Moment Matching of Old-Keynesian Model: Deterministic and Stochastic Cases *

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Abstract

This paper mainly focuses on replicating partial results from Franke (2012). To be specific, we focused on the Old-Keynesian Model in the original paper. We tried to use method of moments to estimate the unknown parameters in the Old-Keynesian model. We also used a different set of moments to examine if the values of parameters would change dramatically. There are two versions of this Old-Keynesian Model. The first is a deterministic version, and the second is a stochastic version with random shocks. For the deterministic version of the model, we applied Generalized Method of Moments in our estimation. For the stochastic version of the model, we employed Simulated Method of Moments to simulate the random shocks.

keywords: Sentiment dynamics, simulated method of moments, general method of moments, old-Keynesian Model, Great Inflation.

JEL classification: C52; E32; E37

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1 Introduction

In his research, Franke (2012) used General Method of Moments (GMM) and Simulated Method of Moments (SMM) to estimate the parameters of Old-Keynesian Model and found the Old Keynesian Model can match the empirical data well. In this paper, we attempted to replicate his methods and investigated whether we can achieve a similar result. Our work indicates that it is possible to get a set of estimated parameters that are close to Franke's estimation in the deterministic settings. We would like to stress, though, that the initial guess plays an important role in the determination of local optimal point. We also extended the original model and used a different set of 13 moments for estimating the parameters in the deterministic settings. Finally, we showed that our 79-moment deterministic model fits these outside 13 moments fairly well.

In our next step, we briefly investigated the performance of stochastic versions of the models. However, we were not able to investigate whether the stochastic models exhibits better fits.

The paper complements the current literature as it proposed a different set of moments that are easy to interpret in the Old-Keynesian Model. The paper also provides more information about the data source than the original paper and is highly replicable. Future research can build on this paper and test for other moments that researchers deem important.

2 Model

The model we used in this paper is the Old-Keynesian Model from Franke (2012). This Old-Keynesian model mainly focuses on three aggregate economic variables: Output gap, Inflation rate, and Interest rate. In the original paper, the author introduced two climate variables to describe the dynamics of the three target variables mentioned above. The first climate variable is the business sentiment. This climate variable describes aggregate confidence of the firms in the market toward the future investment environment. The second climate variable is the inflation climate. This inflation climate is not the same as the expected short-run inflation. In fact, it measures the firms aggregate long time assessment or evaluation of the inflation rate.

This Old-Keynesian model consists of two parts: the static state part and the dynamic part. In the static state part, the values of the climate variables in the current period were used to determine the static state values of output gap, inflation rate, and interest rate in the current period. In the dynamic part, the two climate variables are updated accordingly, so we can use the next period climate variables to calculate the next period output gap, inflation rate, and interest rate. The followings are the static state part of the model.

y_t=ηb_t+$\epsilon_{y,t}$ (1)

π_t=π_t^c+κy_t+$\epsilon_{\pi,t}$ (2)

i_t=μ_i $it-1$+$(1-\mu_i)[i^* + \mu_\pi(\pi_t - \pi^*)$+$\mu_y$ $y_t]$+$\epsilon i, t$ (3)

where b_t is the business sentiment and π_t^c is the inflation climate. From these three equations, we can see that the algorithm of generating model data is quite simple. Given the parameters and the values of b_t and pi_t^c, we use equation (1) to obtain the value of y_t. Then we use y_t and equation (2) to obtain the value of π_t. At last, we use the values of y_t and $_t$ and equation (3) to obtain the value for interest rate i_t.

Next, we shift our focus to the dynamic part of this model. In the dynamic part, we will update the two climate variables, b_t and π_t^c.

