CHAPTER 16: DECISION ANALYSIS

16.2-1.

Phillips Petroleum Company developed a decision analysis tool named DISCOVERY to evaluate available investment opportunities and decide on the participation levels. The need for a systematic decision analysis tool arose from the uncertainty associated with various alternatives, the lack of a consistent risk measure across the organization and the scarcity of capital resources. The notion of risk is incorporated in the model with the use of risk-averse exponential utility function. The objective is to maximize expected utility rather than expected return. DISCOVERY provides a decision-tree display of available alternatives at various participation levels. A simple version of the problem is one where Phillips needs to decide first on the participation level and second on whether to drill or not. The exploration of petroleum when drilled is uncertain. The analysis is performed for different levels of risk-aversion and the sensitivity of the decisions to the risk-aversion level is observed. When additional seismic information is available at a cost, the value of information is computed.

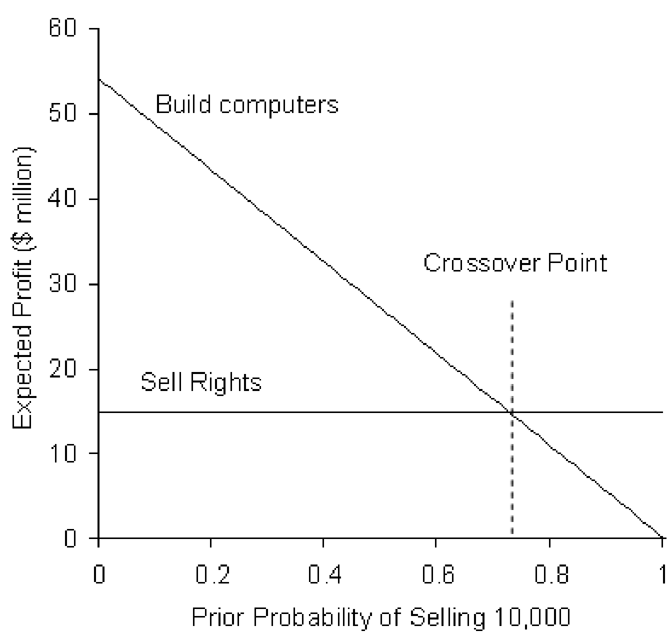
This study "has increased management's awareness of risk and risk tolerance, provided insight into the financial risks associated with its set of investment opportunities, and provided the company a formalized decision model for allocating scarce capital" [p. 55]. The software package developed has been a valuable aid in decision making. It provided a systematic treatment of risk and uncertainty. Other petroleum exploration firms started to use DISCOVERY in analyzing decisions, too.

16.2-2.

(a)

	State of Nature	
Alternative	Sell 10, 000	Sell 100, 000
Build Computers	0	54
Sell Rights	15	15

(b)



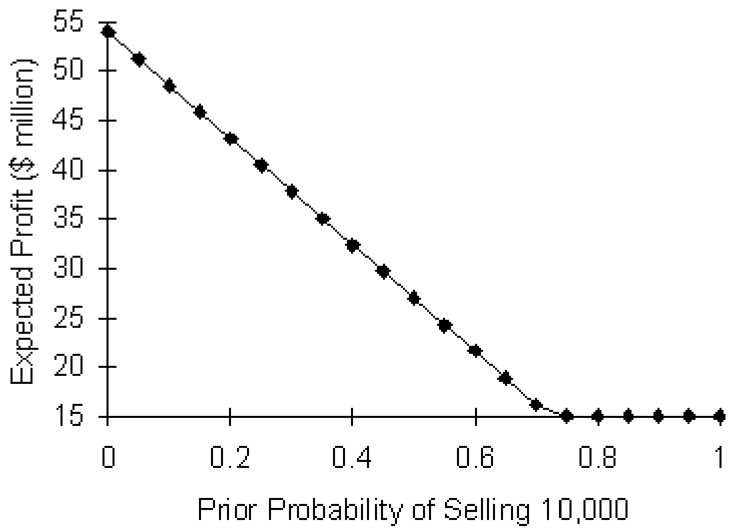
(c) Let p be the prior probability of selling 10,000 computers.

Build: EP $= p(0) + (1-p)(54) = -54p + 54$

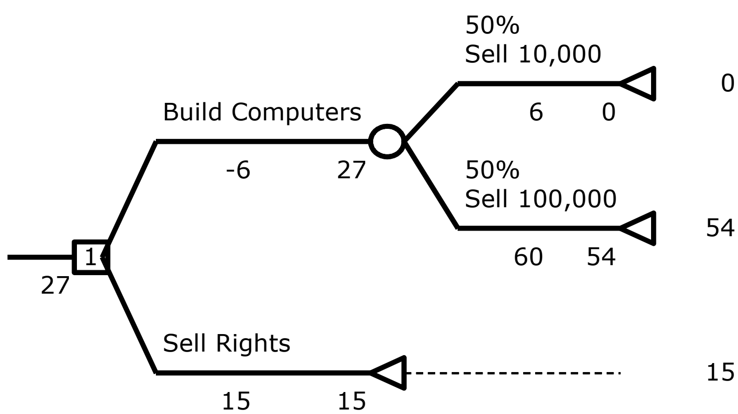
Sell: EP $= p(15) + (1-p)(15) = 15$

The expected profit for Build and Sell is the same when $-54p + 54 = 15 \Rightarrow p = 0.722$. They should build when $p \leq 0.722$ and sell if $p \geq 0.722$.

(d)



(e)



Building computers should be chosen, since it has an expected payoff of $27 million.

16.2-3.

(a)

	State of Nature			
Alternative	Sell 12 Cases	Sell 13 Cases	Sell 14 Cases	Sell 15 Cases
Buy 12 Cases	132	132	132	132
Buy 13 Cases	125	143	143	143
Buy 14 Cases	118	136	154	154
Buy 15 Cases	111	129	147	165
Prior Probability	0.1	0.3	0.4	0.2

(b) According to the maximin payoff criterion, Jean should purchase 12 cases.

	State of Nature				
Alternative	Sell 12 Cases	Sell 13 Cases	Sell 14 Cases	Sell 15 Cases	Min
Buy 12 Cases	132	132	132	132	132
Buy 13 Cases	125	143	143	143	125
Buy 14 Cases	118	136	154	154	118
Buy 15 Cases	111	129	147	165	111
Prior Probability	0.1	0.3	0.4	0.2	

(c) She will be able to sell 14 cases with highest probability and the maximum possible profit from selling 14 cases is earned when she buys 14 cases. Hence, according to the maximum likelihood criterion, Jean should purchase 14 cases.

(d) According to Bayes' decision rule, Jean should purchase 14 cases.

	State of Nature				Exp.
Alternative	Sell 12 Cases	Sell 13 Cases	Sell 14 Cases	Sell 15 Cases	Profit
Buy 12 Cases	132	132	132	132	132
Buy 13 Cases	125	143	143	143	141.2
Buy 14 Cases	118	136	154	154	145
Buy 15 Cases	111	129	147	165	141.6
Prior Probability	0.1	0.3	0.4	0.2	

(e) <u>0.2 and 0.5:</u> Jean should purchase 14 cases.

	State of Nature				Exp.
Alternative	Sell 12 Cases	Sell 13 Cases	Sell 14 Cases	Sell 15 Cases	Profit
Buy 12 Cases	132	132	132	132	132
Buy 13 Cases	125	143	143	143	141.2
Buy 14 Cases	118	136	154	154	146.8
Buy 15 Cases	111	129	147	165	143.4
Prior Probability	0.1	0.2	0.5	0.2	

<u>0.4 and 0.3:</u> Jean should purchase 14 cases.

	State of Nature				Exp.
Alternative	Sell 12 Cases	Sell 13 Cases	Sell 14 Cases	Sell 15 Cases	Profit
Buy 12 Cases	132	132	132	132	132
Buy 13 Cases	125	143	143	143	141.2
Buy 14 Cases	118	136	154	154	143.2
Buy 15 Cases	111	129	147	165	139.8
Prior Probability	0.1	0.4	0.3	0.2	

<u>0.5 and 0.2:</u> Jean should purchase 14 cases.

