CHAPTER 15: GAME THEORY

15.1-1.

Let player 1 be the labor union with strategy i being to decrease the wage demand by $10(i-1)$¢ and player 2 be the management with strategy i being to increase the offer by $10(i-1)$¢. The payoff matrix is:

	1	2	3	4	5	6
1	1.35	1.2	1.3	1.4	1.5	1.6
2	1.5	1.35	1.3	1.4	1.5	1.6
3	1.4	1.4	1.35	1.4	1.5	1.6
4	1.3	1.3	1.3	1.35	1.5	1.6
5	1.2	1.2	1.2	1.2	1.35	1.6
6	1.1	1.1	1.1	1.1	1.1	1.35

where the rows represent the strategy of player 1 and the columns the strategy of player 2.

15.1-2.

Label the products as A and B respectively. The strategies for each manufacturer are:

1- Normal development of both products
2- Crash development of product A
3- Crash development of product B.

Let $p_{ij} = \frac{1}{2}$[(% increase to manufacturer 1 from A) $+$ (% increase to manufacturer 1 from B)] when manufacturer 1 uses strategy i and manufacturer 2 uses strategy j. The payoff matrix is:

	1	2	3	row min
1	8	10	10	**8**
2	4	-4	13	-4
3	4	13	-4	-4
col max	**8**	13	13	8

The rows correspond to the strategy of manufacturer 1 and the columns to the strategy of manufacturer 2. The minimum of the column maxima and the maximum of the row minima is 8, so both manufacturers should use strategy 1, namely choose normal development of both products. Consequently, manufacturer 1 will increase its share by 8%.

15.1-3.

Each player has the same strategy set. A strategy must specify the first chip chosen, the second and third chips chosen for every choice first chip by the opponent. Denote the white, red and blue chips by W, R and B respectively. Then a strategy is of the form: Choose $i \in \{$W, R, B$\}$ as first chip, if the opponent chooses $j \in \{$W, R, B$\}$, then choose $k_j \in \{$W, R, B$\}\backslash\{i\}$, and let $l_j \in \{$W, R, B$\}\backslash\{i, k\}$. There are three choices of i and for each i, eight choices of second and third chips, so 24 strategies in total. Player 1 can either win all three games, or win one and get a draw in another one, or lose all three.

Hence, the payoff to player 1 can be either 120, 0, or -120. The payoff to player 1 in each possible scenario is given in the table below, where the rows and the columns represent the order of chips played by player 1 and 2 respectively.

	WRB	WBR	RWB	RBW	BWR	BRW
WRB	0	0	0	-120	120	0
WBR	0	0	-120	0	0	120
RWB	0	120	0	0	0	-120
RBW	120	0	0	0	-120	0
BWR	-120	0	0	120	0	0
BRW	0	-120	120	0	0	0

15.2-1.

(a) Strategies 4, 5, and 6 of each player are dominated by their strategy 3. Then strategy 1 can be eliminated, since it is dominated by strategy 3 for each player. Once these are eliminated, strategy 2 of each is dominated by strategy 3. Thus, the best strategy of the labor union is to decrease its demand by 20¢ and the best for the management if to increase its offer by 20¢. The resulting wage is $1.35.

(b)

	1	2	3	4	5	6	row min
1	1.35	1.2	1.3	1.4	1.5	1.6	1.2
2	1.5	1.35	1.3	1.4	1.5	1.6	1.3
3	1.4	1.4	1.35	1.4	1.5	1.6	**1.35**
4	1.3	1.3	1.3	1.35	1.5	1.6	1.3
5	1.2	1.2	1.2	1.2	1.35	1.6	1.2
6	1.1	1.1	1.1	1.1	1.1	1.35	1.1
col max	1.5	1.4	**1.35**	1.4	1.5	1.6	1.35

15.2-2.

Strategy 3 of player 1 is dominated by strategy 2.
Strategy 3 of player 2 is dominated by strategy 1.
Strategy 1 of player 1 is dominated by strategy 2.
Strategy 2 of player 2 is dominated by strategy 1.

Therefore, the optimal strategy is strategy 2 for player 1 and strategy 1 for player 2 and the resulting payoff is 1 to player 1.

15.2-3.

Strategy 1 of player 2 is dominated by strategy 3.
Strategy 4 of player 2 is dominated by strategy 2.
Strategies 1 and 2 of player 1 are dominated by strategy 3.
Strategy 2 of player 2 is dominated by strategy 3.

