Unit #5 Quiz

(1) $V = \text{span}(S)$

$S = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 \,\middle|\, x_1 + x_2 = 1.0 \right\}$

since $x_1 + x_2 = 1$

$x_1 = 1.0 - x_2$

any vector in S can be written as

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} x_2 \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} x_3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$v_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \quad v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

- all 3 vectors are linearly independent since no vector can be expressed as a combination of the other two
- the vectors span the subspace V as any vector in S that satisfies $x_1 + x_2 = 1.0$ can be written as these 3 basic vectors

final basis for V:

$\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$

(2) Eigenvectors are special vectors that maintain their direction when a linear transformation is applied. This is crucial for analyzing transformations. Eigenvalues determine the scaling effect on eigenvectors indicating whether they stretch, shrink or remain unchanged.

Eigenvectors provide insight into the intrinsic behavior of the system making them essential for decomposing transformations

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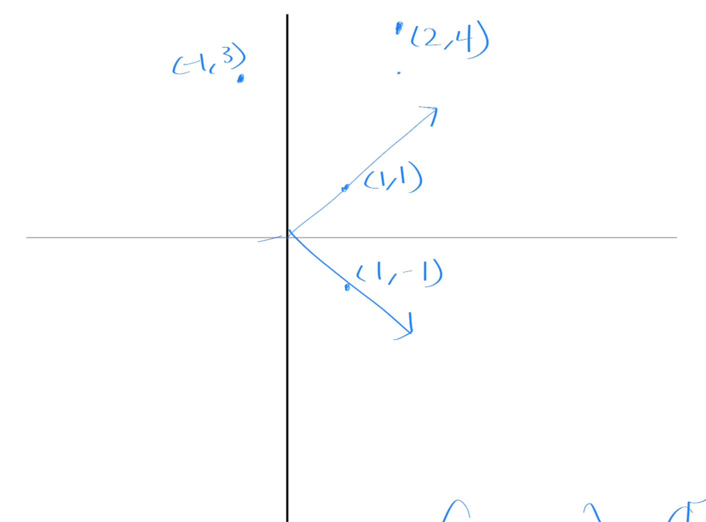
(3) $v_1 = e_1 - e_2 \quad v_2 = e_1 + e_2 \quad e_1 = 1 \quad e_2 = -1$

$v_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$



Given vectors:

$v_1 = e_1 - e_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

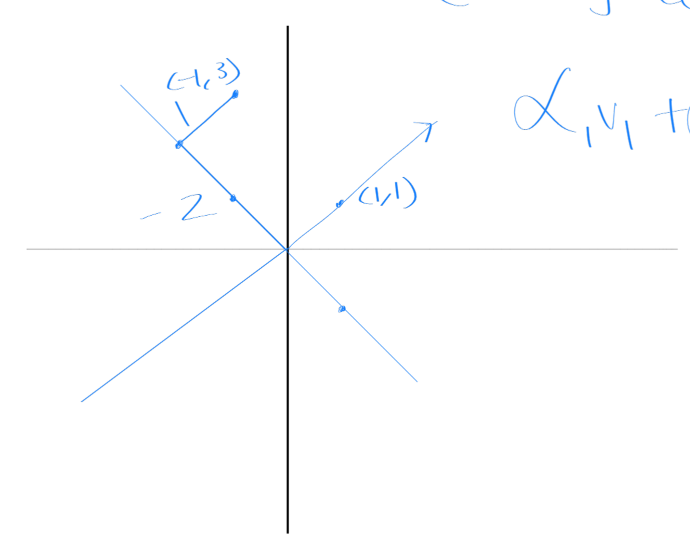
$v_2 = e_1 + e_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(1) Linearly independent

(2) span the entire $\mathbb{R}^2$ space

any vector in $\mathbb{R}^2$ can be uniquely expressed as a linear combination of v_1 and v_2

$\{e_1, e_2\} \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$



$\alpha_1 v_1 + \alpha_2 v_2 = \begin{bmatrix} -1 \\ 3 \end{bmatrix} \quad \alpha_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \alpha_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$

$\alpha_1 + \alpha_2 = -1$

$\alpha_1 + \alpha_2 = 3$

$\alpha = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

-2 in v_1 direction

1 in v_2 direction