	State of Nature				Exp.
Alternative	Sell 12 Cases	Sell 13 Cases	Sell 14 Cases	Sell 15 Cases	Profit
Buy 12 Cases	132	132	132	132	132
Buy 13 Cases	125	143	143	143	141.2
Buy 14 Cases	118	136	154	154	141.4
Buy 15 Cases	111	129	147	165	138
Prior Probability	0.1	0.5	0.2	0.2	

16.2-4.

(a) The optimal (maximin) actions are conservative and countercyclical investments, both incur a loss of \$10 million in the worst case.

(b) The economy is most likely to be stable and the alternative with the highest profit in this state of nature is to make a speculative investment. According to the maximum likelihood criterion, Warren should choose speculative investment.

(c) To maximize his expected payoff, Warren should make a countercyclical investment.

	State of Nature			Exp.
Alternative	Improving	Stable	Worsening	Profit
Conservative	30	5	−10	1.5
Speculative	40	10	−30	−3
Countercyclical	−10	0	15	5
Prior Probability	0.1	0.5	0.4	

16.2-5.

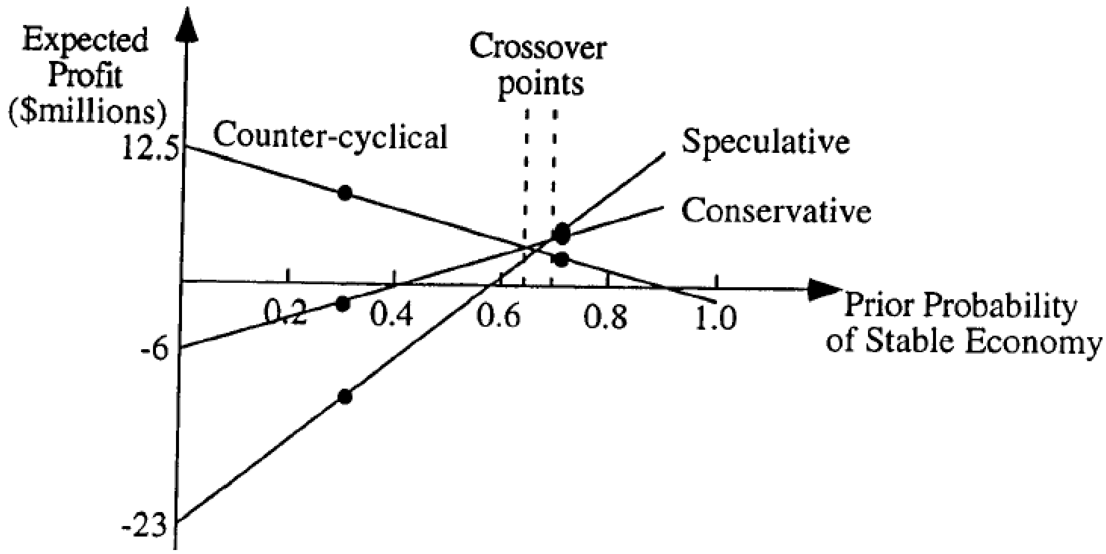
(a) Warren should make a countercyclical investment.

	State of Nature			Exp.
Alternative	Improving	Stable	Worsening	Profit
Conservative	30	5	−10	−1.5
Speculative	40	10	−30	−11
Countercyclical	−10	0	15	8
Prior Probability	0.1	0.3	0.6	

(b) Warren should make a speculative investment.

	State of Nature			Exp.
Alternative	Improving	Stable	Worsening	Profit
Conservative	30	5	−10	4.5
Speculative	40	10	−30	5
Countercyclical	−10	0	15	2
Prior Probability	0.1	0.7	0.2	

(c) The expected profit from countercyclical and conservative investments is the same when $p \approx 0.62$. The expected profit lines for conservative and speculative investments cross at $p \approx 0.68$. Those for countercyclical and speculative investments cross at $p \approx 0.65$; however, this crossover point does not result in a decision shift.



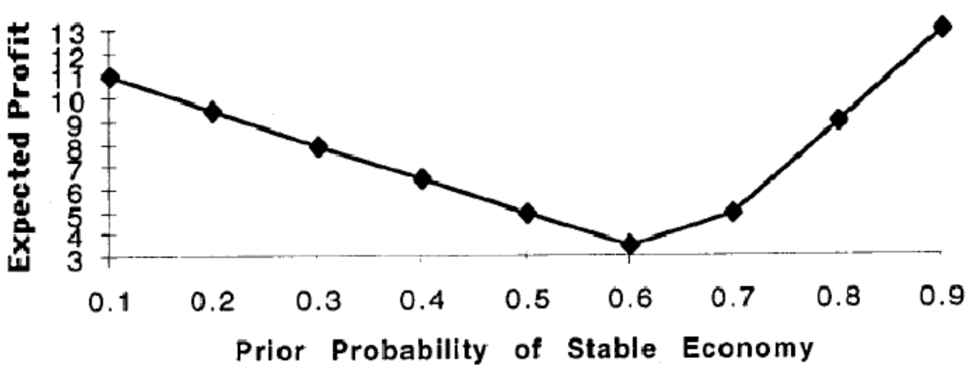
(d) Let p be the prior probability of stable economy.

Conservative:	$\text{EP} = (0.1)(30) + p(5) + (1 - 0.1 - p)(-10) = 15p - 6$
Speculative:	$\text{EP} = (0.1)(40) + p(10) + (1 - 0.1 - p)(-30) = 40p - 23$
Countercyclical:	$\text{EP} = (0.1)(-10) + p(0) + (1 - 0.1 - p)(15) = -15p + 12.5$

Countercyclical and conservative cross when $-15p + 12.5 = 15p - 6 \Rightarrow p = 0.617$.
Conservative and speculative cross when $15p - 6 = 40p - 23 \Rightarrow p = 0.68$.

Accordingly, Warren should choose countercyclical investment when $p < 0.617$, conservative investment when $0.617 \leq p < 0.68$ and speculative investment when $p \geq 0.68$.

(e)



16.2-6.

(a) A_2 should be chosen.

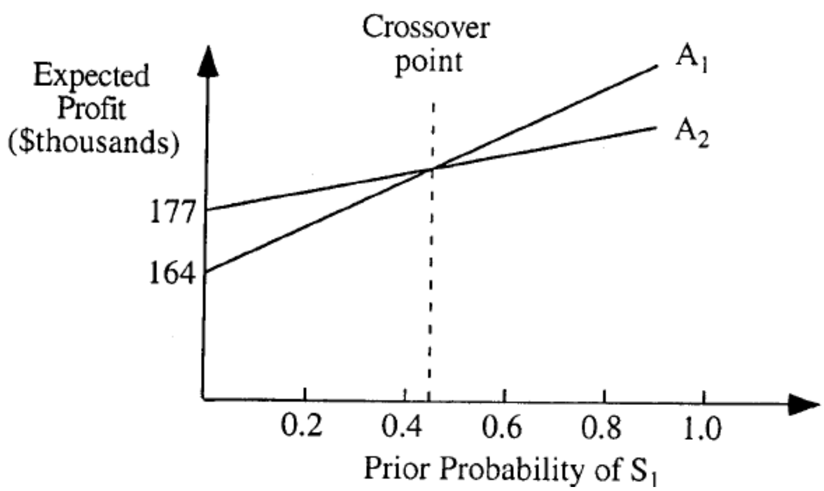
	State of Nature			
Alternative	S_1	S_2	S_3	Min
A_1	220	170	110	110
A_2	200	180	150	150
Prior Probability	0.6	0.3	0.1	

(b) The most likely state of nature is S_1 and the alternative with highest profit in this state is A_1.

(c) A_1 should be chosen.

