Important note: Transcripts are **not** substitutes for textbook assignments.

Decision Analysis:

Making Decisions in the Face of Uncertainty

Methods of Choosing an Alternative

- Maximin Payoff Criterion
- Maximum Likelihood Criterion
- Bayes' Decision Rule

Experimentation

- Prior and posterior probabilities
- EVPI: Expected Value of Perfect Information
- EVE: Expected Value of Experimentation

Decision Trees

- Decision and event nodes
- Branches

Decision analysis is the science of making decisions in the face of uncertainty. In contrast to game theory, the opponent is known as nature. The states of which occur purely by chance rather than in attempt to defeat the decision maker. In common with game theory however, a payoff table is used to organize the information.

For the decision maker there are three different methods of choosing an alternative that will result in a payoff that is optimal in some way. With the maximin payoff criterion, the alternative that provides the maximum of the minimum payoff is chosen. Basically like the maximin criterion in game theory. This method is probably the least desirable because it tends to be overly cautious.

The maximum likelihood criterion says to pick the maximum payoff for the state of nature with the highest probability of occurring. This method is a little better than the maximin payoff criterion because it takes into account the probabilities of the states of nature, but still doesn't make use of as much information as the Bayes' Decision Rule. With Bayes' Decision Rule, the alternative with the maximum expected payoff is chosen. Experimentation is something that can be done to learn more about the states of nature, particularly in providing an improved prior probability for the occurrence of each. Posterior probabilities are computed from the initial prior probabilities along with probabilities associated with the experimentation.

These probabilities are a crucial component of making decisions in the face of uncertainty, because uncertainty is characterized by probability. With experimentation the expected value of perfect information and the expected value of experimentation can be computed. Probability tree diagrams are a great way to enumerate outcomes and compute posterior probabilities. And similarly, decision trees are a great way to visualize the information in the decision analysis problem. The ASPE add-in for Excel has software to build and solve decision trees.

Formulating a Decision Analysis Problem

Problem 16.2-2 will be used to illustrate how to formulate a simple decision analysis problem.

Decision Alternatives:
- Build computers themselves
- Sell the rights to the chip

States of Nature:
- Sales of 10,000 computers
- Sales of 100,000 computers

Other information:
- Selling the rights earns $15 million
- Production set-up cost is $6 million
- $600 profit on each computer sold

We'll begin by looking at exercise 16.2-2 to illustrate the basic formulation of a decision analysis problem. You'll have to read the exercise in the textbook to get the full context, but the basic details are shown here.

Question 1

Self-Assessment Question

Decision making cannot be done without experimentation.

- True
- False

Correct answers can be found at the end of this transcript.

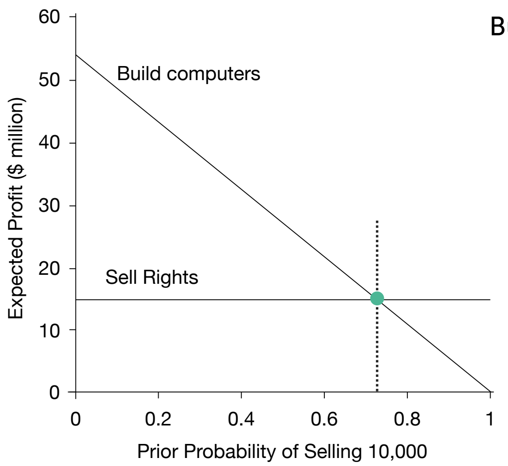
Problem 16.2-2 Payoff Table

	State of Nature	
Alternative	Sell 10,000	Sell 100,000
Build Computers	0	54
Sell rights to chip	15	15

The payoffs to the decision maker in millions of dollars are shown for each combination of decision alternative and state of nature. If Silicon Dynamics decides to build the computers, and only end up selling 10,000 at a $600 profit per computer, they'll bring in $6 million, just enough to cover the cost of setting up the assembly line. So they'll end up with a zero payoff. On the other hand, if they sell 100,000 computers, they'll earn a total of $60 million minus the $6 million in setup costs, for a total of $54 million.

Granted, the actual number of computers sold has more than just two alternatives, but for the sake of this example these values will do for now. Furthermore, the probabilities of each are not known. If they decide to sell the rights to the chip, they earn $15 million no matter how many computers are sold, because they'll be sold by someone else.