Therefore, the optimal strategy is strategy 3 for each player and the resulting payoff is 1 to player 2.

15.2-4.

	1	2	3	row min
1	1	−1	1	−1
2	−2	0	3	−2
3	3	1	2	**1**
col max	3	**1**	3	1

The best strategy is strategy 3 for player 1 and strategy 2 for player 2, the resulting payoff is 1 to player 1. The game is stable with a saddle point $(3, 2)$, since the minimax value equals the maximin value.

15.2-5.

	1	2	3	4	row min
1	3	−3	−2	−4	−4
2	−4	−2	−1	1	−4
3	1	−1	2	0	−**1**
col max	1	−**1**	2	1	−1

The best strategy is strategy 3 for player 1 and strategy 2 for player 2, the resulting payoff is 1 to player 2. The game is stable with a saddle point $(3, 2)$.

15.2-6.

(a)

	1	2	3	row min
1	2	3	1	**1**
2	1	4	0	0
3	3	−2	−1	−2
col max	3	4	**1**	1

The best strategy is strategy 1 for player 1 and strategy 3 for player 2, the resulting payoff is 1 to player 1. The game is stable with a saddle point $(1, 3)$.

(b) Strategy 1 of player 2 is dominated by strategy 3.
Strategy 3 of player 1 is dominated by strategies 1 and 2.
Strategy 2 of player 2 is dominated by strategy 3.
Strategy 2 of player 1 is dominated by strategy 1.

The optimal strategy is strategy 1 for player 1 and strategy 3 for player 2, with a payoff of 1 to player 1.

15.2-7.

(a)

	1	2	3	row min
1	7	−1	3	−1
2	1	0	2	**0**
3	−5	−3	1	−5
col max	7	**0**	**1**	0

The best strategy is to use issue 2 for each politician, with zero payoff to each.

(b) Let p_{ij} be the probability that politician 1 wins the election or the election results in a tie when politician 1 chooses issue i and politician 2 issue j. Then the new payoff matrix is:

	1	2	3
1	1	0	3/5
2	1/5	0	2/5
3	0	0	1/5

Strategies 2 and 3 of politician 1 are dominated by strategy 2.
Strategies 1 and 3 of politician 2 are dominated by strategy 2.

Hence, by eliminating dominated strategies, one gets issue 1 as the best strategy for politician 1 and issue 2 for politician 2, the payoff is zero. Thus, politician 2 can prevent politician 1 from winning or getting a tie.

(c) Let $p_{ij} = \begin{cases} 1 & \text{if politician 1 will win or tie} \\ 0 & \text{if politician 2 will win} \end{cases}$

Then the payoff matrix becomes:

	1	2	3
1	1	0	0
2	0	0	0
3	0	0	0

where the minimax of the columns and the maximin of the rows both equal zero, i.e., politician 1 cannot win. Politician 1 can use any use, politician 2 can choose issue 2 or 3; however, since issue 1 offers politician 1 his only chance of winning, he should use that one and hope that politician 2 chooses issue 1 by mistake.

15.2-8.

<u>Advantages:</u> It provides the best possible guarantee on what the worst outcome can be, regardless of how skillfully the opponent plays the game and hence, reduces the possibility of undesirable outcomes to a minimum.

<u>Disadvantages:</u> Since it aims at eliminating worst cases, it is conservative and may yield payoffs that are far from the best ones.

15.3-1.

(a)

	1	2	row min
1	1	−1	−1
2	−1	1	−1
col max	1	**1**	

The minimax payoff is not the same as the maximin payoff, so the game does not have a saddle point.

(b) Expected payoff for player 1: $(x_1y_1 + x_2y_2) - (x_1y_2 + x_2y_1)$

$x_1 + x_2 = y_1 + y_2 = 1$

(i) $y_1 = 1, y_2 = 0$: $x_1 - x_2 = x_1 - (1 - x_1) = 2x_1 - 1$

(ii) $y_1 = 0, y_2 = 1$: $x_2 - x_1 = (1 - x_1) - x_1 = 1 - 2x_1$

(ii) $y_1 = \frac{1}{2}, y_2 = \frac{1}{2}$: 0

(c) Expected payoff for player 1: $(x_1y_2 + x_2y_1) - (x_1y_1 + x_2y_2)$

$x_1 + x_2 = y_1 + y_2 = 1$

(i) $y_1 = 1, y_2 = 0$: $x_2 - x_1 = (1 - x_1) - x_1 = 1 - 2x_1$

(ii) $y_1 = 0, y_2 = 1$: $x_1 - x_2 = x_1 - (1 - x_1) = 2x_1 - 1$

(ii) $y_1 = \frac{1}{2}, y_2 = \frac{1}{2}$: 0

15.3-2.

