_i \in \mathbb{R}_0^+$).

$$H_i \ge \text{size}_i^h \cdot x_{ik}^h, \qquad \forall i \in \mathbb{N}, \forall k \in \mathbb{K} \tag{18}$$

$$y_i + H_i \le y_j + C^h \cdot (1 - l_{ij}), \qquad \forall i, j (i \ne j) \in \mathbb{N} \tag{19}$$

$$y_i \le C^h - H_i, \qquad \forall i \in \mathbb{N} \tag{20}$$

$$V_i = \sum_{k \in \mathbb{K}} z_{ik} \cdot C_k^v, \qquad \forall i \in \mathbb{N} \tag{21}$$

$$h_i \le P_k + (1 - z_{ik}) \cdot \sum_{k \in \mathbb{K}} C_k^v, \qquad \forall i \in \mathbb{N}, \forall k \in \mathbb{K} \tag{22}$$

$$h_i \ge z_{ik} \cdot (P_k + C_k^v) - V_i \qquad \forall i \in \mathbb{N}, \forall k \in \mathbb{K} \tag{23}$$

$$h_i + V_i \le h_j + \sum_{k \in \mathbb{K}} C_k^v \cdot (1 - b_{ij}), \qquad \forall i, j (i \ne j) \in \mathbb{N} \tag{24}$$

$$l_{ij} + l_{ji} + b_{ij} + b_{ji} \ge 1, \qquad \forall i, j (i \ne j) \in \mathbb{N} \tag{25}$$

$$l_{ij}, b_{ij} \in \{0, 1\}, \qquad \forall i, j \in \mathbb{N} \tag{26}$$

$$H_i, y_i, V_i, h_i \in \mathbb{R}_0^+, \qquad \forall i \in \mathbb{N} \tag{27}$$

Constraints (18)–(20) determine the horizontal position of product i and ensure that it has the same horizontal space on all shelves. Constraints (19) ensure that products i and j do not overlap horizontally, and Constraints (20) guarantee that items of product i do not cross the border of the shelf space.

Constraints (21)–(24) ensure that product i is allocated to contiguous shelves. Constraints (21) define the total shelf height occupied by product i and, together with Constraints (22) and (23), ensure that product i is allocated to contiguous shelves (P_k represents the vertical position of shelf k). Constraints (24) ensure that products above product i have a vertical position higher than the position of product i plus the total vertical space occupied by product i.

Lastly, constraints (26) and (27) define the new auxiliary decision variables.

Dimension of the model (including merchandising rules). Considering that $\mathbb{N}$ is the number of products and $\mathbb{K}$ is the number of shelves, the number of integer variables of the model is $2 \times \mathbb{N} \times \mathbb{K} + 2 \times \mathbb{N}$, the number of binary variables is $2 \times \mathbb{N} \times \mathbb{K} + 2 \times \mathbb{N} \times \mathbb{N}$, the number of continuous variables is $4 \times \mathbb{N}$, and the number of constraints is $4 \times \mathbb{N} \times \mathbb{K} + 3 \times \mathbb{N} \times \mathbb{N} + 5 \times \mathbb{N} + 3 \times \mathbb{K}$.

5. Conclusion and directions for future research

In this paper, we reviewed the literature on the shelf space planning problem and presented a classification framework that systematised the widely differing approaches to the problem. We also distributed the existing literature across the different problems proposed in the framework in order to identify the main research gaps. This study revealed the following suggestions for future research, which we divided into four main directions:

1. Generalizing demand modelling. Fig. 5 presents the distribution of the publications across our framework. The bar charts indicate the amount of research addressing the problem. The figure shows that the basic problem, with and without cross-elasticity effects, has been most frequently addressed in the literature. It also reveals that some elements of the framework have received less attention.