	State of Nature			Exp.
Alternative	S_1	S_2	S_3	Payoff
A_1	220	170	110	194
A_2	200	180	150	189
Prior Probability	0.6	0.3	0.1	

(d)

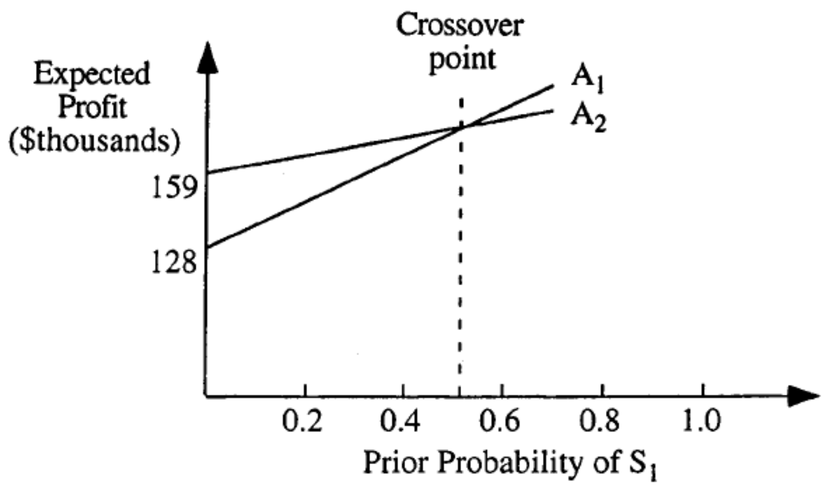


Let p be the prior probability of S_1.

A_1: $\quad \text{EP} = p(220) + (1 - 0.1 - p)(170) + (0.1)(110) = 50p + 164$
A_2: $\quad \text{EP} = p(200) + (1 - 0.1 - p)(180) + (0.1)(150) = 20p + 177$

A_1 and A_2 cross when $50p + 164 = 20p + 177 \Rightarrow p = 0.433$. They should choose A_2 when $p \leq 0.433$ and A_1 if $p > 0.433$.

(e)

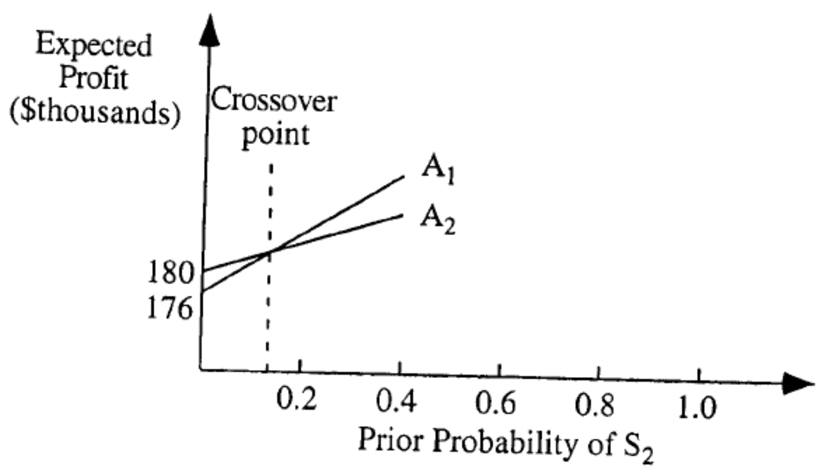


Let p be the prior probability of S_1.

A$_1$: EP $= p(220) + (0.3)(170) + (1 - 0.3 - p)(110) = 110p + 128$
A$_2$: EP $= p(200) + (0.3)(180) + (1 - 0.3 - p)(150) = 50p + 159$

A$_1$ and A$_2$ cross when $110p + 128 = 50p + 159 \Rightarrow p = 0.517$. They should choose A$_2$ when $p \leq 0.517$ and A$_1$ if $p > 0.517$.

(f)



Let p be the prior probability of S_2.

A$_1$: EP $= (0.6)(220) + p(170) + (1 - 0.6 - p)(110) = 60p + 176$
A$_2$: EP $= (0.6)(200) + p(180) + (1 - 0.6 - p)(150) = 30p + 180$

A$_1$ and A$_2$ cross when $60p + 176 = 30p + 180 \Rightarrow p = 0.133$. They should choose A$_2$ when $p \leq 0.133$ and A$_1$ if $p > 0.133$.

(g) A$_1$ should be chosen.

16.2-7.

(a)

	State of Nature		
Alternative	Dry	Moderate	Damp
Crop 1	20	35	40
Crop 2	22.5	30	45
Crop 3	30	25	25
Crop 4	20	20	20
Prior Probability	0.3	0.5	0.2

(b) Grow Crop 1.

	State of Nature			Exp.
Alternative	Dry	Moderate	Damp	Payoff
Crop 1	20	35	40	31.5
Crop 2	22.5	30	45	30.75
Crop 3	30	25	25	26.5
Crop 4	20	20	20	20
Prior Probability	0.3	0.5	0.2	

(c) Prior probability of moderate weather = 0.2: Grow Crop 2.

	State of Nature			Exp.
Alternative	Dry	Moderate	Damp	Payoff
Crop 1	20	35	40	33
Crop 2	22.5	30	45	36.25
Crop 3	30	25	25	26.5
Crop 4	20	20	20	20
Prior Probability	0.3	0.2	0.5	

Prior probability of moderate weather = 0.3: Grow Crop 2.

	State of Nature			Exp.
Alternative	Dry	Moderate	Damp	Payoff
Crop 1	20	35	40	32.5
Crop 2	22.5	30	45	33.75
Crop 3	30	25	25	26.5
Crop 4	20	20	20	20
Prior Probability	0.3	0.3	0.4	

Prior probability of moderate weather = 0.4: Grow Crop 2.

	State of Nature			Exp.
Alternative	Dry	Moderate	Damp	Payoff
Crop 1	20	35	40	32
Crop 2	22.5	30	45	32.25
Crop 3	30	25	25	26.5
Crop 4	20	20	20	20
Prior Probability	0.3	0.4	0.3	

Prior probability of moderate weather $= 0.6$: Grow Crop 1.

	State of Nature			Exp.
Alternative	Dry	Moderate	Damp	Payoff
Crop 1	20	35	40	31
Crop 2	22.5	30	45	29.25
Crop 3	30	25	25	26.5
Crop 4	20	20	20	20
Prior Probability	0.3	0.6	0.1	

16.2-8.

The prior distribution is $\text{P}\{\theta = \theta_1\} = 2/3$, $\text{P}\{\theta = \theta_2\} = 1/3$.

Order 15: $-\text{EP} = 2/3(1.155 \cdot 10^7) + 1/3(1.414 \cdot 10^7) = 1.241 \cdot 10^7$
Order 20: $-\text{EP} = 2/3(1.012 \cdot 10^7) + 1/3(1.207 \cdot 10^7) = 1.077 \cdot 10^7$
Order 25: $-\text{EP} = 2/3(1.047 \cdot 10^7) + 1/3(1.135 \cdot 10^7) = 1.076 \cdot 10^7$

The maximum expected profit, or equivalently the minimum expected cost is that of ordering 25, so the optimal decision under Bayes' decision rule is to order 25.

16.3-1.

This article describes the use of decision analysis at the Workers' Compensation Board of British Columbia (WCB), which is "responsible for the occupational health and safety, rehabilitation, and compensation interests of British Columbia's workers and employers" [p. 15]. The focus of the study is on the short-term disability claims that can later turn into long-term disability claims and can be very costly for the WCB. First, logistic regression is employed to estimate the probability of conversion for each claim. Then using decision analysis, a threshold is determined to classify the claims as high- and low-risk claims. For any fixed conversion probability, the problem consists of a simple decision tree. First the WCB chooses between classifying the claim as high risk or low risk and then whether the claim converts or not determines the actual cost. If the claim is identified as a high-risk claim, the WCB intervenes. The early intervention lowers the costs and ensures faster rehabilitation. The expected total cost is computed for various cutoff points and the point with minimum expected cost is identified as the optimal threshold.

The new policy offers accurate predictions of high-risk claims. As a result, future costs are reduced and injured workers start working sooner. This study is expected to save the WCB $4.7 per year. The scorecard system developed to implement the new policy improved the efficiency of claim management and the productivity of staff. Overall, the benefits accrued from this study paved the way for the WCB's adoption of operations research in other aspects of the organization.