π_{t+1}^c=π_t^c+α_π $[\gamma\pi^*$+$(1-\gamma)$ π_t-π_t^c $]$ (4)

b_{t+1}=b_t+(1-b_t) $prob_t^{-+}$-(1+b_t) $prob_t^{+-}$ (5)

$prob_t^{-+}$=$prob^{-+}$ (f_t)=min1,α_b exp(f_t)

$prob_t^{+-}$=$prob^{+-}$ (f_t)=min1,α_b exp($-f_t$) (6)

f_t=ϕ_b b_t-ϕ_i (i_t-π_t-r^*) (7)

From equation (4), we can tell that the updating process of π_{t+1}^c is fairly simple. By plugging in the current value of π_t^c and π_t, we can obtain the value for π_{t+1}^c directly. On the contrary, the updating process for b_{t+1} is more complicated. First of all, we need to define $prob_t^{-+}$, $prob_t^{+-}$, and f_t. According to the original paper, $prob_t^{-+}$ and $prob_t^{+-}$ are the probabilities that the firms switch from pessimism to optimism and from optimism to pessimism, respectively. f_t is a feedback index measuring transition probability. We can think of f_t as the arrival of new information. After defining these variables, we are able to compute $b_(t+1)$. First of all, from equation (7) and the current value of b_t, we can obtain f_t. Then, we use equation (6) to compute the switching probabilities, $prob_t^{-+}$ and $prob_t^{+-}$. Finally, we could use equation (5) to update b_{t+1}. Notice that equation (5) produces the cyclical feature in b_t. Since y_t is proportional to b_t, and π_t and i_t depend directly on y_t, π_t and i_t depend directly on b_t. Thus, y_t, π_t, and i_t all have cyclical feature.

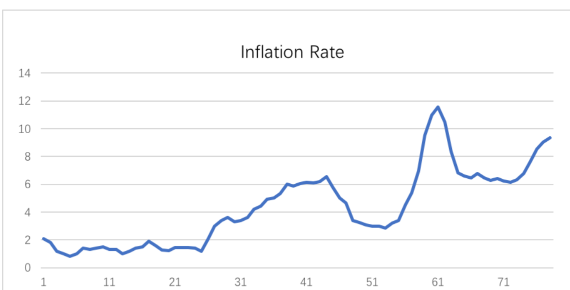
3 Data

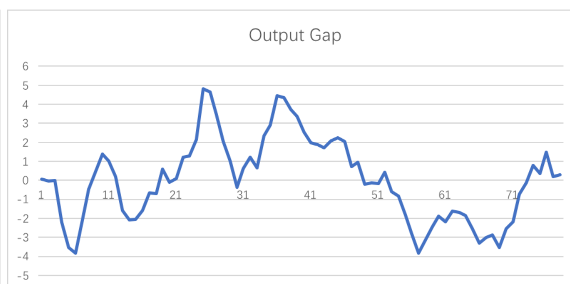
To replicate the methodology of Franke (2012), we collected data from Q1 1960 to Q2 1979 (the Great Inflation period). As Franke (2012) mentioned, the data from the Great Inflation period exhibit substantially different characteristics from the subsequent periods. In alignment with Franke (2012), we collected data on output gap, Federal Funds Rate, and inflation rate. The data on output gap, Federal Funds Rate, and inflation rate are from the Fair-Parke (FP) program section on Ray Fair's website, Federal Reserves, and Bureau of Labor Statistics, respectively. A difficulty that we have encountered in our data collection process is that we could not find readily available data on business climate (b_t) and initial inflation climate (π^e), two important pieces of the original model. To make the matter worse, the author did not elaborate where his data came from or how to estimate them. To deal with these problems, we rescaled the business confidence index and took a 5-year average prior to January 1960 to obtain a new measure that ranges between -1 and 1, which generated the initial guess for b_t. Our approach assumes that companies plan relatively long term and base their initial decisions on medium-term historical data. Our initial value for b_t is 0.28, demonstrating a slight positive sentiment towards the business environment. Moreover, we have used the 10-year average inflation rate prior to January 1960 to generate an initial value for π^e. We assume that businesses forecast future inflation rates according to the inflation rate data from a long period. The average inflation rate between January 1950 and December 1959 was 2.25%. Clearly, businesses would have underestimated the inflation rate during the Great Inflation period if they overemphasized the historical data in their prediction.

The following is the descriptive data of output gap, inflation rate, and interest rates in our sample period. As we can see from the table, the average inflation rate increased substantially during the Great Inflation period to 4.32% from the average of 2.25% in the 1950s. Inflation rate was more volatile than interest rate during this period.

