CHAPTER 7: LINEAR PROGRAMMING UNDER UNCERTAINTY

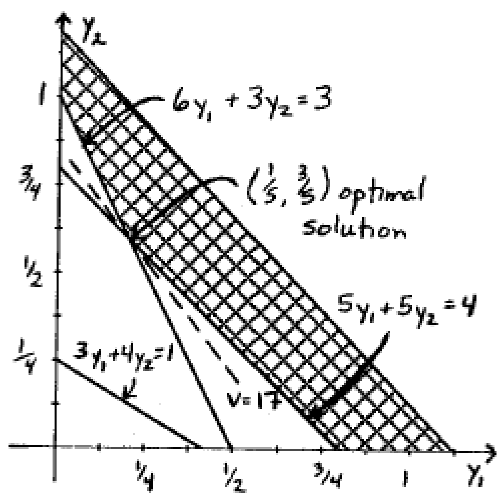
7.1-1.

(a) $(x_1, x_2, x_3) = (5/3, 0, 3)$, $Z = 17$

(b) minimize $W = 25y_1 + 20y_2$

subject to
$$\begin{aligned} 6y_1 + 3y_2 &\geq 3 \\ 3y_1 + 4y_2 &\geq 1 \\ 5y_1 + 5y_2 &\geq 4 \\ y_1, y_2 &\geq 0 \end{aligned}$$

(c) Optimal Solution: $(y_1, y_2) = (1/5, 3/5)$, $W = 17$



(d) Since the new dual constraint $2y_1 + 3y_2 \geq 3$ is violated by $(y_1, y_2) = (1/5, 3/5)$, the current solution is no longer optimal.

(e) New x_2 column:

$$\begin{pmatrix} \frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{5} & \frac{2}{5} \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -\frac{1}{3} \\ \frac{4}{5} \end{pmatrix}$$

(f) The new primal variable adds a constraint to the dual, $3y_1 + 2y_2 \geq 2$, which is not satisfied by $(y_1, y_2) = (1/5, 3/5)$, so the current solution is no longer optimal.

(g) $\overline{c}_{\text{new}} = \begin{pmatrix} \frac{1}{5} & \frac{3}{5} \end{pmatrix} \begin{pmatrix} 3 \\ 2 \end{pmatrix} - 2 = -\frac{1}{5}$, new column: $\begin{pmatrix} \frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{5} & \frac{2}{5} \end{pmatrix} \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} \\ \frac{1}{5} \end{pmatrix}$

7.1-2.

(a) $\Delta b_1 = -15$, $\Delta b_2 = 0$

$$\Rightarrow \Delta Z^* = \begin{pmatrix} \frac{1}{5} & \frac{3}{5} \end{pmatrix} \begin{pmatrix} -15 \\ 0 \end{pmatrix} = -3$$

$$\Delta b_1^* = \begin{pmatrix} \frac{1}{3} & -\frac{1}{3} \end{pmatrix} \begin{pmatrix} -15 \\ 0 \end{pmatrix} = -5$$

$$\Delta b_2^* = \begin{pmatrix} -\frac{1}{5} & \frac{2}{5} \end{pmatrix} \begin{pmatrix} -15 \\ 0 \end{pmatrix} = 3$$

New Tableau:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	Right side
Z	0	1	0	2	0	0.2	0.6	14
X1	1	0	1	-0.33	0	0.333	-0.33	-3.33
X3	2	0	0	1	1	-0.2	0.4	6

The current basic solution $(-10/3, 0, 6, 0, 0)$ is infeasible and superoptimal.

(b) $\Delta b_1 = 0$, $\Delta b_2 = -10$

$$\Rightarrow \Delta Z^* = \begin{pmatrix} \frac{1}{5} & \frac{3}{5} \end{pmatrix} \begin{pmatrix} 0 \\ -10 \end{pmatrix} = -6$$

$$\Delta b_1^* = \begin{pmatrix} \frac{1}{3} & -\frac{1}{3} \end{pmatrix} \begin{pmatrix} 0 \\ -10 \end{pmatrix} = 10/3$$

$$\Delta b_2^* = \begin{pmatrix} -\frac{1}{5} & \frac{2}{5} \end{pmatrix} \begin{pmatrix} 0 \\ -10 \end{pmatrix} = -4$$

New Tableau:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	Right side
Z	0	1	0	2	0	0.2	0.6	11
X1	1	0	1	-0.33	0	0.333	-0.33	5
X3	2	0	0	1	1	-0.2	0.4	-1

The current basic solution $(5, 0, -1, 0, 0)$ is infeasible and superoptimal.

(c) $\Delta c_2 = 2 \Rightarrow \Delta(z_2^* - c_2) = -2$

New Tableau:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	Right side
Z	0	1	0	0	0	0.2	0.6	17
X1	1	0	1	-0.33	0	0.333	-0.33	1.667
X3	2	0	0	1	1	-0.2	0.4	3

The current basic solution $(5/3, 0, 3, 0, 0)$ stays optimal.

(d) $\Delta c_3 = -2 \Rightarrow \Delta(z_3^* - c_3) = 2$

New Tableau:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	Right side
Z	0	1	0	2	2	0.2	0.6	17
X1	1	0	1	-0.33	0	0.333	-0.33	1.667
X3	2	0	0	1	1	-0.2	0.4	3

Proper Form:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	Right side
Z	0	1	0	0	0	0.6	-0.2	11
X1	1	0	1	-0.33	0	0.333	-0.33	1.667
X3	2	0	0	1	1	-0.2	0.4	3

The current basic solution $(5/3, 0, 3, 0, 0)$ stays optimal.

(e) $\Delta a_{12} = 0, \Delta a_{22} = -2$

$$\Rightarrow \Delta(z_2^* - c_2) = \begin{pmatrix} \frac{1}{5} & \frac{3}{5} \end{pmatrix} \begin{pmatrix} 0 \\ -2 \end{pmatrix} = -\frac{6}{5}$$

$$\Delta a_{12}^* = \begin{pmatrix} \frac{1}{3} & -\frac{1}{3} \end{pmatrix} \begin{pmatrix} 0 \\ -2 \end{pmatrix} = \frac{2}{3}$$

$$\Delta a_{22}^* = \begin{pmatrix} -\frac{1}{5} & \frac{2}{5} \end{pmatrix} \begin{pmatrix} 0 \\ -2 \end{pmatrix} = -\frac{4}{5}$$

New Tableau:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	Right side
Z	0	1	0	0.8	0	0.2	0.6	17
X1	1	0	1	0.333	0	0.333	-0.33	1.667
X3	2	0	0	0.2	1	-0.2	0.4	3

The current basic solution $(5/3, 0, 3, 0, 0)$ is feasible and optimal.

(f) $\Delta a_{11} = 2, \Delta a_{21} = 0$

$$\Rightarrow \Delta(z_1^* - c_1) = \begin{pmatrix} \frac{1}{5} & \frac{3}{5} \end{pmatrix} \begin{pmatrix} 2 \\ 0 \end{pmatrix} = \frac{2}{5}$$

$$\Delta a_{11}^* = \begin{pmatrix} \frac{1}{3} & -\frac{1}{3} \end{pmatrix} \begin{pmatrix} 2 \\ 0 \end{pmatrix} = \frac{2}{3}$$

$$\Delta a_{21}^* = \begin{pmatrix} -\frac{1}{5} & \frac{2}{5} \end{pmatrix} \begin{pmatrix} 2 \\ 0 \end{pmatrix} = -\frac{2}{5}$$

New Tableau:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	Right side
Z	0	1	0.4	2	0	0.2	0.6	17
X1	1	0	1.667	-0.33	0	0.333	-0.33	1.667
X3	2	0	-0.4	1	1	-0.2	0.4	3

Proper Form:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	Right side
Z	0	1	0	2.08	0	0.12	0.68	16.6
X1	1	0	1	-0.2	0	0.2	-0.2	1
X3	2	0	0	0.92	1	-0.12	0.32	3.4

The current basic solution $(0.71, 0, 3.57, 0, 0)$ is feasible and optimal.

7.1-3.

(a) $\Delta b_1 = -2, \Delta b_2 = 1$

$$\Rightarrow \Delta Z^* = \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} -2 \\ 1 \end{pmatrix} = -1$$

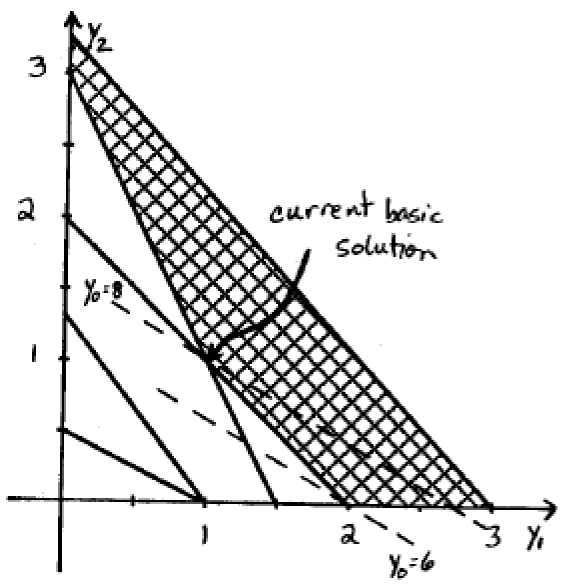
$$\Delta b_1^* = \begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} -2 \\ 1 \end{pmatrix} = -3$$

$$\Delta b_2^* = \begin{pmatrix} -1 & 2 \end{pmatrix} \begin{pmatrix} -2 \\ 1 \end{pmatrix} = 4$$

New Tableau:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	3	0	2	0	1	1	8
X2	1	0	1	1	-1	0	1	-1	-2
X4	2	0	2	0	3	1	-1	2	7

From the tableau, we see that the primal basic solution is feasible, but not optimal.



From the graph, we can see the current basic solution is feasible, but not optimal.

(b) $\Delta c_1 = -1 \Rightarrow \Delta(z_1^* - c_1) = 1$

$\Delta c_2 = 2 \Rightarrow \Delta(z_2^* - c_2) = -2$

$\Delta c_3 = 1 \Rightarrow \Delta(z_3^* - c_3) = -1$

$\Delta c_4 = 1 \Rightarrow \Delta(z_4^* - c_4) = -1$

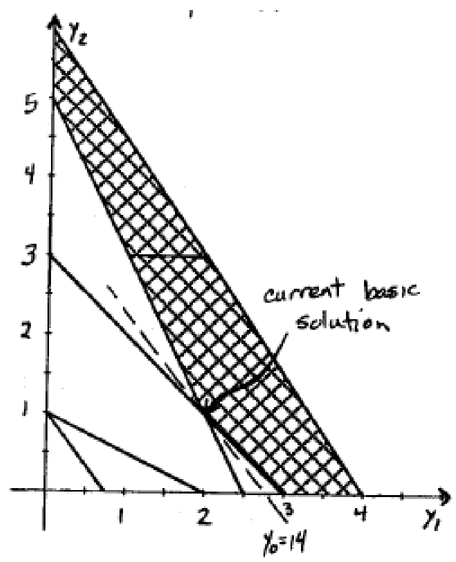
New Tableau:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	4	-2	1	-1	1	1	9
X2	1	0	1	1	-1	0	1	-1	1
X4	2	0	2	0	3	1	-1	2	3

Proper Form:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	8	0	2	0	2	1	14
X2	1	0	1	1	-1	0	1	-1	1
X4	2	0	2	0	3	1	-1	2	3

The primal basic solution is both feasible and optimal.



From the graph, we see that the current basic solution is feasible and optimal.

(c) $\Delta a_{11} = -2, \Delta a_{21} = 1$

$$\Delta c_1 = 3 \Rightarrow \Delta(z_1^* - c_1) = -3 + (1 \quad 1)\begin{pmatrix} -2 \\ 1 \end{pmatrix} = -4$$

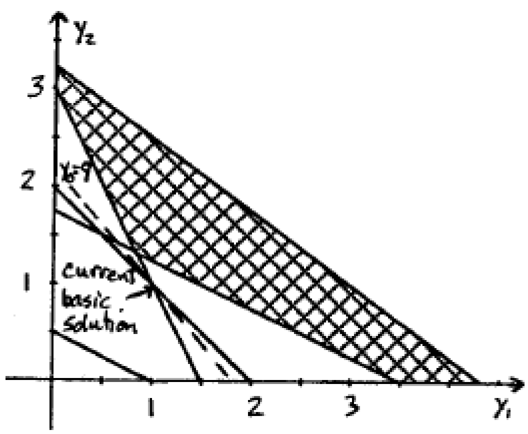
$$\Delta a_{11}^* = (1 \quad -1)\begin{pmatrix} -2 \\ 1 \end{pmatrix} = -3$$

$$\Delta a_{21}^* = (-1 \quad 2)\begin{pmatrix} -2 \\ 1 \end{pmatrix} = 4$$

New Tableau:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	-1	0	2	0	1	1	9
X2	1	0	-2	1	-1	0	1	-1	1
X4	2	0	6	0	3	1	-1	2	3

The primal basic solution is infeasible, but satisfies the optimality criterion.



From the graph, the current basic solution is infeasible and superoptimal.

(d) $\Delta a_{12} = 3, \Delta a_{22} = 1$

$$\Delta c_2 = 7 \Rightarrow \Delta(z_2^* - c_2) = -7 + (1 \quad 1)\begin{pmatrix}3\\1\end{pmatrix} = -3$$

$$\Delta a_{12}^* = (1 \quad -1)\begin{pmatrix}3\\1\end{pmatrix} = 2$$

$$\Delta a_{22}^* = (-1 \quad 2)\begin{pmatrix}3\\1\end{pmatrix} = -1$$

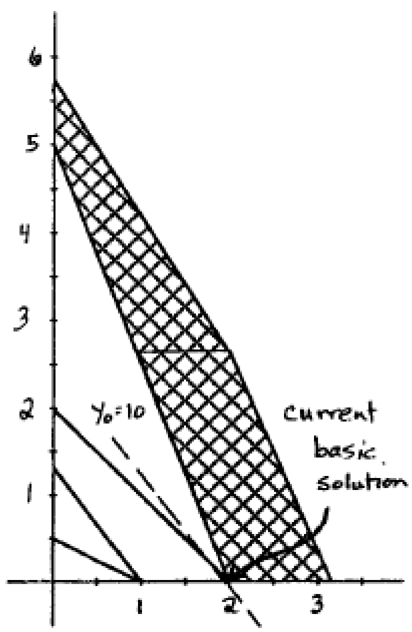
New Tableau:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	3	-3	2	0	1	1	9
X2	1	0	1	3	-1	0	1	-1	1
X4	2	0	2	-1	3	1	-1	2	3

Proper Form:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	4	0	1	0	2	0	10
X2	1	0	0.333	1	-0.33	0	0.333	-0.33	0.333
X4	2	0	2.333	0	2.667	1	-0.67	1.667	3.333

The primal basic solution is feasible and optimal.



From the graph, the current basic solution is feasible and optimal.

7.2-1.

The model Ep(x) is developed to identify a long-term management plan that satisfies the legal requirements and optimizes PALCO's operations and profitability. The model consists of a linear program with the objective of maximizing present net worth subject to harvest-flow constraints, political and environmental constraints. Detailed sensitivity analysis is performed to "determine the optimal mix of habitat types within each of individual watersheds" [p. 93]. Many instances of the LP problem are run with varying parameters.

The financial benefits of this study include an increase of over $398 million in present net worth and of over $29 million in average yearly net revenues. Sustained-yield annual-harvest levels have increased. The habitat mix is improved in accordance with political and environmental regulations. A more profitable long-term plan paved the way for improved short- and mid-term plans. Sensitivity analysis enabled PALCO to improve its knowledge base of the ecosystem and to adjust its plans quickly when a change in costs or in regulations occurs. Since its decisions are now justified through a systematic approach, PALCO is able to obtain better terms from banks. The study did not only affect PALCO and the habitat controlled by PALCO. It has also "shown that the forest product industries can coexist with wildlife and contribute to their habitats" [p. 104] and "increased quality of life for future generations" [p. 105].

7.2-2.

(a) $\Delta b_1 = 10, \Delta b_2 = 0$

$$\Rightarrow \Delta Z^* = (5 \quad 0)\begin{pmatrix} 10 \\ 0 \end{pmatrix} = 50$$

$$\Delta b_1^* = (1 \quad 0)\begin{pmatrix} 10 \\ 0 \end{pmatrix} = 10$$

$$\Delta b_2^* = (-4 \quad 1)\begin{pmatrix} 10 \\ 0 \end{pmatrix} = -40$$

New Tableau:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	Right side
Z	0	1	0	0	2	5	0	150
X2	1	0	-1	1	3	1	0	30
X5	2	0	16	0	-2	-4	1	-30

The current basic solution is infeasible and superoptimal.

(b) $\Delta b_1 = 0, \Delta b_2 = -20$

$$\Rightarrow \Delta Z^* = \begin{pmatrix} 5 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ -20 \end{pmatrix} = 0$$

$$\Delta b_1^* = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ -20 \end{pmatrix} = 0$$

$$\Delta b_2^* = \begin{pmatrix} -4 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ -20 \end{pmatrix} = -20$$

New Tableau:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	Right side
Z	0	1	0	0	2	5	0	100
X2	1	0	-1	1	3	1	0	20
X5	2	0	16	0	-2	-4	1	-10

The current basic solution is infeasible and superoptimal.

(c) $\Delta b_1 = -10, \Delta b_2 = 10$

$$\Rightarrow \Delta Z^* = \begin{pmatrix} 5 & 0 \end{pmatrix} \begin{pmatrix} -10 \\ 10 \end{pmatrix} = -50$$

$$\Delta b_1^* = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} -10 \\ 10 \end{pmatrix} = -10$$

$$\Delta b_2^* = \begin{pmatrix} -4 & 1 \end{pmatrix} \begin{pmatrix} -10 \\ 10 \end{pmatrix} = 50$$

New Tableau:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	Right side
Z	0	1	0	0	2	5	0	50
X2	1	0	-1	1	3	1	0	10
X5	2	0	16	0	-2	-4	1	60

The current basic solution is feasible and optimal.

(d) $\Delta c_3 = -5 \Rightarrow \Delta(z_3^* - c_3) = 5$

New Tableau:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	Right side
Z	0	1	0	0	7	5	0	100
X2	1	0	-1	1	3	1	0	20
X5	2	0	16	0	-2	-4	1	10

The current basic solution is feasible and optimal.

(e) $\Delta a_{11} = 1, \Delta a_{21} = -7$

$$\Delta c_1 = 3 \Rightarrow \Delta(z_1^* - c_1) = -3 + (5 \quad 0)\begin{pmatrix} 1 \\ -7 \end{pmatrix} = 2$$

$$\Delta a_{11}^* = (1 \quad 0)\begin{pmatrix} 1 \\ -7 \end{pmatrix} = 1$$

$$\Delta a_{21}^* = (-4 \quad 1)\begin{pmatrix} 1 \\ -7 \end{pmatrix} = -11$$

New Tableau:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	Right side
Z	0	1	2	0	2	5	0	100
X2	1	0	0	1	3	1	0	20
X5	2	0	5	0	-2	-4	1	10

The current basic solution is feasible and optimal.

(f) $\Delta a_{12} = 1, \Delta a_{22} = 1$

$$\Delta c_2 = 1 \Rightarrow \Delta(z_2^* - c_2) = -1 + (5 \quad 0)\begin{pmatrix}1\\1\end{pmatrix} = 4$$

$$\Delta a_{12}^* = (1 \quad 0)\begin{pmatrix}1\\1\end{pmatrix} = 1$$

$$\Delta a_{22}^* = (-4 \quad 1)\begin{pmatrix}1\\1\end{pmatrix} = -3$$

New Tableau:

Bas Var	Eq No	Z	Coefficient of X1	X2	X3	X4	X5	Right side
Z	0	1	0	4	2	5	0	100
X2	1	0	-1	2	3	1	0	20
X5	2	0	16	-3	-2	-4	1	10

Proper Form:

Bas Var	Eq No	Z	Coefficient of X1	X2	X3	X4	X5	Right side
Z	0	1	2	0	-4	3	0	60
X2	1	0	-0.5	1	1.5	0.5	0	10
X5	2	0	14.5	0	2.5	-2.5	1	40

The current basic solution is feasible, but not optimal.

(g) $\Delta a_{16} = 3, \Delta a_{26} = 5$

$$\Delta c_6 = 10 \Rightarrow \Delta(z_6^* - c_6) = -10 + (5 \quad 0)\begin{pmatrix}3\\5\end{pmatrix} = 5$$

$$\Delta a_{16}^* = (1 \quad 0)\begin{pmatrix}3\\5\end{pmatrix} = 3$$

$$\Delta a_{26}^* = (-4 \quad 1)\begin{pmatrix}3\\5\end{pmatrix} = -7$$

New Tableau:

Bas Var	Eq No	Z	Coefficients of X1	X2	X3	X4	X5	X6	Right side
z	0	1	0	0	2	5	0	5	100
X2	1	0	-1	1	3	1	0	3	20
X5	2	0	16	0	-2	-4	1	-7	10

The current basic solution is feasible and optimal.

(h) New Tableau and Proper Form:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	0	0	2	5	0	0	100
X2	1	0	-1	1	3	1	0	0	20
X5	2	0	16	0	-2	-4	1	0	10
X6	3	0	2	3	5	0	0	1	50

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	0	0	2	5	0	0	100
X2	1	0	-1	1	3	1	0	0	20
X5	2	0	16	0	-2	-4	1	0	10
X6	3	0	5	0	-4	-3	0	1	-10

The current basic solution is infeasible and superoptimal.

(i) $\Delta a_{11} = 0, \Delta a_{21} = -2$

$$\Delta c_1 = 0 \Rightarrow \Delta(z_1^* - c_1) = 0 + (5 \quad 0)\begin{pmatrix} 0 \\ -2 \end{pmatrix} = 0$$

$$\Delta a_{11}^* = (1 \quad 0)\begin{pmatrix} 0 \\ -2 \end{pmatrix} = 0$$

$$\Delta a_{21}^* = (-4 \quad 1)\begin{pmatrix} 0 \\ -2 \end{pmatrix} = -2$$

$\Delta a_{12} = 0, \Delta a_{22} = 1$

$$\Delta c_2 = 0 \Rightarrow \Delta(z_2^* - c_2) = 0 + (5 \quad 0)\begin{pmatrix} 0 \\ 1 \end{pmatrix} = 0$$

$$\Delta a_{12}^* = (1 \quad 0)\begin{pmatrix} 0 \\ 1 \end{pmatrix} = 0$$

$$\Delta a_{22}^* = (-4 \quad 1)\begin{pmatrix} 0 \\ 1 \end{pmatrix} = 1$$

$\Delta b_1 = 0, \Delta b_2 = 10$

$$\Rightarrow \Delta Z^* = (5 \quad 0)\begin{pmatrix} 0 \\ 10 \end{pmatrix} = 0$$

$$\Delta b_1^* = (1 \quad 0)\begin{pmatrix} 0 \\ 10 \end{pmatrix} = 0$$

$$\Delta b_2^* = (-4 \quad 1)\begin{pmatrix} 0 \\ 10 \end{pmatrix} = 10$$

New Tableau:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	Right side
Z	0	1	0	0	2	5	0	100
X2	1	0	-1	1	3	1	0	20
X5	2	0	14	1	-2	-4	1	20

Proper Form:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	Right side
Z	0	1	0	0	2	5	0	100
X2	1	0	-1	1	3	1	0	20
X5	2	0	15	0	-5	-5	1	0

7.2-3.

$$\Delta b_1 = 2\theta, \Delta b_2 = -\theta$$

$$\Rightarrow \Delta Z^* = (5 \quad 0)\begin{pmatrix} 2\theta \\ -\theta \end{pmatrix} = 10\theta$$

$$\Delta b_1^* = (1 \quad 0)\begin{pmatrix} 2\theta \\ -\theta \end{pmatrix} = 2\theta$$

$$\Delta b_2^* = (-4 \quad 1)\begin{pmatrix} 2\theta \\ -\theta \end{pmatrix} = -9\theta$$

$$\Rightarrow Z = 100 + 10\theta$$

$$b_1^* \geq 0 \Leftrightarrow 20 + 2\theta \geq 0$$

$$b_2^* \geq 0 \Leftrightarrow 10 - 9\theta \geq 0$$

$$\Leftrightarrow -10 \leq \theta \leq 10/9$$

7.2-4.