(a) <u>Strategies for player1:</u> 1- Pass on heads or tails
2- Bet on heads or tails
3- Pass on heads, bet on tails
4- Bet on heads, pass on tails

<u>Strategies for player 2:</u> 1- If player 1 bets, call.
2- If player 1 bets, pass.

(b)

	1	2
1	-5	-5
2	0	5
3	-7.5	0
4	2.5	0

Strategies 1 and 3 of player 1 are dominated bye strategy 2. Upon eliminating them, the table is reduced to:

	1	2
2	0	5
4	2.5	0

(c)

	1	2	row min
1	-5	-5	-5
2	0	5	$\mathbf{0}$
3	-7.5	0	-7.5
4	2.5	0	$\mathbf{0}$
col max	$\mathbf{2.5}$	5	

The minimum of the column maxima is not equal to the maximum of the row minima, there is no saddle point. If either player chooses a pure strategy, the other one can choose a strategy to cause the first player to change his strategy. One needs mixed strategies to find an equilibrium.

(d) The dominated strategies will not be chosen. Let x_2 and x_4 be the probabilities that player 1 uses strategy 2 and 4 respectively, y_1 and y_2 be the probabilities that player 2 uses strategy 1 and 2 respectively. Hence, $x_2 + x_4 = 1$ and $y_1 + y_2 = 1$ and the expected payoff can be expressed as $p_{21}x_2y_1 + p_{22}x_2y_2 + p_{41}x_4y_1 + p_{42}x_4y_2$.

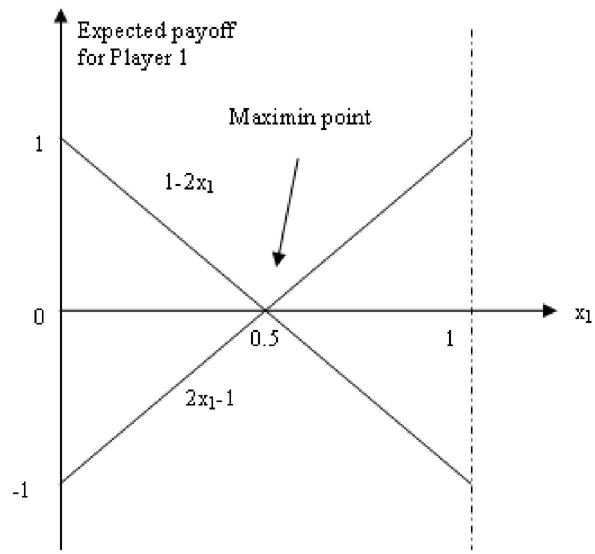
Case (i): $y_1 = 1$, $y_2 = 0$ $\Rightarrow 2.5x_4 = 2.5(1 - x_2)$

Case (ii): $y_1 = 0$, $y_2 = 1$ $\Rightarrow 5x_2 = 5(1 - x_4)$

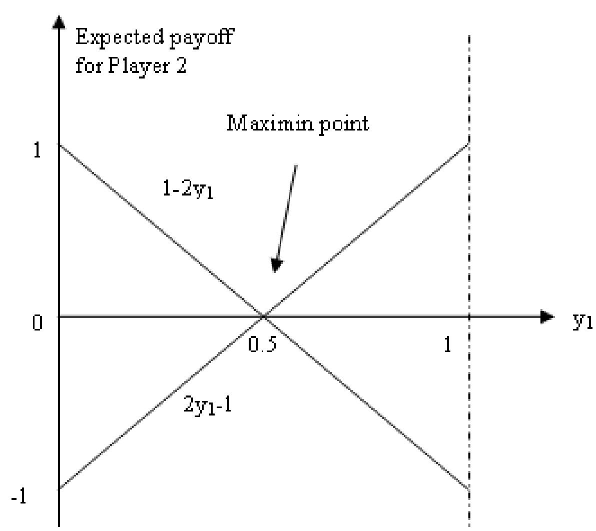
Case (iii): $y_1 = y_2 = 0.5$ $\Rightarrow 5x_2\left(\frac{1}{2}\right) + 2.5x_4\left(\frac{1}{2}\right) = 0.25x_2 + 1.25$

15.4-1.