Table 1: Descriptive Data on output gap, interest rate, and inflation during the Great Inflation period

Percentage	Output Gap	Interest Rate	Inflation Rate
Mean	3.83	5.47	4.32
Max	8.64	12.09	11.53
Min	0.00	1.68	0.80
Variance	4.66	5.88	7.56





Here we have shown graphs of historical output gap and inflation rate from 1960 Q1 to 1979 Q2. The output gap shows a cyclical pattern in this period, indicating that the Old-Keynesian Model that predicts a cyclical feature may be satisfatory. Moreover, the output gap is clearly affected by random shocks to generate some short-term trends. On the other hand, the inflation rate does not show much cyclicality over the sample period. In the next section, we will show details of our estimation strategy to estimate the Old-Keynesian model. The first model is a deterministic model that does not have random shocks, so the model simulates a pure cyclical pattern for output gap and inflation and does not generate much short-term trend. The second model has a stochastic random shock component in it, so it may simulate the time series data better than the deterministic model.

4 Estimation

4.1 Choice of Estimation Methodology

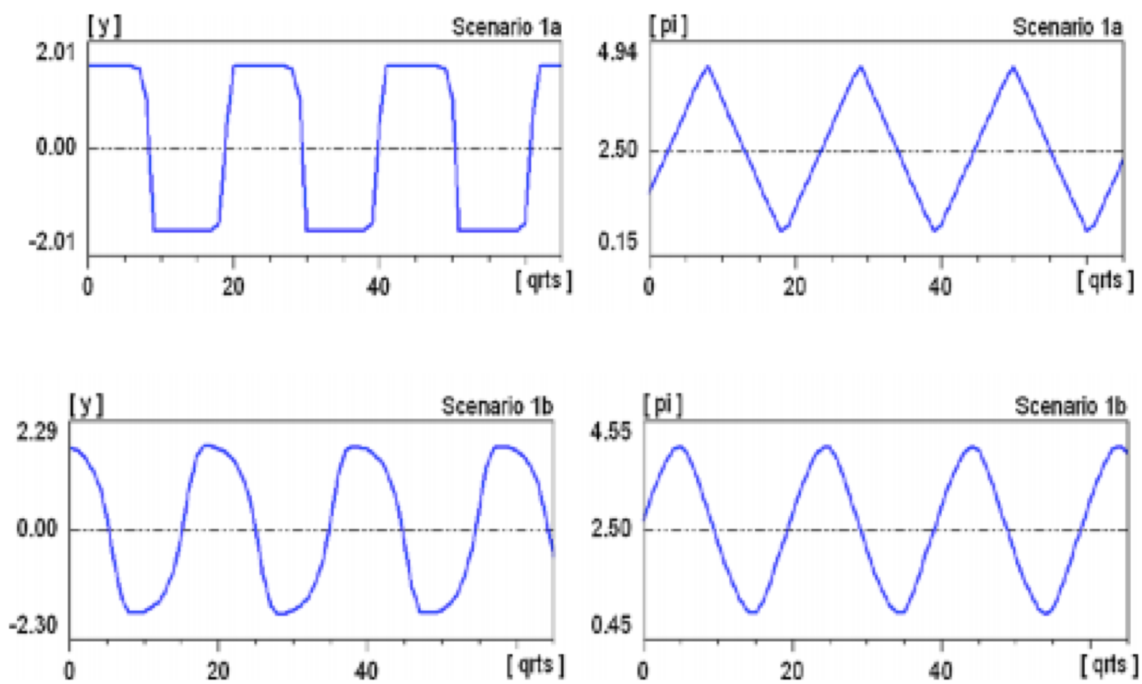
Our choice of estimation strategy is influenced by several factors. Since the Old-Keynesian model is a nonlinear model, we expected that using Maximum Likelihood Estimation (MLE) will be very difficult. Consequently, we did not choose MLE as our principal estimation approach. On the contrary, General Method of Moments (GMM) is suitable for nonlinear models and the approach does not require too many assumptions about the data generating properties. Furthermore, GMM allows us to select two different sets of moments to match and compare the results. Therefore, we decided to use GMM for the deterministic version of the Old-Keynesian model. When the model takes the stochastic random shock into account, Simulated Methods of Moments (SMM) emerged as a natural candidate.