Problem 16.2-2 Graphing Expected Payoffs



Build: $EP = p(0) + (1-p)(54) = -54p + 54$

Sell: $EP = p(15) + (1-p)(15) = 15$

$-54p + 54 = 15$

$p = 0.722$

The prior probabilities for the states of nature selling 10,000 or 100,000 computers are not known in this exercise. But we can let p represent the probability of selling 10,000 computers. Since there are only two states, 1 minus p will represent the probability of selling 100,000 computers. Using probability theory for discrete random variables, the expected payoffs can be found for each decision alternative by summing over the product of each payoff and its probability. This results in a linear equation between the expected payoff and the prior probability for each decision alternative.

The expected payoff equals negative 54 times p plus 54 when they build the computers themselves, and the constant ep equals 15 when they sell the rights to the chip. These equations can be placed on the same graph and used to decide which alternative results in a higher expected payoff for a given set of probabilities. The points or points where the choice of alternative changes are called crossover points. On this graph, there's just one crossover point right here. Algebraically, setting the expected payoff equations equal to each other and solving for p will yield a prior probability of 0.722 for selling 10,000 computers.

So if Silicon Dynamics finds that the probability of selling 10,000 computers is less than 0.722, they would choose to build the computers themselves. Because their expected payoff is highest with that alternative. If p exceeds 0.722, then they should sell the rights to the chip to maximize their expected profit. If p equals 0.722, then either alternative will yield the same expected profit of $15 million.

Another Example: Three Methods of Choosing

	State of Nature		
Alternative	S_1	S_2	S_3
A_1	5	2	1
A_2	1	3	0
A_3	4	1	3
Prior probability	0.25	0.40	0.35

For a simple decision analysis payoff table, there are three methods for choosing an alternative. Maximin payoff criterion, maximum likelihood criterion, and Bayes' Decision Rule. Each of these three will be demonstrated using the payoff table given here. This payoff table will also be used to illustrate the expected value of perfect information, the computation of posterior probabilities, the expected value of experimentation, and the decision tree in the videos that follow. This scenario will be used for all of the remaining examples.

Maximin Payoff Criterion

	State of Nature			
Alternative	S_1	S_2	S_3	Minimum
A_1	5	2	1	1
A_2	1	3	0	0
A_3	4	1	3	1
Prior probability	0.25	0.40	0.35	

The maximin payoff criterion is really rather conservative. Especially considering that nature isn't actively trying to bit you, like an opponent in game theory would. According to the maximin criterion, we would choose the alternative with the largest of these minimum payoffs.

But in decision analysis, it is assumed that the decision maker is indifferent to two alternatives with the same payoff. So in this case, the maximum payoff criterion says that the decision maker would be just as well off choosing A1 or A3. This criterion for decision making doesn't take into account the probabilities of the states of nature, which is important information.

Maximum Likelihood Criterion

	State of Nature		
Alternative	S_1	S_2	S_3
A_1	5	2	1
A_2	1	3	0
A_3	4	1	3
Prior probability	0.25	0.40	0.35

The maximum likelihood criterion says to pick the maximum payoff for the state of nature with the highest probability of occurring. Here S2 is the most likely state of nature at a 40% chance of occurring. Alternative A2 has the highest payoff at 3. So the decision maker should choose A2. The maximum likelihood criterion is a step in the right direction, but the Bayes' Decision Rule takes into account even more information from the payoff table to provide an even more informed decision.

Question 2

Self-Assessment Question

Choosing the decision alternative with the maximum expected payoff out of all possible decision alternatives using the best available estimates of the probabilities of the respective states of nature is called:

- Bayes' decision rule
- The maximum likelihood criterion
- The maximin payoff criterion
- Dealing with experimentation
- Irrational

Correct answers can be found at the end of this transcript.

Bayes' Decision Rule

	State of Nature			
Alternative	S_1	S_2	S_3	Expected Payoff
A_1	5	2	1	$5(0.25) + 2(0.40) + 1(0.35) = 2.40$
A_2	1	3	0	$1(0.25) + 3(0.40) + 0(0.35) = 1.45$
A_3	4	1	3	$4(0.25) + 1(0.40) + 3(0.35) = 2.45$
Prior probability	0.25	0.40	0.35	

With Bayes' Decision Rule, the alternative with the maximum expected payoff is chosen. For discrete random variables such as these, the expected value is the sum of the products of each payoff multiplied by its probability. And these can be computed for each decision alternative.

