UNIT 1 QUIZ

#1

$$2x_1 + 2x_2 + x_3 = 20$$
$$4x_1 + x_2 + 0 = 12$$
$$5x_1 + 0x_2 + 2x_3 = 16$$

$$\begin{pmatrix} 2 & 2 & 1 \\ 4 & 1 & 0 \\ 5 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 20 \\ 12 \\ 16 \end{pmatrix}$$

matrix A

$$Ax = b$$

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 4 & 1 & 0 \\ 5 & 0 & 2 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad b = \begin{bmatrix} 20 \\ 12 \\ 16 \end{bmatrix}$$

#2

$$\begin{bmatrix} 10 & 2 & 22 \\ 4 & 15 & 66 \\ 5 & 42 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$C = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad C^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$C_{1,1} = (10 \times x_1) + (2 \times x_2) + (22 \times x_3)$$
$$C_{2,1} = (4 \times x_1) + (15 \times x_2) + (66 \times x_3)$$
$$C_{3,1} = (5 \times x_1) + (42 \times x_2) + (1 \times x_3)$$

$$C = \begin{bmatrix} (10x_1 + 2x_2 + 22x_3) \\ (4x_1 + 15x_2 + 66x_3) \\ (5x_1 + 42x_2 + x_3) \end{bmatrix}$$

#3

$$\begin{bmatrix} 1 \times -1 + 0 \times 2 \\ 2 \times -1 + 1 \times 2 \end{bmatrix} \begin{pmatrix} 1 \times -4 + 0 \times 3 \\ 2 \times -4 + 1 \times 3 \end{pmatrix} = \begin{bmatrix} -1 & -4 \\ 0 & -5 \end{bmatrix}$$

Determinant: $ad - bc = (-1)(-5) - (0)(-4) = 5$

inverse $(AB)^{-1} = \frac{1}{5}\begin{bmatrix} -5 & 4 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 4/5 \\ 0 & -1/5 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1 & -4 \\ 2 & 3 \end{bmatrix}$$

#4

$$\begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 4(1)+1(3) & 4(2)+1(2) \\ 2(1)+3(3) & 2(2)+3(2) \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 11 & 10 \end{bmatrix}$$

$\det(A) = ad - bc = 70 - 110 = 40$

the product has linear independent columns, because the determinant is non zero

#5

$$4x_1 - 2x_2 = 22$$
$$-x_1 + 3x_2 = -23$$
$$5x_1 + x_2 = 3$$

$$Ax = b \quad A = \begin{bmatrix} 4 & -2 \\ -1 & 3 \\ 5 & 1 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad b = \begin{bmatrix} 22 \\ -23 \\ 3 \end{bmatrix}$$

$A' = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$ $\det(A') = (4)(3) - (-2)(-1) = 12 - 2 = 10$

det is non-zero, the columns of A are linearly independent

$$A'x = b'$$

$$A'^{-1} = \frac{1}{\det(A')}\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} \quad \frac{1}{10}\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$$

$$x = A'^{-1}b'$$

$$x = \frac{1}{10}\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 22 \\ -23 \end{bmatrix}$$

$$x_1 = 2 \quad x_2 = -7$$

- vector b can be expressed as a linear combination of the columns of A
- the columns of A are linearly independent as proven by the non-zero determinant
- b is a linear combination of independent vectors

system has a unique solution $(x_1, x_2) = (2, -7)$

#6

The system of linear equations having a unique solution requires the columns of A to be linearly independent, not just b being a linear combination of the columns of A

If the columns of A are linearly dependent, there could be multiple solutions or no solutions at all