4.2 Choice of moments

As Franke (2012) emphasized in his research, researchers need to select moments that are easy to interpret. According to him, second moments "provide similar information to the impulse-response functions of the three types of shocks in the models, in which (or just one of them) many New-Keynesian studies take a greater interest." Moreover, because the US business cycles vary between five to ten years, Franke argues that the lag horizon should not be too long. Therefore, he chose 8 quarters of lag and matched 78 moments (81 moments minus 3 to avoid double-counting).

In our analysis, we used two sets of moments to estimate the deterministic and stochastic versions of the model.

In the spirit of the original article, we used the same 78 moments consists of autocovariances of output gap, inflation rate, and interest rate as moments to match. According to the original paper, using the 78 moments will cause the transition probability (f_t) to be too large, causing the output gap to stay at ceiling and floor for an extended period, and the simulated inflation rate series has sharp tent-like behavior. The simulated time series of the output gap and inflation rate using 78 moments in the original paper are shown in the graph below on the top. As a result, the author decided to punish the excess transition probability by adding an extra moment. With 79 moments, the simulated time series of output gap and inflation rate both exhibit smooth oscillation, and the transition probability revert to a smaller number. The simulated output gap and inflation rate using 79 moments in the original paper are shown in the following graph on the bottom.



In our second set of moments, we used 12 moments consists of mean, variance, and auto-covariances of output gap, inflation rate, and interest rate. We believe our 12 moments may have advantages over the original 78 moments, because mean and variance are straightforward to understand and using 12 moments does not run the risk of over-identifying parameters.

This second approach also serves as a test for our 79-moment model. To perform this test, we used the parameters estimated from the 79-moment model to compute model moments and compared them with the 12 data moments to judge their performance.

For the SMM estimation, we used the same two sets of models as we did in the GMM estimation. As our computing power is not enough, we deviated from the original article and used S=100 to facilitate a faster calculation process.

4.3 Choice of Weighting Matrix

While the article used block bootstrap method for their 78-moment model, applying the method is beyond our ability. In our replication, we used a simple identity weighting matrix as our weighting matrix. Our choice of identity matrix is due to the fact that the 78 autocovariances are likely to be dependent with each other, causing difficulties to reply on the second-step weighting matrix.

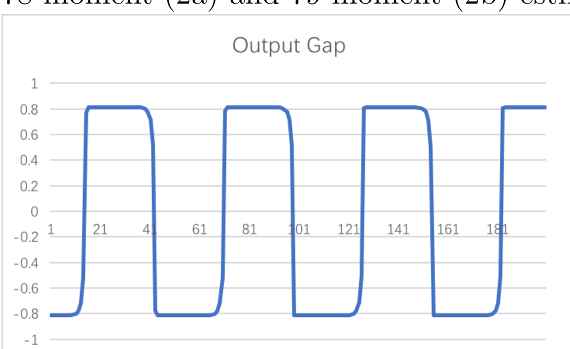
4.4 Determination of Goodness of Fit

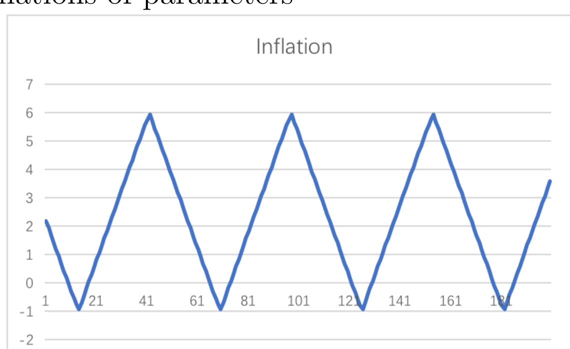
Our measure of goodness-of-fit is mainly based on the value of the criterion function in our estimation. Moreover, we want to make sure the estimated parameters do not exhibit behaviors that are inconsistent with economic facts, such as a very large transition probability that the original paper encountered. Finally, we compare our simulated time series with the actual time series to determine if there are any irregularities in our estimated parameters.