Original Final Tableau:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	Right side
Z	0	1	0	1	1	0	2	20
X4	1	0	0	-1	5	1	-1	20
X1	2	0	1	4	-1	0	1	10

(a) $\Delta b_1 = -10, \Delta b_2 = 20$

$$\Rightarrow \Delta Z^* = (0 \quad 2)\begin{pmatrix} -10 \\ 20 \end{pmatrix} = 40$$

$$\Delta b_1^* = (1 \quad -1)\begin{pmatrix} -10 \\ 20 \end{pmatrix} = -30$$

$$\Delta b_2^* = (0 \quad 1)\begin{pmatrix} -10 \\ 20 \end{pmatrix} = 20$$

Revised Final Tableau:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	Right side
Z	0	1	0	1	1	0	2	60
X4	1	0	0	-1	5	1	-1	-10
X1	2	0	1	4	-1	0	1	30

The current basic solution is superoptimal, but infeasible.

Revised Final Tableau After Reoptimization (Dual Simplex Method):

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	Right side
Z	0	1	0.333	0	12.33	2.333	0	46.67
X2	1	0	0.333	1	1.333	0.333	0	6.667
X5	2	0	-0.33	0	-6.33	-1.33	1	3.333

(b) $\Delta a_{13} = -1, \Delta a_{23} = -1$

$$\Delta c_3 = 1 \Rightarrow \Delta(z_3^* - c_3) = -1 + (0 \quad 2)\begin{pmatrix} -1 \\ -1 \end{pmatrix} = -3$$

$$\Delta a_{13}^* = (1 \quad -1)\begin{pmatrix} -1 \\ -1 \end{pmatrix} = 0$$

$$\Delta a_{23}^* = (0 \quad 1)\begin{pmatrix} -1 \\ -1 \end{pmatrix} = -1$$

Revised Final Tableau:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	Right side
Z	0	1	0	1	-2	0	2	20
X4	1	0	0	-1	5	1	-1	20
X1	2	0	1	4	-2	0	1	10

The current basic solution is feasible, but not optimal.

Revised Final Tableau After Reoptimization (Simplex Method):

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	Right side
Z	0	1	0	0.6	0	0.4	1.6	28
X3	1	0	0	-0.2	1	0.2	-0.2	4
X1	2	0	1	3.6	0	0.4	0.6	18

(c) $\Delta a_{11} = 2, \Delta a_{21} = 1$

$$\Delta c_1 = 2 \Rightarrow \Delta(z_2^* - c_2) = -2 + \begin{pmatrix} 0 & 2 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = 0$$

$$\Delta a_{11}^* = \begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = 1$$

$$\Delta a_{21}^* = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = 1$$

Revised Final Tableau:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	Right side
Z	0	1	0	1	1	0	2	20
X4	1	0	1	-1	5	1	-1	20
X1	2	0	2	4	-1	0	1	10

Revised Final Tableau After Converting to Proper Form:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	Right side
Z	0	1	0	1	1	0	2	20
X4	1	0	0	-3	5.5	1	-1.5	15
X1	2	0	1	2	-0.5	0	0.5	5

The current basic solution is feasible and optimal.

(d) $\Delta a_{16} = 1, \Delta a_{26} = 2$

$$\Delta c_6 = -3 \Rightarrow \Delta(z_6^* - c_6) = 3 + (\,0 \quad 2\,)\begin{pmatrix}1\\2\end{pmatrix} = 7$$

$$\Delta a_{16}^* = (\,1 \quad -1\,)\begin{pmatrix}1\\2\end{pmatrix} = -1$$

$$\Delta a_{26}^* = (\,0 \quad 1\,)\begin{pmatrix}1\\2\end{pmatrix} = 2$$

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	0	1	1	0	2	7	20
X4	1	0	0	-1	5	1	-1	-1	20
X1	2	0	1	4	-1	0	1	2	10

The current basic solution is feasible and optimal.

(e) $\Delta c_1 = -1 \Rightarrow \Delta(z_1^* - c_1) = 1$

$$\Delta c_2 = -2 \Rightarrow \Delta(z_2^* - c_2) = 2$$

$$\Delta c_3 = 1 \Rightarrow \Delta(z_3^* - c_3) = -1$$

Revised Final Tableau:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	Right side
Z	0	1	1	3	0	0	2	20
X4	1	0	0	-1	5	1	-1	20
X1	2	0	1	4	-1	0	1	10

Revised Final Tableau After Converting to Proper Form:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	Right side
Z	0	1	0	-1	1	0	1	10
X4	1	0	0	-1	5	1	-1	20
X1	2	0	1	4	-1	0	1	10

The current basic solution is feasible, but not optimal.

Revised Final Tableau After Reoptimization (Simplex Method):

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	Right side
Z	0	1	0.25	0	0.75	0	1.25	12.5
X4	1	0	0.25	0	4.75	1	-0.75	22.5
X2	2	0	0.25	1	-0.25	0	0.25	2.5

(f) New Tableau:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	0	1	1	0	2	0	20
X4	1	0	0	-1	5	1	-1	0	20
X1	2	0	1	4	-1	0	1	0	10
X6	3	0	3	2	3	0	0	1	25

Proper Form:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	0	1	1	0	2	0	20
X4	1	0	0	-1	5	1	-1	0	20
X1	2	0	1	4	-1	0	1	0	10
X6	3	0	0	-10	6	0	-3	1	-5

The current basic solution is infeasible and superoptimal.

Tableau After Reoptimization:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	0	0	1.6	0	1.7	0.1	19.5
X4	1	0	0	0	4.4	1	-0.7	-0.1	20.5
X2	2	0	0	1	-0.6	0	0.3	-0.1	0.5
X1	3	0	1	0	1.4	0	-0.2	0.4	8

(g) $\Delta a_{22} = -2, \Delta a_{23} = 3$

$$\Rightarrow \Delta(z_2^* - c_2) = (0 \quad 2)\begin{pmatrix} 0 \\ -2 \end{pmatrix} = -4$$

$$\Delta a_{12}^* = (1 \quad -1)\begin{pmatrix} 0 \\ -2 \end{pmatrix} = 2$$

$$\Delta a_{22}^* = (0 \quad 1)\begin{pmatrix} 0 \\ -2 \end{pmatrix} = -2$$

$$\Rightarrow \Delta(z_3^* - c_3) = (0 \quad 2)\begin{pmatrix} 0 \\ 3 \end{pmatrix} = 6$$

$$\Delta a_{13}^* = (1 \quad -1)\begin{pmatrix} 0 \\ 3 \end{pmatrix} = -3$$

$$\Delta a_{23}^* = (0 \quad 1)\begin{pmatrix} 0 \\ 3 \end{pmatrix} = 3$$

$\Delta b_1 = 0, \Delta b_2 = 25$

$$\Rightarrow \Delta Z^* = (0 \quad 2)\begin{pmatrix} 0 \\ 25 \end{pmatrix} = 50$$

$$\Delta b_1^* = (1 \quad -1)\begin{pmatrix} 0 \\ 25 \end{pmatrix} = -25$$

$$\Delta b_2^* = (0 \quad 1)\begin{pmatrix} 0 \\ 25 \end{pmatrix} = 25$$

Revised Final Tableau:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	Right side
Z	0	1	0	-3	7	0	2	70
X4	1	0	0	1	2	1	-1	-5
X1	2	0	1	2	2	0	1	35

The current basic solution is neither feasible nor optimal.

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	Right side
Z	0	1	0.333	0	12.33	2.333	0	70
X2	1	0	0.333	1	1.333	0.333	0	10
X5	2	0	0.333	0	-0.67	-0.67	1	15

7.2-5.

$\Delta b_1 = 3\theta, \Delta b_2 = -\theta$

$\Rightarrow \Delta Z^* = (0 \quad 2)\begin{pmatrix} 3\theta \\ -\theta \end{pmatrix} = -2\theta$

$\Delta b_1^* = (1 \quad -1)\begin{pmatrix} 3\theta \\ -\theta \end{pmatrix} = 4\theta$

$\Delta b_2^* = (0 \quad 1)\begin{pmatrix} 3\theta \\ -\theta \end{pmatrix} = -\theta$

$Z^*(\theta) = 20 - 2\theta$

$(x_1, x_2, x_3, x_4, x_5) = (10-\theta, 0, 0, 20+4\theta, 0)$ is feasible if $-5 \leq \theta \leq 10$.

7.2-6.

Original Final Tableau:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	0	0	2	1	1	0	18
X2	1	0	0	1	5	1	3	0	24
X6	2	0	0	0	2	0	1	1	7
X1	3	0	1	0	4	1	2	0	21

(a) $\Delta b_1 = -5, \Delta b_2 = 1, \Delta b_3 = -2$

$\Rightarrow \Delta Z^* = (1 \quad 1 \quad 0)\begin{pmatrix} -5 \\ 1 \\ -2 \end{pmatrix} = -4$

$\Delta b_1^* = (1 \quad 3 \quad 0)\begin{pmatrix} -5 \\ 1 \\ -2 \end{pmatrix} = -2$

$\Delta b_2^* = (0 \quad 1 \quad 1)\begin{pmatrix} -5 \\ 1 \\ -2 \end{pmatrix} = -1$

$\Delta b_3^* = (1 \quad 2 \quad 0)\begin{pmatrix} -5 \\ 1 \\ -2 \end{pmatrix} = -3$

```
Revised Final Tableau:

Bas|Eq|                Coefficient of                | Right
Var|No| Z|   X1     X2     X3     X4     X5     X6   |  side
___|__|__|___________________________________________|_______
   |  |  |                                           |
 Z | 0| 1|    0      0      2      1      1      0   |    14
 X2| 1| 0|    0      1      5      1      3      0   |    22
 X6| 2| 0|    0      0      2      0      1      1   |     6
 X1| 3| 0|    1      0      4      1      2      0   |    18
```

The current basic solution is feasible and optimal.

(b) $\Delta c_3 = 1 \Rightarrow \Delta(z_3^* - c_3) = -1$

```
Revised Final Tableau:

Bas|Eq|                Coefficient of                | Right
Var|No| Z|   X1     X2     X3     X4     X5     X6   |  side
___|__|__|___________________________________________|_______
   |  |  |                                           |
 Z | 0| 1|    0      0      1      1      1      0   |    18
 X2| 1| 0|    0      1      5      1      3      0   |    24
 X6| 2| 0|    0      0      2      0      1      1   |     7
 X1| 3| 0|    1      0      4      1      2      0   |    21
```

The current basic solution remains feasible and optimal.

(c) $\Delta c_1 = 3 \Rightarrow \Delta(z_1^* - c_1) = -3$

```
Revised Final Tableau:

Bas|Eq|                Coefficient of                | Right
Var|No| Z|   X1     X2     X3     X4     X5     X6   |  side
___|__|__|___________________________________________|_______
   |  |  |                                           |
 Z | 0| 1|   -1      0      2      1      1      0   |    18
 X2| 1| 0|    0      1      5      1      3      0   |    24
 X6| 2| 0|    0      0      2      0      1      1   |     7
 X1| 3| 0|    1      0      4      1      2      0   |    21
```

```
Revised Final Tableau After Converting to Proper Form:

Bas|Eq|                Coefficient of                | Right
Var|No| Z|   X1     X2     X3     X4     X5     X6   |  side
___|__|__|___________________________________________|_______
   |  |  |                                           |
 Z | 0| 1|    0      0      6      2      3      0   |    39
 X2| 1| 0|    0      1      5      1      3      0   |    24
 X6| 2| 0|    0      0      2      0      1      1   |     7
 X1| 3| 0|    1      0      4      1      2      0   |    21
```

The current basic solution is feasible and optimal.

(d) $\Delta a_{13} = 1,\ \Delta a_{23} = 1,\ \Delta a_{33} = 0$

$$\Delta c_3 = 3 \Rightarrow \Delta(z_3^* - c_3) = -3 + \begin{pmatrix} 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = -1$$

$$\Delta a_{13}^* = \begin{pmatrix} 1 & 3 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 4$$

$$\Delta a_{23}^* = \begin{pmatrix} 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 1$$

$$\Delta a_{33}^* = \begin{pmatrix} 1 & 2 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 3$$

Revised Final Tableau:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	0	0	1	1	1	0	18
X2	1	0	0	1	9	1	3	0	24
X6	2	0	0	0	3	0	1	1	7
X1	3	0	1	0	7	1	2	0	21

The current basic solution remains feasible and optimal.

(e) $\Delta a_{11} = -2,\ \Delta a_{21} = -1,\ \Delta a_{31} = 2$

$$\Delta c_1 = -1 \Rightarrow \Delta(z_1^* - c_1) = 1 + \begin{pmatrix} 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix} = -2$$

$$\Delta a_{11}^* = \begin{pmatrix} 1 & 3 & 0 \end{pmatrix} \begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix} = -5$$

$$\Delta a_{21}^* = \begin{pmatrix} 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix} = 1$$

$$\Delta a_{31}^* = \begin{pmatrix} 1 & 2 & 0 \end{pmatrix} \begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix} = -4$$

$\Delta a_{12} = 0,\ \Delta a_{22} = 2,\ \Delta a_{32} = 3$

$$\Delta c_2 = -1 \Rightarrow \Delta(z_2^* - c_2) = 1 + \begin{pmatrix} 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} = 3$$

$$\Delta a_{12}^* = \begin{pmatrix} 1 & 3 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} = 6$$

$$\Delta a_{22}^* = \begin{pmatrix} 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} = 5$$

$$\Delta a_{32}^* = \begin{pmatrix} 1 & 2 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} = 4$$

Revised Final Tableau:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	-2	3	2	1	1	0	18
X2	1	0	-5	7	5	1	3	0	24
X6	2	0	1	5	2	0	1	1	7
X1	3	0	-3	4	4	1	2	0	21

Revised Final Tableau After Converting to Proper Form:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	0	0	1	1	0	0	15
X2	1	0	0	1	-5	-2	-1	0	-33
X6	2	0	0	0	35	13	8	1	223
X1	3	0	1	0	-8	-3	-2	0	-51

The current basic solution is superoptimal, but infeasible.

Revised Final Tableau After Reoptimization (Dual Simplex Method):

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	0	4.4	0	0	0.4	0.6	3.6
X1	1	0	1	-0.2	0	0	-0.2	0.2	0.2
X3	2	0	0	2.6	1	0	0.6	0.4	3.4
X4	3	0	0	-7	0	1	-1	-1	8

(f) $\Delta c_1 = 3 \Rightarrow \Delta(z_1^* - c_1) = -3$

$\Delta c_2 = 2 \Rightarrow \Delta(z_2^* - c_2) = -2$

$\Delta c_3 = 2 \Rightarrow \Delta(z_3^* - c_3) = -2$

Revised Final Tableau:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	-3	-2	0	1	1	0	18
X2	1	0	0	1	5	1	3	0	24
X6	2	0	0	0	2	0	1	1	7
X1	3	0	1	0	4	1	2	0	21

Revised Final Tableau After Converting to Proper Form:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	0	0	22	6	13	0	129
X2	1	0	0	1	5	1	3	0	24
X6	2	0	0	0	2	0	1	1	7
X1	3	0	1	0	4	1	2	0	21

The current basic solution is feasible and optimal.

(g) $\Delta a_{11} = -1,\ \Delta a_{21} = 0,\ \Delta a_{31} = 0$

$$\Rightarrow \Delta(z_1^* - c_1) = \begin{pmatrix} 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} = -1$$

$$\Delta a_{11}^* = \begin{pmatrix} 1 & 3 & 0 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} = -1$$

$$\Delta a_{21}^* = \begin{pmatrix} 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} = 0$$

$$\Delta a_{31}^* = \begin{pmatrix} 1 & 2 & 0 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} = -1$$

$\Delta a_{12} = 1,\ \Delta a_{22} = 0,\ \Delta a_{32} = 0$

$$\Rightarrow \Delta(z_2^* - c_2) = \begin{pmatrix} 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 1$$

$$\Delta a_{12}^* = \begin{pmatrix} 1 & 3 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 1$$

$$\Delta a_{22}^* = \begin{pmatrix} 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 0$$

$$\Delta a_{32}^* = \begin{pmatrix} 1 & 2 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 1$$

$\Delta a_{13} = 2,\ \Delta a_{23} = 0,\ \Delta a_{33} = 0$

$$\Rightarrow \Delta(z_3^* - c_3) = \begin{pmatrix} 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} = 2$$

$$\Delta a_{13}^* = \begin{pmatrix} 1 & 3 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} = 2$$

$$\Delta a_{23}^* = \begin{pmatrix} 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} = 0$$

$$\Delta a_{33}^* = \begin{pmatrix} 1 & 2 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} = 2$$

$\Delta b_1 = -3, \Delta b_2 = 0, \Delta b_3 = 0$

$$\Rightarrow \Delta Z^* = \begin{pmatrix} 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} -3 \\ 0 \\ 0 \end{pmatrix} = -3$$

$$\Delta b_1^* = \begin{pmatrix} 1 & 3 & 0 \end{pmatrix} \begin{pmatrix} -3 \\ 0 \\ 0 \end{pmatrix} = -3$$

$$\Delta b_2^* = \begin{pmatrix} 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} -3 \\ 0 \\ 0 \end{pmatrix} = 0$$

$$\Delta b_3^* = \begin{pmatrix} 1 & 2 & 0 \end{pmatrix} \begin{pmatrix} -3 \\ 0 \\ 0 \end{pmatrix} = -3$$

Revised Final Tableau:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	-1	1	4	1	1	0	15
X2	1	0	-1	2	7	1	3	0	21
X6	2	0	0	0	2	0	1	1	7
X1	3	0	0	1	6	1	2	0	18

Revised Final Tableau After Converting to Proper Form:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	Right side
Z	0	1	0	0	3	1	0	0	12
X2	1	0	0	1	6	1	2	0	18
X6	2	0	0	0	2	0	1	1	7
X1	3	0	1	0	5	1	1	0	15

The current basic solution is feasible and optimal.

(h)

New Tableau:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	X7	Right side
Z	0	1	0	0	2	1	1	0	0	18
X2	1	0	0	1	5	1	3	0	0	24
X6	2	0	0	0	2	0	1	1	0	7
X1	3	0	1	0	4	1	2	0	0	21
X7	4	0	2	1	3	0	0	0	1	60

Proper Form:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	X7	Right side
Z	0	1	0	0	2	1	1	0	0	18
X2	1	0	0	1	5	1	3	0	0	24
X6	2	0	0	0	2	0	1	1	0	7
X1	3	0	1	0	4	1	2	0	0	21
X7	4	0	0	0	-10	-3	-7	0	1	-6

The current basic solution is infeasible and superoptimal.

Tableau After Reoptimization:

```
Bas|Eq|                    Coefficient of                          | Right
Var|No| Z|   X1     X2     X3     X4     X5     X6     X7           |  side
___|__|__|__________________________________________________________|______
   |  |  |                                                          |
 Z | 0| 1|    0      0 0.571  0.571      0      0 0.143             | 17.14
X2| 1| 0|    0      1 0.714  -0.29      0      0 0.429              | 21.43
X5| 2| 0|    0      0 1.429  0.429      1      0 -0.14              | 0.857
X1| 3| 0|    1      0 1.143  0.143      0      0 0.286              | 19.29
X6| 4| 0|    0      0 0.571  -0.43      0      1 0.143              | 6.143
```

7.2-7.

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$11	Solution F1-DC	50	-200	300	200	1E+30
$C$11	Solution F2-DC	30	0	400	100	1E+30
$D$11	Solution F1-W1	30	0	700	1E+30	200
$E$11	Solution F2-W1	40	0	900	1E+30	100
$F$11	Solution DC-W1	30	0	200	200	1E+30
$G$11	Solution DC-W2	50	-100	400	100	1E+30

(a) F2-DC, F2-W1 and DC-W2 have the smallest margins for error (100). The greatest effort in estimating the unit shipping costs should be placed on these lanes.

(b)

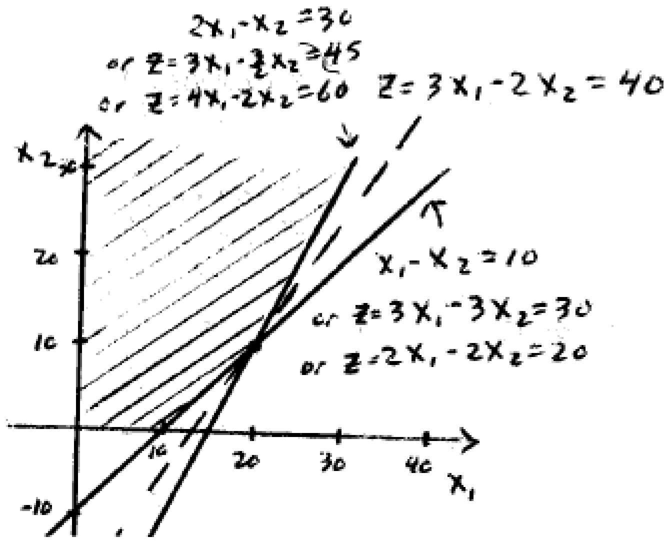
Cost	Allowable Range
$C_{\text{F1-DC}}$	≤ 500
$C_{\text{F2-DC}}$	≤ 500
$C_{\text{F1-W1}}$	≥ 500
$C_{\text{F2-W1}}$	≥ 800
$C_{\text{DC-W1}}$	≤ 400
$C_{\text{DC-W2}}$	≤ 500

(c) The range of optimality for each unit shipping cost indicates how much that shipping cost can change before the optimal shipping quantities change.

(d) Use the 100% rule for simultaneous changes in the objective function coefficients. If the sum of the percentage changes does not exceed 100%, the optimal solution will remain optimal. If it exceeds 100%, then it may or may not be optimal for the new problem.

7.2-8.

(a)



The allowable range for c_1 is $2 \le c_1 \le 4$ and the one for c_2 is $-3 \le c_2 \le -3/2$.

(b) Increasing c_1 by Δc_1 ($c_1 = 3 + \Delta c_1$)causes the coefficient of x_1 in row 0 of the final tableau to become $-\Delta c_1$. To make it 0, add Δc_1 times row 2 to row 0:

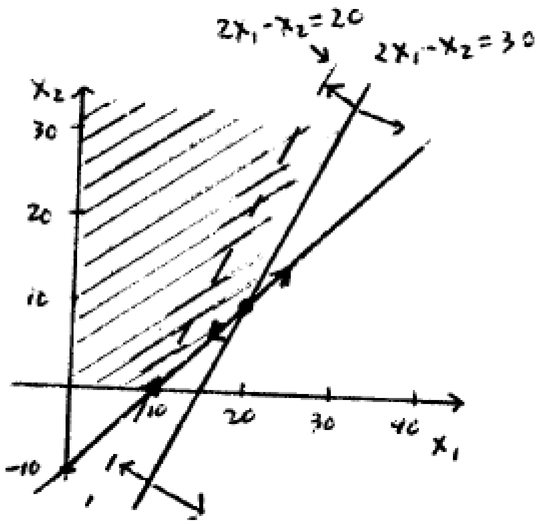
$$(\,-\Delta c_1 \quad 0 \quad 1 \quad 1\,) + \Delta c_1(1 \quad 0 \quad 1 \quad -1\,) = (\,0 \quad 0 \quad 1 + \Delta c_1 \quad 1 - \Delta c_1\,).$$

For optimality, we need $1 + \Delta c_1 \ge 0$ and $1 - \Delta c_1 \ge 0$, so $-1 \le \Delta c_1 \le 1$. Hence, the allowable range for c_1 is $3 - 1 = 2 \le c_1 \le 3 + 1 = 4$. Similarly, increasing c_2 by Δc_2 ($c_2 = -2 + \Delta c_2$)causes the coefficient of x_2 in row 0 of the final tableau to become $-\Delta c_2$. To make it 0, add Δc_2 times row 1 to row 0:

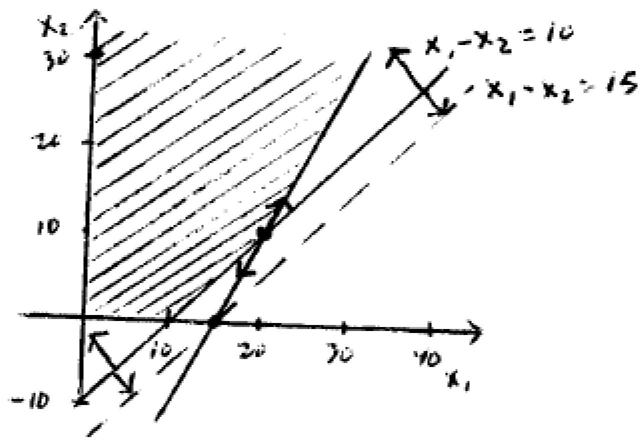
$$(\,0 \quad -\Delta c_2 \quad 1 \quad 1\,) + \Delta c_2(0 \quad 1 \quad 1 \quad -2\,) = (\,0 \quad 0 \quad 1 + \Delta c_2 \quad 1 - 2\Delta c_2\,).$$

For optimality, we need $1 + \Delta c_2 \ge 0$ and $1 - 2\Delta c_2 \ge 0$, so $-1 \le \Delta c_2 \le 1/2$. Hence, the allowable range for c_2 is $-2 - 1 = -3 \le c_2 \le -2 + 1/2 = -3/2$.