16.3-2.

(a)

	State of Nature	
Alternative	Sell $10,000$	Sell $100,000$
Build Computers	0	54
Sell Rights	15	15
Prior Probability	0.5	0.5
Maximum Payoff	15	54

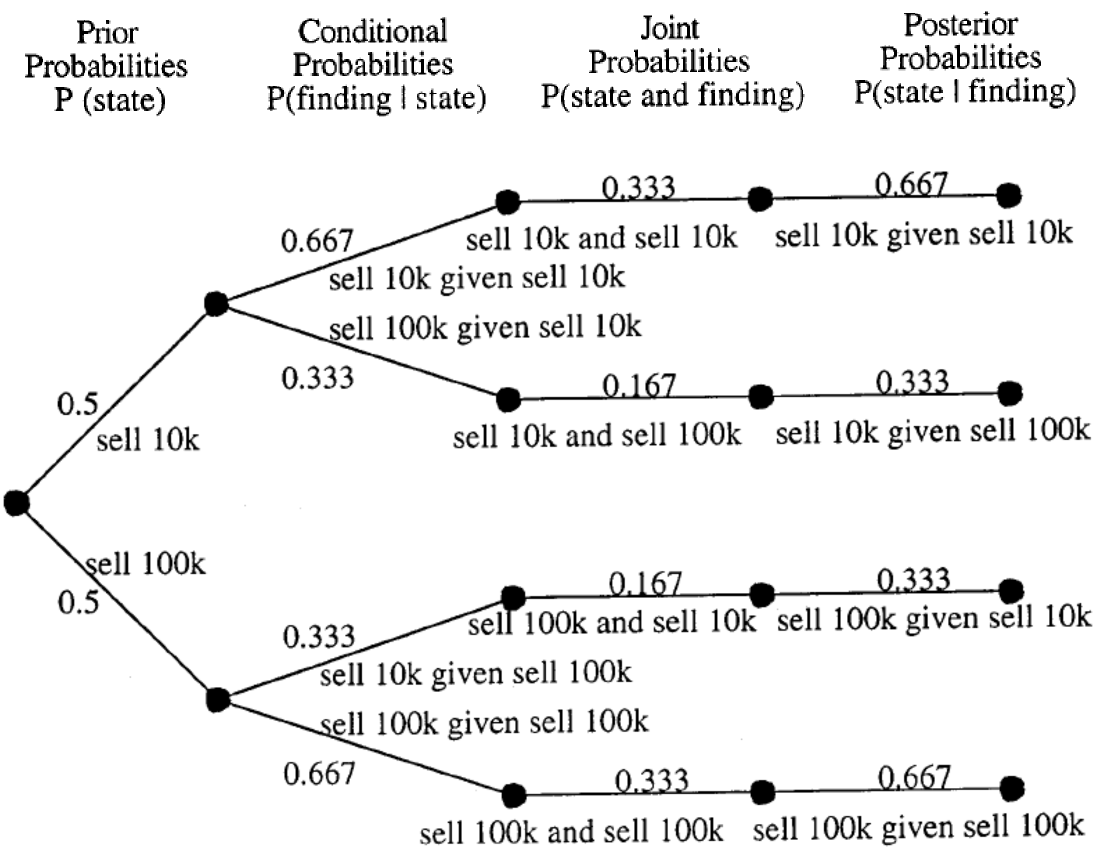
Expected Payoff with Perfect Information: $0.5(15) + 0.5(54) = 34.5$

Expected Payoff without Information: $0.5(0) + 0.5(54) = 27$

EVPI $= 34.5 - 27 =$ \$7.5 million

(b) Since the market research will cost \$1 million, it might be worthwhile to perform it.

(c)



(d)

Data:		P(Finding | State)	
State of	Prior	Finding	
Nature	Probability	Predict Sell 10,000	Predict Sell 100,000
Sell 10,000	0.5	0.667	0.333
Sell 100,000	0.5	0.333	0.667

Posterior		P(State | Finding)	
Probabilities:		State of Nature	
Finding	P(Finding)	Sell 10,000	Sell 100,000
Predict Sell 10,000	0.5	0.667	0.333
Predict Sell 100,000	0.5	0.333	0.667

(e) $\text{EVE} = [0.5(1800) + 0.5(3600)] - 2700 = 0$, so performing the market research is not worthwhile.

16.3-3.

(a) Choose A_2 with expected payoff \$1000.

	State of Nature			Exp.
Alternative	S_1	S_2	S_3	Payoff
A_1	4	0	0	0.8
A_2	0	2	0	1.0
A_3	3	0	1	0.9
Prior Probability	0.2	0.5	0.3	

(b)

	State of Nature		
Alternative	S_1	S_2	S_3
A_1	4	0	0
A_2	0	2	0
A_3	3	0	1
Prior Probability	0.2	0.5	0.3
Maximum Payoff	4	2	1

Expected Payoff with Perfect Information: $0.2(4) + 0.5(2) + 0.3(1) = 2.1$
Expected Payoff without Information: 1.0
$\text{EVPI} = 2.1 - 1.0 = \$1.1$ thousand.

(c) Since the information will cost \$1000 and the value is \$1100, it might be worthwhile to spend the money.

16.3-4.

(a) Choose A_1 with expected payoff \$35.

	State of Nature			Exp.
Alternative	S_1	S_2	S_3	Payoff
A_1	50	100	-100	35
A_2	0	10	-10	1
A_3	20	40	-40	14
Prior Probability	0.5	0.3	0.2	

(b)

	State of Nature		
Alternative	S_1	S_2	S_3
A_1	50	100	-100
A_2	0	10	-10
A_3	20	40	-40
Prior Probability	0.5	0.3	0.2
Maximum Payoff	50	100	-10

Expected Payoff with Perfect Information: $0.5(50) + 0.3(100) + 0.2(-10) = 53$
Expected Payoff without Information: 35
EVPI $= 53 - 35 = \$18$

(c) Betsy should consider spending up to \$18 to obtain more information.

16.3-5.

(a) Choose A_3 with expected payoff $\$35,000$.

	State of Nature			Exp.
Alternative	S_1	S_2	S_3	Payoff
A_1	-100	10	100	33
A_2	-10	20	50	29
A_3	10	10	60	35
Prior Probability	0.2	0.3	0.5	

(b) If S_1 occurs for certain, then choose A_3 with expected payoff $\$10,000$. If S_1 does not occur for certain, then the probability that S_2 will occur is $\frac{3}{8}$ and the probability that S_3 will occur is $\frac{5}{8}$.

$$\begin{aligned}
A_1&: \quad (\tfrac{3}{8})(10) + (\tfrac{5}{8})(100) = 66.25\\
A_2&: \quad (\tfrac{3}{8})(20) + (\tfrac{5}{8})(50) = 38.75\\
A_3&: \quad (\tfrac{3}{8})(10) + (\tfrac{5}{8})(60) = 41.25
\end{aligned}$$

Hence, choose A_1 which offers an expected payoff of $\$66,250$.

Expected Payoff with Information: $0.2(10) + 0.8(66.25) = 55$
Expected Payoff without Information: 35
EVI $= 55 - 35 = \$20$ thousand

The maximum amount that should be paid for the information is $\$20,000$. The decision with this information will be to choose A_3 if the state of nature is S_1 and A_1 otherwise.

(c) If S_2 occurs for certain, then choose A_2 with expected payoff $\$20,000$. If S_2 does not occur for certain, then the probability that S_1 will occur is $\frac{2}{7}$ and the probability that S_3 will occur is $\frac{5}{7}$.

$$\begin{aligned}
A_1&: \quad (\tfrac{2}{7})(-100) + (\tfrac{5}{7})(100) = 42.857\\
A_2&: \quad (\tfrac{2}{7})(-10) + (\tfrac{5}{7})(50) = 32.857\\
A_3&: \quad (\tfrac{2}{7})(10) + (\tfrac{5}{7})(60) = 45.714
\end{aligned}$$

Hence, choose A_3 which offers an expected payoff of $\$45,714$.