5 Results and Interpretation

5.1 Deterministic Model with 78 and 79 moments

First of all, we used 78 moments to estimate the deterministic Old-Keynesian model without performing the global annealing method that the original paper conducted. Our initial values of the parameters are the estimated parameter value from the original 78-moment model. The simulated output gaps and inflation rate at local optimal point exhibits the similar patterns as the original paper's simulated time series using 78 moments. The output gap stays at the extreme levels for too long, and the turn of inflation is sharp. Moreover, our output gap has a cycle of roughly 55 quarters, which is a little high because normal business cycle are 5 to 10 years. Our model also got a f_t that was fairly large, implying a significant possibility to switch the business sentiment. This result is undesirable to us, because businesses simply do not switch their investment decisions too frequently. We noticed that our model is highly sensitive to the selection of initial guesses. If we started from other initial guesses, the local optimum point could be different. Here, we only present the results based on the initial guess from the original paper's 78-moment estimated parameters. In the following table, we compare our 78-moment estimation (1a) with original model's 78-moment (2a) and 79-moment (2b) estimations of parameters





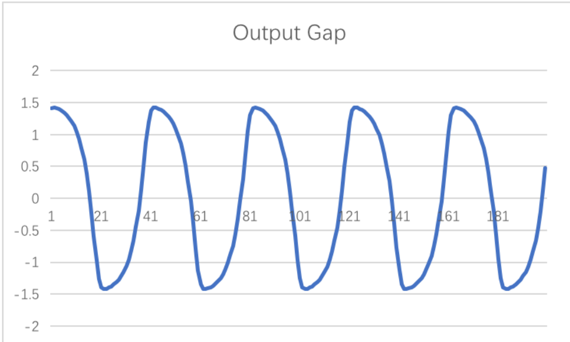
As we can see, our model has a much smaller value of $J^{(78)}$ than the original paper's results. The difference between our result and the original paper's result may come from the different weighting matrix that we used in our paper.

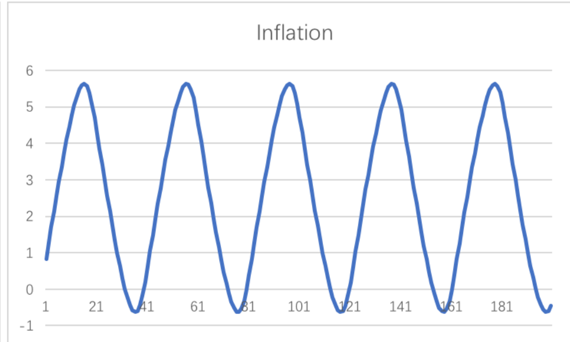
Table 2: Comparison of estimated parameters

	1a	1b	2a	2b
ϕ_b	4.94	1.49	4.94	0.73
ϕ_i	13.87	2.42	13.83	2.96
η	0.82	1.49	1.55	1.84
κ	0.32	0.31	0.24	0.24
α_π	0.96	0.99	1.00	1.00
γ	0.00	0.00	0.00	0.00
μ_i	0.77	0.63	0.76	0.72
μ_π	1.19	1.11	1.48	1.46
μ_y	0.85	0.58	0.79	0.63
$J^{(78)}$	206.10	205.86	626.75	658.19
$J^{(79)}$	999.99	205.86	999.99	658.16

Because we encountered the same problem as the original paper using the 78-moment approach, we adopted the approach in Franke (2012)79th moment to limit the excess transition. Franke (2012) assigned W79 in the weighting matrix to be 1000, which we changed to 100 in our paper. We also adopted the original paper's estimated parameters in its 79-moment model as our initial guess of parameters.