Expected payoff for player 1: (i) $y_1 = 1, y_2 = 0$: $2x_1 - 1$
(ii) $y_1 = 0, y_2 = 1$: $1 - 2x_1$



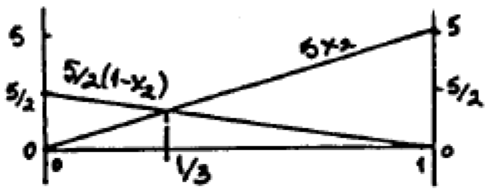
Expected payoff for player 2: (i) $x_1 = 1, x_2 = 0$: $1 - 2y_1$
(ii) $x_1 = 0, x_2 = 1$: $2y_1 - 1$



The corresponding value of the game is zero.

15.4-2.

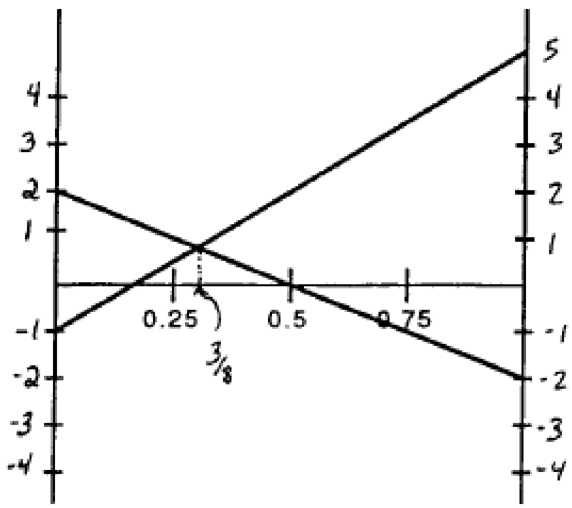
(y_1, y_2)	Expected Payoff
$(1, 0)$	$2.5(1 - x_2)$
$(0, 1)$	$5x_2$



$2.5(1 - x_2) = 5x_2 \;\Rightarrow\; (x_1^*, x_2^*, x_3^*, x_4^*) = (0, 1/3, 0, 2/3)$ and $v = 5/3$.
$2.5y_1^*(1 - x_2) + 5y_2^* x_2 = 5/3$ for $0 \leq x_2 \leq 1 \;\Rightarrow\; 2.5y_1^* = 5/3$ and $5y_2^* = 5/3$
$\Rightarrow (y_1^*, y_2^*) = (2/3, 1/3)$.

15.4-3.

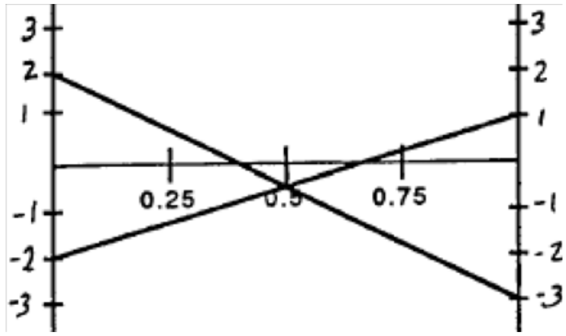
(y_1, y_2)	Expected Payoff
$(1, 0)$	$3x_1 - 1(1 - x_1) = \quad 4x_1 - 1$
$(0, 1)$	$-2x_1 + 2(1 - x_1) = -4x_1 + 2$



$4x_1 - 1 = -4x_1 + 2 \;\Rightarrow\; (x_1^*, x_2^*) = (\frac{3}{8}, \frac{5}{8})$ and $v = 4(\frac{3}{8}) - 1 = \frac{1}{2}$.
$3y_1^* - 2y_2^* = \frac{1}{2}$ and $-y_1^* + 2y_2^* = \frac{1}{2} \;\Rightarrow\; (y_1^*, y_2^*) = (\frac{1}{2}, \frac{1}{2})$.

