For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good Luck!

1. Shreyas the summation-master is struggling to solve this strenuous stumper. Surpass Shreyas is summation solving by evaluating $\sum_{n=1}^{\infty} \frac{2n}{3^n}$.

   (A)$\frac{1}{2}$ (B)1   (C)$\frac{3}{2}$ (D)2   (E) NOTA

2. Given regular octagon ABCDEFGH has side length 4, what is the perimeter of triangle BEH?

   (A) $8 + 12\sqrt{2}$ (B) $12 + 8\sqrt{2}$ (C) $8 + 8\sqrt{2} + 4\sqrt{1 + \sqrt{2}}$
   (D) $8 + 8\sqrt{2} + 4\sqrt{2 + \sqrt{2}}$ (E) NOTA

3. If $x + \frac{1}{x} = 2$, what is $x^5 + \frac{1}{x^5}$?

   (A) 2   (B) 4   (C) 32   (D) 8   (E) NOTA

4. Find the smallest value of $\frac{z^2 + 2z + 2}{z + 1}$ for real values of z greater than -1.

   (A) -1   (B) 1   (C) 2   (D) 0   (E) NOTA

5. If you have a right triangle with legs of length a and b, hypotenuse of length 24, and an area of 12, what is $(a + b)^2$?

   (A) 624   (B) 144   (C) 576   (D) 720   (E) NOTA

6. Rohan is a radical surfer, but he cannot reduce this rambunctious radical. Rectify this situation and relieve Rohan from this radical root by evaluating this radical for him.

   $$\sqrt{(45)(47)(51)(53) + 36}$$

   (A) 2500   (B) 2401   (C) 2391   (D) 2304   (E) NOTA

7. Vishnav needs to pick up Chanda from her spaceship. Chanda's spaceship lands between 2 am and 3 am. Chanda will only wait 10 minutes before taking an tUber and Vishnav will only wait 20 minutes before going home. Given that they both arrive at some point between 2 am and 3 am, what's the probability they will meet each other?

   (A) $\frac{19}{36}$ (B) $\frac{1}{2}$ (C) $\frac{41}{72}$ (D) $\frac{31}{72}$ (E) NOTA

8. Given polynomial $x^3 + 9x^2 + 7x + 1$ has roots r_1, r_2, and r_3, what is $(r_1 + 1)(r_2 + 1)(r_3 + 1)$?

   (A) -1   (B) -2   (C) 18   (D) 1   (E) NOTA

9. If $x^5 = 1$ and $x \neq 1$, what is $x^4 + x^3 + x^2 + x + 1$?

   (A) 0   (B) 1   (C) 2   (D) 3   (E) NOTA

10. A cow is attached to the outside corner of a 4 ft by 12 ft rectangular barn with 16 ft rope. What is the the total area it can roam?

    (A) 128π (B) 301π (C) 262π (D) 232π (E) NOTA

11. How many 3-digit positive integers are there such that the sum of its digits is 9?

    (A) 90   (B) 55   (C) 165   (D) 45   (E) NOTA

12. What is the sum of the non-real roots of the polynomial $x^5 + x^4 + x^3 + x^2 + x + 1 = 0$?

    (A) 0   (B) -1   (C) -2   (D) 3   (E) NOTA

13. Tanvi is running late and needs to find a way to get from car pick-up to her first period class without getting caught by a teacher. How many ways are there for her to get from car pick-up, located at $(0,0)$, to her first period, located at $(3,3)$, if she cannot go through the teacher check points, which are found at $(1,1)$ and $(2,2)$, and given she can only move up or to the right 1 unit at a time?

(A) 10   (B) 4   (C) 6   (D) 8   (E) NOTA

14. Triangle ABC has coordinates A (0,1) and B (-4,-5). Given triangle ABC has a right angle at C, find the equation of the possible coordinates for C.