As we reported in the table above, the new moment, weighting matrix, and initial guesses improved the model performance. The comparison demonstrates that a punishment of excess transitioning is necessary and would generate a superior result. As we demonstrated in the table above, the improvement in $J^{(79)}$ is considerable, as the estimated ϕ_b and ϕ_i in our 78-moment estimation were quite large. Moreover, from the graph below, we can tell the problem that haunted our 78-moment estimation has been solved. The smooth oscillation is a desirable feature in our graphs, and the cycle reduced to roughly 40 quarters.



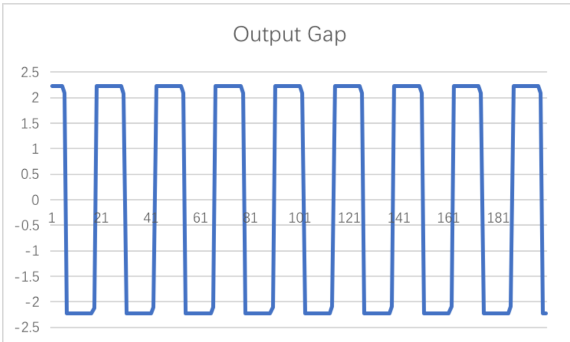


Two differences between our approach and Frankes approach are worth mentioning. First, we did not use the global search method to find the global minimum. Therefore, our 78-moment estimation probably only found a local minimum, which is evidence by the fact that the 79-moment (1b) estimation also returned a lower J^78. Second, our weighting matrix is the identity matrix, while the original paper used block bootstrap method. These two differences may explain why the original paper had a more desirable result where their 78-moment estimation had a lower J^78 while their 79-moment estimation generated a lower J^79.

5.2 Deterministic Model with 12 and 13 moments

Because we only used 12 moments in our estimation, we fixed α_π and γ at 1 and 0 respectively, so that there are only 7 parameters to match. For initial guesses, we used the original paper's 78-moment parameter estimations. The simulated time series for output gap and inflation are shown below. The graphs look very similar to the 78-moment estimation from the original paper. The output gap creeps on the ceilings and floors longer than we desired, and the tent-shaped inflation rate seems have sharp turns. The parameters also imply a high f_t, which is the same problem that Frankes 78-moment method encountered. Under our 12-moment model, the business cycle in this graph is about 5 years, which is within the range that we desire.

From the table below, we can see that our estimated parameters are fairly close to the original 78-parameter model. We doubt that the selection of initial value may have a great impact on which minimum value we land. Unfortunately, without performing a global search, we were unable to determine if we are on the local minimum or a global minimum.



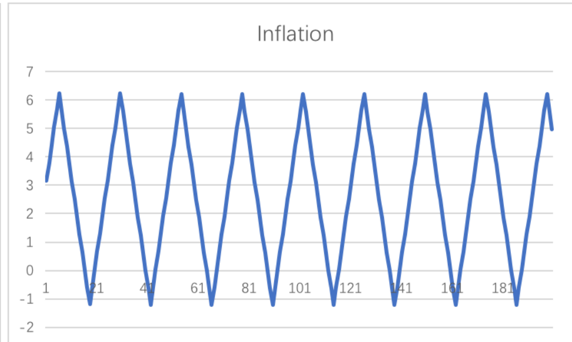
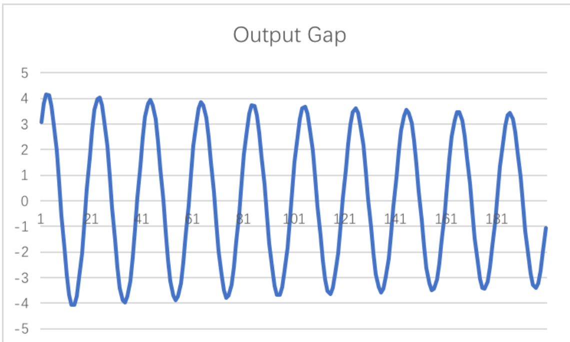
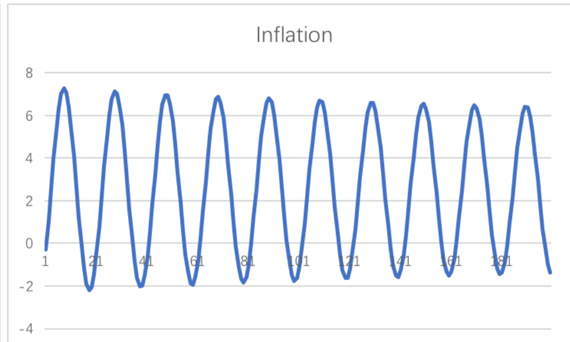