The largest of these here is 2.45. And so alternative 3 would be chosen according to the Bayes' Decision Rule. Bayes' Decision Rule has the advantage of taking into account all of the information available in the payoff table.

EVPI

Expected Value of Perfect Information

EVPI = expected payoff w/ perfect info – expected payoff w/out experimentation

$$EVPI = [5(0.25) + 3(0.40) + 3(0.35)] - 2.45$$

$$EVPI = 3.5 - 2.45 = 1.05$$

The expected value of perfect information is an upper bound on the potential value of the experiment. Not counting the cost of the experiment, and assuming the experiment removes all uncertainty. The expected payoff with perfect information is found by weighting the maximum payoff under each state of nature with the prior probability of that state occurring. The expected payoff without experimentation is the maximum expected payoff from the Bayes' Decision Criterion. The difference between these two is the value of perfect information.

This is the upper bound that one might be willing to pay if it's possible for experimentation to determine perfect information. So if these payoffs are in terms of thousands of dollars for example, up to $1,050 could be allocated to finding perfect information. The phrase perfect information sounds a bit optimistic. There is another measure for the value of experimentation that we can compute, but more information about the experimentation is needed and posterior probabilities for the states of nature must be computed. The coming videos will address these things.

Bayes' Theorem and Posterior Probabilities

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \rightarrow P(A \cap B) = P(B|A) \cdot P(A)$$

$$P(B) = P(A \cap B) + P(A^C \cap B) = P(B|A) \cdot P(A) + P(B|A^C) \cdot P(A^C)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A) \cdot P(A)}{P(B|A) \cdot P(A) + P(B|A^C) \cdot P(A^C)}$$

$$P(\text{state } i|\text{finding } j) = \frac{P(\text{finding } j|\text{state } i) \cdot P(\text{state } i)}{\sum_{k=1}^{n} P(\text{finding } j|\text{state } k) \cdot P(\text{state } k)}$$

If you've ever had a course in probability, this may be very helpful for you to understand how the posterior probabilities are eventually obtained. If you've never studied probability, this may be confusing for you. Not to worry, there's an Excel spreadsheet set up to do this computation for you. But this will still help you understand how it's set up and why it works. You may recall the standard definition of conditional probability of event A, given that event B has already occurred.

For any two events generically labeled as A and B is the probability of the intersection of and B divided by the probability of the given event B since A and B are arbitrary labels, the probability of B given A is the probability of the intersection of A and B divided by the probability of the given event A. So the probability of the intersection of A and B can be written as the conditional probability of B given A times the marginal probability of event A. According to the law of total probability, the probability of event B is the sum of event B intersected with all of the mutually exclusive partitions of event A, shown as A and A complement here for simplicity. But A could be segmented into any number of mutually exclusive partitions. The intersection probabilities can be rewritten as products of the appropriate conditional and marginal probabilities.

Substituting into the initial definition of conditional probability gives this formula for the probability of A given B. In the decision analysis setting, think about the probability of the state of nature being in state I in place of event A. The probability of the finding of the experimentation being finding j in the place of event B.

Bayes' Theorem is applied here to reverse the conditioning from finding given state to state given finding. The probabilities of finding given state are typically easier to come by, and so this formula is required. The initial probabilities of each state of nature are called prior probabilities, and these conditional probabilities of n states of nature given the finding from experimentation are called posterior probabilities.

Findings from Experimentation

	State of Nature		
Alternative	S_1	S_2	S_3
$P(\text{finding} = F_1 \mid \text{state} = S_i)$	0.65	0.90	0.55
$P(\text{finding} = F_2 \mid \text{state} = S_i)$	0.35	0.10	0.45

F_1 = Experimentation yields favorable information
F_2 = Experimentation yields unfavorable information

Continuing with the second example given previously, the one called another example, suppose that there are two findings from an experiment that can be done to gather more information about the favorability of each state of nature, given that that state of nature happens to be the one that's true. These are conditional probabilities. But in order to get the posterior probabilities for each state of nature, the order of conditioning will have to be reversed using Bayes' Theorem.