The payoff matrix for player 2 is:

	1	2
1	-3	2
2	1	-2

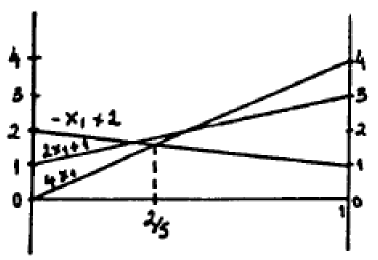


(x_1, x_2)	Expected Payoff
$(1,0)$	$-3y_1 + 2(1-y_1) = -5y_1 + 2$
$(0,1)$	$y_1 - 2(1-y_1) = 3y_1 - 2$

$-5y_1 + 2 = 3y_1 - 2 \;\Rightarrow\; y_1^* = y_2^* = \frac{1}{2}$.

15.4-4.

(y_1, y_2, y_3)	Expected Payoff
$(1,0,0)$	$4x_1$
$(0,1,0)$	$3x_1 + (1-x_1) = 2x_1 + 1$
$(0,0,1)$	$x_1 + 2(1-x_1) = -x_1 + 2$



$4x_1 = -x_1 + 2 \;\Rightarrow\; (x_1^*, x_2^*) = (2/5, 3/5)$ and $v = 8/5$.
$y_1^*(4x_1) + y_3^*(-x_1+2) = 8/5$ for $0 \le x_1 \le 1 \;\Rightarrow\; 2y_3^* = 8/5$ and $4y_1^* + y_3^* = 3/5$
$\Rightarrow (y_1^*, y_2^*, y_3^*) = (1/5, 0, 4/5)$.

15.4-5.

(a) Strategies for A.J. Team:
1- John does not swim butterfly.
2- John does not swim backstroke.
3- John does not swim breaststroke.

Strategies for G.N. Team:
1- Mark does not swim butterfly.
2- Mark does not swim backstroke.
3- Mark does not swim breaststroke.

Let the payoff entries be the total points earned in all three events by A.J. Team when a given pair of strategies are chosen by the teams. Then the payoff matrix becomes:

	1	2	3
1	14	13	12
2	13	12	12
3	12	12	13

Strategy 2 of A.J. Team is dominated by strategy 1 and strategy 1 of G.N. Team is dominated by strategy 2. When we eliminate these strategies we obtain the table:

	2	3
1	13	12
3	12	13

(y_1, y_2)	Expected Payoff
$(1, 0)$	$13x_1 + 12(1 - x_1) = x_1 + 12$
$(0, 1)$	$12x_1 + 13(1 - x_1) = -x_1 + 13$

$x_1 + 12 = -x_1 + 13 \;\Rightarrow\; (x_1^*, x_2^*, x_3^*) = (0.5, 0, 0.5)$ and $v = 12.5$.
$y_2^*(x_1 + 12) + y_3^*(-x_1 + 13) = 12.5$ for $0 \le x_1 \le 1 \;\Rightarrow\; 12y_2^* + 13y_3^* = 12.5$ and $13y_2^* + 12y_3^* = 12.5 \Rightarrow (y_1^*, y_2^*, y_3^*) = (0, 0.5, 0.5)$.

Hence, John should always swim backstroke and should swim butterfly and breaststroke each with probability $1/2$. Also, Mark should always swim butterfly and should swim backstroke and breaststroke each with probability $1/2$. Consequently, A.J. Team can expect to get 12.5 points on average in three events.

(b) The strategies for the two teams are the same as in (a). If p_{ij} denotes the total points earned by A.J. Team, let p'_{ij} be the new payoff that is defined as:

$$p'_{ij} = \begin{cases} 1/2 & \text{if } p_{ij} \ge 13\text{, i.e., if A.J. Team wins} \\ -1/2 & \text{if } p_{ij} < 13\text{, i.e., if A.J. Team loses} \end{cases}.$$

Then, the new payoff matrix becomes:

	1	2	3
1	$1/2$	$1/2$	$-1/2$
2	$1/2$	$-1/2$	$-1/2$
3	$-1/2$	$-1/2$	$1/2$

where strategy 2 of A.J. Team is dominated by strategy 1 and strategy 1 of G.N. Team is dominated by strategy 2. After eliminating these, the reduced payoff matrix is:

	2	3
1	$1/2$	$-1/2$
3	$-1/2$	$1/2$

Adding the constant 12.5 to every entry does not change the optimal strategies. Furthermore, the payoff matrix in (a) is obtained by doing so. Hence, the best strategies found in (a) are still optimal, the new payoff is $v' = 12.5 - 12.5 = 0$.