Expected Payoff with Information: $0.3(20) + 0.7(45.714) = 38$
Expected Payoff without Information: 35
EVI $= 38 - 35 =$ \$3 thousand

The maximum amount that should be paid for the information is \$3000. The decision with this information will be to choose A_2 if the state of nature is S_2 and A_3 otherwise.

(d) If S_3 occurs for certain, then choose A_1 with expected payoff \$100,000. If S_3 does not occur for certain, then the probability that S_1 will occur is $\frac{2}{5}$ and the probability that S_2 will occur is $\frac{3}{5}$.

$$
\begin{array}{lll}
A_1: & (\frac{2}{5})(-100) + (\frac{3}{5})(100) & = -34 \\
A_2: & (\frac{2}{5})(-10) + (\frac{3}{5})(20) & = \quad 8 \\
A_3: & (\frac{2}{5})(10) + (\frac{3}{5})(10) & = \quad 10
\end{array}
$$

Hence, choose A_3 which offers an expected payoff of \$10,000.

Expected Payoff with Information: $0.5(100) + 0.5(10) = 55$
Expected Payoff without Information: 35
EVI $= 55 - 35 =$ \$20 thousand

The maximum amount that should be paid for the information is \$4,000. The decision with this information will be to choose A_1 if the state of nature is S_3 and A_3 otherwise.

(e) Expected Payoff with Perfect Information: $0.2(10) + 0.3(20) + 0.5(100) = 58$

Expected Payoff without Information: 35

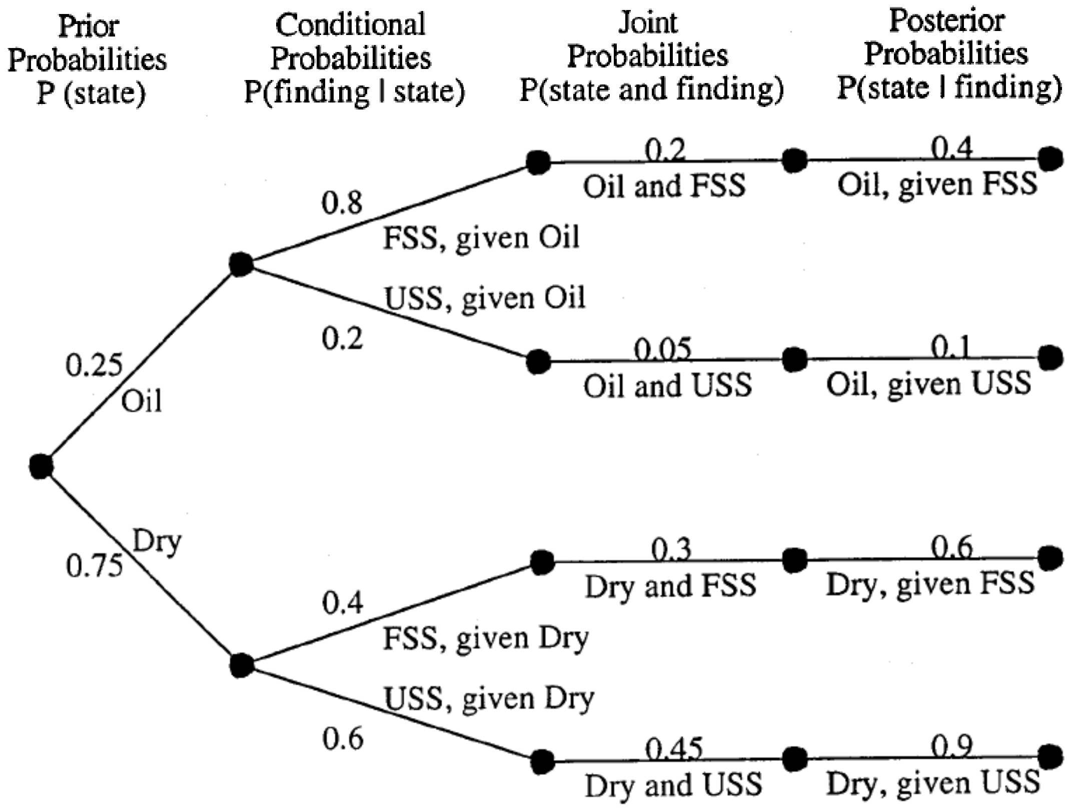
EVPI $= 58 - 35 =$ \$23 thousand

The maximum amount that should be paid for the information is \$23,000. The decision with this information will be to choose A_3 if the state of nature is S_1, A_2 if the state of nature is S_2 and A_1 otherwise.

(f) The maximum amount that should be paid for testing is \$23,000, since any additional information cannot add more value than perfect information.

16.3-6.

(a)



(b)

Data:		P(Finding | State)	
State of	Prior	Finding	
Nature	Probability	FSS	USS
Oil	0.25	0.8	0.2
Dry	0.75	0.4	0.6

Posterior		P(State | Finding)	
Probabilities:		State of Nature	
Finding	P(Finding)	Oil	Dry
FSS	0.5	0.4	0.6
USS	0.5	0.1	0.9

(c) The optimal policy is to do a seismic survey, to drill if favorable seismic surroundings are obtained, and to sell if unfavorable surroundings are obtained.

16.3-7.

(a) Choose A_1 with expected payoff \$100.

	State of Nature		Exp.
Alternative	S_1	S_2	Payoff
A_1	400	-100	100
A_2	0	100	60
Prior Probability	0.4	0.6	

(b)

	State of Nature	
Alternative	S_1	S_2
A_1	400	-100
A_2	0	100
Prior Probability	0.4	0.6
Maximum Payoff	400	100

Expected Payoff with Perfect Information: $0.4(400) + 0.6(100) = 220$

Expected Payoff without Information: 100

EVPI $= 220 - 100 = \$120$, so it might be worthwhile to do the research.

(c) Let X denote the state of nature and Y denote the prediction. From Bayes' Rule,

$$\mathrm{P}(X = x \text{ and } Y = y) = \mathrm{P}(X = x)\mathrm{P}(Y = y|X = x).$$

(i) $\mathrm{P}(X = \mathrm{S}_1 \text{ and } Y = \mathrm{S}_1) = (0.4)(0.6) = 0.24$
(ii) $\mathrm{P}(X = \mathrm{S}_1 \text{ and } Y = \mathrm{S}_2) = (0.4)(0.4) = 0.16$
(iii) $\mathrm{P}(X = \mathrm{S}_2 \text{ and } Y = \mathrm{S}_1) = (0.6)(0.2) = 0.12$
(iv) $\mathrm{P}(X = \mathrm{S}_2 \text{ and } Y = \mathrm{S}_2) = (0.6)(0.8) = 0.48$

(d) $\mathrm{P}(\mathrm{S}_1) = 0.24 + 0.12 = 0.36$, $\mathrm{P}(\mathrm{S}_2) = 0.16 + 0.48 = 0.64$

(e) Bayes' Rule: $\mathrm{P}(X = x|Y = y) = \frac{\mathrm{P}(X=x \text{ and } Y=y)}{\mathrm{P}(X=x)}$

$\mathrm{P}(\mathrm{S}_1|\mathrm{S}_1) = 0.24/0.36 = 0.667$
$\mathrm{P}(\mathrm{S}_1|\mathrm{S}_2) = 0.16/0.64 = 0.25$
$\mathrm{P}(\mathrm{S}_2|\mathrm{S}_1) = 0.12/0.36 = 0.333$
$\mathrm{P}(\mathrm{S}_2|\mathrm{S}_2) = 0.48/0.64 = 0.75$

(f)

Data:		P(Finding | State)	
State of	Prior	Finding	
Nature	Probability	Predict S1	Predict S2
Actual S1	0.4	0.6	0.4
Actual S2	0.6	0.2	0.8

Posterior		P(State | Finding)	
Probabilities:		State of Nature	
Finding	P(Finding)	Actual S1	Actual S2
Predict S1	0.36	0.667	0.333
Predict S2	0.64	0.250	0.750

(g) If S_1 is predicted, then choose A_1 with expected payoff \$233.33.