Computing Posterior Probabilities

$$P(S_1|F_1) = \frac{P(F_1|S_1) \cdot P(S_1)}{P(F_1|S_1) \cdot P(S_1) + P(F_1|S_2) \cdot P(S_2) + P(F_1|S_3) \cdot P(S_3)}$$

$$P(S_1|F_1) = \frac{0.65 \cdot 0.25}{0.65 \cdot 0.25 + 0.90 \cdot 0.4 + 0.55 \cdot 0.35} = \frac{0.1625}{0.715} = 0.227$$

$P(F_1) = 0.715$ and so $P(F_2) = 0.285$

$P(S_1) = 0.25$	$P(S_2) = 0.40$	$P(S_3) = 0.35$
$P(S_1\|F_1) = \frac{0.1625}{0.715} = 0.227$	$P(S_2\|F_1) = \frac{0.360}{0.715} = 0.503$	$P(S_3\|F_1) = \frac{0.1925}{0.715} = 0.269$
$P(S_1\|F_2) = \frac{0.0875}{0.285} = 0.307$	$P(S_2\|F_2) = \frac{0.040}{0.285} = 0.140$	$P(S_3\|F_2) = \frac{0.1575}{0.285} = 0.553$

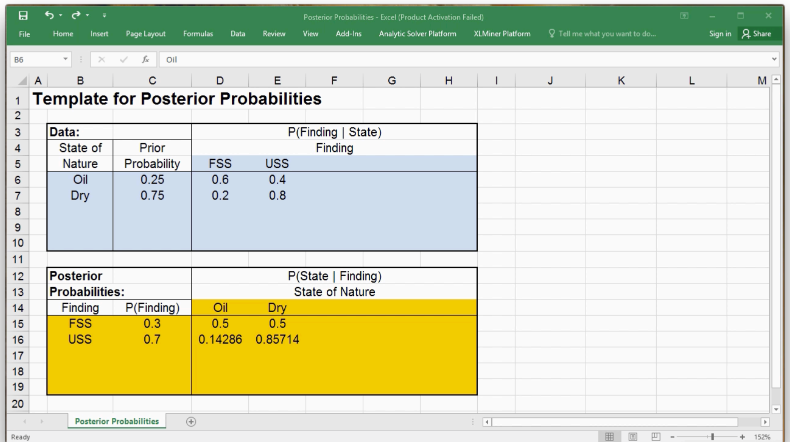
Here's the computation for the first of six posterior probabilities of state given finding. Using Bayes' Theorem, the posterior probability that state 1 is the case, given that the experimentation results in finding 1, which is a favorable finding, is seen here to be 0.227. Note the denominator 0.715. This is the total probability that finding 1 occurs when experimentation is conducted, regardless of which state of nature is true. Since there are only two possible findings to this particular experiment, the other finding must occur with probability 0.285.

These are the denominators in the posterior probability calculations. Applying the formula to the other combinations of state in finding, the remaining posterior probabilities for state given finding are as follows. You may recall that in the original example, the prior probabilities for the state of nature were 0.25 for state 1, 0.4 for state 2, and 0.35 for state 3. They are called prior probabilities because they are the probabilities assigned to the states of nature prior to experimentation. Testing to gain more information about which state of nature may be the one that occurs.

Posterior probabilities are the conditional probabilities of each state of nature after experimentation, and are conditioned on the outcome of the testing. The posterior probabilities can be interpreted this way. When the experimentation comes back favorable, the probability of nature being in state 1 drop slightly from 0.25 to 0.227. The probability of being in state 2 increases from 0.4 to 0.503, and the probability of being in state 3 is revised downward from 0.35 to 0.269.

If on the other hand the experimentation yields finding 2, giving unfavorable information, the probability of nature being in state 1 increases from 0.25 to 0.307, the probability of being in state 2 plummets from 0.4 to 0.14, and the probability of being in state 3 increases from 0.35 to 0.553. A probability tree diagram such as the one shown in figure 16.2 on page 692 of the textbook, is a great way to show all of the combinations of states and findings and organize the probabilities to facilitate the calculation of posterior probabilities.

Excel Template for Posterior Probabilities



Template for Posterior Probabilities

Data: P(Finding \| State)

State of Nature	Prior Probability	FSS	USS
Oil	0.25	0.6	0.4
Dry	0.75	0.2	0.8

Posterior Probabilities: P(State \| Finding)

Finding	P(Finding)	Oil	Dry
FSS	0.3	0.5	0.5
USS	0.7	0.14286	0.85714

Question 3

Self-Assessment Question

An excellent way to organize the outcomes along with prior, conditional, joint, and posterior probabilities is by using a:

- Probability tree diagram
- Calculator
- Slide rule
- Computer
- Notebook

Correct answers can be found at the end of this transcript.