(c) Since John and Mark are the best swimmers of their teams, they will always swim in two events. Their teams cannot do better if they do not swim or if they swim in only one event. Hence, if either one of them does not swim in the first event, namely butterfly, he will surely swim the last two events. Accordingly, the strategies for A.J. Team are:

1- John swims butterfly and then backstroke regardless of whether Mark swims butterfly.
2- John swims butterfly and then backstroke if Mark swims butterfly, breaststroke else.
3- John swims butterfly and then breaststroke if Mark swims butterfly, backstroke else.
4- John swims butterfly and then breaststroke regardless of whether Mark swims butterfly.
5- John does not swim butterfly, swims both backstroke and breaststroke.

The strategies for G.N. Team are the same but with the roles of John and Mark are reversed. The associated payoff matrix is:

	1	2	3	4	5
1	$1/2$	$1/2$	$-1/2$	$-1/2$	$-1/2$
2	$1/2$	$1/2$	$-1/2$	$-1/2$	$1/2$
3	$-1/2$	$-1/2$	$-1/2$	$-1/2$	$-1/2$
4	$-1/2$	$-1/2$	$-1/2$	$-1/2$	$1/2$
5	$-1/2$	$1/2$	$-1/2$	$1/2$	$1/2$

	3
1	$-1/2$
2	$-1/2$
3	$-1/2$
4	$-1/2$
5	$-1/2$

Strategy 3 of G.N. Team dominates all others, by eliminating them, we obtain the payoff matrix on the right. It shows that if G.N. Team uses strategy 3, it will win regardless of what strategy is employed by A.J. Team.

(d) Strategy 2 of A.J. Team dominates strategies 1, 3, and 4. Thus, if the coach of G.N. Team may choose any of their strategies at random, the coach of A.J. Team should choose either strategy 2 or 5. After eliminating the dominated strategies, the payoff matrix becomes:

	1	2	3	4	5
2	$1/2$	$1/2$	$-1/2$	$-1/2$	$1/2$
5	$-1/2$	$1/2$	$-1/2$	$1/2$	$1/2$

The two rows are identical except for columns 1 and 4. Thus, if the coach of A.J. team knows that the other coach has a tendency to enter Mark in butterfly and backstroke more often than breaststroke, that means column 1 is more likely to be chosen than column 4, so the coach of A.J. team should choose strategy 2.

15.5-1.

(a) Player 1: maximize x_3

subject to
$$\begin{aligned} x_1 - x_2 - x_3 &\geq 0 \\ -x_1 + x_2 - x_3 &\geq 0 \\ x_1 + x_2 &= 1 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Player 2: minimize y_3

subject to
$$\begin{aligned} y_1 - y_2 - y_3 &\leq 0 \\ -y_1 + y_2 - y_3 &\leq 0 \\ y_1 + y_2 &= 1 \\ y_1, y_2 &\geq 0 \end{aligned}$$

(b) Optimal Solution: $x_1 = x_2 = y_1 = y_2 = 0.5, x_3 = y_3 = 0$

15.5-2.

After adding 3 to the entries of Table 15.6, the payoff table becomes:

	1	2	3
1	3	1	5
2	8	7	0

The new linear programming problem for player 1 is:

$$\begin{array}{ll} \text{maximize} & x_3 \\ \text{subject to} & 3x_1 + 8x_2 - x_3 \geq 0 \\ & x_1 + 7x_2 - x_3 \geq 0 \\ & 5x_1 - x_3 \geq 0 \\ & x_1 + x_2 = 1 \\ & x_1, x_2, x_3 \geq 0 \end{array}$$

The new linear programming problem for player 2 is:

$$\begin{array}{ll} \text{maximize} & y_4 \\ \text{subject to} & 3y_1 + y_2 + 5y_3 - y_4 \leq 0 \\ & 8y_1 + 7y_2 - y_4 \leq 0 \\ & y_1 + y_2 + y_3 = 1 \\ & y_1, y_2, y_3, y_4 \geq 0 \end{array}$$

Based on the information given in Section 15.5, the optimal solutions for these new models are:

$$\begin{aligned} (x_1^*, x_2^*, x_3^*) &= (7/11, 4/11, 35/11) \\ (y_1^*, y_2^*, y_3^*, y_4^*) &= (0, 5/11, 6/11, 35/11). \end{aligned}$$

Note that $x_3^* = y_4^* = v + 3$ where v is the value for the original version of the game.