	State of Nature		Exp.
Alternative	S_1	S_2	Payoff
A_1	400	-100	233.5
A_2	0	100	33.3
Prior Probability	0.667	0.333	

(h) If S_2 is predicted, then choose A_2 with expected payoff \$75.

	State of Nature		Exp.
Alternative	S_1	S_2	Payoff
A_1	400	-100	25
A_2	0	100	75
Prior Probability	0.25	0.75	

(i) Given that the research is done, the expected payoff is

$$(0.36)(233.33) + (0.64)(75) - 100 = \$32.$$

(j) The optimal policy is to not do research and to choose A_1.

16.3-8.

(a) $\text{EVPI} = [(2/3)(-1.012 \cdot 10^7) + (1/3)(-1.135 \cdot 10^7)] - (-1.076 \cdot 10^7)$
$= \$230,000.$

(b)

$$\text{P}(\theta = 21|\text{ 30 spares required}) = \frac{\text{P(30 spares required }|\theta{=}21)\text{P}(\theta{=}21)}{\text{P(30 spares required }|\theta{=}21)\text{P}(\theta{=}21)+\text{P(30 spares required }|\theta{=}24)\text{P}(\theta{=}24)}$$

$$= \frac{(0.013)(2/3)}{(0.013)(2/3)+(0.036)(1/3)} = 0.419$$

$\text{P}(\theta = 24|\text{ 30 spares required}) = 1 - 0.419 = 0.581$

Order 15: $\quad \text{EP} = 0.419(-1.155 \cdot 10^7) + 0.581(-1.414 \cdot 10^7) = -1.305 \cdot 10^7$
Order 20: $\quad \text{EP} = 0.419(-1.012 \cdot 10^7) + 0.581(-1.207 \cdot 10^7) = -1.125 \cdot 10^7$
Order 25: $\quad \text{EP} = 0.419(-1.047 \cdot 10^7) + 0.581(-1.135 \cdot 10^7) = -1.098 \cdot 10^7$

The optimal alternative is to order 25.

16.3-9.

(a)

	State of Nature		
Alternative	Poor Risk	Average Risk	Good Risk
Extend Credit	$-15,000$	$10,000$	$20,000$
Not Extend Credit	0	0	0
Prior Probability	0.2	0.5	0.3

(b) Choose to extend credit with expected payoff \$8,000.

	State of Nature			Exp.
Alternative	Poor Risk	Average Risk	Good Risk	Payoff
Extend Credit	$-15,000$	$10,000$	$20,000$	$8,000$
Not Extend Credit	0	0	0	0
Prior Probability	0.2	0.5	0.3	

(c)

	State of Nature		
Alternative	Poor Risk	Average Risk	Good Risk
Extend Credit	$-15,000$	$10,000$	$20,000$
Not Extend Credit	0	0	0
Prior Probability	0.2	0.5	0.3
Maximum Payoff	0	$10,000$	$20,000$

Expected Payoff with Perfect Information:

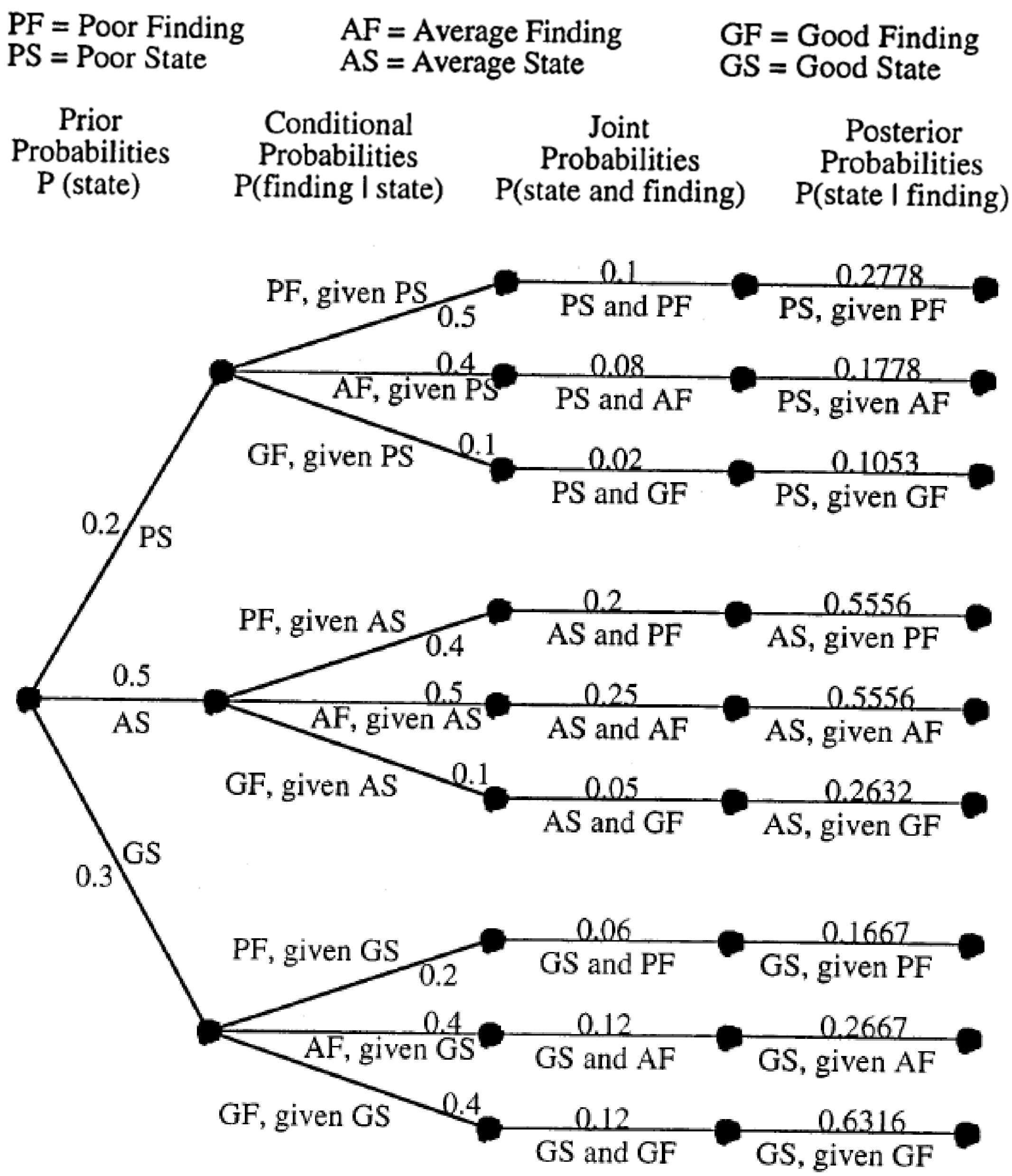
$$0.2(0) + 0.3(10,000) + 0.4(20,000) = 11,000$$

Expected Payoff without Information: $8,000$

EVPI $= 11,000 - 8,000 = \$3,000$

Hence, the credit-rating organization should not be used.