(A) $(x + 2)^2 + (y + 2)^2 = 52$ (B) $(x - 2)^2 + (y - 2)^2 = 52$

(C) $(x + 2)^2 + (y + 2)^2 = 13$ (D) $(x - 2)^2 + (y - 2)^2 = 13$ (E) NOTA

15. What is the area of the circle inscribed in a triangle with side lengths of 13,14,15?

(A) 4π (B) 16π (C) 84π (D) $\frac{65}{8}\pi$ (E) NOTA

16. What is the area of a rectangle with perimeter $4\sqrt{13}$ inscribed in a circle with radius $\frac{7}{2}$?

(A) 10   (B) 13   (C) $\frac{49}{4}$ (D) $\frac{3}{2}$ (E) NOTA

17. Tanusri often gets triggered by trigonometry. Teach Tanusri how to terrify trigonometry by simplifying $\sin^2 x + \cos^2 x$?

(A) 0   (B) 2   (C) $\frac{1}{2}$ (D) 1   (E) NOTA

18. Triangle ABC has cevian $\overline{AD}$ with point D on $\overline{BC}$. The length of $\overline{BC}$ is 14 and the length of $\overline{BD}$ is 2. What is the ratio of the area of triangle ABC to the area of triangle ADC?

(A) $\frac{1}{6}$ (B) 3   (C) $\frac{3}{2}$ (D) $\frac{7}{6}$ (E) NOTA

19. The value $\sqrt{8 - 4\sqrt{3}}$ can be expressed in the form of $\sqrt{a} - \sqrt{b}$, where a and b are integers. What is a+b?

(A) 3   (B) 7   (C) 56   (D) 8   (E) NOTA

20. Triangle ABC has coordinates A (0,0), B (2,3) and C (3,2). What is the slope of the angle bisector going through point A?

(A) 1   (B) $\frac{13}{12}$ (C) $\frac{1}{2}$ (D) -1   (E) NOTA

21. How many of the roots of $x^3 - x^2 - x + 1$ are complex?

(A) 0   (B) 3   (C) 1   (D) 2   (E) NOTA

22. Shubham aspires to be a smart summation-master as well. Shreyas is showing him a sample sum, but this time, Shubham is the one who is stumped. Help Shubham and become a summation-master by computing the following.

$$\sum_{x=3}^{\infty} \frac{-1}{x^2 - 3x + 2}$$

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(A) 1   (B) 0   (C) -1   (D) $\frac{1}{2}$ (E) NOTA

23. Whats the sum of the digits of $2^{2018} \cdot 5^{2020}$?

(A) 7   (B) 2500   (C) 5   (D) 4   (E) NOTA

24. Calculate $(\log_2 9)(\log_3 8)(10^{\log 4})$.

    (A) 60   (B) 24   (C) 6   (D) 12   (E) NOTA

25. The triangle ABC is partitioned into 6 non-overlapping triangles by drawing all of its medians. What is the ratio of the area of one of the 6 triangles to the area of triangle ABC?

    (A) Cannot be Determined   (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{1}{6}$
    (E) NOTA

26. Completely factor $x^6 - 64^2$ over $\mathbb{R}$.

    (A) $(x^3 + 64)(x^3 - 64)$ (B) $(x + 4)(x - 4)(x + 2)(x - 2)(x + 2)(x - 2)$
    (C) $(x + 4)(x - 4)(x + 2)(x - 2)(x + 2)(x + 2)$ (D) $(x + 4)(x^2 - 4x + 16)(x - 4)(x^2 + 4x + 16)$
    (E) NOTA

27. Given that a and b are positive real numbers, $a^2 + b^2 = 9$, and $ab = 8$, what is $a^3 + b^3$?

    (A) 5   (B) 85   (C) 125   (D) 729   (E) NOTA

28. What is the sum of the product of the roots of $3x^4 + 3x^3 + 4x^2 - 7x - 1$ taken 3 at a time?

    (A) $\frac{7}{3}$ (B) $-\frac{7}{3}$ (C) $\frac{4}{3}$ (D) $-\frac{4}{3}$ (E) NOTA

29. How many distinct ways can you arrange the letters in THETA?

    (A) 120   (B) 20   (C) 60   (D) 40   (E) NOTA

30. What is $12 \div 2(2 + 1)$?

    (A) 2   (B) $\frac{12}{5}$ (C) 1   (D) 4   (E) NOTA