First of all, most of the current literature on shelf space planning disregards allocation and location decisions. As a result, existing formulations are not capable of being directly transferred to planograms. Furthermore, we have revealed the importance of

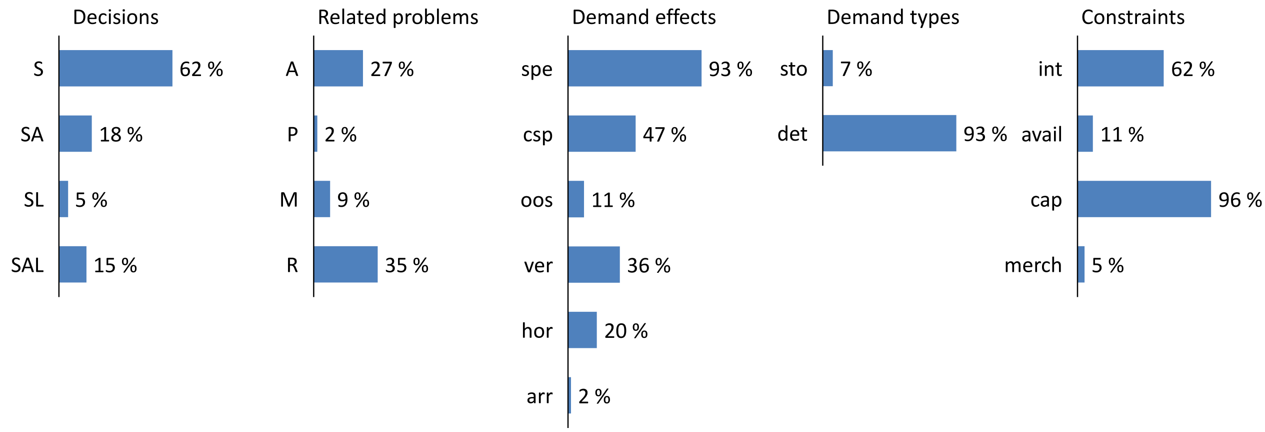

Fig. 5. Distribution of the publications across the framework, in percentage of all publications identified in Table 3 (notation is present in Table 2).

merchandising rules and strengthened the need to consider product families, as this is a key practical requirement from most retailers in practice. Until now, family groups have been used as constraints and given as inputs to the models. Shelf space models could be extended to identify the product arrangement that should be used to group products, seeking to increase display attractiveness. To the best of our knowledge, the quantification of the impact of this design complexity on demand is yet to be studied and experimental studies need to be conducted. A further extension could be the analysis of product positioning (within the shelf and the store) with regard to the customers' shopping behaviour (e.g. purchasing tree, shopping paths).

Most current models centre on a deterministic and stationary demand for a single period. This should be further relaxed, as retail sales are usually stochastic, seasonal and non-stationary (high variability). Extensions with this regard may result in multi-period problems and the related short-term adjustment of shelf space. This requires approaches of dynamic programming, in particular for settings with over- and understock of perishable products. Analysing the effects of daily or continuously changing planograms from customer and costs perspectives is an open question. Further efforts should be put into improving demand forecasting, which is a core input to the model. Predictive and prescriptive analytics (based on big data or machine learning techniques, for example) could be applied. Moreover, using store traffic predictions and the in-store shopping pathways of customers can enhance shelf space planning capabilities.

2. Extending the shelf space planning formulation. Besides exploring the impact that merchandising rules have on demand, the integration of merchandising rules as constraints to the problem needs to be further investigated. This is a very recent topic that is still very poorly explored in the literature, with very few papers approaching it so far. This includes exploring other relevant business rules, such as pre-defined family sequences or the combination of complementary products.

Most literature on shelf space planning considers that the shelf design (such as the number of shelves, layers, height, etc.) is defined beforehand. The reasons behind this assumption come mostly from the observation that retailers are not likely to change shelf positions during operations due to the high cost that such an endeavour would represent. However, the placement of shelves is an important decision to consider during the opening and refurbishment of the stores, as well as during significant assortment modifications. This problem extension is therefore not just a valuable practical contribution but also a scientific contribution, given the challenge that it holds. As stores usually have different types of shelf fixtures (e.g. pegboards, bins, tables, and polygonal shelves), the study and integration of these are also potential research topics. Pegboards are a particularly engaging fixture type for future studies because of their inherent complexity and the fact that they have scarcely been studied in the literature. It would also be worth studying shelf problems where products are displayed in a two-dimensional manner, e.g. fresh food or fashion items.

Other problem extension is related with partnerships with suppliers for co-marketing activities, a topic that has received increased attention in category management over the last years. These partnerships may result in additional business rules and reserved slots that need to be taken into consideration in the shelf space planning formulations.