(c)



The allowable range for b_1 is $b_1 \geq 20$.



The allowable range for b_2 is $b_2 \leq 15$.

(d) If we increase b_1 by Δb_1, the final right-hand side becomes:

$$S^*\overline{b} = \begin{pmatrix} 1 & -2 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 30 + \Delta b_1 \\ 10 \end{pmatrix} = \begin{pmatrix} 10 \\ 20 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} \Delta b_1.$$

In order to preserve feasibility, $\Delta b_1 \geq -10$, so the allowable range for b_1 is $b_1 \geq 20$. Similarly, if b_2 is increased by Δb_2, the final right-hand side becomes:

$$S^*\overline{b} = \begin{pmatrix} 1 & -2 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 30 \\ 10 + \Delta b_2 \end{pmatrix} = \begin{pmatrix} 10 \\ 20 \end{pmatrix} + \begin{pmatrix} -2 \\ -1 \end{pmatrix} \Delta b_2.$$

In order to preserve feasibility, $\Delta b_2 \leq 5$, so the allowable range for b_2 is $b_2 \leq 15$.

(e) (in MPL)

```
MAX 3x1-2x2;

SUBJECT TO
2x1-x2<=30;
x1-x2<=10;
```

Variable Name	Coefficient	Lower Range	Upper Range
x1	3.0000	2.0000	4.0000
x2	-2.0000	-3.0000	-1.5000

RANGES RHS

PLAIN CONSTRAINTS

Constraint Name	RHS Value	Lower Bound	Upper Bound
c1	30.0000	20.0000	1E+020
c2	10.0000	-1E+020	15.0000

7.2-9.

If we increase b_i by Δb_i, the final right-hand side becomes:

$$b^* = S^*\overline{b} = \begin{pmatrix} 1 & 0 & 0 \\ -\frac{3}{4} & 0 & \frac{1}{4} \\ \frac{9}{4} & 1 & -\frac{3}{4} \end{pmatrix} \begin{pmatrix} 4 + \Delta b_1 \\ 24 + \Delta b_2 \\ 18 + \Delta b_3 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \\ \frac{3}{2} \\ \frac{39}{2} \end{pmatrix} + \begin{pmatrix} 1 \\ -\frac{3}{4} \\ \frac{9}{4} \end{pmatrix} \Delta b_1 + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \Delta b_2 + \begin{pmatrix} 0 \\ \frac{1}{4} \\ -\frac{3}{4} \end{pmatrix} \Delta b_3.$$

Assuming $\Delta b_2 = \Delta b_3 = 0$, Δb_1 must satisfy:

$$4 + \Delta b_1 \geq 0 \Leftrightarrow \Delta b_1 \geq -4$$

$$\tfrac{3}{2} - \tfrac{3}{4}\Delta b_1 \geq 0 \Leftrightarrow \Delta b_1 \leq 2$$

$$\tfrac{39}{2} + \tfrac{9}{4}\Delta b_1 \geq 0 \Leftrightarrow \Delta b_1 \geq -\tfrac{78}{9}$$

$$\Leftrightarrow \; -4 \leq \Delta b_1 \leq 2 \Leftrightarrow 0 \leq b_1 \leq 6.$$

Assuming $\Delta b_1 = \Delta b_3 = 0$, Δb_2 must satisfy:

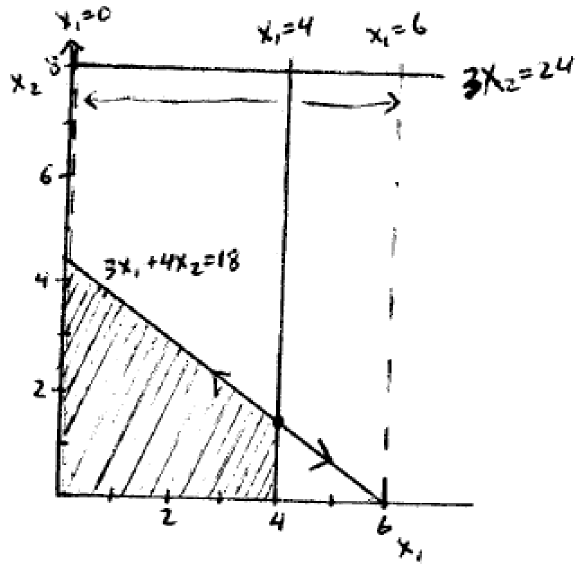
$$\tfrac{39}{2} + \Delta b_2 \geq 0 \Leftrightarrow \Delta b_2 \geq -\tfrac{39}{2} \Leftrightarrow b_2 \geq \tfrac{9}{2}.$$

Assuming $\Delta b_1 = \Delta b_2 = 0$, Δb_3 must satisfy:

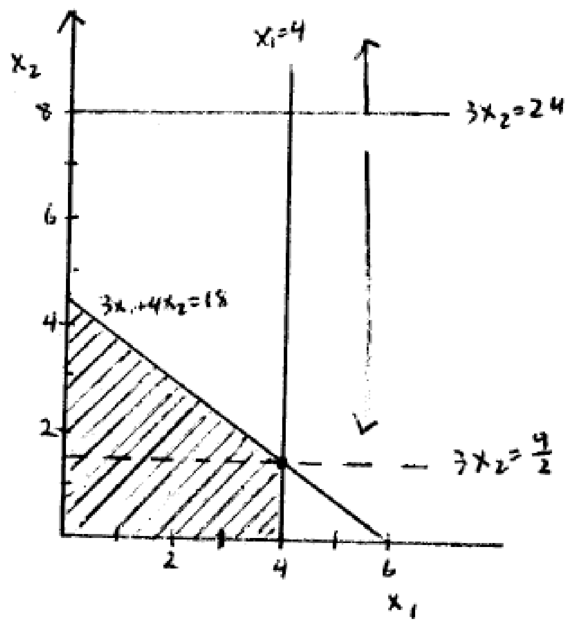
$$\tfrac{3}{2} + \tfrac{1}{4}\Delta b_3 \geq 0 \Leftrightarrow \Delta b_3 \geq -6$$

$$\tfrac{39}{2} - \tfrac{3}{4}\Delta b_3 \geq 0 \Leftrightarrow \Delta b_3 \leq 26$$

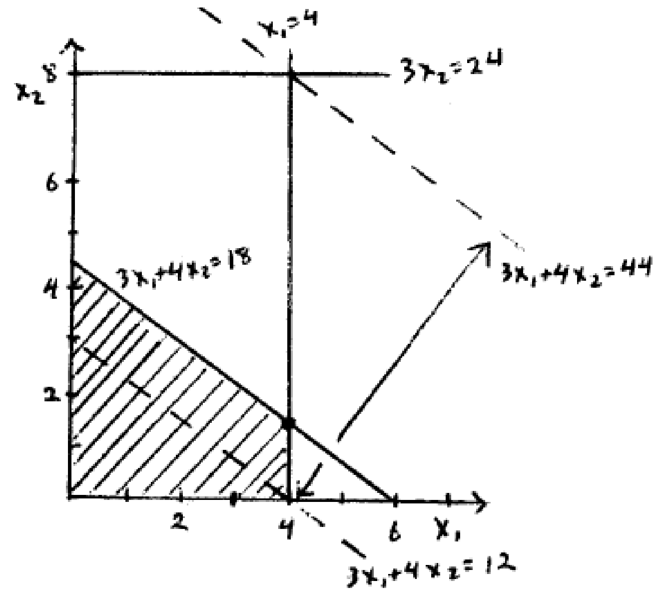
$$\Leftrightarrow 12 \leq b_3 \leq 44.$$



The allowable range for b_1 is $0 \leq b_1 \leq 6$.



The allowable range for b_2 is $9/2 \leq b_2$.



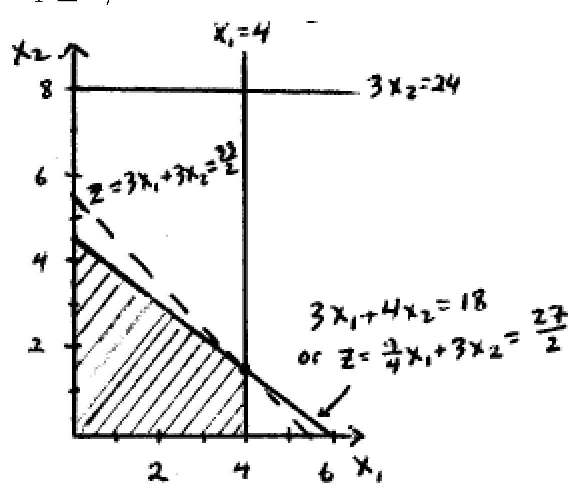
The allowable range for b_3 is $12 \leq b_3 \leq 44$.

7.2-10.

If we increment c_1 by Δc_1 ($c_1 = 3 + \Delta c_1$), the coefficient of x_1 in row 0 of the final tableau becomes $-\Delta c_1$. Add Δc_1 times row 1 to row 0 to get:

$$\begin{pmatrix} -\Delta c_1 & 0 & \frac{3}{4} & 0 & \frac{3}{4} \end{pmatrix} + \Delta c_1 \begin{pmatrix} 1 & 0 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & \frac{3}{4} + \Delta c_1 & 0 & \frac{3}{4} \end{pmatrix}.$$

For optimality, we need $(3/4) + \Delta c_1 \geq 0$, so $\Delta c_1 \geq -3/4$. Hence, the allowable range for c_1 is $c_1 \geq 9/4$.



The allowable range for c_1 is $c_1 \geq 9/4$. No matter how large c_1 gets, $(4, 3/2)$ stays optimal as long as $c_1 \geq 9/4$.

7.2-11.

If we increment c_2 by Δc_2 ($c_2 = 5 + \Delta c_2$), the coefficient of x_2 in row 0 of the final tableau becomes $-\Delta c_2$. Add Δc_2 times row 2 to row 0 to get:

$$\left(\tfrac{9}{2} \quad -\Delta c_2 \quad 0 \quad 0 \quad \tfrac{5}{2}\right) + \Delta c_2\left(\tfrac{3}{2} \quad 1 \quad 0 \quad 0 \quad \tfrac{1}{2}\right) = \left(\tfrac{9}{2} + \tfrac{3}{2}\Delta c_2 \quad 0 \quad 0 \quad 0 \quad \tfrac{5}{2} + \tfrac{1}{2}\Delta c_2\right).$$

For optimality, we need $(9/2) + (3/2)\Delta c_2 \geq 0$ and $(5/2) + (1/2)\Delta c_2 \geq 0$, so $\Delta c_2 \geq -3$, so the allowable range for c_2 is $c_2 \geq 2$. Looking at Figure 6.3, we see that if $c_2 = 2$, $Z = 3x_1 + 2x_2 = 18$ lies exactly on the constraint boundary. Thus, if c_2 is decreased any more, $(0, 9)$ does not remain optimal and the optimal solution becomes $(4, 3)$. On the other hand, as c_2 increases, the objective function gets closer to the horizontal line $Z = x_2 = 9$, so for any $c_2 \geq 2$, $(0, 9)$ stays optimal.

7.2-12.

(a) $$b^* = \begin{pmatrix} 1 & \frac{1}{3} & -\frac{1}{3} \\ 0 & \frac{1}{2} & 0 \\ 0 & -\frac{1}{3} & \frac{1}{3} \end{pmatrix} \begin{pmatrix} \Delta b_1 \\ 0 \\ 0 \end{pmatrix} \geq 0 \Leftrightarrow \begin{pmatrix} 2 \\ 6 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \Delta b_1 \geq 0$$

$$\Leftrightarrow \Delta b_1 \geq -2 \Leftrightarrow b_1 \geq 2$$

$$b^* = \begin{pmatrix} 1 & \frac{1}{3} & -\frac{1}{3} \\ 0 & \frac{1}{2} & 0 \\ 0 & -\frac{1}{3} & \frac{1}{3} \end{pmatrix} \begin{pmatrix} 0 \\ \Delta b_2 \\ 0 \end{pmatrix} \geq 0 \Leftrightarrow \begin{pmatrix} 2 \\ 6 \\ 2 \end{pmatrix} + \begin{pmatrix} \frac{1}{3} \\ \frac{1}{2} \\ -\frac{1}{3} \end{pmatrix} \Delta b_2 \geq 0$$

$$\Leftrightarrow -6 \leq \Delta b_2 \leq 6 \Leftrightarrow 6 \leq b_2 \leq 18$$

$$b^* = \begin{pmatrix} 1 & \frac{1}{3} & -\frac{1}{3} \\ 0 & \frac{1}{2} & 0 \\ 0 & -\frac{1}{3} & \frac{1}{3} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \Delta b_3 \end{pmatrix} \geq 0 \Leftrightarrow \begin{pmatrix} 2 \\ 6 \\ 2 \end{pmatrix} + \begin{pmatrix} -\frac{1}{3} \\ 0 \\ \frac{1}{3} \end{pmatrix} \Delta b_3 \geq 0$$

$$\Leftrightarrow -6 \leq \Delta b_3 \leq 6 \Leftrightarrow 12 \leq b_3 \leq 24$$

(b) $(\text{Row } 0) + \Delta c_1(\text{Row } 3) \geq 0 \Leftrightarrow \frac{3}{2} - \frac{1}{3}\Delta c_1 \geq 0$ and $1 + \frac{1}{3}\Delta c_1 \geq 0$

$$\Leftrightarrow -3 \leq \Delta c_1 \leq \tfrac{9}{2} \Leftrightarrow 0 \leq c_1 \leq \tfrac{15}{2}$$

$(\text{Row } 0) + \Delta c_2(\text{Row } 2) \geq 0 \Leftrightarrow \frac{3}{2} + \frac{1}{2}\Delta c_2 \geq 0$

$$\Leftrightarrow -3 \leq \Delta c_2 \Leftrightarrow 2 \leq c_2$$

(c) (in MPL)

```
MAX 3x1+5x2;

SUBJECT TO
x1<=4;
2x2<=12;
3x1+2x2<=18;
```

PLAIN CONSTRAINTS

Constraint Name	RHS Value	Lower Bound	Upper Bound
c1	4.0000	2.0000	1E+020
c2	12.0000	6.0000	18.0000
c3	18.0000	12.0000	24.0000

PLAIN VARIABLES

Variable Name	Coefficient	Lower Range	Upper Range
x1	3.0000	0.0000	7.5000
x2	5.0000	2.0000	1E+020

7.2-13.

(a) $$b^* = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} \\ 0 & 1 & 0 & -\frac{2}{3} \\ 0 & 0 & 1 & -\frac{2}{3} \end{pmatrix} \begin{pmatrix} 4+\Delta b_1 \\ 24 \\ 18 \\ 24 \end{pmatrix} \geq 0 \Leftrightarrow \begin{pmatrix} 4 \\ 8 \\ 8 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \Delta b_1 \geq 0$$

$\Leftrightarrow \Delta b_1 \geq -4 \Leftrightarrow b_1 \geq 0$

$$b^* = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} \\ 0 & 1 & 0 & -\frac{2}{3} \\ 0 & 0 & 1 & -\frac{2}{3} \end{pmatrix} \begin{pmatrix} 4 \\ 24+\Delta b_2 \\ 18 \\ 24 \end{pmatrix} \geq 0 \Leftrightarrow \begin{pmatrix} 4 \\ 8 \\ 8 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \Delta b_2 \geq 0$$

$\Leftrightarrow \Delta b_2 \geq -8 \Leftrightarrow b_2 \geq 16$

$$b^* = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} \\ 0 & 1 & 0 & -\frac{2}{3} \\ 0 & 0 & 1 & -\frac{2}{3} \end{pmatrix} \begin{pmatrix} 4 \\ 24 \\ 18+\Delta b_3 \\ 24 \end{pmatrix} \geq 0 \Leftrightarrow \begin{pmatrix} 4 \\ 8 \\ 8 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \Delta b_3 \geq 0$$

$\Leftrightarrow \Delta b_3 \geq -2 \Leftrightarrow b_3 \geq 16$

$$b^* = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} \\ 0 & 1 & 0 & -\frac{2}{3} \\ 0 & 0 & 1 & -\frac{2}{3} \end{pmatrix} \begin{pmatrix} 4 \\ 24 \\ 18 \\ 24+\Delta b_4 \end{pmatrix} \geq 0 \Leftrightarrow \begin{pmatrix} 4 \\ 8 \\ 8 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{3} \\ -\frac{2}{3} \\ -\frac{2}{3} \end{pmatrix} \Delta b_4 \geq 0$$

$\Leftrightarrow \Delta b_4 \geq -24,\ \Delta b_4 \leq 12,\ \Delta b_4 \leq 3 \Leftrightarrow 0 \leq b_4 \leq 27$

(b) Incrementing c_1 by Δc_1, the coefficient of x_1 in row 0 of the final tableau becomes $(1/3) - \Delta c_1$. In order for the solution to remain optimal, $(1/3) - \Delta c_1 \geq 0$, so

$$c_1 \leq 3 + \tfrac{1}{3} = \tfrac{10}{3}.$$

Incrementing c_2 by Δc_2, the coefficient of x_2 in row 0 of the final tableau becomes $-\Delta c_2$. Using row 2 to eliminate this coefficient, we get:

$$\begin{aligned}&\left(\tfrac{1}{3} \quad -\Delta c_2 \quad 0 \quad 0 \quad 0 \quad \tfrac{5}{3}\right) + \Delta c_2\left(\tfrac{2}{3} \quad 1 \quad 0 \quad 0 \quad 0 \quad \tfrac{1}{3}\right)\\ &= \left(\tfrac{1}{3} + \tfrac{2}{3}\Delta c_2 \quad 0 \quad 0 \quad 0 \quad 0 \quad \tfrac{5}{3} + \tfrac{1}{3}\Delta c_2\right).\end{aligned}$$

To keep optimality, we need:

$$\tfrac{1}{3} + \tfrac{2}{3}\Delta c_2 \geq 0 \text{ and } \tfrac{5}{3} + \tfrac{1}{3}\Delta c_2 \geq 0 \Leftrightarrow \Delta c_2 \geq -\tfrac{1}{2} \Leftrightarrow c_2 \geq \tfrac{9}{2}.$$

(c) (in MPL)

```
MAX 3x1+5x2;

SUBJECT TO
x1<=4;
2x2<=12;
3x1+2x2<=18;
2x1+3x2<=24;
```

PLAIN CONSTRAINTS

Constraint Name	RHS Value	Lower Bound	Upper Bound
c1	4.0000	2.0000	1E+020
c2	12.0000	6.0000	14.4000
c3	18.0000	12.0000	21.0000
c4	24.0000	22.0000	1E+020

PLAIN VARIABLES

Variable Name	Coefficient	Lower Range	Upper Range
x1	3.0000	0.0000	7.5000
x2	5.0000	2.0000	1E+020

7.2-14.

$$\Delta c_2 = 4 \Rightarrow \Delta(z_2^* - c_2) = -4$$

$$\Delta c_3 = 1 \Rightarrow \Delta(z_3^* - c_3) = -1$$

$$\Delta b_3 = -1 \Rightarrow \Delta Z^* = \begin{pmatrix} 2 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} = -1$$

$$\Delta b_1^* = \begin{pmatrix} 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} = 1$$

$$\Delta b_2^* = \begin{pmatrix} 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} = 0$$

$$\Delta b_3^* = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} = -1$$

New Tableau:

Bas Var	Eq No	Z	x_1	x_2	x_3	$\bar{x}_4$	x_5	$\bar{x}_6$	x_7	Right side
Z	0	1	0	1	-1	M+2	0	M	1	7
x_1	1	0	1	-1	0	1	0	0	-1	2
x_5	2	0	0	3	0	1	1	-1	0	2
x_3	3	0	0	2	1	0	0	0	1	1

Proper Form:

Bas Var	Eq No	Z	x_1	x_2	x_3	$\bar{x}_4$	x_5	$\bar{x}_6$	x_7	Right side
Z	0	1	0	3	0	M+2	0	M	2	8
x_1	1	0	1	-1	0	1	0	0	-1	2
x_5	2	0	0	3	0	1	1	-1	0	2
x_3	3	0	0	2	1	0	0	0	1	1

The current basic solution is feasible and optimal.

7.3-1.

(a)

	Activity 1	Activity 2			
Unit Profit	$2.00	$5.00			
	Resource Usage		Totals		Available
Resource 1	1	2	10	<=	10
Resource 2	1	3	12	<=	12
					Total Profit
Solution	6	2			$22.00

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$9	Solution X1	6	0	2	0.5	0.333333333
$C$9	Solution X2	2	0	5	1	1

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$5	Constraint 1 Totals	10	1	10	2	2
$D$6	Constraint 2 Totals	12	1	12	3	2

(b) The optimal solution is $(0, 4)$ if the unit profit for Activity 1 is \$1.

	Activity 1	Activity 2			
Unit Profit	\$1.00	\$5.00			
	Resource Usage		Totals		Available
Resource 1	1	2	8	<=	10
Resource 2	1	3	12	<=	12
					Total Profit
Solution	0	4			\$20.00

The optimal solution is $(10, 0)$ if the unit profit for Activity 1 is \$3.

	Activity 1	Activity 2			
Unit Profit	\$3.00	\$5.00			
	Resource Usage		Totals		Available
Resource 1	1	2	10	<=	10
Resource 2	1	3	10	<=	12
					Total Profit
Solution	10	0			\$30.00

(c) The optimal solution is $(10, 0)$ if the unit profit for Activity 2 is \$2.50.

	Activity 1	Activity 2			
Unit Profit	\$2.00	\$2.50			
	Resource Usage		Totals		Available
Resource 1	1	2	10	<=	10
Resource 2	1	3	10	<=	12
					Total Profit
Solution	10	0			\$20.00

The optimal solution is $(0, 4)$ if the unit profit for Activity 2 is \$7.50.

	Activity 1	Activity 2			
Unit Profit	\$2.00	\$7.50			
	Resource Usage		Totals		Available
Resource 1	1	2	8	<=	10
Resource 2	1	3	12	<=	12
					Total Profit
Solution	0	4			\$30.00

(d)

Unit Profit for	Solution		
Activity 1	Activity 1	Activity 2	Total Profit
$1.00	0	4	$20.00
$1.20	0	4	$20.00
$1.40	0	4	$20.00
$1.60	0	4	$20.00
$1.80	6	2	$20.80
$2.00	6	2	$22.00
$2.20	6	2	$23.20
$2.40	6	2	$24.40
$2.60	10	0	$26.00
$2.80	10	0	$28.00
$3.00	10	0	$30.00

Unit Profit for	Solution		
Activity 2	Activity 1	Activity 2	Total Profit
$2.50	10	0	$20.00
$3.00	10	0	$20.00
$3.50	10	0	$20.00
$4.00	6	2	$20.00
$4.50	6	2	$21.00
$5.00	6	2	$22.00
$5.50	6	2	$23.00
$6.00	0	4	$24.00
$6.50	0	4	$26.00
$7.00	0	4	$28.00
$7.50	0	4	$30.00

The allowable range for the unit profit of Activity 1 is approximately between $1.60 and $1.80 up to between $2.40 and $2.60. The allowable range for the unit profit of Activity 2 is between $3.50 and $4 up to between $5.50 and $6.

(e) The allowable range for the unit profit of Activity 1 is approximately between $1.67 and $2.50. The allowable range for the unit profit of Activity 2 is between $4 and $6.