Table 3: Comparison of estimated parameters

	3a	3b	2a	2b
ϕ_b	5.00	1.75	4.94	0.73
ϕ_i	13.94	2.18	13.83	2.96
η	2.23	13.42	1.55	1.84
κ	0.28	0.35	0.24	0.24
α_π	1.00	1.00	1.00	1.00
γ	0.00	0.00	0.00	0.00
μ_i	0.77	0.60	0.76	0.72
μ_π	1.44	1.03	1.48	1.46
μ_y	0.78	0.59	0.79	0.63
$J^{(12)}$	30.11	25.65	87.01	91.57
$J^{(13)}$	999.99	25.65	999.99	658.16

Considering the caveats of the 12-moment model, we speculated that it may be useful to include the additional moment to punish the excessive transition probability. We were hopeful that the additional moment can help us solve the problem again. For our 13-moment estimation, we used the original paper's 79-moment estimation as our initial guess for parameters. The graphs of simulated output gap and inflation initially made us worried, as the amplitude of output gap seems to be shrinking. However, we drew the simulated time series for 2000 periods, and found that the amplitude stabilized later. Therefore, the initial decrease in amplitude is acceptable to us. From the table above, we can tell that our 13-moment model improved the performance of $J^(12)$ and $J^(13)$. Besides improving $J^{(12)}$, the model also caused the excess transition probability to disappear. Comparing the parameters in 12-moment model (3a) and 13-moment model (3b), it is clear that ϕ_b and ϕ_i decreased considerable, demonstrating the effect of the 13th moment. Additionally, the business cycle in the 13-moment model remains at roughly 5 years.





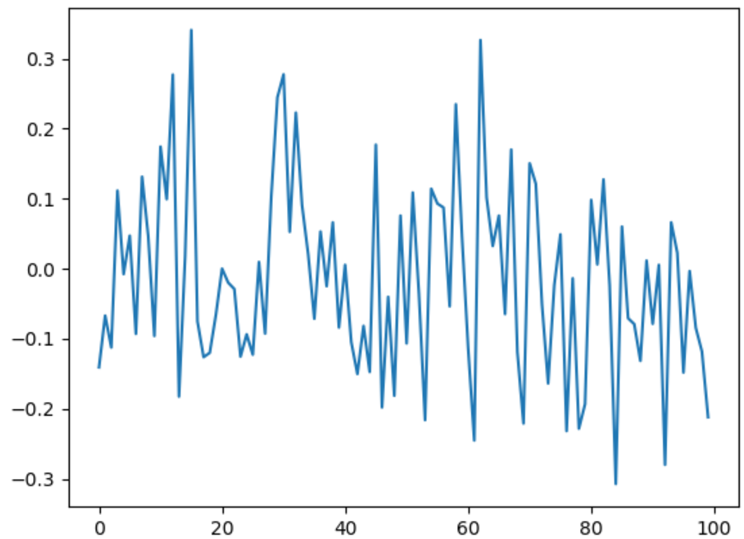
After estimating the parameters with two sets of moments, we are curious how well the parameters of the 79-moment model fits the 13 data moments and vice versa. Plugging in the parameter of our 79-moment model in $J^{(13)}$, the criterion function value is 43.94, which is slightly worse than 12-moment and 13-moment models, but better than the original 78-moment and 79-moment models. The results shows that our 79-moment model has a fairly strong explanatory power. In fact, when we examined the fitting of data moments, we can also tell that our 79-moment model performed fairly well.