Expected Payoffs After Experimentation

$$E[\text{Payoff}(A_1|F_1)] = 5(0.227) + 2(0.503) + 1(0.269) = 2.410$$
$$E[\text{Payoff}(A_2|F_1)] = 1(0.227) + 3(0.503) + 0(0.269) = 1.736$$
$$E[\text{Payoff}(A_3|F_1)] = 4(0.227) + 1(0.503) + 3(0.269) = 2.218$$

$$E[\text{Payoff}(A_1|F_2)] = 5(0.307) + 1(0.140) + 1(0.553) = 2.368$$
$$E[\text{Payoff}(A_2|F_2)] = 1(0.307) + 3(0.140) + 0(0.553) = 0.727$$
$$E[\text{Payoff}(A_3|F_2)] = 4(0.307) + 1(0.140) + 3(0553) = 3.027$$

After experimentation, information gathered from the testing can be used to improve the probability estimates for each state of nature. Using these posterior probabilities, the expected payoff for each decision alternative can be computed, and the Bayes' Decision Criterion can be used to choose the alternative with the highest expected payoff for each finding of the experimentation. For this scenario, the optimal decision alternative when the experimentation results in favorable finding F1, is A1, with an expected payoff of 2.41. However, if the experimentation results in finding F2, then A3 has the highest expected payoff at 3.027. So the experimentation has an effect on the decision.

So to review, the maximin criterion suggested that the decision alternatives A1 and A3 were equally optimal. The maximum likelihood criterion recommends A2. Bayes' Decision Rule recommends A3, using only prior probabilities. And Bayes' Decision Rule with experimentation says to choose A1 if the testing results in F1, and A3 if the finding is F2.

Of these results, Bayes' Decision Rule with experimentation takes into account the most information, and so should provide the best decision. Next, we compute the expected value of experimentation for this scenario to estimate just how much value the experimentation is providing.

EVE

Expected Value of Experimentation

EVE = expected payoff w/experimentation – expected payoff w/out experimentation

$$EVE = [0.715(2.410) + 0.285(3.027)] - 2.45$$

$$EVE = 2.586 - 2.45 = 0.136$$

While the expected value of perfect information is an upper bound for the value of experimentation, the expected value of experimentation or EVE, is a more direct estimate of what the experimentation is worth. EVE is the difference between the expected payoff with experimentation, found by multiplying each maximum payoff under each finding by the probability that that finding is the one that occurs, and the expected value without experimentation, according to the Bayes' Decision Criterion. Here in the formula for the expected value with experimentation as seen in previous videos, the payoff from A1 is 2.41 when F1 is the finding, which occurs with probability 0.715. And the maximum payoff under finding F2, which occurs with probability 0.285 was 3.027 when A3 was the finding. If these payoff values are in thousands, then the expected value of experimentation in this case ends up being $136.

On the other hand, if our payoffs are in millions, then we should be willing to pay around $136,000 for this experimentation. If the right probabilities are known or can be estimated, all of these computations are made before any action is taken. So this is a guide to how much you should be willing to pay for the experimentation.

Question 4

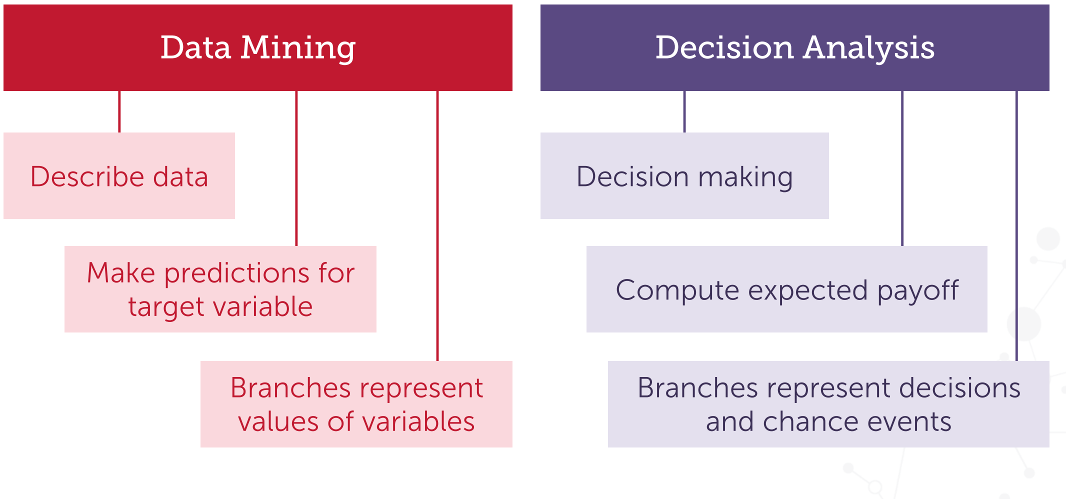
Self-Assessment Question

The difference between the expected payoff with perfect information and the expected payoff without experimentation is called the expected value of perfect information (EVPI).