Objective Coefficient

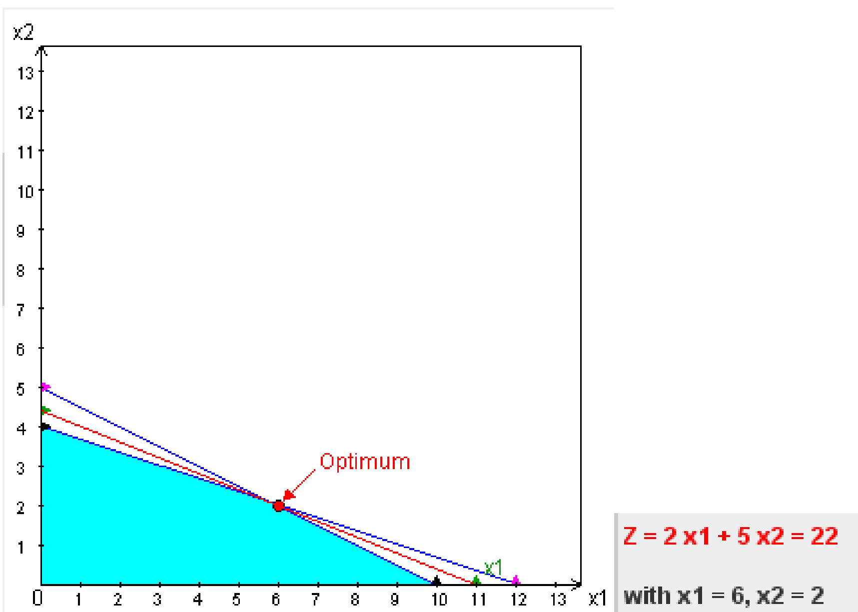
Current Value	Allowable Range to Stay Optimal	
	Minimum	Maximum
2	1.67	2.5
5	4	6

(f) The allowable range for the unit profit of Activity 1 is approximately between $1.67 and $2.50. The allowable range for the unit profit of Activity 2 is between $4 and $6.

(g)

Total Profit	Unit Profit for Activity 2										
Unit Profit for Activity 1	$2.50	$3.00	$3.50	$4.00	$4.50	$5.00	$5.50	$6.00	$6.50	$7.00	$7.50
$1.00	$11.00	$12.00	$14.00	$16.00	$18.00	$20.00	$22.00	$24.00	$26.00	$28.00	$30.00
$1.20	$12.20	$13.20	$14.20	$16.00	$18.00	$20.00	$22.00	$24.00	$26.00	$28.00	$30.00
$1.40	$14.00	$14.40	$15.40	$16.40	$18.00	$20.00	$22.00	$24.00	$26.00	$28.00	$30.00
$1.60	$16.00	$16.00	$16.60	$17.60	$18.60	$20.00	$22.00	$24.00	$26.00	$28.00	$30.00
$1.80	$18.00	$18.00	$18.00	$18.80	$19.80	$20.80	$22.00	$24.00	$26.00	$28.00	$30.00
$2.00	$20.00	$20.00	$20.00	$20.00	$21.00	$22.00	$23.00	$24.00	$26.00	$28.00	$30.00
$2.20	$22.00	$22.00	$22.00	$22.00	$22.20	$23.20	$24.20	$25.20	$26.20	$28.00	$30.00
$2.40	$24.00	$24.00	$24.00	$24.00	$24.00	$24.40	$25.40	$26.40	$27.40	$28.40	$30.00
$2.60	$26.00	$26.00	$26.00	$26.00	$26.00	$26.00	$26.60	$27.60	$28.60	$29.60	$30.60
$2.80	$28.00	$28.00	$28.00	$28.00	$28.00	$28.00	$28.00	$28.80	$29.80	$30.80	$31.80
$3.00	$30.00	$30.00	$30.00	$30.00	$30.00	$30.00	$30.00	$30.00	$31.00	$32.00	$33.00

(h) Keeping the unit profit of Activity 2 fixed, the unit profit of Activity 1 cannot be changed to less than 1.67 or more than 2.5 without changing the optimal solution. Similarly if the unit profit of Activity 1 is fixed at 1, the unit profit of Activity 2 needs to stay between 4 and 6 so that the optimal solution remains the same. Otherwise, the objective function line becomes either too flat or too steep and the optimal solution becomes $(0, 4)$ or $(10, 0)$.



7.3-2.

(a) The original model:

	Activity 1	Activity 2			
Unit Profit	$2.00	$5.00			
	Resource Usage		Totals		Available
Resource 1	1	2	10	<=	10
Resource 2	1	3	12	<=	12
					Total Profit
Solution	6	2			$22.00

With one additional unit of resource 1:

	Activity 1	Activity 2			
Unit Profit	$2.00	$5.00			
	Resource Usage		Totals		Available
Resource 1	1	2	11	<=	11
Resource 2	1	3	12	<=	12
					Total Profit
Solution	9	1			$23.00

The shadow price (the increase in total profit) is $1.

(b) The shadow price of $1 is valid in the range of 8 to 12.

Available	Solution		Total	Incremental
Resource 1	Activity 1	Activity 2	Profit	Profit
5	0	2.5	$12.50	
6	0	3	$15.00	$2.50
7	0	3.5	$17.50	$2.50
8	0	4	$20.00	$2.50
9	3	3	$21.00	$1.00
10	6	2	$22.00	$1.00
11	9	1	$23.00	$1.00
12	12	0	$24.00	$1.00
13	12	0	$24.00	$0.00
14	12	0	$24.00	$0.00
15	12	0	$24.00	$0.00

(c) With one additional unit of resource 2:

	Activity 1	Activity 2			
Unit Profit	$2.00	$5.00			
	Resource Usage		Totals		Available
Resource 1	1	2	10	<=	10
Resource 2	1	3	13	<=	13
					Total Profit
Solution	4	3			$23.00

The shadow price (the increase in total profit) is $1.

(d) The shadow price of $1 is valid in the range of 10 to 15.

Available	Solution		Total	Incremental
Resource 2	Activity 1	Activity 2	Profit	Profit
6	6	0	$12.00	
7	7	0	$14.00	$2.00
8	8	0	$16.00	$2.00
9	9	0	$18.00	$2.00
10	10	0	$20.00	$2.00
11	8	1	$21.00	$1.00
12	6	2	$22.00	$1.00
13	4	3	$23.00	$1.00
14	2	4	$24.00	$1.00
15	0	5	$25.00	$1.00
16	0	5	$25.00	$0.00
17	0	5	$25.00	$0.00
18	0	5	$25.00	$0.00

(e) From the sensitivity report, the shadow prices for both constraints are $1. According to the allowable increase and decrease, the allowable range for the right-hand side of the first constraint is 8 to 12. Similarly, the allowable range for the right-hand side of the second constraint is 10 to 15.

Variable Cells

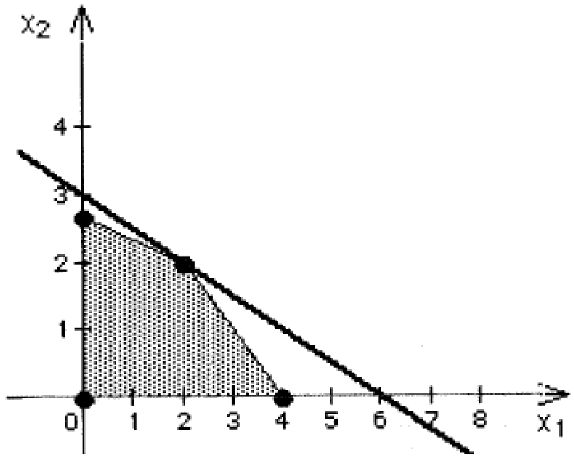
Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$9	Solution Resource Usage	6	0	2	0.5	0.333333333
$C$9	Solution Activity 2	2	0	5	1	1

Constraints

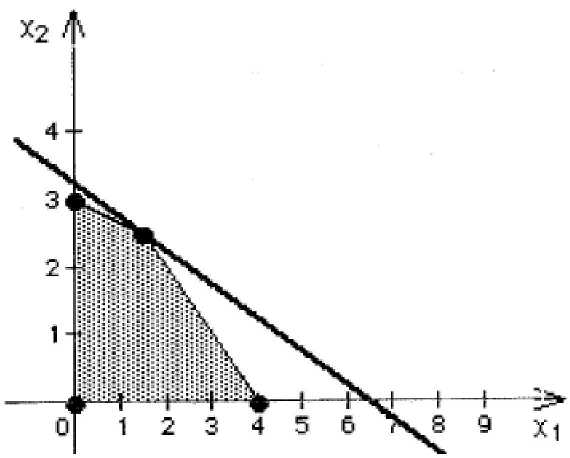
Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$5	Resource 1 Totals	10	1	10	2	2
$D$6	Resource 2 Totals	12	1	12	3	2

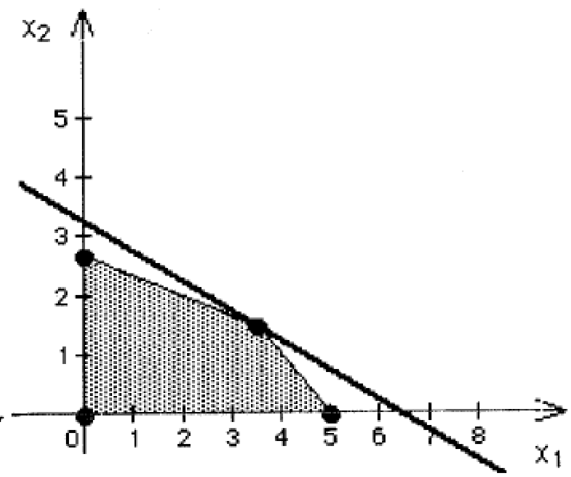
7.3-3.

(a) Optimal Solution: $(x_1, x_2) = (2, 2)$, with profit 6.



(b) When one unit is added to resource 1, the profit increases to 6.5, so the shadow price for resource 1 is 0.5. When one unit is added to resource 2, the profit increases to 6.5, so the shadow price for resource 2 is 0.5.





(c) The original model:

	Activity 1	Activity 2			
Unit Profit	1	2			
	Resource Usage		Totals		Available
Resource 1	1	3	8	<=	8
Resource 2	1	1	4	<=	4
					Total Profit
Solution	2	2			6

The shadow price for resource 1 is 0.5.

	Activity 1	Activity 2			
Unit Profit	1	2			
	Resource Usage		Totals		Available
Resource 1	1	3	9	<=	9
Resource 2	1	1	4	<=	4
					Total Profit
Solution	1.5	2.5			6.5

The shadow price for resource 2 is 0.5.

	Activity 1	Activity 2			
Unit Profit	1	2			
	Resource Usage		Totals		Available
Resource 1	1	3	8	<=	8
Resource 2	1	1	5	<=	5
					Total Profit
Solution	3.5	1.5			6.5

(d) The allowable range for the right-hand side of the resource 1 constraint is approximately between 4 (or less) and 12.

Available	Solution		Total	Incremental
Resource 1	**Activity 1**	**Activity 2**	**Profit**	**Profit**
4	4	0	4	
5	3.5	0.5	4.5	0.5
6	3	1	5	0.5
7	2.5	1.5	5.5	0.5
8	2	2	6	0.5
9	1.5	2.5	6.5	0.5
10	1	3	7	0.5
11	0.5	3.5	7.5	0.5
12	0	4	8	0.5
13	0	4	8	0
14	0	4	8	0

The allowable range for the right-hand side of the resource 2 constraint is approximately between 3 and 8.

Available	Solution		Total	Incremental
Resource 2	Activity 1	Activity 2	Profit	Profit
0	0	0	0	
1	0	1	2	2
2	0	2	4	2
3	0.5	2.5	5.5	1.5
4	2	2	6	0.5
5	3.5	1.5	6.5	0.5
6	5	1	7	0.5
7	6.5	0.5	7.5	0.5
8	8	0	8	0.5
9	8	0	8	0
10	8	0	8	0

(e) The shadow price for both resources is 0.5. The allowable range for the right-hand side of the first resource is between 4 and 12 and that of the second resource is between 2.667 and 8.

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$9	Solution Resource Usage	2	0	1	1	0.33333
$C$9	Solution Activity 2	2	0	2	1	1

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$5	Resource 1 Totals	8	0.5	8	4	4
$D$6	Resource 2 Totals	4	0.5	4	4	1.33333

(f) These shadow prices tell management that for each additional unit of resource, the profit increases by 0.5 (for small changes). Management is then able to evaluate whether or not to change the available amount of resources.

7.3-4.

(a)

	Toys	Subassemblies			
Unit Profit	$3.00	-$2.50			
	Resource Usage		Used		Available
Subassembly A	2	-1	3,000	<=	3,000
Subassembly B	1	-1	1,000	<=	1,000
	Toys	Subassemblies			Total Profit
Production	2,000	1,000			$3,500

(b)

Unit Profit for Toys	Optimal Production Rates		Total
	Toys	Subassemblies	Profit
$2.00	1000	0	$2000
$2.50	1000	0	$2500
$3.00	2000	1000	$3500
$3.50	2000	1000	$4500
$4.00	2000	1000	$5500

The estimate of the unit profit for toys can be off by something between 0 and 0.50 before the optimal solution changes. There is no change in the solution for an increase in the unit profit for toys, at least for an increase up to $1.

(c)

Unit Profit for Subassemblies	Optimal Production Rates		Total
	Toys	Subassemblies	Profit
-$3.50	1000	0	$3000
-$3.00	1000	0	$3000
-$2.50	2000	1000	$3500
-$2.00	2000	1000	$4000
-$1.50	2000	1000	$4500

The estimate of the unit profit for subassemblies can be off by something between 0 and 0.50 before the optimal solution changes. There is no change in the solution for an increase in the unit profit for subassemblies, at least for an increase up to $1.

(d) A parameter analysis report for the change in unit profit for toys as in (b):

Unit Profit for Toys	Toys	Subassemblies	Total Profit
$2.00	1,000	0	$2,000
$2.25	1,000	0	$2,250
$2.50	1,000	0	$2,500
$2.75	2,000	1,000	$3,000
$3.00	2,000	1,000	$3,500
$3.25	2,000	1,000	$4,000
$3.50	2,000	1,000	$4,500
$3.75	2,000	1,000	$5,000
$4.00	2,000	1,000	$5,500

A parameter analysis report for the change in unit profit for subassemblies as in (c):

Unit Profit for Subassemblies	Toys	Subassemblies	Total Profit
-$3.50	1,000	0	$3,000
-$3.25	1,000	0	$3,000
-$3.00	1,000	0	$3,000
-$2.75	2,000	1,000	$3,250
-$2.50	2,000	1,000	$3,500
-$2.25	2,000	1,000	$3,750
-$2.00	2,000	1,000	$4,000
-$1.75	2,000	1,000	$4,250
-$1.50	2,000	1,000	$4,500

(e) The unit profit for toys can vary between $2.50 and $5 before the solution changes. For subassemblies, the unit profit can change between -$3 and -1.50 before the solution changes.

(f) The allowable range of the unit profit for toys is $2.50 to $5 whereas that for subassemblies is -$3 to -$1.50.

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$9	Production Toys	2,000	0	3	2	0.5
$C$9	Production Subassemblies	1,000	0	-2.5	1	0.5

(g)

Total Profit	Unit Profit for Subassemblies								
Unit Profit Toys	-$3.50	-$3.25	-$3.00	-$2.75	-$2.50	-$2.25	-$2.00	-$1.75	-$1.50
$2.00	$2,000	$2,000	$2,000	$2,000	$2,000	$2,000	$2,000	$2,250	$2,500
$2.25	$2,250	$2,250	$2,250	$2,250	$2,250	$2,250	$2,500	$2,750	$3,000
$2.50	$2,500	$2,500	$2,500	$2,500	$2,500	$2,750	$3,000	$3,250	$3,500
$2.75	$2,750	$2,750	$2,750	$2,750	$3,000	$3,250	$3,500	$3,750	$4,000
$3.00	$3,000	$3,000	$3,000	$3,250	$3,500	$3,750	$4,000	$4,250	$4,500
$3.25	$3,250	$3,250	$3,500	$3,750	$4,000	$4,250	$4,500	$4,750	$0
$3.50	$3,500	$3,750	$4,000	$4,250	$4,500	$4,750	$5,000	$5,250	$0
$3.75	$4,000	$4,250	$4,500	$4,750	$5,000	$5,250	$5,500	$0	$0
$4.00	$4,500	$4,750	$5,000	$5,250	$5,500	$5,750	$6,000	$0	$0

(h) As long as the sum of the percentage change of the unit profit for subassemblies does not exceed 100% (where the allowable range is given in part (f)), the solution does not change.

7.3-5.

(a)

	Toys	Subassemblies			
Unit Profit	$3.00	-$2.50			
	Resource Usage		Used		Available
Subassembly A	2	-1	3,000	<=	3,000
Subassembly B	1	-1	1,000	<=	1,000
	Toys	Subassemblies			Total Profit
Production	2,000	1,000			$3,500
	<=				
	2500				

(b)

	Toys	Subassemblies			
Unit Profit	$3.00	-$2.50			
	Resource Usage		Used		Available
Subassembly A	2	-1	3,001	<=	3,001
Subassembly B	1	-1	1,000	<=	1,000
	Toys	Subassemblies			Total Profit
Production	2,001	1,001			$3,500.50
	<=				
	2500				

The shadow price for subassembly A is $0.50, which is the maximum premium that the company should be willing to pay.

(c)

	Toys	Subassemblies			
Unit Profit	$3.00	-$2.50			
	Resource Usage		Used		Available
Subassembly A	2	-1	3,000	<=	3,000
Subassembly B	1	-1	1,001	<=	1,001
	Toys	Subassemblies			Total Profit
Production	1,999	998			$3,502.00
	<=				
	2500				

The shadow price for subassembly B is $2, which is the maximum premium that the company should be willing to pay.

(d)

Availabile	Solution		Total	Incremental
Subassembly A	Toys	Subassemblies	Profit	Profit
3,000	2,000	1,000	$3,500	
3,100	2,100	1,100	$3,550	$50
3,200	2,200	1,200	$3,600	$50
3,300	2,300	1,300	$3,650	$50
3,400	2,400	1,400	$3,700	$50
3,500	2,500	1,500	$3,750	$50
3,600	2,500	1,500	$3,750	$0
3,700	2,500	1,500	$3,750	$0
3,800	2,500	1,500	$3,750	$0
3,900	2,500	1,500	$3,750	$0
4,000	2,500	1,500	$3,750	$0

The shadow price is still valid until the maximum supply of subassembly A is at least 3,500.

(e)

Availabile	Solution		Total	Incremental
Subassembly B	Toys	Subassemblies	Profit	Profit
1,000	2,000	1,000	$3,500	
1,100	1,900	800	$3,700	$200
1,200	1,800	600	$3,900	$200
1,300	1,700	400	$4,100	$200
1,400	1,600	200	$4,300	$200
1,500	1,500	0	$4,500	$200
1,600	1,500	0	$4,500	$0
1,700	1,500	0	$4,500	$0
1,800	1,500	0	$4,500	$0
1,900	1,500	0	$4,500	$0
2,000	1,500	0	$4,500	$0

The shadow price is still valid until the maximum supply of subassembly A is at least 1,500.

(f)

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$9	Production Toys	2000	0	3	2	0.5
$C$9	Production Subassemblies	1000	0	-2.5	1	0.5

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$5	Subassembly A Used	3000	0.5	3000	500	1000
$D$6	Subassembly B Used	1000	2	1000	500	500

As shown in the sensitivity report, the shadow price is $0.50 for subassembly A and $2 for subassembly B. According to the allowable increase and decrease, the allowable range for the right-hand side of the subassembly A constraint is 2,000 to 3,500. The allowable range for the right-hand side of the subassembly B constraint is 500 to 1,500.

7.3-6.

Original Solution:

	6am-2pm Shift	8am-4pm Shift	Noon-8pm Shift	4pm-midnight Shift	10pm-6am Shift			
Cost per Shift	$170	$160	$175	$180	$195			
						Total		Minimum
Time Period	Shift Works Time Period? (1=yes, 0=no)					Working		Needed
6am-8am	1	0	0	0	0	48	>=	48
8am-10am	1	1	0	0	0	79	>=	79
10am- 12pm	1	1	0	0	0	79	>=	65
12pm-2pm	1	1	1	0	0	118	>=	87
2pm-4pm	0	1	1	0	0	70	>=	64
4pm-6pm	0	0	1	1	0	82	>=	73
6pm-8pm	0	0	1	1	0	82	>=	82
8pm-10pm	0	0	0	1	0	43	>=	43
10pm-12am	0	0	0	1	1	58	>=	52
12am-6am	0	0	0	0	1	15	>=	15
	6am-2pm Shift	8am-4pm Shift	Noon-8pm Shift	4pm-midnight Shift	10pm-6am Shift			Total Cost
Number Working	48	31	39	43	15			$30,610

(a) The optimal solution does not change.

	6am-2pm Shift	8am-4pm Shift	Noon-8pm Shift	4pm-midnight Shift	10pm-6am Shift			
Cost per Shift	$170	$165	$175	$180	$195			
						Total		Minimum
Time Period	Shift Works Time Period? (1=yes, 0=no)					Working		Needed
6am-8am	1	0	0	0	0	48	>=	48
8am-10am	1	1	0	0	0	79	>=	79
10am- 12pm	1	1	0	0	0	79	>=	65
12pm-2pm	1	1	1	0	0	118	>=	87
2pm-4pm	0	1	1	0	0	70	>=	64
4pm-6pm	0	0	1	1	0	82	>=	73
6pm-8pm	0	0	1	1	0	82	>=	82
8pm-10pm	0	0	0	1	0	43	>=	43
10pm-12am	0	0	0	1	1	58	>=	52
12am-6am	0	0	0	0	1	15	>=	15
	6am-2pm Shift	8am-4pm Shift	Noon-8pm Shift	4pm-midnight Shift	10pm-6am Shift			Total Cost
Number Working	48	31	39	43	15			$30,765

(b) The optimal solution changes.

	6am-2pm Shift	8am-4pm Shift	Noon-8pm Shift	4pm-midnight Shift	10pm-6am Shift			
Cost per Shift	$170	$160	$175	$170	$195			
						Total		Minimum
Time Period		Shift Works Time Period? (1=yes, 0=no)				Working		Needed
6am-8am	1	0	0	0	0	48	>=	48
8am-10am	1	1	0	0	0	79	>=	79
10am- 12pm	1	1	0	0	0	79	>=	65
12pm-2pm	1	1	1	0	0	112	>=	87
2pm-4pm	0	1	1	0	0	64	>=	64
4pm-6pm	0	0	1	1	0	82	>=	73
6pm-8pm	0	0	1	1	0	82	>=	82
8pm-10pm	0	0	0	1	0	49	>=	43
10pm-12am	0	0	0	1	1	64	>=	52
12am-6am	0	0	0	0	1	15	>=	15
	6am-2pm Shift	8am-4pm Shift	Noon-8pm Shift	4pm-midnight Shift	10pm-6am Shift			Total Cost
Number Working	48	31	33	49	15			$30,150

(c) The optimal solution changes.

	6am-2pm Shift	8am-4pm Shift	Noon-8pm Shift	4pm-midnight Shift	10pm-6am Shift			
Cost per Shift	$170	$165	$175	$170	$195			
						Total		Minimum
Time Period		Shift Works Time Period? (1=yes, 0=no)				Working		Needed
6am-8am	1	0	0	0	0	48	>=	48
8am-10am	1	1	0	0	0	79	>=	79
10am- 12pm	1	1	0	0	0	79	>=	65
12pm-2pm	1	1	1	0	0	112	>=	87
2pm-4pm	0	1	1	0	0	64	>=	64
4pm-6pm	0	0	1	1	0	82	>=	73
6pm-8pm	0	0	1	1	0	82	>=	82
8pm-10pm	0	0	0	1	0	49	>=	43
10pm-12am	0	0	0	1	1	64	>=	52
12am-6am	0	0	0	0	1	15	>=	15
	6am-2pm Shift	8am-4pm Shift	Noon-8pm Shift	4pm-midnight Shift	10pm-6am Shift			Total Cost
Number Working	48	31	33	49	15			$30,305

(d) The optimal solution does not change.

	Shift	Shift	Shift	Shift	Shift			
Cost per Shift	$166	$164	$171	$184	$199			
						Total		Minimum
Time Period		Shift Works Time Period? (1=yes, 0=no)				Working		Needed
6am-8am	1	0	0	0	0	48	>=	48
8am-10am	1	1	0	0	0	79	>=	79
10am- 12pm	1	1	0	0	0	79	>=	65
12pm-2pm	1	1	1	0	0	118	>=	87
2pm-4pm	0	1	1	0	0	70	>=	64
4pm-6pm	0	0	1	1	0	82	>=	73
6pm-8pm	0	0	1	1	0	82	>=	82
8pm-10pm	0	0	0	1	0	43	>=	43
10pm-12am	0	0	0	1	1	58	>=	52
12am-6am	0	0	0	0	1	15	>=	15
	6am-2pm	8am-4pm	Noon-8pm	4pm-midnight	10pm-6am			
	Shift	Shift	Shift	Shift	Shift			Total Cost
Number Working	48	31	39	43	15			$30,618

(e) The optimal solution does not change.