Moment	0	1	2	3	4	5	6	7	8	9	10	11	12
Data Moment	3.83	2.15	4.32	2.73	5.47	2.41	0.24	(0.67)	4.76	4.30	5.40	7.26	0.00
13-Moment	0.01	2.08	2.51	2.38	5.01	2.48	0.90	0.76	5.91	4.13	5.86	5.42	0.00
79-Moment	0.00	1.10	2.50	2.17	5.00	2.41	0.19	0.18	5.23	1.19	5.73	4.66	0.00

When we plug in our 13-moment model in $J^{(79)}$, it seems that the criterion value increased considerably. Therefore, our 13-moment model did not perform well in the outside moment test.

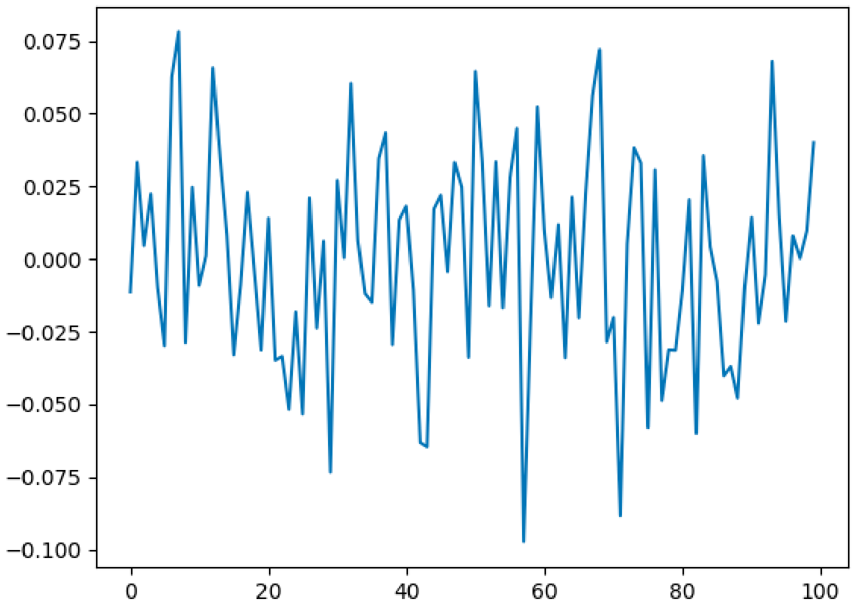
5.3 Stochastic Model with 78 moments

Now we incorporate the demand shocks $\epsilon_{y,t}$, $\epsilon_{\pi,t}$, and $\epsilon_{i,t}$ in the equation (7), (8), (9). We need to estimate additional parameters, σ_y, σ_π, σ_i. The simulated time series for output gap is shown below



5.4 Stochastic Model with 12 moments

We also added stochastic shocks in our 12-moment model. The simulated time series for output gap is presented here. As we can see, both the stochastic 78-moment model and 12-moment model exhibits substantial randomness. In the model with random shocks, the apparent cycle in the deterministic model disappeared. It is then difficult to judge the goodness of fit for the stochastic models.



6 Conclusion

From the analysis above, we can see that the estimations for the deterministic version with 78 and 79 moments produce similar results as the original paper. This implies that our estimations are correct in the sense that they obtain at least local minimum. It is obvious that our results did not obtain global minimum due to two factors. The first reason is that we did not conduct the global search. Therefore, we can not pin down the ranges of parameters under which they can obtain the global minimum. The second reason is the choice of weighting matrix. As we mentioned above, we only applied identity matrix in the estimations. This dramatically changes the objective function to be minimized. These two factors may distort our estimations and lead to results that differ from the original article.

While we cannot perfectly replicate the original paper, we extended the original paper by introducing different moments into the estimations. We can clearly see that the deterministic version of the Old-Keynesian model with 12 and 13 moments also produce similar graphs as the original paper. However, the estimated values of the parameters are different from both the original paper and our previous estimations with 78 and 79 moments. Our interpretation is that the disparities may be due to the differences in the objective functions. It is reasonable to believe that different objective functions will lead to different results. Again, the similarity between the simulated time series from 12 and 13 moments and the time series from the original paper indicates that our estimation with 12 and 13 moments achieved local minimum.

7 Reference

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