(d)



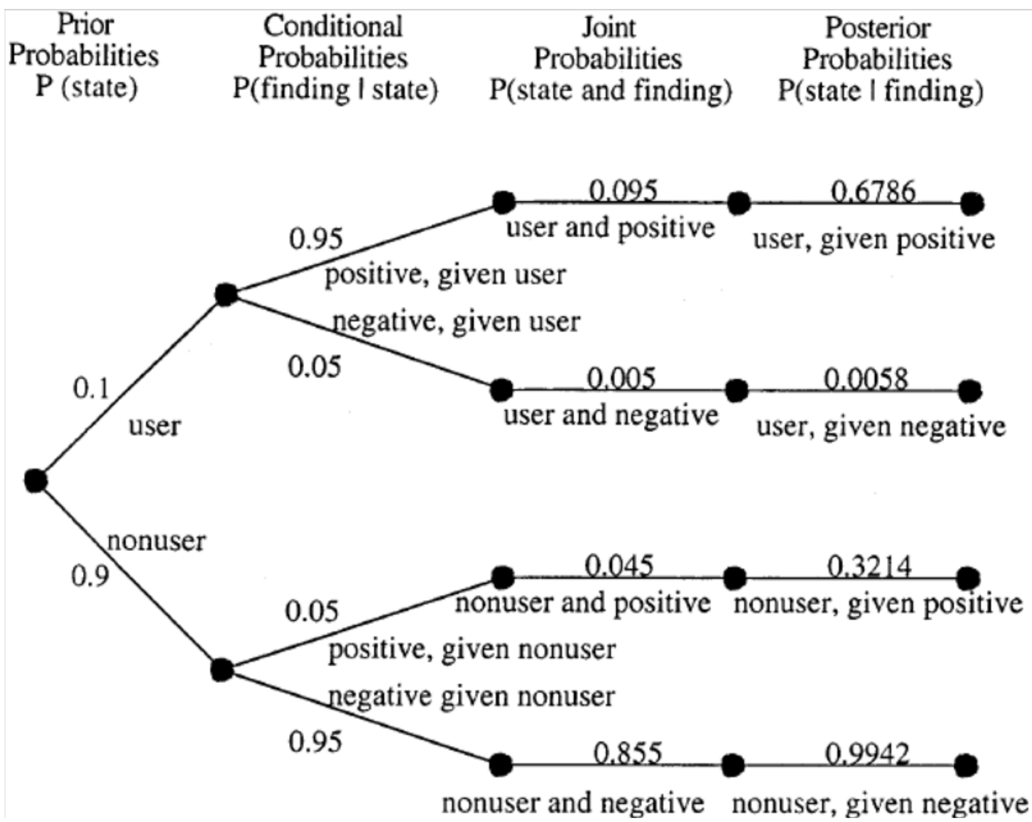
(e)

Data:		P(Finding | State)		
State of	Prior	Finding		
Nature	Probability	Poor	Average	Good
Poor	0.2	0.5	0.4	0.1
Average	0.5	0.4	0.5	0.1
Good	0.3	0.2	0.4	0.4

Posterior		P(State | Finding)		
Probabilities:		State of Nature		
Finding	P(Finding)	Poor	Average	Good
Poor	0.360	0.278	0.556	0.167
Average	0.450	0.178	0.556	0.267
Good	0.190	0.105	0.263	0.632

(f) Vincent should not get the credit rating and extend credit.

16.3-10.



(a) Given that the test is positive, the athlete is a drug user with probability 0.6786.

(b) Given that the test is positive, the athlete is not a drug user with probability 0.3214.

(c) Given that the test is negative, the athlete is a drug user with probability 0.0058.

(d) Given that the test is negative, the athlete is not a drug user with probability 0.9942.

(e) The answers in Excel agree with those found in parts (a), (b), (c), and (d).

Data:		P(Finding | State)	
State of	Prior	Finding	
Nature	Probability	Positive	Negative
User	0.1	0.95	0.05
Nonuser	0.9	0.05	0.95

Posterior		P(State | Finding)	
Probabilities:		State of Nature	
Finding	P(Finding)	User	Nonuser
Positive	0.14	0.6786	0.3214
Negative	0.86	0.0058	0.9942

16.3-11.

(a)

	State of Nature	
Alternative	Successful	Unsuccessful
Develop New Product	$1,500,000$	$-1,800,000$
Not Develop New Product	0	0
Prior Probability	0.667	0.333

(b) Choose to develop new product with expected payoff $\$400,000$.

	State of Nature		Exp.
Alternative	Successful	Unsuccessful	Payoff
Develop New Product	$1,500,000$	$-1,800,000$	$400,000$
Not Develop New Product	0	0	0
Prior Probability	0.667	0.333	

(c)

	State of Nature	
Alternative	Successful	Unsuccessful
Develop New Product	$1,500,000$	$-1,800,000$
Not Develop New Product	0	0
Prior Probability	0.667	0.333
Maximum Payoff	$1,500,000$	0

Expected Payoff with Perfect Information: $0.667(1,500,000) + 0.333(0) = 1,000,000$

Expected Payoff without Information: $400,000$

EVPI $= 1,000,000 - 400,000 = \$600,000$

This indicates that consideration should be given to conducting the market survey.

(d)

Data:		P(Finding | State)	
State of	Prior	Finding	
Nature	Probability	Predict Successful	Predict Unsuccessful
Successful	0.667	0.8	0.2
Unsuccessful	0.333	0.3	0.7

Posterior		P(State | Finding)	
Probabilities:		State of Nature	
Finding	P(Finding)	Successful	Unsuccessful
Predict Successful	0.633	0.8421	0.1579
Predict Unsuccessful	0.367	0.3636	0.6364

(e)

Action	Prediction	Expected Payoff
Develop product	Successful	$[0.8421(1.5) + 0.1579(-1.8)] \cdot 10^6 = \$979,000$
Not develop product	Successful	0
Develop product	Unsuccessful	$[0.3636(1.5) + 0.6364(-1.8)] \cdot 10^6 = -\$600,000$
Not develop product	Unsuccessful	0

It is optimal to develop the product if it is predicted to be successful and to not develop otherwise. Let S be the event that the product is predicted to be successful. Then,

$$\mathrm{P(S)} = \mathrm{P(S}|\theta_1)\mathrm{P}(\theta_1) + \mathrm{P(S}|\theta_2)\mathrm{P}(\theta_2) = 0.8(2/3) + 0.2(1/3) = 0.6.$$

The expected payoff given the information is $0.6(979,000) + 0.4(0) = \$587,000$, so

$$\text{EVE} = 587,000 - 400,000 = \$187,000 < \$300,000 = \text{Cost of survey.}$$

Hence, the optimal strategy is to not conduct the market survey, and to market the product.

16.3-12.

(a)

	State of Nature	
Alternative	$p = 0.05$	$p = 0.25$
Screen	$-1,500$	$-1,500$
Not Screen	-750	$-3,750$
Prior Probability	0.8	0.2

(b) Choose to not screen with expected loss $\$1,350$.

	State of Nature		Exp.
Alternative	$p = 0.05$	$p = 0.25$	Payoff
Screen	$-1,500$	$-1,500$	$-1,500$
Not Screen	-750	$-3,750$	$-1,350$
Prior Probability	0.8	0.2	

(c)

	State of Nature	
Alternative	$p = 0.05$	$p = 0.25$
Screen	$-1,500$	$-1,500$
Not Screen	-750	$-3,750$
Prior Probability	0.8	0.2
Maximum Payoff	-750	$-1,500$

Expected Payoff with Perfect Information: $0.8(-750) + 0.2(-1,500) = -900$

Expected Payoff without Information: $-1,350$

EVPI $= -900 - (-1,350) = \$450$

This indicates that consideration should be given to inspecting the single item.

(d)

Data:		P(Finding | State)	
State of	Prior	Finding	
Nature	Probability	Defective	Nondefective
p = 0.05	0.8	0.05	0.95
p = 0.25	0.2	0.25	0.75

Posterior		P(State | Finding)	
Probabilities:		State of Nature	
Finding	P(Finding)	p = 0.05	p = 0.25
Defective	0.09	0.4444	0.5556
Nondefective	0.91	0.8352	0.1648

(e) P(defective) $= (0.05)(0.8) + (0.25)(0;2) = 0.09$ and P(nondefective) $= 0.91$

EVE $= [(0.09)(-1500) + (0.91)(-1245)] - (-1350) = 82.05$

Since the cost of the inspection is $\$125 > \$82.05$, inspecting the single item is not worthwhile.

(f) If defective:

$$\text{EP(screen, } \theta|\text{defective)} = 0.444(-1500) + 0.556(-1500) = -1500$$
$$\text{EP(no screen, } \theta|\text{defective)} = 0.444(-750) + 0.556(-3750) = -2418$$

If nondefective:

$$\text{EP(screen, } \theta|\text{defective)} = -1500$$
$$\text{EP(no screen, } \theta|\text{defective)} = 0.835(-750) + 0.165(-3750) = -1245$$

Hence, the optimal policy with experimentation is to screen if defective is found and not screen if nondefective is found. On the other hand, from part (e), inspecting a single item, in other words experimenting is not worthwhile. Using part (b), the overall optimal policy is to not inspect the single items, to not screen each item in the lot, instead, rework each item that is ultimately found to be defective.