	6am-2pm	8am-4pm	Noon-8pm	4pm-midnight	10pm-6am			
	Shift	Shift	Shift	Shift	Shift			
Cost per Shift	$173.40	$163.20	$178.50	$183.60	$198.90			
						Total		Minimum
Time Period		Shift Works Time Period? (1=yes, 0=no)				Working		Needed
6am-8am	1	0	0	0	0	48	>=	48
8am-10am	1	1	0	0	0	79	>=	79
10am- 12pm	1	1	0	0	0	79	>=	65
12pm-2pm	1	1	1	0	0	118	>=	87
2pm-4pm	0	1	1	0	0	70	>=	64
4pm-6pm	0	0	1	1	0	82	>=	73
6pm-8pm	0	0	1	1	0	82	>=	82
8pm-10pm	0	0	0	1	0	43	>=	43
10pm-12am	0	0	0	1	1	58	>=	52
12am-6am	0	0	0	0	1	15	>=	15
	6am-2pm	8am-4pm	Noon-8pm	4pm-midnight	10pm-6am			
	Shift	Shift	Shift	Shift	Shift			Total Cost
Number Working	48	31	39	43	15			$31,222

(f)

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$C$21	Number Working Shift	48	0	170	1E+30	10
$D$21	Number Working Shift	31	0	160	10	160
$E$21	Number Working Shift	39	0	175	5	175
$F$21	Number Working Shift	43	0	180	1E+30	5
$G$21	Number Working Shift	15	0	195	1E+30	195

Part (a): The optimal solution does not change (within the allowable increase of $10).

Part (b): The optimal solution does change (outside the allowable decrease of $5).

Part (c): Percentage of allowable increase for shift 2: $(165 - 160)/10 = 50\%$

Percentage of allowable decrease for shift 4: $(180 - 170)/5 = 200\%$

Sum: 250%

The optimal solution may or may not change.

Part (d): Percentage of allowable decrease for shift 1: $(170 - 166)/10 = 40\%$

Percentage of allowable increase for shift 2: $(164 - 160)/10 = 40\%$

Percentage of allowable decrease for shift 3: $(175 - 171)/175 = 2\%$

Percentage of allowable increase for shift 4: $(184 - 180)/\infty = 0\%$

Percentage of allowable increase for shift 5: $(199 - 194)/\infty = 0\%$

Sum: 82%

The optimal solution does not change.

Part (e): Percentage of allowable increase for shift 1: $(173.40 - 170)/\infty = 0\%$

Percentage of allowable increase for shift 2: $(163.20 - 160)/10 = 32\%$

Percentage of allowable increase for shift 3: $(178.50 - 175)/5 = 70\%$

Percentage of allowable increase for shift 4: $(183.60 - 180)/\infty = 0\%$

Percentage of allowable increase for shift 5: $(198.90 - 195)/\infty = 0\%$

Sum: 102%

The optimal solution may or may not change.

(g)

Cost per Shift (6a-2p)	6a-2p	8a-4p	12p-10p	4p-12a	10p-6a	Total Cost
$155	54	25	39	43	15	$29,860
$158	54	25	39	43	15	$30,022
$161	48	31	39	43	15	$30,178
$164	48	31	39	43	15	$30,322
$167	48	31	39	43	15	$30,466
$170	48	31	39	43	15	$30,610
$173	48	31	39	43	15	$30,754
$176	48	31	39	43	15	$30,898
$179	48	31	39	43	15	$31,042
$182	48	31	39	43	15	$31,186
$185	48	31	39	43	15	$31,330

Cost per Shift (8a-4p)	6a-2p	8a-4p	12p-10p	4p-12a	10p-6a	Total Cost
$145	48	31	39	43	15	$30,145
$148	48	31	39	43	15	$30,238
$151	48	31	39	43	15	$30,331
$154	48	31	39	43	15	$30,424
$157	48	31	39	43	15	$30,517
$160	48	31	39	43	15	$30,610
$163	48	31	39	43	15	$30,703
$166	48	31	39	43	15	$30,796
$169	48	31	39	43	15	$30,889
$172	54	25	39	43	15	$30,970
$175	54	25	39	43	15	$31,045

Cost per Shift (12p-10p)	6a-2p	8a-4p	12p-10p	4p-12a	10p-6a	Total Cost
$160	48	31	39	43	15	$30,025
$163	48	31	39	43	15	$30,142
$166	48	31	39	43	15	$30,259
$169	48	31	39	43	15	$30,376
$172	48	31	39	43	15	$30,493
$175	48	31	39	43	15	$30,610
$178	48	31	39	43	15	$30,727
$181	48	31	33	49	15	$30,838
$184	48	31	33	49	15	$30,937
$187	48	31	33	49	15	$31,036
$190	48	31	33	49	15	$31,135

Cost per Shift (4p-12a)	6a-2p	8a-4p	12p-10p	4p-12a	10p-6a	Total Cost
$165	48	31	33	49	15	$29,905
$168	48	31	33	49	15	$30,052
$171	48	31	33	49	15	$30,199
$174	48	31	33	49	15	$30,346
$177	48	31	39	43	15	$30,481
$180	48	31	39	43	15	$30,610
$183	48	31	39	43	15	$30,739
$186	48	31	39	43	15	$30,868
$189	48	31	39	43	15	$30,997
$192	48	31	39	43	15	$31,126
$195	48	31	39	43	15	$31,255

Cost per Shift (10p-6a)	6a-2p	8a-4p	12p-10p	4p-12a	10p-6a	Total Cost
$180	48	31	39	43	15	$30,385
$183	48	31	39	43	15	$30,430
$186	48	31	39	43	15	$30,475
$189	48	31	39	43	15	$30,520
$192	48	31	39	43	15	$30,565
$195	48	31	39	43	15	$30,610
$198	48	31	39	43	15	$30,655
$201	48	31	39	43	15	$30,700
$204	48	31	39	43	15	$30,745
$207	48	31	39	43	15	$30,790
$210	48	31	39	43	15	$30,835

7.3-7.

	6am-2pm Shift	8am-4pm Shift	Noon-8pm Shift	4pm-midnight Shift	10pm-6am Shift			
Cost per Shift	$170	$160	$175	$180	$195			
						Total		Minimum
Time Period	Shift Works Time Period? (1=yes, 0=no)					Working		Needed
6am-8am	1	0	0	0	0	48	>=	48
8am-10am	1	1	0	0	0	79	>=	79
10am- 12pm	1	1	0	0	0	79	>=	65
12pm-2pm	1	1	1	0	0	118	>=	87
2pm-4pm	0	1	1	0	0	70	>=	64
4pm-6pm	0	0	1	1	0	82	>=	73
6pm-8pm	0	0	1	1	0	82	>=	82
8pm-10pm	0	0	0	1	0	43	>=	43
10pm-12am	0	0	0	1	1	58	>=	52
12am-6am	0	0	0	0	1	15	>=	15
	6am-2pm Shift	8am-4pm Shift	Noon-8pm Shift	4pm-midnight Shift	10pm-6am Shift			Total Cost
Number Working	48	31	39	43	15			$30,610

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$C$21	Number Working Shift	48	0	170	1E+30	10
$D$21	Number Working Shift	31	0	160	10	160
$E$21	Number Working Shift	39	0	175	5	175
$F$21	Number Working Shift	43	0	180	1E+30	5
$G$21	Number Working Shift	15	0	195	1E+30	195

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$H$8	6am-8am Working	48	10	48	6	48
$H$9	8am-10am Working	79	160	79	1E+30	6
$H$10	10am- 12pm Working	79	0	65	14	1E+30
$H$11	12pm-2pm Working	118	0	87	31	1E+30
$H$12	2pm-4pm Working	70	0	64	6	1E+30
$H$13	4pm-6pm Working	82	0	73	9	1E+30
$H$14	6pm-8pm Working	82	175	82	1E+30	6
$H$15	8pm-10pm Working	43	5	43	6	6
$H$16	10pm-12am Working	58	0	52	6	1E+30
$H$17	12am-6am Working	15	195	15	1E+30	6

(a) The following shifts can be increased by the indicated amounts without increasing the total cost:
Serve 10-12 a.m. → 14
Serve 12-2 p.m. → 31
Serve 2-4 p.m. → 6
Serve 4-6 p.m. → 9
Serve 10-12 p.m. → 6.

(b) For each of the following shifts, the total cost increases by the amount indicated per unit increase. These costs hold for the indicated increases.

Shift	Increased Cost	Valid for Increase
Serve 6-8 a.m.	\$10	6
Serve 8-10 a.m.	\$160	8
Serve 6-8 p.m.	\$175	8
Serve 8-10 p.m.	\$5	6
Serve 12-6 a.m.	\$195	8

(c) Percentage of allowable increase for 6-8 a.m.: $(49-48)/6 = 16.7\%$
Percentage of allowable increase for 8-10 a.m.: $(80-79)/\infty = 0\%$
Percentage of allowable increase for 6-8 p.m.: $(83-82)/\infty = 0\%$
Percentage of allowable increase for 8-10 p.m.: $(44-43)/6 = 16.7\%$
Percentage of allowable increase for 12-6 a.m.: $(16-15)/\infty = 0\%$
Sum: 33.4%

The shadow prices are still valid.

(d) Percentage of allowable increase for 6-8 a.m.: $(49-48)/6 = 16.7\%$
Percentage of allowable increase for 8-10 a.m.: $(80-79)/\infty = 0\%$
Percentage of allowable increase for 10-12 a.m.: $(66-65)/14 = 7.1\%$
Percentage of allowable increase for 12-2 p.m.: $(88-87)/31 = 3.2\%$
Percentage of allowable increase for 2-4 p.m.: $(65-64)/6 = 16.7\%$
Percentage of allowable increase for 4-6 p.m.: $(74-73)/9 = 11.1\%$
Percentage of allowable increase for 6-8 p.m.: $(83-82)/\infty = 0\%$
Percentage of allowable increase for 8-10 p.m.: $(44-43)/6 = 16.7\%$
Percentage of allowable increase for 10-12 p.m.: $(53-52)/6 = 16.7\%$
Percentage of allowable increase for 12-6 a.m.: $(16-15)/\infty = 0\%$
Sum: 88.2%

The shadow prices are still valid.

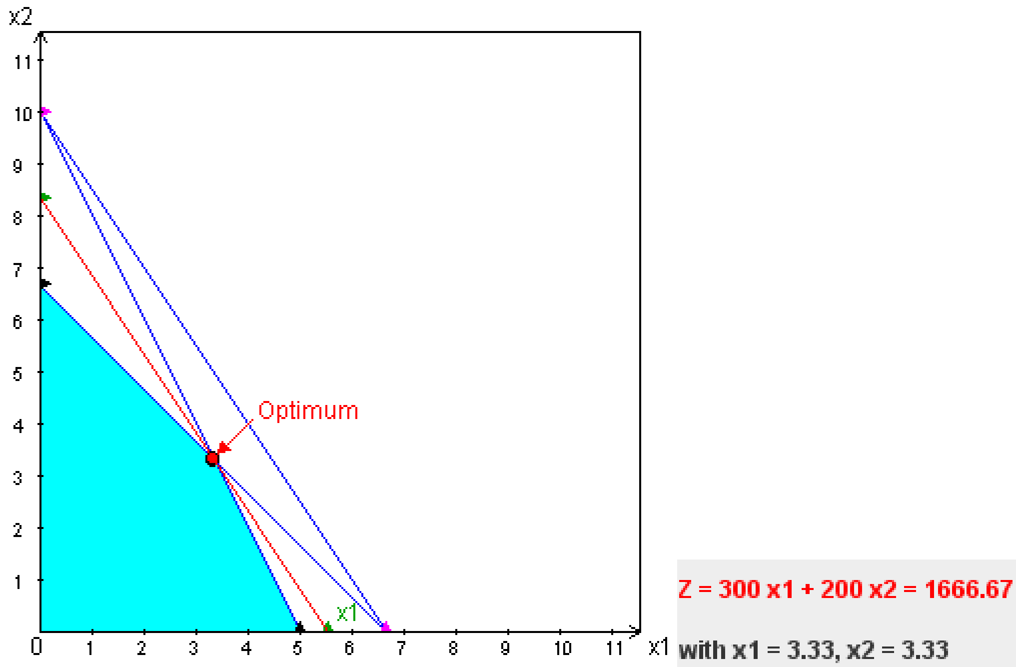
(e) All numbers can be increased by $100/88.2 \approx 1.13$ hours before it is no longer definite that the shadow prices remain valid.

7.3-8.

(a) Let x_G and x_W be the number of grandfather and wall clocks produced respectively.

$$\begin{array}{lrl} \text{maximize} & 300x_G + 200x_W & \\ \text{subject to} & 6x_G + \;\; 4x_W & \leq 40 \\ & 8x_G + \;\; 4x_W & \leq 40 \\ & 3x_G + \;\; 3x_W & \leq 20 \\ \text{and} & x_G, x_W & \geq 0 \end{array}$$

(b) Optimal Solution: $(x_G, x_W) = (3.33, 3.33)$, $Z^* = 1666.67$



Objective Coefficient

Current Value	Allowable Range to Stay Optimal	
	Minimum	Maximum
300	200	400
200	150	300

The unit profit for grandfather clocks is allowed to vary between \$200 and \$400, so if it changed from \$300 to \$375, the optimal solution would remain the same, provided that there are no other changes in the model. However, if in addition to this, the unit profit for wall clocks is changed to \$175, the optimal solution becomes $(5, 0)$.

(c) Using Excel Solver:

	Grandfather Clock	Wall Clock			
Unit Profit	\$300	\$200			
			Hours		Hours
	Time Required		Used		Available
Assembly (David)	6	4	33	<=	40
Carving (LaDeana)	8	4	40	<=	40
Shipping (Lydia)	3	3	20	<=	20
	Grandfather Clock	Wall Clock			Total Profit
Production	3.33	3.33			\$1,667

(d) Changing the unit profit of grandfather clocks to $375 does not change the optimal solution.

	Grandfather	Wall			
	Clock	Clock			
Unit Profit	$375	$200			
			Hours		Hours
	Time Required		Used		Available
Assembly (David)	6	4	33	<=	40
Carving (LaDeana)	8	4	40	<=	40
Shipping (Lydia)	3	3	20	<=	20
	Grandfather	Wall			
	Clock	Clock			Total Profit
Production	3.33	3.33			$1,917

If we also change the unit profit of wall clocks to $175, then the optimal solution changes to reflect the fact that it is now more profitable to produce only grandfather clocks.

	Grandfather	Wall			
	Clock	Clock			
Unit Profit	$375	$175			
			Hours		Hours
	Time Required		Used		Available
Assembly (David)	6	4	30	<=	40
Carving (LaDeana)	8	4	40	<=	40
Shipping (Lydia)	3	3	15	<=	20
	Grandfather	Wall			
	Clock	Clock			Total Profit
Production	5	0			$1,875

(e) From the parameter analysis report, the allowable range to stay optimal for the unit profit of grandfather clocks is roughly in the interval [$210, $390].

Unit Profit for Grandfather Clocks	Production of Grandfather Clocks	Production of Wall Clocks	Total Profit
$150	0.00	6.67	$1,333
$170	0.00	6.67	$1,333
$190	0.00	6.67	$1,333
$210	3.33	3.33	$1,367
$230	3.33	3.33	$1,433
$250	3.33	3.33	$1,500
$270	3.33	3.33	$1,567
$290	3.33	3.33	$1,633
$310	3.33	3.33	$1,700
$330	3.33	3.33	$1,767
$350	3.33	3.33	$1,833
$370	3.33	3.33	$1,900
$390	3.33	3.33	$1,967
$410	5.00	0.00	$2,050
$430	5.00	0.00	$2,150
$450	5.00	0.00	$2,250

From the parameter analysis report, the allowable range to stay optimal for the unit profit of wall clocks is roughly in the interval [$170, $290].

Unit Profit for Wall Clocks	Production of Grandfather Clocks	Production of Wall Clocks	Total Profit
$50	5.00	0.00	$1,500
$70	5.00	0.00	$1,500
$90	5.00	0.00	$1,500
$110	5.00	0.00	$1,500
$130	5.00	0.00	$1,500
$150	5.00	0.00	$1,500
$170	3.33	3.33	$1,567
$190	3.33	3.33	$1,633
$210	3.33	3.33	$1,700
$230	3.33	3.33	$1,767
$250	3.33	3.33	$1,833
$270	3.33	3.33	$1,900
$290	3.33	3.33	$1,967
$310	0.00	6.67	$2,067
$330	0.00	6.67	$2,200
$350	0.00	6.67	$2,333

(f)

Total Profit	Unit Profit (Grandfather Clock)						
Unit Profit (Wall Clock)	$150	$200	$250	$300	$350	$400	$450
$50	$750	$1,000	$1,250	$1,500	$1,750	$2,000	$2,250
$100	$833	$1,000	$1,250	$1,500	$1,750	$2,000	$2,250
$150	$1,000	$1,167	$1,333	$1,500	$1,750	$2,000	$2,250
$200	$1,333	$1,333	$1,500	$1,667	$1,833	$2,000	$2,250
$250	$1,667	$1,667	$1,667	$1,833	$2,000	$2,167	$2,333
$300	$2,000	$2,000	$2,000	$2,000	$2,167	$2,333	$2,500
$350	$2,333	$2,333	$2,333	$2,333	$2,333	$2,500	$2,667

(g) If David is available to work a maximum of 45 hours, the optimal solution and the total profit do not change. Even when he is available for 40 hours, he is required to use less.

	Grandfather Clock	Wall Clock			
Unit Profit	$300	$200			
			Hours		Hours
	Time Required		Used		Available
Assembly (David)	6	4	33	<=	45
Carving (LaDeana)	8	4	40	<=	40
Shipping (Lydia)	3	3	20	<=	20
	Grandfather Clock	Wall Clock			Total Profit
Production	3.33	3.33			$1,667

If LaDeana is available for 5 more hours every week, the optimal number of grandfather clocks to be produced increases whereas the optimal number of wall clocks to be produced decreases. The total profit increases by $125.

	Grandfather Clock	Wall Clock			
Unit Profit	$300	$200			
			Hours		Hours
	Time Required		Used		Available
Assembly (David)	6	4	36	<=	40
Carving (LaDeana)	8	4	45	<=	45
Shipping (Lydia)	3	3	20	<=	20
	Grandfather Clock	Wall Clock			Total Profit
Production	4.58	2.08			$1,792

Finally, if Lydia increases her availability by 5 hours, the optimal number of grandfather clocks to be produced decreases whereas the optimal number of wall clocks to be produced increases. The optimal total profit increases by $166, which is more than the increase caused by increasing LaDeana's working hours by the same amount.

	Grandfather Clock	Wall Clock			
Unit Profit	$300	$200			
			Hours		Hours
	Time Required		Used		Available
Assembly (David)	6	4	37	<=	40
Carving (LaDeana)	8	4	40	<=	40
Shipping (Lydia)	3	3	25	<=	25
	Grandfather Clock	Wall Clock			Total Profit
Production	1.67	6.67			$1,833

Note that in each case, the binding constraints remain the same.

(h)

Assembly Time Available	Grandfather Clocks	Wall Clocks	Total Profit
35	3.33	3.33	$1,667
37	3.33	3.33	$1,667
39	3.33	3.33	$1,667
41	3.33	3.33	$1,667
43	3.33	3.33	$1,667
45	3.33	3.33	$1,667

Carving Time Available	Grandfather Clocks	Wall Clocks	Total Profit
35	2.08	4.58	$1,542
37	2.58	4.08	$1,592
39	3.08	3.58	$1,642
41	3.58	3.08	$1,692
43	4.08	2.58	$1,742
45	4.58	2.08	$1,792

Shipping Time Available	Grandfather Clocks	Wall Clocks	Total Profit
15	5.00	0.00	$1,500
17	4.33	1.33	$1,567
19	3.67	2.67	$1,633
21	3.00	4.00	$1,700
23	2.33	5.33	$1,767
25	1.67	6.67	$1,833

(i) The unit profit for grandfather clocks should stay in the interval $[200, 400]$ and that for wall clocks should stay in $[150, 300]$ for the optimal solution to remain unchanged.

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$12	Production Clock	3.33	0.00	300	100	100
$C$12	Production Clock	3.33	0.00	200	100	50

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$6	Assembly (David) Used	33	0	40	1E+30	6.667
$D$7	Carving (LaDeana) Used	40	25	40	13.333	13.333
$D$8	Shipping (Lydia) Used	20	33.33	20	10	5

Provided that the maximum number of hours David is available is more than 33.334, the binding constraints stay the same. LaDeana's number of available hours can differ from 40 only by 13.333. Lydia's maximum number of hours is allowed to vary between 15 and 30.

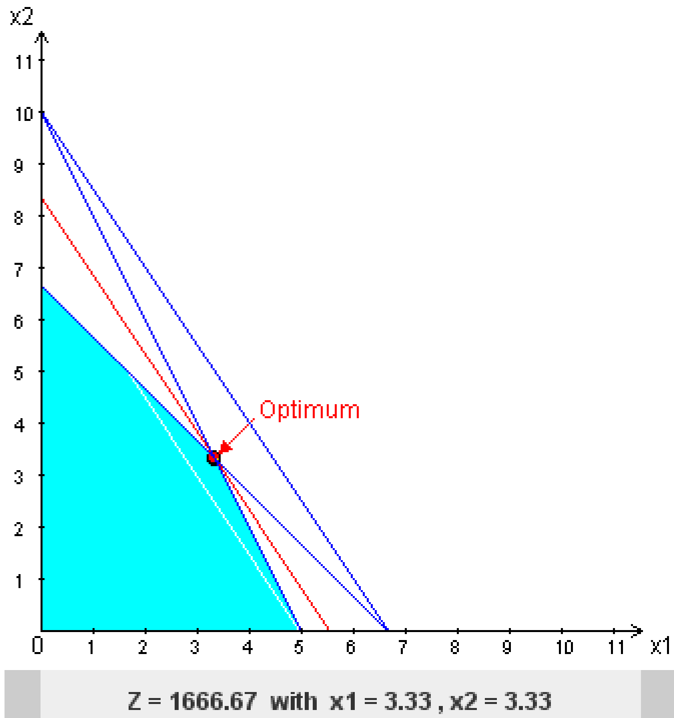
(j) The constraint associated with Lydia has the highest shadow price, so Lydia should be the one to increase the maximum number of hours available to work per week.

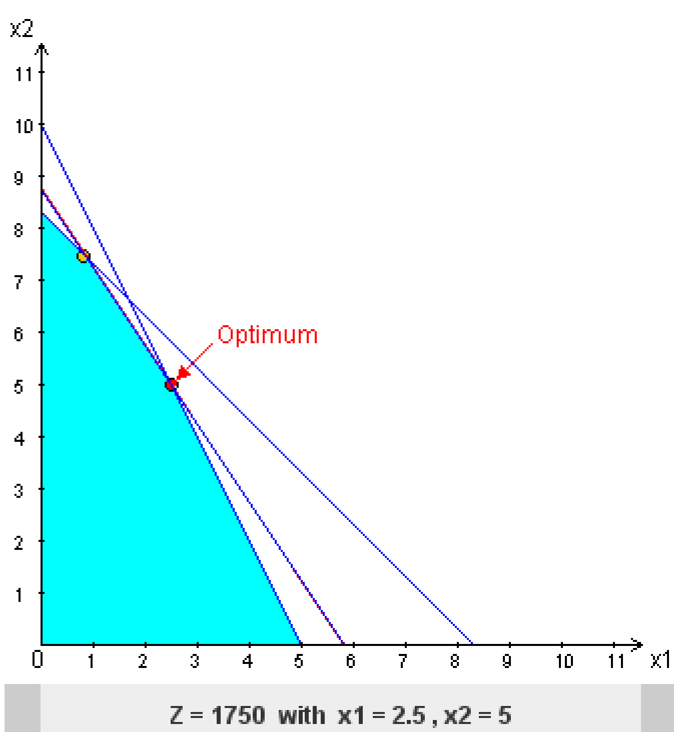
(k) The constraint associated with David is not binding in the optimal solution. In other words, David is required to work less than the maximum number of hours he is available. Hence increasing his availability does not improve the profit unless the other partners offer more time as well, so the shadow price of his constraint is equal to zero.

(l) The allowable increase for Lydia's hours is 10, so this shadow price can be used for an increase of 5. If Lydia increases her available hours from 20 to 25, the total profit is improved by approximately $5 \times 33.333 = \$166.665$, which is pretty close to what was found in part (g). The difference is due to rounding.