- True
- False

Correct answers can be found at the end of this transcript.

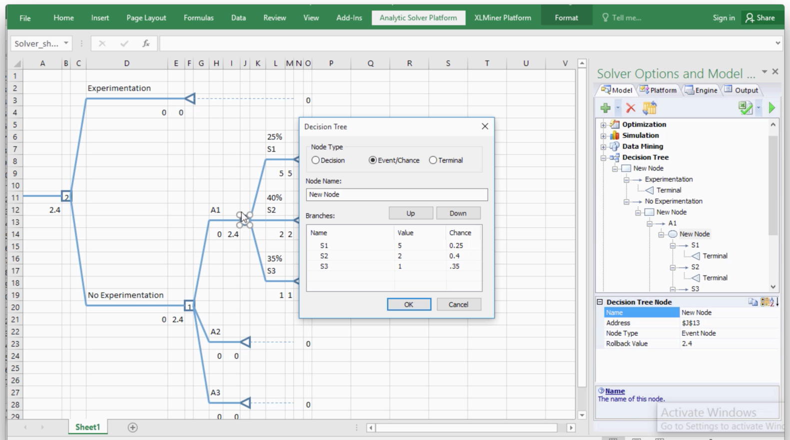
Decision Trees or Decision Trees



You may recall using decision trees in the data mining course. And indeed you did use them there. You may have also referred to them as regression trees or classification trees. The decision trees used here in the realm of decision analysis are different.

In data mining, decision trees are used to describe data. In decision analysis they're a tool for decision making. In data mining, the tree is used to make predictions for a target variable. In decision analysis, it's used to compute an expected payoff. In data mining, branches represent values of particular variables. In decision analysis, branches represent decisions and chance events.

The Decision Tree



Question 5

Self-Assessment Question

Decision analysis provides a methodology for rational decision making when outcomes are uncertain.

- True
- False

Correct answers can be found at the end of this transcript.

Question 6

Self-Assessment Question

In a decision tree, a circle represents a decision node, indicating that a decision needs to be made at that point in the process.

- True
- False

Correct answers can be found at the end of this transcript.

Question 1 Answer

DS775 - Decision Analysis Part One

Feedback for Self Assessment

✓ Correct!

Decision making cannot be done without experimentation.

Your answer:
True

Correct answer:
True

Feedback:
Read section 16.2.

Question 2 Answer

DS775 - Decision Analysis Part One

Feedback for Self Assessment

✓ Correct!

Choosing the decision alternative with the maximum expected payoff out of all possible decision alternatives using the best available estimates of the probabilities of the respective states of nature is called:

Your answer:
Bayes' decision rule

Correct answer:
Bayes' decision rule

Question 3 Answer

DS775 - Decision Analysis Part One

Feedback for Self Assessment

✓ Correct!

An excellent way to organize the outcomes along with prior, conditional, joint, and posterior probabilities is by using a:

Your answer:
probability tree diagram

Correct answer:
probability tree diagram

Feedback:
Oh yeah.

Question 4 Answer

DS775 - Decision Analysis Part One

Feedback for Self Assessment

✓ Correct!

The difference between the expected payoff with perfect information and the expected payoff without experimentation is called the expected value of perfect information (EVPI).

Your answer:
True

Correct answer:
True

Question 5 Answer

DS775 - Decision Analysis Part One

Feedback for Self Assessment

✓ Correct!

Decision analysis provides a methodology for rational decision making when outcomes are uncertain.

Your answer:
True

Correct answer:
True

Question 6 Answer

DS775 - Decision Analysis Part One

Feedback for Self Assessment

✓ Correct!

In a decision tree, a circle represents a decision node, indicating that a decision needs to be made at that point in the process.

Your answer:
False

Correct answer:
False

Feedback:
FEEDBACK-B