3. Developing efficient solution methods and transferring them to practice. The literature review in Section 3.2 has shown that early shelf space planning literature mainly focused on developing formulations and exact mathematical approaches, whereas the more recent literature proposes increasingly more heuristics and meta-heuristics to deal with the complexity of the problems encountered. Our summary in Section 3.2.1 and qualitative analysis show that many approaches are simplistic and lack key practical features, while others are complex and often require expensive parameter estimation requirements, compromising their use in practice. Moreover, there is not yet any comparison and benchmark study available that tests the effectiveness of various solution approaches. Further investigating heuristic solution approaches, both in terms of solution quality and run-time as well as more generalized models, would be a beneficial area of research. Efficient solution approaches for the related general OR problems (i.e. knapsack, assignment and scheduling problems) could be a valid starting point.

Most computational experiments have used instances with a very limited number of products and larger problems are usually randomly generated. Hence, it is crucial to gather a set of benchmark instances that can be used to compare solution approaches covering realistic features and sizes. One area of research should continue investigating the gap between state-of-the-art models and their implementation in practice/software packages. Besides the need for efficient algorithms able to cope with a large number of products, the expensive estimation requirements for parameters is also a major barrier for a better alignment between science and software applications. Nevertheless, there are other ways of improving the use of shelf space planning theory in practice. One example is to study the creation of shelf plans taking into account the current planograms implemented in the stores, by considering the trade off between the potential profit and the costs of changes (e.g. handling costs). Furthermore, retailers usually operate with a multi-store network. This requires replication and clustering assessment, models and solution approaches for shelf space planning for multi-store retailers with differing demand across stores or different store sizes/types. This also goes along with an investigation of different store settings (e.g. small stores for daily supply vs. hypermarkets) and varying customer preferences (e.g. urban vs. rural customers).

4. Integrating related areas into shelf space planning problems. The interdependency with other retail planning activities was also highlighted throughout this review. Our review indicates a need for more comprehensive and integrated decision models in terms of macro-shelf space planning, assortment, pricing, promotion and replenishment planning.

The solutions generated using any shelf space model strongly depend on the product assortment previously defined, considered as input to these models. Some authors have already tackled assortment and shelf space jointly, but usually, they disregard shelf space planning effects (typical in shelf space literature) or substitution effects (typical in assortment literature). Supplier–retailer relations in shelf space planning could also form another set of studies. Further studies might investigate the impact of pricing and promotion policies as well as other in-store marketing instruments on demand estimates and the required shelf space assignment. Investigating dynamic pricing and promotion effects on shelf space and inventory requirements is a laudable pathway. The current research focuses in particular on regular shelves with a defined assortment. Retailers often use dedicated areas for promotions, e.g. separate shelves, gondola-end. The space allocation of these shelf types, as well as the effect on other shelves, should be analysed.

Moreover, shelf space planning would benefit from a deeper analysis of inventory-related concerns with a focus on replenishment synergies. Both problems are interdependent, as shelf replenishment operations have expensive handling costs and are limited to the shelf merchandisers available to immediately fill the shelves after stockout. The inclusion of the concept of target service level and further service constraints would be an interesting approach. A further area lies in the integration of backroom inventory and backroom replenishment processes into shelf space planning. This also holds true for the integration of delivery frequency and its alignment with shelf management. Moreover, approaches to determine appropriate safety stocks are necessary to hedge against the uncertainties in consumer demand and warehouse supply. The inventory questions are especially relevant for all types of perishable products. Combining shelf space planning with waste management can be an interesting opportunity. This requires addressing best-before problems, inventory and replenishment issues, discounting and dynamic shelf life questions (e.g. discounted non-fresh items and their respective shelf share), out-of-stock situations and options for substitutions and donations to food banks. Furthermore, the role of shelf management in omnichannel retailing needs to be assessed. Retail stores can be integrated into online channels as further stocking and shipping points, whereas online channels can be leveraged as virtual shelf extensions to expand the assortment. Shelf space management also plays a role in supplier–retailer relations. This can be part of modelling certain category captain roles or supporting negotiations between both.

Our literature review, the unified modelling approach and the framework developed build the foundation for this ongoing research and will foster the creation of further advanced models and solution approaches for shelf space planning problems.

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