(m) Percentage of Lydia's allowable increase used $= (25 - 20)/10 = 50\%$.
Percentage of David's allowable decrease used $= (40 - 35)/6.6667 = 75\%$.
The sum is 125%, so by the 100% rule, the shadow prices may or may not be valid, and hence should not be used to determine the effect on total profit.

(n)


x2
11
10
9
8
7
6
5
4
3
2
1
0
1
2
3
4
5
6
7
8
9
10
11
x1
Optimum
Z = 1666.67 with x1 = 3.33 , x2 = 3.33


x2
11
10
9
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7
6
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3
2
1
0
1
2
3
4
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6
7
8
9
10
11
x1
Optimum
Z = 1750 with x1 = 2.5 , x2 = 5

7.4-1

(a) Applying the procedure for robust optimization with independent parameters, the model is as follows:

Maximize $Z = 2.75x_1 + 4.75x_2$

subject to

$1.1x_1 \quad \leq 3.8$

$2.1x_2 \quad \leq 11.5$

$3.25x_1 + 2.25x_2 \leq 17$

and

$x_1 \geq 0, x_2 \geq 0.$

The optimal solution is $x_1 = 1.44, x_2 = 5.48,$ and $Z = 29.971$ (a total profit of \$29,971 per week).

(b) The resulting solution provides \$4971 more profit per week than the solution of $x_1 = 1$ and $x_2 = 5$. Assuming there is only a small chance that the production rates would fall below the guaranteed minimum amounts, this additional profit would be worth the small chance of needing to pay a \$5000 penalty.

7.4-2

(a) The model using the estimates of the parameters is:

Maximize $Z = 3x_1 + 4x_2$

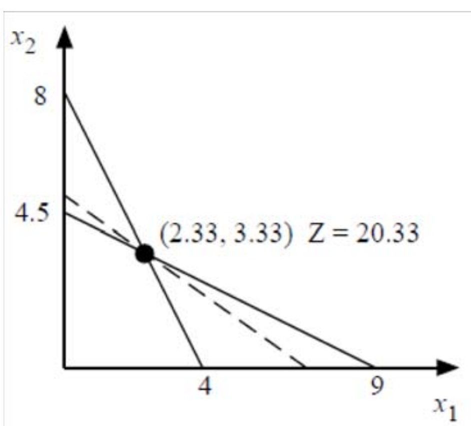
subject to

$x_1 + 2x_2 \leq 9$

$2x_1 + x_2 \leq 8$

and

$x_1 \geq 0, x_2 \geq 0.$



The optimal solution is $x_1 = 2.33, x_2 = 3.33,$ and $Z = 20.33.$

(b) Using robust optimization to formulate a conservative version, the model is:

Maximize $Z = 2.7x_1 + 3.6x_2$

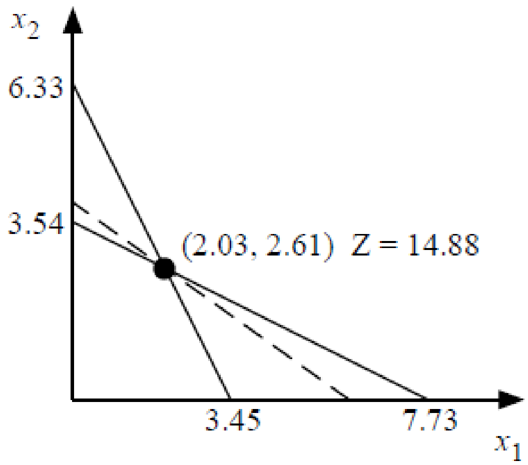
subject to

$1.1x_1 + 2.4x_2 \leq 8.5$

$2.2x_1 + 1.2x_2 \leq 7.6$

and

$x_1 \geq 0, x_2 \geq 0.$



The optimal solution is $x_1 = 2.03, x_2 = 2.61,$ and $Z = 14.88,$ or a (5.45/20.33) = 26.8% change in the value of Z from part (a).

7.4-3

(a) The model using the estimates of the parameters is:

Maximize $Z = 20x_1 + 15x_2$

subject to

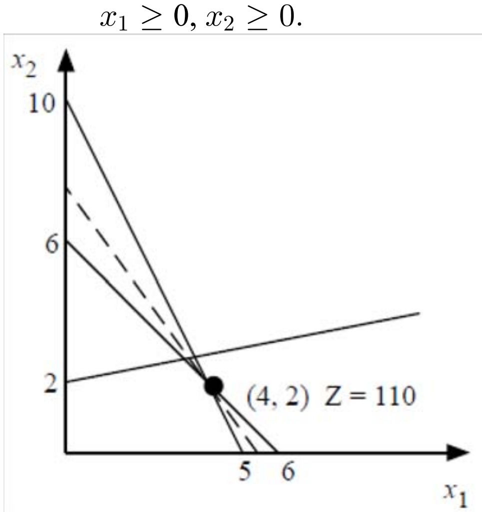
$10x_1 + 5x_2 \leq 50$

$-2x_1 + 10x_2 \leq 20$

$5x_1 + 5x_2 \leq 30$

and

$x_1 \geq 0, x_2 \geq 0.$



The optimal solution is $x_1 = 4, x_2 = 2,$ and $Z = 110.$

(b) Using robust optimization to formulate a conservative version, the model is:

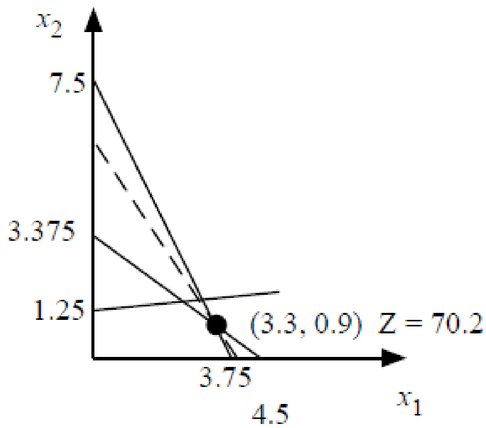
Maximize $Z = 18x_1 + 12x_2$,

subject to

$12x_1 + 6x_2 \leq 45$
$-x_1 + 12x_2 \leq 15$
$6x_1 + 8x_2 \leq 27$

and

$x_1 \geq 0, x_2 \geq 0.$



The optimal solution is $x_1 = 3.30, x_2 = 0.90,$ and $Z = 70.2$, or a (39.8/110) = 36.2% reduction in the value of Z from part (a).

7.4-4

(a) The model using the estimates of the parameters is:

Maximize $Z = 5x_1 - 8x_2 + 4x_3$

subject to

$4x_1 - 3x_2 + 2x_3 \leq 30$
$3x_1 - x_2 - x_3 \leq 20$
$2x_1 - 4x_2 - 3x_3 \leq 20$

and

$x_1 \geq 0, x_2 \geq 0.$

The optimal solution is $x_1 = 5.71, x_2 = 0, x_3 = 2.86,$ and $Z = 40.$

(b) The model using a conservative version of the model is:

Maximize $Z = 5x_1 - 9x_2 + 3x_3$

subject to

$4.4x_1 - 3x_2 + 2x_3 \leq 27$
$3x_1 - 0.6x_2 = x_3 \leq 19$
$2x_1 - 4x_2 = 3.5x_3 \leq 19$

and

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$

The optimal solution is $x_1 = 4.96, x_2 = 0, x_3 = 2.60,$ and $Z = 32.57.$

7.5-1

(a) The three chance constraints are

$P(x_1 \leq b_1) \geq 0.99$

$P(2x_2 \leq b_2) \geq 0.99$

$P(3x_1 + 2x_2 \leq b_3) \geq 0.99$

The deterministic equivalents of these chance constraints are

$x_1 \leq 4 - 2.33(0.1) = 3.767$

$2x_2 \leq 12 - 2.33(0.25) = 11.418$

$3x_1 + 2x_2 \leq 18 - 2.33(0.5) = 16.835$

(b) The optimal solution is $x_1 = 1.44, x_2 = 5.476,$ and $Z = 31.7$ ($31,700 profit per week).

7.5-2

(a) The chance constraint is $P(30x_1 + 20x_2 \leq b) \geq \alpha.$
The deterministic equivalent is $30x_1 + 20x_2 \leq \mu - K_\alpha \sigma.$

(b)

σ	α	K_α	Adjusted RHS
10	0.9	-1.28	$100 - 1.28(10) = 88$
10	0.95	-1.645	$100 - 1.645(10) = 83.55$
10	0.975	-1.96	$100 - 1.96(10) = 80.4$
10	0.99	-2.33	$100 - 2.33(10) = 76.7$
10	0.99865	-3.00	$100 - 3.00(10) = 70.0$
2	0.9	-1.28	$100 - 1.28(2) = 97.44$
2	0.95	-1.645	$100 - 1.645(2) = 96.71$
2	0.975	-1.96	$100 - 1.96(2) = 96.08$
2	0.99	-2.33	$100 - 2.33(2) = 95.34$
2	0.99865	-3.00	$100 - 3.00(2) = 94$

7.5-3

(a) Lower bound $= (0.95)^{20} = 0.358$
Upper bound $= (0.95)^{10} = 0.599$

(b) Lower bound $= (0.99)^{20} = 0.818$
Upper bound $= (0.99)^{10} = 0.904$

(c) When $\alpha = 0.998$, the lower bound is $(0.998)^{20} = 0.961,$ thus guaranteeing a probability of at least 0.95 that the optimal solution will be feasible.

7.5-4

(a) When $x_1 = 7, x_2 = 22,$ and $x_3 = 19,$ the left-hand-sides of the three constraints are 84, 140, and 168, respectively, yielding z-scores of $-2, -1.67,$ and -1.33. This leads to probabilities of 0.977, 0.953, and 0.908, respectively, that the three constraints will be satisfied.

(b) The three chance constraints are

$$P\{3x_1 = 2x_2 + x_3 \leq b_1\} \geq 0.975$$
$$P\{2x_1 + 4x_2 + 2x_3 \leq b_2\} \geq 0.95$$
$$P\{x_1 + 3x_2 + 5x_3 \leq b_3\} \geq 0.90$$

The deterministic equivalents of the three chance constraints are

$$3x_1 + 2x_2 + x_3 \leq 90 - 1.96(3) = 84.12$$
$$2x_1 + 4x_2 + 2x_3 \leq 150 - 1.645(6) = 140.13$$
$$x_1 + 3x_2 + 5x_3 \leq 180 - 1.28(9) = 168.48$$

The optimal solution is $x_1 = 7.03, x_2 = 21.96, x_3 = 19.11,$ and $Z = 1277.24$.

(c) All three constraints are satisfied with equality, so the probability the optimal solution will turn out to be feasible is (0.975)(0.95)(0.90) = 0.834.

7.6-1

The revised model is

Maximize $Z = 0.5(3x_1 + 5x_{21}) + 0.5(3x_1 + x_{22}) = 3x_1 + 2.5x_{21} + 0.5x_{22}$

subject to

$$\begin{array}{llll} x_1 & & & \leq 4 \\ & 2x_{21} & & \leq 12 \\ & & 2x_{22} & \leq 12 \\ 3x_1 & + 2x_{21} & & \leq 18 \\ 3x_1 & & + 2x_{22} & \leq 18 \end{array}$$

and

$$x_1 \geq 0, x_{21} \geq 0, x_{22} \geq 0.$$

The optimal solution is $x_1 = 2, x_{21} = 6, x_{22} = 2,$ and $Z = 22$. In words, the optimal plan is to produce 2 batches of product 1 per week; produce 6 batches of product 2 only if scenario 1 occurs; produce 2 batches of the modified version of product 2 per week only if scenario 2 occurs.

7.6-2

When the probability is 0.65 or more, the optimal plan presented in Sec. 7.6 is still optimal. When the probability is less than 0.65, the optimal solution becomes $x_1 = 2, x_{21} = 6,$ and $x_{22} = 2$.

7.6-3

Let x_1 = test market advertising

x_{21} = national campaign advertising if test market is very favorable

x_{22} = national campaign advertising if test market is barely favorable

x_{23} = national campaign advertising if test market is unfavorable

(all decision variables in units of $millions)

Then the revised model is

$$\begin{aligned}\text{Maximize } Z &= 0.25(0.5x_1 + 2x_{21}) + 0.25(0.5x_1 + 0.2x_{22}) + 0.5(0.5x_1) - 40 \\ &= 0.5x_1 + 0.5x_{21} + 0.05x_{22} - 40\end{aligned}$$

subject to

$$\begin{array}{llll} x_1 & & & \geq 5 \\ x_1 & & & \leq 10 \\ x_1 + x_{21} & & & \leq 100 \\ x_1 & + x_{22} & & \leq 100 \\ x_1 & & + x_{23} & \leq 100 \end{array}$$

and

$x_1 \geq 0, x_{21} \geq 0, x_{22} \geq 0,$ and $x_{23} \geq 0.$

The optimal solution is $x_1 = 5, x_{21} = 95, x_{22} = 95, x_{23} = 0,$ and $Z = 14.75$. In words, they should spend $5 million in advertising for the test market. If the test market is very favorable or barely favorable, then they should spend $95 million advertising the drink nationally. If the test market is unfavorable, they should drop the product. The expected net profit is $14.75 million.

7.6-4

The stochastic programming model is

$$\begin{aligned}\text{Minimize } Z &= 0.333(5x_1 + 4x_{21}) + 0.33(5x_1 + 6x_{22}) + 0.333(5x_1 + 3x_{23}) \\ &= 5x_1 + 1.333x_{21} + 2x_{22} + x_{23}\end{aligned}$$

subject to

$$\begin{aligned} 3x_1 + 2x_{21} &\geq 60 \\ 3x_1 + 3x_{22} &\geq 60 \\ 3x_1 + 2x_{23} &\geq 60 \\ 2x_1 + 3x_{21} &\geq 60 \\ 2x_1 + 4x_{22} &\geq 60 \\ 2x_1 + 1x_{23} &\geq 60 \end{aligned}$$

and

$x_1 \geq 0, x_{21} \geq 0, x_{22} \geq 0,$ and $x_{23} \geq 0.$

The optimal solution is $x_1 = 12, x_{21} = 12, x_{22} = 9, x_{23} = 36,$ and $Z = 130$. In words, they should do activity 1 at level 12, and then activity 2 at level 12, 9, or 36 under scenario 1, 2, or 3, respectively.

Case 7.1

a)

	A	B	C	D	E	F	G	H	I	J
1		Taller Smokestacks		Filters		Better Fuels				
2		Blast	Open-Hearth	Blast	Open-Hearth	Blast	Open-Hearth			
3		Furnaces	Furnaces	Furnaces	Furnaces	Furnaces	Furnaces			
4	**Cost ($million)**	8	10	7	6	11	9			
5								Total		Minimum
6								Reduction		Reduction
7	**Pollutant**	Reduction in Emission (Maximum Feasible Use of Abatement Method)						(millions of lbs.)		(millions of lbs.)
8	Particulates	12	9	25	20	17	13	60	>=	60
9	Sulfur oxides	35	42	18	31	56	49	150	>=	150
10	Hydrocarbons	37	53	28	24	29	20	125	>=	125
11										
12		Taller Smokestacks		Filters		Better Fuels				
13		Blast	Open-Hearth	Blast	Open-Hearth	Blast	Open-Hearth			Total Cost
14		Furnaces	Furnaces	Furnaces	Furnaces	Furnaces	Furnaces			($million)
15	**Fraction Used**	100%	62.27%	34.35%	100%	4.76%	100%			32.155
16		<=	<=	<=	<=	<=	<=			
17		100%	100%	100%	100%	100%	100%			

Range Name	Cells
Cost	B4:G4
FractionUsed	B15:G15
MinimumReduction	J8:J10
OneHundredPercent	B17:G17
ReductionInEmission	B8:G10
TotalCost	J15
TotalReduction	H8:H10

	H
5	Total
6	Reduction
7	(millions of lbs.)
8	=SUMPRODUCT(B8:G8,FractionUsed)
9	=SUMPRODUCT(B9:G9,FractionUsed)
10	=SUMPRODUCT(B10:G10,FractionUsed)

	J
13	Total Cost
14	($million)
15	=SUMPRODUCT(Cost,FractionUsed)

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$15	Fraction Taller Smokestack (Blast)	100%	-34%	8	0.336	1E+30
$C$15	Fraction Taller Smokestack (Open Hearth)	62.27%	0.00%	10	0.429	0.667
$D$15	Fraction Filter (Blast)	34.35%	0.00%	7	0.382	2.011
$E$15	Fraction Filter (Open Hearth)	100%	-182%	6	1.816	1E+30
$F$15	Fraction Better Fuel (Blast)	4.76%	0.00%	11	2.975	0.045
$G$15	Fraction Better Fuel (Open Hearth)	100%	-4%	9	0.044	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$H$8	Particulates (millions of lbs.)	60	0.111	60	14.297	7.480
$H$9	Sulfur oxides (millions of lbs.)	150	0.127	150	20.453	1.690
$H$10	Hydrocarbons (millions of lbs.)	125	0.069	125	2.042	21.692

b) The right-hand-side of each constraint with a non-zero shadow price is sensitive, since changing its value will impact the total cost. All three required reductions in emission rates are sensitive parameters. All of the objective coefficients have an allowable range to stay optimal around them, and thus are not as sensitive. However, for some, the allowable change is small—in particular, the cost of the two better fuel options (with an allowable increase of only 0.045 and an allowable decrease of 0.044, respectively) are fairly sensitive. Thus, all five of these parameters should be estimated more closely, if possible.

c) The sensitivity report and, in particular, the allowable range for the objective coefficients can be used to determine whether the solution will change. The following table shows in which cases the optimal solution will change.

Abatement Method	Current Value	10% Less Value	Solution Changes ?	10% More Value	Solution Changes ?
Taller Smoke (Blast)	8	7.2	No	8.8	Yes
Taller Smoke (Open H)	10	9	Yes	11	Yes
Filter (Blast)	7	6.3	No	7.7	Yes
Filter (Open H)	6	5.4	No	6.6	No
Better Fuel (Blast)	11	9.9	Yes	12.1	No
Better Fuel (Open H)	9	8.1	No	9.9	Yes

This suggests that focus should be put on estimating all of the costs except the filter for the open hearth furnaces, since it's optimal solution will not change with a 10% increase or decrease. Special consideration should be given to the estimate of the cost of the taller smokestack for the open hearth furnaces, since it affects the optimal solution for both an increase and a decrease. Special consideration should also be given to the estimate of the cost of the better fuel options, since the allowable decrease (for the blast furnace) or allowable decrease (for the open hearth furnace) is so small.

d) Below is the corresponding dual problem.

	Benefit Contribution Per Unit of Abatement Method								
	Taller Smokestacks		Filters		Better Fuels				
Constraints	Blast	Open-earth	Blast	Open-earth	Blast	Open-earth	Dual Variables		Accepatabl Level
reduce particulates	12	9	25	20	17	13	y1	<= 0	60
reduce sulfer-oxides	35	42	18	31	56	49	y2	<= 0	150
reduce hydrocarbons	37	53	28	24	29	20	y3	<= 0	125
smokestacks - blast	1	0	0	0	0	0	y4	>= 0	1
smokestacks - open-earth	0	1	0	0	0	0	y5	>= 0	1
filters - blast	0	0	1	0	0	0	y6	>= 0	1
filters - open-earth	0	0	0	1	0	0	y7	>= 0	1
fuels - blast	0	0	0	0	1	0	y8	>= 0	1
fuels - open-earth	0	0	0	0	0	1	y9	>= 0	1
Totals									
	>=	>=	>=	>=	>=	>=			
Unit Cost	-8	-10	-7	-6	-11	-9			

This is the sensitivity report for the primal (maximization problem)

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$C$16	Solution Blast	1.00	0.00	-8	1E+30	0.336210968
$D$16	Solution Open-earth	0.62	0.00	-10	0.666961637	0.429446294
$E$16	Solution Blast	0.34	0.00	-7	2.01145997	0.381632659
$F$16	Solution Open-earth	1.00	0.00	-6	1E+30	1.816085017
$G$16	Solution Blast	0.05	0.00	-11	0.044638358	2.975225225
$H$16	Solution Open-earth	1.00	0.00	-9	1E+30	0.044161638

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$I$6	reduce particulates Totals	60	-0.111046969	60	14.29714286	7.48
$I$7	reduce sulfer-oxides Totals	150	-0.126817108	150	20.453125	1.689655172
$I$8	reduce hydrocarbons Totals	125	-0.069325636	125	2.041666667	21.69195612
$I$9	smokestacks - blast Totals	1	0.336210968	1	0.246231156	0.748477435
$I$10	smokestacks - open-earth Totals	0.623	0.000	1	1E+30	0.377302545
$I$11	filters - blast Totals	0.343	0.000	1	1E+30	0.656520598
$I$12	filters - open-earth Totals	1	1.816085017	1	0.110609481	1
$I$13	fuels - blast Totals	0.0476	0.0000	1	1E+30	0.952427184
$I$14	fuels - open-earth Totals	1	0.044161638	1	0.048086359	0.962708538

The dual variables are the shadow prices of the constraints. If the primal had been left in minimization form, the sign of the dual variables would be the opposite. The dual would be the same except that the "sign" constraints on the dual variables changes from ≤ to ≥ and vice versa, and the dual functional constraints all change from ≤ to ≥. It would also be a maximization problem instead of minimization.

e)

Pollutant	Rate that cost changes ($million)	Maximum increase before rate changes (million lb.)	Maximum decrease before rate changes (million lb.)
Particulates	0.111	14.297	7.480
Sulfur oxides	0.127	20.453	1.690
Hydrocarbons	0.069	2.042	21.692

f) Particulates and sulfur oxides:
For each unit increase in particulate reduction, cost will increase by \$0.111 million.
For each unit decrease in sulfur oxide reduction, cost will decrease by \$0.127 million.
Thus, cost will remain equal if for each unit increase in particulate reduction, the sulfur oxide reduction is reduced by \$0.111 / \$0.127 = 0.874 units.

Particulates and hydrocarbons:
For each unit increase in particulate reduction, cost will increase by \$0.111 million.
For each unit decrease in hydrocarbon reduction, cost will decrease by \$0.069 million.
Thus, cost will remain equal if for each unit increase in particulate reduction, the hydrocarbon reduction is reduced by \$0.111 / \$0.069 = 1.609 units.

Particulates and both sulfur oxides and hydrocarbons:
For each unit increase in particulate reduction, cost will increase by \$0.111 million.
For each simultaneous unit decrease in sulfur oxide and hydrocarbon reduction, cost will decrease by \$0.127 + \$0.069 = \$0.196.
Thus, cost will remain equal if for each unit increase in particulate reduction, the sulfur oxide and hydrocarbon reduction are each reduced by \$0.111 / \$0.196 = 0.566 units.

Case 7.2

a) The decisions to be made are how much acreage should be planted in each of the crops and how many cows and hens to have for the coming year. The constraints on these decisions are amount of labor hours available, the investment funds available, the number of acres available, the space available in the barn and chicken house, the minimum requirements for feed to be planted. The overall measure of performance is monetary worth, which is to be maximized.

b & c)

	A	B	C	D	E	F	G
1	**Plantings**				Planting		
2		Soybeans	Corn	Wheat	Totals		
3	W&S Hours Required	1	0.9	0.6	537		
4	S&F Hours Required	1.4	1.2	0.7	736		
5	Net Value	$70	$60	$40	$37,300		
6							
7	Acres Planted	450	30	100	580		
8			>=	>=			
9			30	100			
10			1	0.05			
11			acre/cow	acre/hen			
12							
13	**Livestock**			Livestock			
14		Cows	Hens	Totals			
15	Hours Required per Month	10	0.05	400			
16	Grazing Land Required	2	0	60			
17	Net Annual Cash Income	$850	$4	$34,000			
18							
19	Beginning Value (Current Livestock)	$35,000	$5,000				
20	Decrease in Value per Year	10%	25%				
21	End Value (Current Livestock)	$31,500	$3,750	$35,250		Investment	
22						Fund	
23	Cost of New Livestock	$1,500	$3	$0	<=	$20,000	
24	End Value (New Livestock)	$1,350	$2	$0			
25							
26	Current Livestock	30	2,000				
27	New Livestock	0	0				
28	Total Livestock	30	2,000				
29		<=	<=				
30	Barn/House Limits	42	5,000				
31							
32	**Neighboring Farm Work**			Neighbor			
33		W&S	S&F	Totals			
34	Wage	$5	$5.50	$12,817.00			
35							
36	Hours Worked	1063	1364	2427			
37							
38	**Totals**	Plantings	Livestock	Neighbor	Total		Available
39	W&S Hours	537	2,400	1,063	4,000	<=	4,000
40	S&F Hours	736	2,400	1,364	4,500	<=	4,500
41	Acreage	580	60	0	640	<=	640
42							
43	Net Income	$37,300	$34,000	$12,817	$84,117		
44	End of Year Value		$35,250		$35,250		
45	Leftover Investment Fund		$20,000		$20,000		
46	Living Expenses				-$40,000		
47	Total Monetary Worth				$99,367		

This model predicts that the family's monetary worth at the end of the coming year will be $99, 367.