16.3-13.

(a) Say coin 1 tossed: $\text{EP} = 0.6(0) + 0.4(-1) = -0.4$
Say coin 2 tossed: $\text{EP} = 0.6(-1) + 0.4(0) = -0.6$

The optimal alternative is to say coin 1 is tossed.

(b) If the outcome is heads (H):

$$\text{P(coin 1|H)} = \frac{\text{P(H|coin 1)P(coin 1)}}{\text{P(H|coin 1)P(coin 1)+P(H|coin 2)P(coin 2)}} = \frac{0.3(0.6)}{0.3(0.6)+0.6(0.4)} = \frac{3}{7}$$

$\text{P(coin 2|H)} = \frac{4}{7}$

Say coin 1: $\text{EP} = \frac{3}{7}(0) + \frac{4}{7}(-1) = -\frac{4}{7}$

Say coin 2: $\text{EP} = \frac{3}{7}(-1) + \frac{4}{7}(0) = -\frac{3}{7}$

The optimal alternative is to say coin 2.

If the outcome is tails (T):

$$\text{P(coin 1|T)} = \frac{\text{P(T|coin 1)P(coin 1)}}{\text{P(T|coin 1)P(coin 1)+P(T|coin 2)P(coin 2)}} = \frac{0.7(0.6)}{0.7(0.6)+0.4(0.4)} = 0.7241$$

$\text{P(coin 2|T)} = 0.2759$

Say coin 1: $\text{EP} = 0.7241(0) + 0.2759(-1) = -0.2759$

Say coin 2: $\text{EP} = 0.7241(-1) + 0.2759(0) = -0.7241$

The optimal alternative is to say coin 1.

16.3-14.

(a)

	State of Nature	
Alternative	Coin 1	Coin 2
Predict 0 H	4	36
Predict 1 H	32	48
Predict 2 H	64	16
Prior probabilities	0.5	0.5

Predict 0 H: $\text{EP} = 0.5(4) + 0.5(36) = 20$
Predict 1 H: $\text{EP} = 0.5(32) + 0.5(48) = 40$
Predict 2 H: $\text{EP} = 0.5(64) + 0.5(16) = 40$

The optimal alternative is to predict one or two heads with expected payoff of $40.

(b)

Data:		P(Finding | State)	
State of	Prior	Finding	
Nature	Probability	Heads	Tails
Coin 1	0.5	0.8	0.2
Coin 2	0.5	0.4	0.6

Posterior		P(State | Finding)	
Probabilities:		State of Nature	
Finding	P(Finding)	Coin 1	Coin 2
Heads	0.6	0.6667	0.3333
Tails	0.4	0.2500	0.7500

(c) If the outcome is heads (H):

$$\begin{array}{lll} \text{Predict 0 H:} & \text{EP} = 0.667(4) + 0.333(36) & = 14.67 \\ \text{Predict 1 H:} & \text{EP} = 0.667(32) + 0.333(48) & = 37.33 \\ \text{Predict 2 H:} & \text{EP} = 0.667(64) + 0.333(16) & = 48 \end{array}$$

The optimal alternative is to predict two heads.

If the outcome is tails (T):

$$\begin{array}{lll} \text{Predict 0 H:} & \text{EP} = 0.25(4) + 0.75(36) & = 28 \\ \text{Predict 1 H:} & \text{EP} = 0.25(32) + 0.75(48) & = 44 \\ \text{Predict 2 H:} & \text{EP} = 0.25(64) + 0.75(16) & = 28 \end{array}$$

The optimal alternative is to predict one heads.

The expected payoff $= 0.6(\$48) + 0.4(\$44) = \$46.40$.

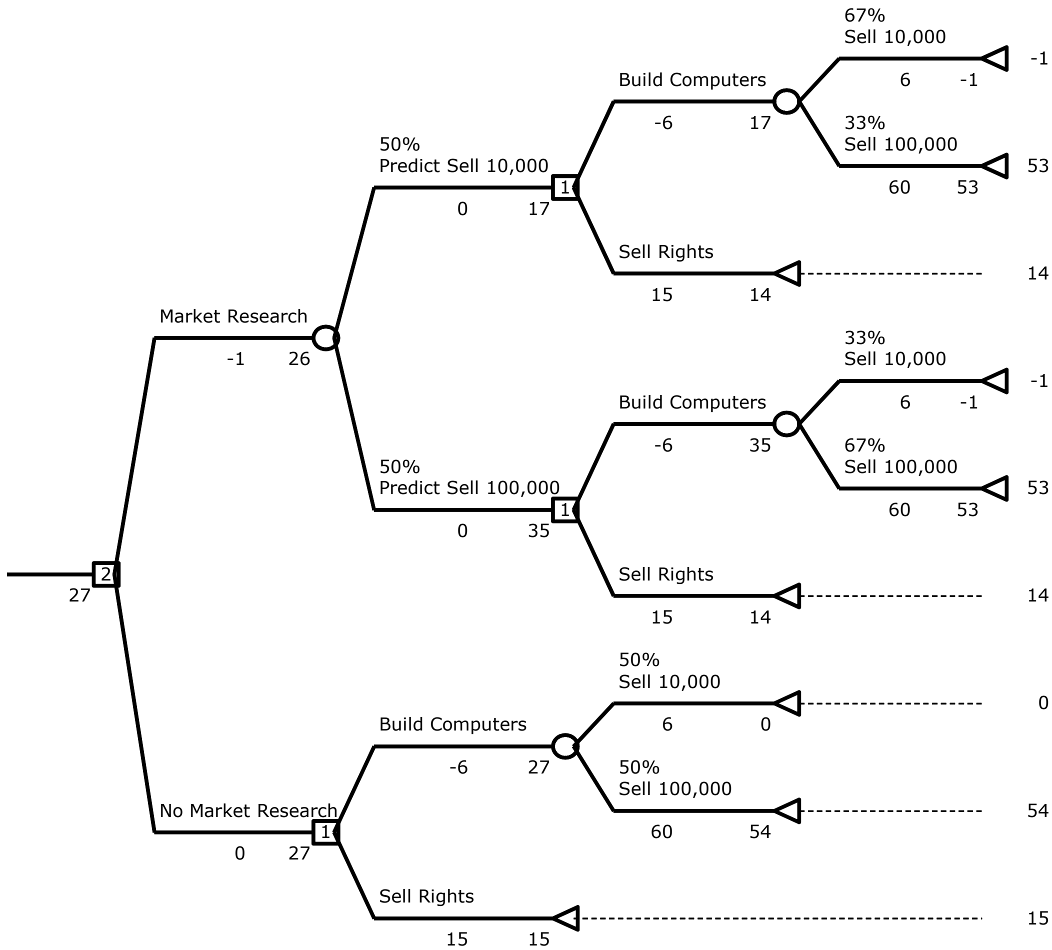
(d) EVE $= \$46.40 - \$40 = \$6.40 < \$30$, so it is better to not pay for the experiment and choose to predict either one or two heads.

16.4-1.

Driven by "the pressure to reduce costs and deliver high-impact technology quickly while justifying investments" [p. 57], Westinghouse initiated this study to evaluate R and D efforts effectively. At any point in time, the firm chooses between launching, delaying and abandoning an innovation. When the launch is delayed, there is a chance of losing the opportunity. R and D is hence treated as a call option with flexibility. The value of the innovation and the optimal decision rule in subsequent stages are found by using dynamic programming. This value is then used in the analysis of the decision tree constructed to find the present value of the project. In this tree, decisions consist of whether to fund or not at different stages and each decision node is followed by a chance node that represents either a technical milestone or strategic fit. Sensitivity analysis is performed to ensure robustness of the model.

As a result of this study, explicit decision rules for funding R and D projects are obtained. Including flexibility in the model yields a more realistic model. The new system helps identifying cost-effective research portfolios with simplified data acquisition and easy implementation.

16.4-2.



The optimal policy is to build the computers without doing market research.

16.4-3.