15.5-3.

(a)

$$\begin{array}{ll} \text{maximize} & x_4 \\ \text{subject to} & 5x_1 + 2x_2 + 3x_3 - x_4 \geq 0 \\ & 4x_2 + 2x_3 - x_4 \geq 0 \\ & 3x_1 + 3x_2 - x_4 \geq 0 \\ & x_1 + 2x_2 + 4x_3 - x_4 \geq 0 \\ & x_1 + x_2 + x_3 = 1 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{array}$$

(b)

```
Optimal Solution
Objective Function: 2.368
 Variable    |   Value
_____________|____________
X1           |0.053
X2           |0.737
X3           |0.211
X4           |2.368
```

15.5-4.

(a) To insure $x_4 \geq 0$, add 3 to each entry of the payoff table.

$$\begin{array}{ll} \text{maximize} & x_4 \\ \text{subject to} & 7x_1 + 2x_2 + 5x_3 - x_4 \geq 0 \\ & 5x_1 + 3x_2 + 6x_3 - x_4 \geq 0 \\ & 6x_2 + x_3 - x_4 \geq 0 \\ & x_1 + x_2 + x_3 = 1 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{array}$$

(b)

Value of the
Objective Function: Z = 3.79166667

Variable	Value
X_1	0.33333
X_2	0.625
X_3	0.04167
X_4	3.79167

15.5-5.

(a) To insure $x_5 \geq 0$, add 4 to each entry of the payoff table.

$$\begin{array}{ll} \text{maximize} & x_5 \\ \text{subject to} & 5x_1 + 6x_2 + 4x_3 - x_5 \geq 0 \\ & x_1 + 7x_2 + 8x_3 + 4x_4 - x_5 \geq 0 \\ & 6x_1 + 4x_2 + 3x_3 + 2x_4 - x_5 \geq 0 \\ & 2x_1 + 7x_2 + x_3 + 6x_4 - x_5 \geq 0 \\ & 5x_1 + 2x_2 + 6x_3 + 3x_4 - x_5 \geq 0 \\ & x_1 + x_2 + x_3 + x_4 = 1 \\ & x_1, x_2, x_3, x_4, x_5 \geq 0 \end{array}$$

(b)

Optimal Solution
Objective Function: 3.981

Variable	Value
X1	0.31
X2	0.266
X3	0.209
X4	0.215
X5	3.981

15.5-6.

Following Table 6.14, the dual of player 1's problem is:

$$\begin{array}{ll} \text{minimize} & y_{n+1} \\ \text{subject to} & p_{11}y'_1 + p_{12}y'_2 + \cdots + p_{1n}y'_n + y_{n+1} \geq 0 \\ & p_{21}y'_1 + p_{22}y'_2 + \cdots + p_{2n}y'_n + y_{n+1} \geq 0 \\ & \vdots \\ & p_{m1}y'_1 + p_{m2}y'_2 + \cdots + p_{mn}y'_n + y_{n+1} \geq 0 \\ & -y'_1 - y'_2 - \cdots - y'_n = 1 \\ & y'_i \leq 0,\ i = 1, 2, \ldots, n;\ (y_{n+1} \text{ free}). \end{array}$$

Now, let $y_i = -y'_i$ for $i = 1, 2, \ldots, n$ to get the linear program for player 2.

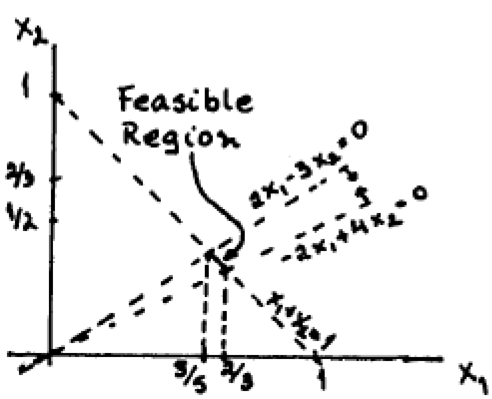
15.5-7.