	A	B	C	D	E
1	**Plantings**				Planting
2		Soybeans	Corn	Wheat	Totals
3	W&S Hours Required	1	0.9	0.6	=SUMPRODUCT(B3:D3,AcresPlanted)
4	S&F Hours Required	1.4	1.2	0.7	=SUMPRODUCT(B4:D4,AcresPlanted)
5	Net Value	70	60	40	=SUMPRODUCT(B5:D5,AcresPlanted)
6					
7	Acres Planted	450	30	100	=SUM(AcresPlanted)
8			>=	>=	
9			=C10*B28	=D10*C28	
10			1	0.05	
11			acre/cow	acre/hen	

	A	B	C	D	E	F
13	**Livestock**			Livestock		
14		Cows	Hens	Totals		
15	Hours Required per Month	10	0.05	=SUMPRODUCT(B15:C15,TotalLivestock)		
16	Grazing Land Required	2	0	=SUMPRODUCT(B16:C16,TotalLivestock)		
17	Net Annual Cash Income	850	4.25	=SUMPRODUCT(B17:C17,TotalLivestock)		
18						
19	Beginning Value (Current Livestock)	35000	5000			
20	Decrease in Value per Year	0.1	0.25			
21	End Value (Current Livestock)	=(1-B20)*B19	=(1-C20)*C19	=SUM(B21:C21)		Investment
22						Fund
23	Cost of New Livestock	1500	3	=SUMPRODUCT(B23:C23,NewLivestock)	<=	20000
24	End Value (New Livestock)	=(1-B20)*B23	=(1-C20)*C23	=SUMPRODUCT(B24:C24,NewLivestock)		
25						
26	Current Livestock	30	2000			
27	New Livestock	0	0			
28	Total Livestock	=CurrentLivestock+NewLivestock	=CurrentLivestock+NewLivestock			
29		<=	<=			
30	Barn/House Limits	42	5000			

	A	B	C	D
32	**Neighboring Farm Work**			Neighbor
33		W&S	S&F	Totals
34	Wage	5	5.5	=SUMPRODUCT(Wage,HoursWorked)
35				
36	Hours Worked	1063	1364	=SUM(HoursWorked)

	A	B	C	D	E	F	G
38	**Totals**	Plantings	Livestock	Neighbor	Total		Available
39	W&S Hours	=E3	=6*D15	=B36	=SUM(B39:D39)	<=	4000
40	S&F Hours	=E4	=6*D15	=C36	=SUM(B40:D40)	<=	4500
41	Acreage	=E7	=D16	0	=SUM(B41:D41)	<=	640
42							
43	Net Income	=E5	=D17	=D34	=SUM(B43:D43)		
44	End of Year Value		=D21+D24		=SUM(B44:D44)		
45	Leftover Investment Fund		=InvestmentFund-D23		=SUM(B45:D45)		
46	Living Expenses				-40000		
47	Total Monetary Worth				=SUM(E43:E46)		

Range Name	Cells
AcresPlanted	B7:D7
Available	G39:G41
BarnHouseLimits	B30:C30
CurrentLivestock	B26:C26
HoursWorked	B36:C36
InvestmentFund	F23
MonetaryWorth	E47
NewLivestock	B27:C27
Total	E39:E41
TotalLivestock	B28:C28
Wage	B34:C34

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$7	Acres Planted Soybeans	450	0	70	1E+30	8.4
$C$7	Acres Planted Corn	30	0	60	8.4	1E+30
$D$7	Acres Planted Wheat	100	0	40	17.15	1E+30
$B$27	New Livestock Cows	0	-53	700	53	1E+30
$C$27	New Livestock Hens	0	-0.857	3.5	0.857	1E+30
$B$36	Hours Worked W&S	1063	0	5	57.3	0.915
$C$36	Hours Worked S&F	1364	0	5.5	34.5	0.930

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$C$7	Acres Planted Corn	30	-8.4	0	450	30
$D$7	Acres Planted Wheat	100	-24.15	0	450	100
$D$23	Cost of New Livestock Totals	$0	$0	20000	1E+30	20000
$B$28	Total Livestock Cows	30	0	42	1E+30	12
$C$28	Total Livestock Hens	2000	0	5000	1E+30	3000
$E$39	W&S Hours Total	4000	5	4000	1E+30	1063
$E$40	S&F Hours Total	4500	5.5	4500	1E+30	1364
$E$41	Acreage Total	640	57.3	640	974.29	450

d) The allowable range for the value per acre planted of soybeans is 61.6 to ∞.
The allowable range for the value per acre planted of corn is $-\infty$ to 68.4.
The allowable range for the value per acre planted of wheat is $-\infty$ to 57.15.

e) Drought

	A	B	C	D	E	F	G
1	**Plantings**				Planting		
2		Soybeans	Corn	Wheat	Totals		
3	W&S Hours Required	1	0.9	0.6	117.8		
4	S&F Hours Required	1.4	1.2	0.7	143.733		
5	Net Value	-$10	-$15	$0	-$630		
6							
7	Acres Planted	0	42	133.33	175.333		
8			>=	>=			
9			42	133.33			
10			1	0.05			
11			acre/cow	acre/hen			
12							
13	**Livestock**			Livestock			
14		Cows	Hens	Totals			
15	Hours Required per Month	10	0.05	553.333333			
16	Grazing Land Required	2	0	84			
17	Net Annual Cash Income	$850	$4	$47,033			
18							
19	Beginning Value (Current Livestock)	$35,000	$5,000				
20	Decrease in Value per Year	10%	25%				
21	End Value (Current Livestock)	$31,500	$3,750	$35,250		Investment	
22						Fund	
23	Cost of New Livestock	$1,500	$3	$20,000	<=	$20,000	
24	End Value (New Livestock)	$1,350	$2	$17,700			
25							
26	Current Livestock	30	2,000				
27	New Livestock	12	667				
28	Total Livestock	42	2,667				
29		<=	<=				
30	Barn/House Limits	42	5,000				
31							
32	**Neighboring Farm Work**			Neighbor			
33		W&S	S&F	Totals			
34	Wage	$5	$5.50	$8,510.47			
35							
36	Hours Worked	562.2	1036.267	1598.46665			
37							
38	**Totals**	Plantings	Livestock	Neighbor	Total		Available
39	W&S Hours	117.8	3,320	562	4,000	<=	4,000
40	S&F Hours	143.7333	3,320	1,036	4,500	<=	4,500
41	Acreage	175.3333	84	0	259.333	<=	640
42							
43	Net Income	-$630	$47,033	$8,510	$54,914		
44	End of Year Value		$52,950		$52,950		
45	Leftover Investment Fund		$0		$0		
46	Living Expenses				-$40,000		
47	Total Monetary Worth				$67,864		

In a drought, the model predicts (under the optimal solution) that the family's monetary worth at the end of the year will be $67,864.

Flood

	A	B	C	D	E	F	G
1	**Plantings**				Planting		
2		Soybeans	Corn	Wheat	Totals		
3	W&S Hours Required	1	0.9	0.6	460.4		
4	S&F Hours Required	1.4	1.2	0.7	600.533		
5	Net Value	$15	$20	$10	$9,787		
6							
7	Acres Planted	0	422.6667	133.33	556		
8			>=	>=			
9			42	133.33			
10			1	0.05			
11			acre/cow	acre/hen			
12							
13	**Livestock**			Livestock			
14		Cows	Hens	Totals			
15	Hours Required per Month	10	0.05	553.333333			
16	Grazing Land Required	2	0	84			
17	Net Annual Cash Income	$850	$4	$47,033			
18							
19	Beginning Value (Current Livestock)	$35,000	$5,000				
20	Decrease in Value per Year	10%	25%				
21	End Value (Current Livestock)	$31,500	$3,750	$35,250		Investment	
22						Fund	
23	Cost of New Livestock	$1,500	$3	$20,000	<=	$20,000	
24	End Value (New Livestock)	$1,350	$2	$17,700			
25							
26	Current Livestock	30	2,000				
27	New Livestock	12	667				
28	Total Livestock	42	2,667				
29		<=	<=				
30	Barn/House Limits	42	5,000				
31							
32	**Neighboring Farm Work**			Neighbor			
33		W&S	S&F	Totals			
34	Wage	$5	$5.50	$4,285.07			
35							
36	Hours Worked	219.6	579.4667	799.067			
37							
38	**Totals**	Plantings	Livestock	Neighbor	Total		Available
39	W&S Hours	460.4	3,320	220	4,000	<=	4,000
40	S&F Hours	600.5333	3,320	579	4,500	<=	4,500
41	Acreage	556	84	0	640	<=	640
42							
43	Net Income	$9,787	$47,033	$4,285	$61,105		
44	End of Year Value		$52,950		$52,950		
45	Leftover Investment Fund		$0		$0		
46	Living Expenses				-$40,000		
47	Total Monetary Worth				$74,055		

In a flood, the model predicts (under the optimal solution) that the family's monetary worth at the end of the year will be $74,055.

Early Frost

	A	B	C	D	E	F	G
1	**Plantings**				Planting		
2		Soybeans	Corn	Wheat	Totals		
3	W&S Hours Required	1	0.9	0.6	537		
4	S&F Hours Required	1.4	1.2	0.7	736		
5	Net Value	$50	$40	$30	$26,700		
6							
7	Acres Planted	450	30	100.00	580		
8			>=	>=			
9			30	100.00			
10			1	0.05			
11			acre/cow	acre/hen			
12							
13	**Livestock**			Livestock			
14		Cows	Hens	Totals			
15	Hours Required per Month	10	0.05	400			
16	Grazing Land Required	2	0	60			
17	Net Annual Cash Income	$850	$4	$34,000			
18							
19	Beginning Value (Current Livestock)	$35,000	$5,000				
20	Decrease in Value per Year	10%	25%				
21	End Value (Current Livestock)	$31,500	$3,750	$35,250		Investment	
22						Fund	
23	Cost of New Livestock	$1,500	$3	$0	<=	$20,000	
24	End Value (New Livestock)	$1,350	$2	$0			
25							
26	Current Livestock	30	2,000				
27	New Livestock	0	0				
28	Total Livestock	30	2,000				
29		<=	<=				
30	Barn/House Limits	42	5,000				
31							
32	**Neighboring Farm Work**			Neighbor			
33		W&S	S&F	Totals			
34	Wage	$5	$5.50	$12,817.00			
35							
36	Hours Worked	1063	1364	2427.000			
37							
38	**Totals**	Plantings	Livestock	Neighbor	Total		Available
39	W&S Hours	537	2,400	1,063	4,000	<=	4,000
40	S&F Hours	736	2,400	1,364	4,500	<=	4,500
41	Acreage	580	60	0	640	<=	640
42							
43	Net Income	$26,700	$34,000	$12,817	$73,517		
44	End of Year Value		$35,250		$35,250		
45	Leftover Investment Fund		$20,000		$20,000		
46	Living Expenses				-$40,000		
47	Total Monetary Worth				$88,767		

In an early frost, the model predicts (under the optimal solution) that the family's monetary worth at the end of the year will be $88,767.

Drought and Early Frost

	A	B	C	D	E	F	G
1	**Plantings**				Planting		
2		Soybeans	Corn	Wheat	Totals		
3	W&S Hours Required	1	0.9	0.6	97.8		
4	S&F Hours Required	1.4	1.2	0.7	120.4		
5	Net Value	-$15	-$20	-$10	-$1,840		
6							
7	Acres Planted	0	42	100.00	142		
8			>=	>=			
9			42	100.00			
10			1	0.05			
11			acre/cow	acre/hen			
12							
13	**Livestock**			Livestock			
14		Cows	Hens	Totals			
15	Hours Required per Month	10	0.05	520			
16	Grazing Land Required	2	0	84			
17	Net Annual Cash Income	$850	$4	$44,200			
18							
19	Beginning Value (Current Livestock)	$35,000	$5,000				
20	Decrease in Value per Year	10%	25%				
21	End Value (Current Livestock)	$31,500	$3,750	$35,250		Investment	
22						Fund	
23	Cost of New Livestock	$1,500	$3	$18,000	<=	$20,000	
24	End Value (New Livestock)	$1,350	$2	$16,200			
25							
26	Current Livestock	30	2,000				
27	New Livestock	12	0				
28	Total Livestock	42	2,000				
29		<=	<=				
30	Barn/House Limits	42	5,000				
31							
32	**Neighboring Farm Work**			Neighbor			
33		W&S	S&F	Totals			
34	Wage	$5	$5.50	$10,838.80			
35							
36	Hours Worked	782.2	1259.6	2041.800			
37							
38	**Totals**	Plantings	Livestock	Neighbor	Total		Available
39	W&S Hours	97.8	3,120	782	4,000	<=	4,000
40	S&F Hours	120.4	3,120	1,260	4,500	<=	4,500
41	Acreage	142	84	0	226	<=	640
42							
43	Net Income	-$1,840	$44,200	$10,839	$53,199		
44	End of Year Value		$51,450		$51,450		
45	Leftover Investment Fund		$2,000		$2,000		
46	Living Expenses				-$40,000		
47	Total Monetary Worth				$66,649		

In a drought and early frost, the model predicts (under the optimal solution) that the family's monetary worth at the end of the year will be $66,649.

Flood and Early Frost

	A	B	C	D	E	F	G
1	**Plantings**				Planting		
2		Soybeans	Corn	Wheat	Totals		
3	W&S Hours Required	1	0.9	0.6	183.6		
4	S&F Hours Required	1.4	1.2	0.7	219.8		
5	Net Value	$10	$10	$5	$1,623		
6							
7	Acres Planted	0	37.33333	250.00	287.333		
8			>=	>=			
9			37.33333	250.00			
10			1	0.05			
11			acre/cow	acre/hen			
12							
13	**Livestock**			Livestock			
14		Cows	Hens	Totals			
15	Hours Required per Month	10	0.05	623.333333			
16	Grazing Land Required	2	0	74.6666667			
17	Net Annual Cash Income	$850	$4	$52,983			
18							
19	Beginning Value (Current Livestock)	$35,000	$5,000				
20	Decrease in Value per Year	10%	25%				
21	End Value (Current Livestock)	$31,500	$3,750	$35,250		Investment	
22						Fund	
23	Cost of New Livestock	$1,500	$3	$20,000	<=	$20,000	
24	End Value (New Livestock)	$1,350	$2	$16,650			
25							
26	Current Livestock	30	2,000				
27	New Livestock	7.333333	3,000				
28	Total Livestock	37.33333	5,000				
29		<=	<=				
30	Barn/House Limits	42	5,000				
31							
32	**Neighboring Farm Work**			Neighbor			
33		W&S	S&F	Totals			
34	Wage	$5	$5.50	$3,353.10			
35							
36	Hours Worked	76.39999	540.2	616.600			
37							
38	**Totals**	Plantings	Livestock	Neighbor	Total		Available
39	W&S Hours	183.6	3,740	76	4,000	<=	4,000
40	S&F Hours	219.8	3,740	540	4,500	<=	4,500
41	Acreage	287.3333	74.66667	0	362	<=	640
42							
43	Net Income	$1,623	$52,983	$3,353	$57,960		
44	End of Year Value		$51,900		$51,900		
45	Leftover Investment Fund		$0		$0		
46	Living Expenses				-$40,000		
47	Total Monetary Worth				$69,860		

In a flood and early frost, the model predicts (under the optimal solution) that the family's monetary worth at the end of the year will be $69,860.

f)

Opt. Sol. Used	Family's monetary worth at year's end if the scenario is actually: Good	Drought	Flood	Early Frost	Drought&EF	Flood&EF
Good Weather	$99,367	$57,117	$70,417	$88,767	$53,717	$67,367
Drought	$76,348	$67,864	$70,668	$74,174	$66,321	$69,581
Flood	$94,962	$57,929	$74,055	$85,175	$54,482	$69,162
Early Frost	$99,367	$57,117	$70,417	$88,767	$53,717	$67,367
Drought & E.F.	$75,009	$67,859	$70,329	$73,169	$66,649	$69,409
Flood & E.F.	$80,476	$67,676	$71,483	$77,230	$64,990	$69,860

Answers will vary. No solution is clearly best. The Good Weather solution is the riskiest, with the highest upside and downside. The Flood solution appears to be a good middle ground. The Drought, Drought&EF, and Flood&EF solutions are the most conservative.

g and h)

The expected net value for each of the crops is calculated as follows:

Soybeans: ($70)(0.4) + (–$10)(0.2) + ($15)(0.1) + ($50)(0.15) + (–$15)(0.1) + ($10)(0.05) = $34,

Corn: ($60)(0.4) + (–$15)(0.2) + ($20)(0.1) + ($40)(0.15) + (–$20)(0.1) + ($10)(0.05) = $27.5,

Wheat: ($40)(0.4) + ($0)(0.2) + ($10)(0.1) + ($30)(0.15) + (–$10)(0.1) + ($5)(0.05) = $20.75.

The resulting spreadsheet solution is shown below:

	A	B	C	D	E	F	G
1	**Plantings**				Planting		
2		Soybeans	Corn	Wheat	Totals		
3	W&S Hours Required	1	0.9	0.6	511.8		
4	S&F Hours Required	1.4	1.2	0.7	700		
5	Net Value	$34.00	$27.50	$20.75	$17,306		
6							
7	Acres Planted	414	42	100.00	556		
8			>=	>=			
9			42	100.00			
10			1	0.05			
11			acre/cow	acre/hen			
12							
13	**Livestock**			Livestock			
14		Cows	Hens	Totals			
15	Hours Required per Month	10	0.05	520			
16	Grazing Land Required	2	0	84			
17	Net Annual Cash Income	$850	$4	$44,200			
18							
19	Beginning Value (Current Livestock)	$35,000	$5,000				
20	Decrease in Value per Year	10%	25%				
21	End Value (Current Livestock)	$31,500	$3,750	$35,250		Investment	
22						Fund	
23	Cost of New Livestock	$1,500	$3	$18,000	<=	$20,000	
24	End Value (New Livestock)	$1,350	$2	$16,200			
25							
26	Current Livestock	30	2,000				
27	New Livestock	12	0				
28	Total Livestock	42	2,000				
29		<=	<=				
30	Barn/House Limits	42	5,000				
31							
32	**Neighboring Farm Work**			Neighbor			
33		W&S	S&F	Totals			
34	Wage	$5	$5.50	$5,581.00			
35							
36	Hours Worked	368.2	680	1048.200			
37							
38	**Totals**	Plantings	Livestock	Neighbor	Total		Available
39	W&S Hours	511.8	3,120	368	4,000	<=	4,000
40	S&F Hours	700	3,120	680	4,500	<=	4,500
41	Acreage	556	84	0	640	<=	640
42							
43	Net Income	$17,306	$44,200	$5,581	$67,087		
44	End of Year Value		$51,450		$51,450		
45	Leftover Investment Fund		$2,000		$2,000		
46	Living Expenses				-$40,000		
47	Total Monetary Worth				$80,537		

This model predicts that the family's monetary worth at the end of the coming year will be (on average) $80,537.

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$7	Acres Planted Soybeans	414	0	34	7.5	0.4
$C$7	Acres Planted Corn	42	0	27.5	4.9	22.5
$D$7	Acres Planted Wheat	100	0.00	20.75	0.4	1E+30
$B$27	New Livestock Cows	12	0	700	1E+30	22.5
$C$27	New Livestock Hens	0	0	3.5	0.02	1E+30
$B$36	Hours Worked W&S	368.2	0	5	0.389	0.071
$C$36	Hours Worked S&F	680	0	5.5	0.395	0.075

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$C$7	Acres Planted Corn	42	-4.9	0	414	42
$D$7	Acres Planted Wheat	100.00	-7.40	0	414	100
$D$23	Cost of New Livestock Totals	$18,000	$0	20000	1E+30	2000
$B$28	Total Livestock Cows	42	22.5	42	1.333	12
$C$28	Total Livestock Hens	2,000	0	5000	1E+30	3000
$E$39	W&S Hours Total	4,000	5	4000	1E+30	368.2
$E$40	S&F Hours Total	4,500	5	4500	1E+30	680
$E$41	Acreage Total	640	21.3	640	368.2	414

i) The shadow price for the investment constraint is zero, indicating that additional investment funds will not increase their total monetary worth at all. Thus, it is not worthwhile to obtain a bank loan. The shadow price would need to be at least $1.10 before a loan at 10% interest would be worthwhile.

j) The expected net value for soybeans can increase up to $7.50 or decrease up to $0.40; for corn can increase up to $4.90 or decrease up to $22.50; for wheat can increase up to $0.40 or decrease any amount without changing the optimal solution. The expected net value for soybeans and wheat should be estimated most carefully.

The solution is sensitive to decreases in the expected value of soybeans and increases in the expected value of wheat. If the *cumulative* decrease in the expected value of soybeans *and* increase in the expected value of wheat exceeds $0.40, then the 100% rule will be violated, and the solution might change.

MonetaryWorth	Corn Net Value				
Wheat Net Value	$18.75	$19.75	$20.75	$21.75	$22.75
$32.00	$79,440	$79,479	$79,519	$79,559	$79,599
$33.00	$79,776	$79,818	$79,860	$79,902	$79,944
$34.00	$80,170	$80,212	$80,254	$80,296	$80,338
$35.00	$80,584	$80,626	$80,668	$80,710	$80,752
$36.00	$80,998	$81,040	$81,082	$81,124	$81,166

k) Answers will vary.

Case 7.3

a)

	A	B	C	D	E	F	G
1	**Data:**	Percentage	Percentage	Percentage			
2		in 6th	in 7th	in 8th	Bussing Cost ($/Student)		
3	Area	Grade	Grade	Grade	School 1	School 2	School 3
4	1	32%	38%	30%	$300	$0	$700
5	2	37%	28%	35%	-	$400	$500
6	3	30%	32%	38%	$600	$300	$200
7	4	28%	40%	32%	$200	$500	-
8	5	39%	34%	27%	$0	-	$400
9	6	34%	28%	38%	$500	$300	$0
10							
11							
12	**Solution:**	Number of Students Assigned			Total		Number of
13		School 1	School 2	School 3	From Area		Students
14	Area 1	0	450	0	450	=	450
15	Area 2	0	422.22	177.78	600	=	600
16	Area 3	0	227.78	322.22	550	=	550
17	Area 4	350	0	0	350	=	350
18	Area 5	366.67	0	133.33	500	=	500
19	Area 6	83.33	0	366.67	450	=	450
20	Total In School	800	1,100	1,000			
21		<=	<=	<=			Total
22	Capacity	900	1,100	1,000			Bussing
23							Cost
24							$555,556
25	**Grade Constraints:**						
26		240	330	300	30%	of total in school	
27		<=	<=	<=			
28	6th Graders	269.33	368.56	339.11			
29	7th Graders	288.00	362.11	300.89			
30	8th Graders	242.67	369.33	360.00			
31		<=	<=	<=			
32		288	396	360	36%	of total in school	

Range Name	Cells
BussingCost	E4:G9
Capacity	B22:D22
NumberOfStudents	G14:G19
PercentageInGrade	B4:D9
Solution	B14:D19
TotalBussingCost	G24
TotalFromArea	E14:E19
TotalInSchool	B20:D20

	E
12	Total
13	From Area
14	=SUM(B14:D14)
15	=SUM(B15:D15)
16	=SUM(B16:D16)
17	=SUM(B17:D17)
18	=SUM(B18:D18)
19	=SUM(B19:D19)

	G
21	Total
22	Bussing
23	Cost
24	=SUMPRODUCT(BussingCost,Solution)

	A	B	C	D
20	Total In School	=SUM(B14:B19)	=SUM(C14:C19)	=SUM(D14:D19)

	A	B	C	D	E	F
25	**Grade Constraints:**					
26		=$E$26*TotalInSchool	=$E$26*TotalInSchool	=$E$26*TotalInSchool	0.3	of total in school
27		<=	<=	<=		
28	6th Graders	=SUMPRODUCT(B14:B19,B4:B9)	=SUMPRODUCT(C14:C19,B4:B9)	=SUMPRODUCT(D14:D19,B4:B9)		
29	7th Graders	=SUMPRODUCT(B14:B19,C4:C9)	=SUMPRODUCT(C14:C19,C4:C9)	=SUMPRODUCT(C4:C9,D14:D19)		
30	8th Graders	=SUMPRODUCT(B14:B19,D4:D9)	=SUMPRODUCT(C14:C19,D4:D9)	=SUMPRODUCT(D14:D19,D4:D9)		
31		<=	<=	<=		
32		=$E$32*TotalInSchool	=$E$32*TotalInSchool	=$E$32*TotalInSchool	0.36	of total in school

b)

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$14	Area 1 School 1	0	177.778	300	1E+30	177.778
$C$14	Area 1 School 2	450	0	0	177.778	1E+30
$D$14	Area 1 School 3	0	266.667	700	1E+30	266.667
$B$15	Area 2 School 1	0	-800.000	0	1E+30	800.000
$C$15	Area 2 School 2	422.22	0	400	34.211	4.545
$D$15	Area 2 School 3	177.78	0	500	4.545	34.211
$B$16	Area 3 School 1	0	11.111	600	1E+30	11.111
$C$16	Area 3 School 2	227.78	0	300	4.545	34.211
$D$16	Area 3 School 3	322.22	0	200	34.211	7.692
$B$17	Area 4 School 1	350	0	200	366.667	2.08E+17
$C$17	Area 4 School 2	0	366.667	500	1E+30	366.667
$D$17	Area 4 School 3	0	-433.333	0	1E+30	433.333
$B$18	Area 5 School 1	366.67	0	0	16.667	108.333
$C$18	Area 5 School 2	0	233.333	0	1E+30	233.333
$D$18	Area 5 School 3	133.33	0	400	108.333	16.667
$B$19	Area 6 School 1	83.33	0	500	33.333	166.667
$C$19	Area 6 School 2	0	200	300	1E+30	200
$D$19	Area 6 School 3	366.67	0	0	166.667	33.333

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$B$30	8th Graders <=	242.67	0.00	0	1E+30	45.333
$C$30	8th Graders <=	369.33	0.00	0	1E+30	26.667
$D$30	8th Graders <=	360.00	-6666.67	0	5.333	0.667
$B$20	Total In School School 1	800	0	900	1E+30	100
$C$20	Total In School School 2	1,100	-178	1100	36.364	3.774
$D$20	Total In School School 3	1,000	-144	1000	42.105	3.883
$B$28	6th Graders <=	269.33	0.00	0	29.333	1E+30
$C$28	6th Graders <=	368.56	0.00	0	38.556	1E+30
$D$28	6th Graders <=	339.11	0.00	0	39.111	1E+30
$B$28	6th Graders <=	269.33	0.00	0	1E+30	18.667
$C$28	6th Graders <=	368.56	0.00	0	1E+30	27.444
$D$28	6th Graders <=	339.11	0.00	0	1E+30	20.889
$B$29	7th Graders <=	288.00	0.00	0	48	1E+30
$C$29	7th Graders <=	362.11	0.00	0	32.111	1E+30
$D$29	7th Graders <=	300.89	0.00	0	0.889	1E+30
$B$29	7th Graders <=	288.00	-2777.78	0	0.258	2.909
$C$29	7th Graders <=	362.11	0.00	0	1E+30	33.889
$D$29	7th Graders <=	300.89	0.00	0	1E+30	59.111
$B$30	8th Graders <=	242.67	0.00	0	2.667	1E+30
$C$30	8th Graders <=	369.33	0.00	0	39.333	1E+30
$D$30	8th Graders <=	360.00	0.00	0	60	1E+30
$E$14	Area 1 From Area	450	177.778	450	3.774	36.364
$E$15	Area 2 From Area	600	577.778	600	3.774	36.364
$E$16	Area 3 From Area	550	477.778	550	3.774	36.364
$E$17	Area 4 From Area	350	311.111	350	72.727	6.452
$E$18	Area 5 From Area	500	-55.556	500	12.903	145.455
$E$19	Area 6 From Area	450	277.778	450	3.226	36.364

c) The bussing cost from area 6 to school 1 can increase $33.33 before the current optimal solution would no longer be optimal. The new solution with a 10% increase ($50) is shown below.

	A	B	C	D	E	F	G
1	**Data:**	Percentage	Percentage	Percentage			
2		in 6th	in 7th	in 8th		Bussing Cost ($/Student)	
3	Area	Grade	Grade	Grade	School 1	School 2	School 3
4	1	32%	38%	30%	$300	$0	$700
5	2	37%	28%	35%	-	$400	$500
6	3	30%	32%	38%	$600	$300	$200
7	4	28%	40%	32%	$200	$500	-
8	5	39%	34%	27%	$0	-	$400
9	6	34%	28%	38%	$550	$300	$0
10							
11							
12	**Solution:**		Number of Students Assigned		Total		Number of
13		School 1	School 2	School 3	From Area		Students
14	Area 1	0	450	0	450	=	450
15	Area 2	0	600	0	600	=	600
16	Area 3	72.73	50	427.27	550	=	550
17	Area 4	350	0	0	350	=	350
18	Area 5	318.18	0	181.82	500	=	500
19	Area 6	59.09	0	390.91	450	=	450
20	Total In School	800	1,100	1,000			
21		<=	<=	<=			Total
22	Capacity	900	1,100	1,000			Bussing
23							Cost
24							$559,318
25	**Grade Constraints:**						
26		240	330	300	30%	of total in school	
27		<=	<=	<=			
28	6th Graders	264.00	381.00	332.00			
29	7th Graders	288.00	355.00	308.00			
30	8th Graders	248.00	364.00	360.00			
31		<=	<=	<=			
32		288	396	360	36%	of total in school	

d) The bussing cost from area 6 to school 2 can increase any amount and the optimal solution from part (a) will still be optimal.

e) If the bussing costs increase 1% from area 6 to all the schools, then:

Percentage of allowable increase for school 1 used = ($505 – $500) / $33.33 = 15%.
Percentage of allowable increase for school 2 used = ($303 – $300) / ∞ = 0%.
Percentage of allowable increase for school 3 used = ($0 – $0) / $166.67 = 0%.
Sum = 15%. Therefore, the bussing costs from area 6 can increase uniformly by (100%/15%)(1%) = 6.67% before 100% will be reached. Beyond that, the solution might change.

If the bussing costs increase 10% from area 6 to all schools, the new solution is:

	A	B	C	D	E	F	G
1	**Data:**	Percentage	Percentage	Percentage			
2		in 6th	in 7th	in 8th	Bussing Cost ($/Student)		
3	Area	Grade	Grade	Grade	School 1	School 2	School 3
4	1	32%	38%	30%	$300	$0	$700
5	2	37%	28%	35%	-	$400	$500
6	3	30%	32%	38%	$600	$300	$200
7	4	28%	40%	32%	$200	$500	-
8	5	39%	34%	27%	$0	-	$400
9	6	34%	28%	38%	$550	$330	$0
10							
11							
12	**Solution:**	Number of Students Assigned			Total		Number of
13		School 1	School 2	School 3	From Area		Students
14	Area 1	0	450	0	450	=	450
15	Area 2	0	600	0	600	=	600
16	Area 3	72.73	50	427.27	550	=	550
17	Area 4	350	0	0	350	=	350
18	Area 5	318.18	0	181.82	500	=	500
19	Area 6	59.09	0	390.91	450	=	450
20	Total In School	800	1,100	1,000			
21		<=	<=	<=			Total
22	Capacity	900	1,100	1,000			Bussing
23							Cost
24							$559,318
25	**Grade Constraints:**						
26		240	330	300	30%	of total in school	
27		<=	<=	<=			
28	6th Graders	264.00	381.00	332.00			
29	7th Graders	288.00	355.00	308.00			
30	8th Graders	248.00	364.00	360.00			
31		<=	<=	<=			
32		288	396	360	36%	of total in school	

f) The shadow price for school 1 is zero. Thus, adding a temporary classroom at school 1 would not save any money, and thus would not be worthwhile.

The shadow price for school 2 is –$177.78. Thus, adding a temporary classroom at school 2 would save ($177.78)(20) = $3,555.60 in bussing cost. This is worthwhile, since it exceeds the $2500 leasing cost.

The shadow price for school 3 is –$144.44. Thus, adding a temporary classroom at school 3 would save ($144.44)(20) = $2,888.80 in bussing cost. This is also worthwhile, since it exceeds the $2500 leasing cost.

g) For school 2, the allowable increase for school capacity is 36. This means the shadow price is only valid for a single additional portable classroom.

For school 3, the allowable increase for school capacity is 42. This means the shadow price is valid for up to two additional portable classrooms.

h) The following combinations do not violate the 100% rule:

Portables to add to school 2	Portables to add to school 3	100%-rule calculation
1	0	(20/36) + (0/42) = 55.6%
0	1	(0/36) + (20/42) = 47.6%
0	2	(0/36) + (40/42) = 95.23%

Each combination yields the following total savings

Portables to add to school 2	Portables to add to school 3	Bussing Cost Savings	Lease Cost	Total Savings
1	0	(\$177.78)(20) = \$3555.60	\$2500	\$1055.60
0	1	(\$144.44)(20) = \$2888.80	\$2500	\$388.80
0	2	(\$144.44)(40) = \$5777.60	\$5000	\$777.60

Of these combinations, adding one portable to school 2 is best in terms of minimizing total cost. The spreadsheet solution is shown below.

	A	B	C	D	E	F	G
1	**Data:**	Percentage	Percentage	Percentage			
2		in 6th	in 7th	in 8th	Bussing Cost ($/Student)		
3	Area	Grade	Grade	Grade	School 1	School 2	School 3
4	1	32%	38%	30%	$300	$0	$700
5	2	37%	28%	35%	-	$400	$500
6	3	30%	32%	38%	$600	$300	$200
7	4	28%	40%	32%	$200	$500	-
8	5	39%	34%	27%	$0	-	$400
9	6	34%	28%	38%	$500	$300	$0
10							
11							
12	**Solution:**	Number of Students Assigned			Total		Number of
13		School 1	School 2	School 3	From Area		Students
14	Area 1	0	450	0	450	=	450
15	Area 2	0	520	80	600	=	600
16	Area 3	0	150	400	550	=	550
17	Area 4	350	0	0	350	=	350
18	Area 5	340	0	160	500	=	500
19	Area 6	90	0	360	450	=	450
20	Total In School	780	1,120	1,000			
21		<=	<=	<=		Bussing Cost	$552,000
22	Capacity	900	1,120	1,000		Leasing Cost	$2,500
23						Total Cost	$554,500
24							
25	**Grade Constraints:**						
26		234	336	300	30%	of total in school	
27		<=	<=	<=			
28	6th Graders	261.20	381.40	334.40			
29	7th Graders	280.80	364.60	305.60			
30	8th Graders	238.00	374.00	360.00			
31		<=	<=	<=			
32		280.8	403.2	360	36%	of total in school	

i) Adding two portables to school 2 yields the following solution. This is the best plan.

	A	B	C	D	E	F	G
1	**Data:**	Percentage	Percentage	Percentage			
2		in 6th	in 7th	in 8th	Bussing Cost ($/Student)		
3	Area	Grade	Grade	Grade	School 1	School 2	School 3
4	1	32%	38%	30%	$300	$0	$700
5	2	37%	28%	35%	-	$400	$500
6	3	30%	32%	38%	$600	$300	$200
7	4	28%	40%	32%	$200	$500	-
8	5	39%	34%	27%	$0	-	$400
9	6	34%	28%	38%	$500	$300	$0
10							
11							
12	**Solution:**	Number of Students Assigned			Total		Number of
13		School 1	School 2	School 3	From Area		Students
14	Area 1	0	450	0	450	=	450
15	Area 2	0	600	0	600	=	600
16	Area 3	0	90	460	550	=	550
17	Area 4	350	0	0	350	=	350
18	Area 5	318.95	0	181.05	500	=	500
19	Area 6	95.26	0	354.74	450	=	450
20	Total In School	764	1,140	996			
21		<=	<=	<=		Bussing Cost	$549,053
22	Capacity	900	1,140	1,000		Leasing Cost	$5,000
23						Total Cost	$554,053
24							
25	**Grade Constraints:**						
26		229.2631579	342	298.7368421	30%	of total in school	
27		<=	<=	<=			
28	6th Graders	254.78	393.00	329.22			
29	7th Graders	275.12	367.80	308.08			
30	8th Graders	234.32	379.20	358.48			
31		<=	<=	<=			
32		275.1157895	410.4	358.4842105	36%	of total in school	

Case 7.4

a) In this case, the decisions to be made are

TV = number of units of advertising on television

PM = number of units of advertising in the printed media

The resulting linear programming model is

Minimize Cost = 1 TV + 2PM (in millions of dollars)

subject to

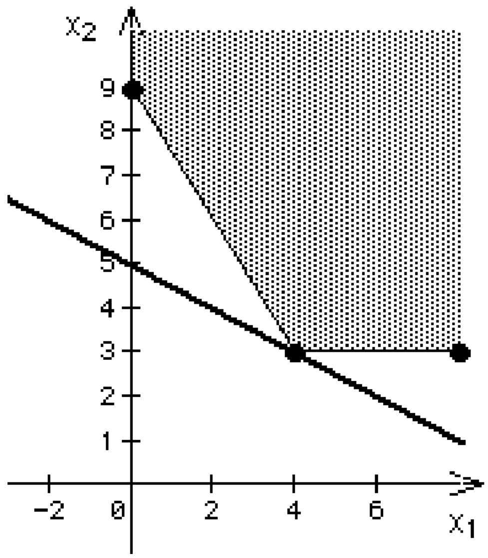
Stain Remover: 1 PM ≥ 3 (in %)

Liquid Detergent: 3 TV + 2PM ≥ 18 (in %)

Powder Detergent: −1TV + 4 PM ≥ 4 (in %)

and TV ≥ 0, PM ≥ 0.

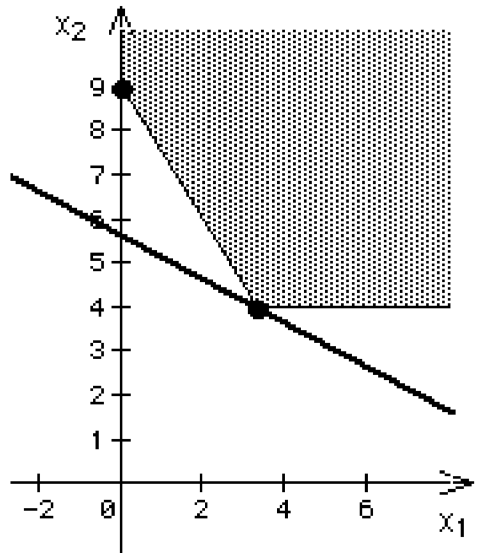
a) Optimal Solution: 4 units of television advertising and 3 units of print media advertising, with a total cost of $10 million.



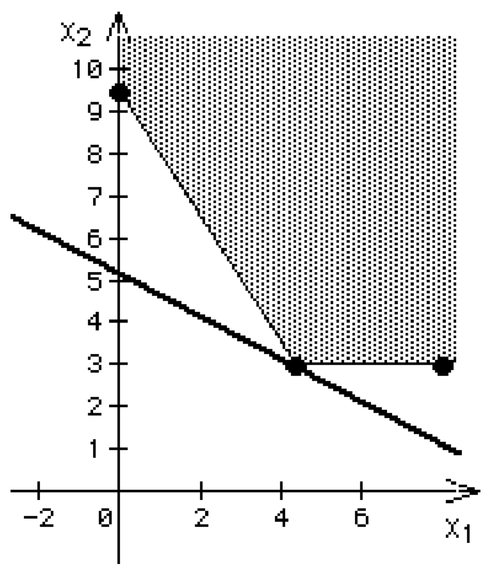
b) The spreadsheet model is shown below. Solver finds to take 4 units of television advertising and 3 units of print media advertising, with a total cost of $10 million.

	B	C	D	E	F	G
3		Television	Print Media			
4	Unit Cost ($millions)	1	2			
5						
6				Increased		Minimum
7		Increase in Sales per Unit of Advertising		Sales		Increase
8	Stain Remover	0%	1%	3%	>=	3%
9	Liquid Detergent	3%	2%	18%	>=	18%
10	Powder Detergent	-1%	4%	8%	>=	4%
11						
12						Total Cost
13		Television	Print Media			($millions)
14	Advertising Units	4	3			10

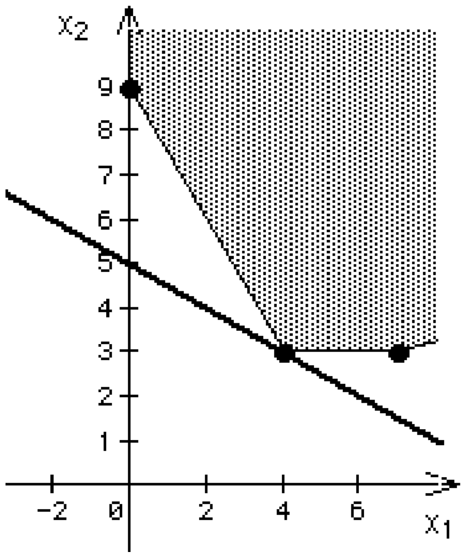
c) Increasing the required minimum increase in sales for Stain Remover by 1% changes the solution to 3.33 units of television advertising and 4 units of print media advertising, and increases the total cost by $1.33 million to $11.33 million.



Increasing the required minimum increase in sales for Liquid Detergent by 1% changes the solution to 4.33 units of television advertising and 3 units of print media advertising,, and increases the total cost by $0.33 million to $10.33 million.



Increasing the required minimmum increase in sales for Powder Detergent by 1% has no impact on the solution nor the total cost.



d) Original Solution:

	B	C	D	E	F	G
3		Television	Print Media			
4	Unit Cost ($millions)	1	2			
5						
6				Increased		Minimum
7		Increase in Sales per Unit of Advertising		Sales		Increase
8	Stain Remover	0%	1%	3%	>=	3%
9	Liquid Detergent	3%	2%	18%	>=	18%
10	Powder Detergent	-1%	4%	8%	>=	4%
11						
12						Total Cost
13		Television	Print Media			($millions)
14	Advertising Units	4	3			10

Increasing the required minimum increase in sales for Stain Remover by 1% increases the total cost by $1.333 million.

	B	C	D	E	F	G
3		Television	Print Media			
4	Unit Cost ($millions)	1	2			
5						
6				Increased		Minimum
7		Increase in Sales per Unit of Advertising		Sales		Increase
8	Stain Remover	0%	1%	4%	>=	4%
9	Liquid Detergent	3%	2%	18%	>=	18%
10	Powder Detergent	-1%	4%	13%	>=	4%
11						
12						Total Cost
13		Television	Print Media			($millions)
14	Advertising Units	3.333	4			11.333

Increasing the required minimum increase in sales for Liquid Detergent by 1% increases the total cost by $0.333 million.

	B	C	D	E	F	G
3		Television	Print Media			
4	Unit Cost ($millions)	1	2			
5						
6				Increased		Minimum
7		Increase in Sales per Unit of Advertising		Sales		Increase
8	Stain Remover	0%	1%	3%	>=	3%
9	Liquid Detergent	3%	2%	19%	>=	19%
10	Powder Detergent	-1%	4%	8%	>=	4%
11						
12						Total Cost
13		Television	Print Media			($millions)
14	Advertising Units	4.333	3			10.333

Increasing the required minimmum increase in sales for Powder Detergent by 1% has no impact on the total cost.

	B	C	D	E	F	G
3		Television	Print Media			
4	Unit Cost ($millions)	1	2			
5						
6				Increased		Minimum
7		Increase in Sales per Unit of Advertising		Sales		Increase
8	Stain Remover	0%	1%	3%	>=	3%
9	Liquid Detergent	3%	2%	18%	>=	18%
10	Powder Detergent	-1%	4%	8%	>=	5%
11						
12						Total Cost
13		Television	Print Media			($millions)
14	Advertising Units	4	3			10

e)

Minimum Increase (Stain Remover)	Television	Print Media	Total Cost	Incremental Cost
0%	4.571	2.143	8.857	
1%	4.571	2.143	8.857	0.000
2%	4.571	2.143	8.857	0.000
3%	4.000	3.000	10.000	1.143
4%	3.333	4.000	11.333	1.333
5%	2.667	5.000	12.667	1.333
6%	2.000	6.000	14.000	1.333

Minimum Increase (Liquid Detergent)	Television	Print Media	Total Cost	Incremental Cost
0%	0.000	3.000	6.000	
1%	0.000	3.000	6.000	0.000
2%	0.000	3.000	6.000	0.000
3%	0.000	3.000	6.000	0.000
4%	0.000	3.000	6.000	0.000
5%	0.000	3.000	6.000	0.000
6%	0.000	3.000	6.000	0.000
7%	0.333	3.000	6.333	0.333
8%	0.667	3.000	6.667	0.333
9%	1.000	3.000	7.000	0.333
10%	1.333	3.000	7.333	0.333
11%	1.667	3.000	7.667	0.333
12%	2.000	3.000	8.000	0.333
13%	2.333	3.000	8.333	0.333
14%	2.667	3.000	8.667	0.333
15%	3.000	3.000	9.000	0.333
16%	3.333	3.000	9.333	0.333
17%	3.667	3.000	9.667	0.333
18%	4.000	3.000	10.000	0.333
19%	4.333	3.000	10.333	0.333
20%	4.667	3.000	10.667	0.333
21%	5.000	3.000	11.000	0.333
22%	5.333	3.000	11.333	0.333
23%	5.667	3.000	11.667	0.333
24%	6.000	3.000	12.000	0.333
25%	6.333	3.000	12.333	0.333
26%	6.667	3.000	12.667	0.333
27%	7.000	3.000	13.000	0.333
28%	7.333	3.000	13.333	0.333
29%	7.667	3.000	13.667	0.333
30%	8.000	3.000	14.000	0.333
31%	8.286	3.071	14.429	0.429
32%	8.571	3.143	14.857	0.429
33%	8.857	3.214	15.286	0.429
34%	9.143	3.286	15.714	0.429
35%	9.429	3.357	16.143	0.429
36%	9.714	3.429	16.571	0.429

Minimum Increase (Powder Detergent)	Television	Print Media	Total Cost	Incremental Cost
0%	4	3	10	
1%	4	3	10	0
2%	4	3	10	0
3%	4	3	10	0
4%	4	3	10	0
5%	4	3	10	0
6%	4	3	10	0
7%	4	3	10	0
8%	4	3	10	0

f) Sensitivity Report:

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$C$14	Advertising Units Television	4	0	1	2	1
$D$14	Advertising Units Print Media	3	0	2	1E+30	1.333

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$E$8	Stain Remover Sales	3%	133.33	0.03	0.06	0.008571429
$E$9	Liquid Detergent Sales	18%	33.33	0.18	0.12	0.12
$E$10	Powder Detergent Sales	8%	0	0.04	0.04	1E+30

The shadow price indicates the increase in total cost (in $millions) per unit increase in the right hand side (i.e., per 100% increase). Thus, a 1% increase in the minimum required increase in sales will only increase the total cost by one hundredth of the shadow price, or $1.33 million for the Stain Remover, $0.33 million for the Liquid Detergent, and $0 million for the Powder Detergent.

The allowable range for the required minimum increase in sales constraint for Stain Remover is 2.15% to 9%.

The allowable range for the required minimum increase in sales constraint for Liquid Detergent is 6% to 30%.

The allowable range for the required minimum increase in sales constraint for Powder Detergent is -∞% to 8%.

These allowable ranges can also be seen in the results from part (c). For Stain Remover, the incremental cost remains $1.33 million for each 1% change above 3%. Similarly, for Liquid Detergent, the incremental cost remains $0.33 million for each 1% change above between 6% and 30%. For Powder Detergent, the incremental cost remains $0 million for each 1% change throughout the parameter analysis report.

g) Suppose that each of the original numbers in MinimumIncrease (G8:G10) is increased by 1%.

Percent of allowable increase for Stain Remover used = (4% – 3%) / 6% = 16.7%.
Percent of allowable increaes for Liquid Detergent used = (19% – 18%) / 12% = 8.3%.
Percent of allowable increase for Powder Detergent used = (5% – 4%) / 4% = 25%.
Sum = 50%.

Thus, if each of the original numbers in MinimumIncrease (G8:G10) is increased by 2%, the sum will be 100%. By the 100% rule, this is the most they can be increased before the shadow prices may no longer be valid.

h) Answers will vary.