Taking the dual of player 1's problem gives:

$$\begin{array}{ll} \text{minimize} & y_4 \\ \text{subject to} & -2y'_2 + 2y'_3 + y_4 \geq 0 \\ & 5y'_1 + 4y'_2 - 3y'_3 + y_4 \geq 0 \\ & -y'_1 - y'_2 - y'_3 = 1 \\ & y'_1, y'_2, y_3 \leq 0;\ (y_4 \text{ free}). \end{array}$$

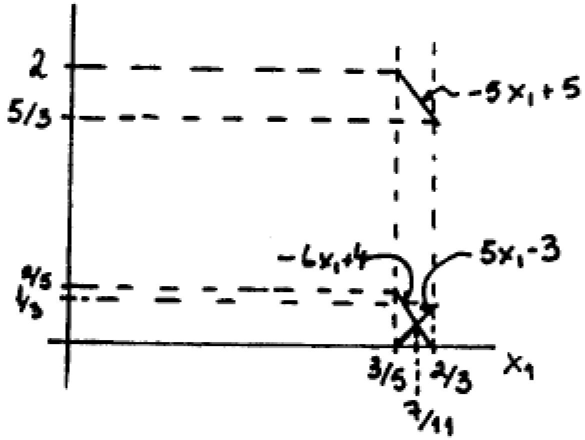
Now, let $y_i = -y'_i$ for $i = 1, 2, 3$ to get the linear program for player 2.

15.5-8.



The feasible region may be algebraically described by: $x_2 = 1 - x_1$ and $3/5 \leq x_1 \leq 2/3$. The restrictions may be rewritten as:

$$\begin{array}{ll} x_3 \leq -5x_1 + 5 & 3/5 \leq x_1 \leq 2/3 \\ x_3 \leq -6x_1 + 4 & 3/5 \leq x_1 \leq 2/3 \\ x_3 \leq 5x_1 - 3 & 3/5 \leq x_1 \leq 2/3 \end{array}$$



$-6x_1 + 4 = 5x_1 - 3 \;\Rightarrow\; x_1 = 7/11.$

Therefore, the algebraic expression for the maximizing value of x_3 for any point in the feasible region is:

$$x_3 = \begin{cases} 5x_1 - 3 & \text{for } 3/5 \leq x_1 \leq 7/11 \\ -6x_1 + 4 & \text{for } 7/11 \leq x_1 \leq 2/3 \end{cases}$$

Hence, the optimal solution is:

$(x_1^*, x_2^*, x_3^*) = (7/11, 1 - 7/11, 5(7/11) - 3) = (7/11, 4/11, 2/11).$

15.5-9.

AUTOMATIC SIMPLEX METHOD: FINAL TABLEAU

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	Right side
									1M	1M	1M	1M	
Z	0	1	0	0	0	0	0.455	0.545	0	-0.45	-0.55	0.182	0.182
X2	1	0	0	1	0	0	-0.09	0.091	0	0.091	-0.09	0.364	0.364
X4	2	0	0	0	0	1	-0.91	-0.09	-1	0.909	0.091	1.636	1.636
X1	3	0	1	0	0	0	0.091	-0.09	0	-0.09	0.091	0.636	0.636
X3	4	0	0	0	1	0	0.455	0.545	0	-0.45	-0.55	0.182	0.182

Optimal primal solution: $(x_1, x_2) = (0.636, 0.364)$ with a payoff of 0.182
Optimal dual solution: $(y_1, y_2, y_3) = (0, 0.455, 0, 545)$

15.5-10.

(a) Since the saddle points can be found by linear programming, (a) follows from (b).

(b) Consider the linear programming formulation of the problem for player 2. The ith and kth constraints are:

$$\begin{aligned} p_{i1}y_1 + p_{i2}y_2 + \cdots + p_{in}y_n &\le y_{n+1} \\ p_{k1}y_1 + p_{k2}y_2 + \cdots + p_{kn}y_n &\le y_{n+1} \end{aligned}$$

If row k weakly dominates row i, then

$$p_{i1}y_1 + p_{i2}y_2 + \cdots + p_{in}y_n \le p_{k1}y_1 + p_{k2}y_2 + \cdots + p_{kn}y_n$$

for every $y_1, \ldots, y_n$. In that case, the ith constraint is redundant, as it is implied by the kth constraint. Hence, eliminating dominated pure strategies for player 1 corresponds to eliminating redundant constraints from the linear program for player 2. Similarly, eliminating dominated strategies of player 2 is equivalent to eliminating redundant constraints of player 1's linear program. Since this process cannot eliminate any feasible solutions or create new ones, all optimal strategies are preserved and no new ones are added.