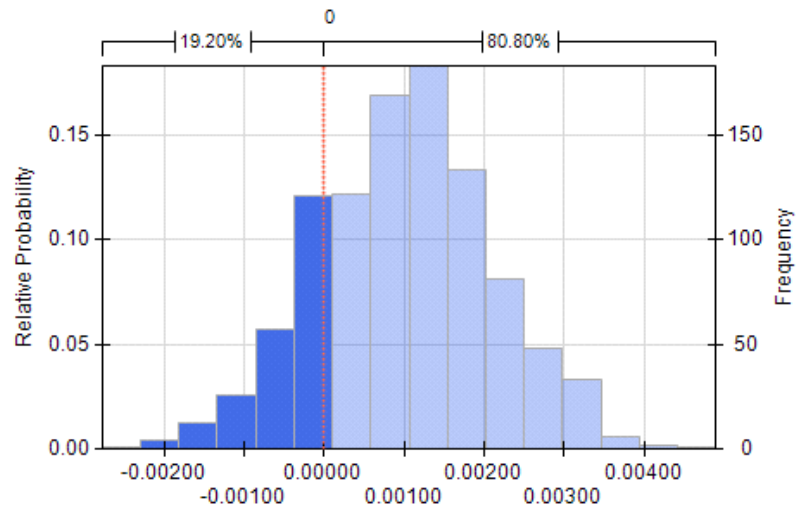


#### 20.6-4.

The chance of negative clearance is approximately 19.2%.

Shaft Radius	1.00122	Triangular(min,likely,max)	1.000	1.001	1.002
Bushing Radius	1.00317	Normal(mean,st.dev.)	1.002	0.001	
Clearance	0.00196				
Mean Clearance	0.00100				

Simulation Results - \$B\$4



20.6-5.

Toss	Die 1	Die 2	Sum	Win?	Lose?	Continue?	Win Game? (1=yes,0=no)
1	6	4	10	No	No	Yes	
2	5	3	8	No	No	Yes	
3	1	2	3	No	No	Yes	1
4	6	6	12	No	No	Yes	
5	3	3	6	No	No	Yes	Mean (Win Game?)
6	3	2	5	No	No	Yes	0.500
7	4	1	5	No	No	Yes	
8	2	2	4	No	No	Yes	
9	5	5	10	Yes	No	No	
10	4	3	7	#N/A	#N/A	#N/A	
11	6	6	12	#N/A	#N/A	#N/A	
12	3	3	6	#N/A	#N/A	#N/A	
13	1	1	2	#N/A	#N/A	#N/A	
14	4	6	10	#N/A	#N/A	#N/A	
15	3	5	8	#N/A	#N/A	#N/A	
16	2	5	7	#N/A	#N/A	#N/A	
17	3	6	9	#N/A	#N/A	#N/A	
18	2	1	3	#N/A	#N/A	#N/A	
19	3	2	5	#N/A	#N/A	#N/A	
20	6	5	11	#N/A	#N/A	#N/A	
21	6	4	10	#N/A	#N/A	#N/A	
22	4	1	5	#N/A	#N/A	#N/A	
23	3	3	6	#N/A	#N/A	#N/A	
24	6	5	11	#N/A	#N/A	#N/A	
25	6	4	10	#N/A	#N/A	#N/A	
26	5	2	7	#N/A	#N/A	#N/A	
27	2	6	8	#N/A	#N/A	#N/A	
28	1	5	6	#N/A	#N/A	#N/A	
29	6	4	10	#N/A	#N/A	#N/A	
30	2	6	8	#N/A	#N/A	#N/A	

(a) Answers will vary. The standard error is approximately 0.05, so the typical values should be between 0.450 and 0.550.

(b) Answers will vary. The standard error is approximately 0.016, so the typical values should be between 0.484 and 0.516.

(c) Answers will vary. The standard error is approximately 0.005, so the typical values should be between 0.495 and 0.505.

(d) Answers will vary. There is a fair amount of variability in the number of wins, so a large number of iterations, say 10,000, is necessary to predict the true probability. With 10,000 iterations, the standard error is 0.005.

**20.6-6.**

The order quantity that maximizes the mean profit is approximately 55.

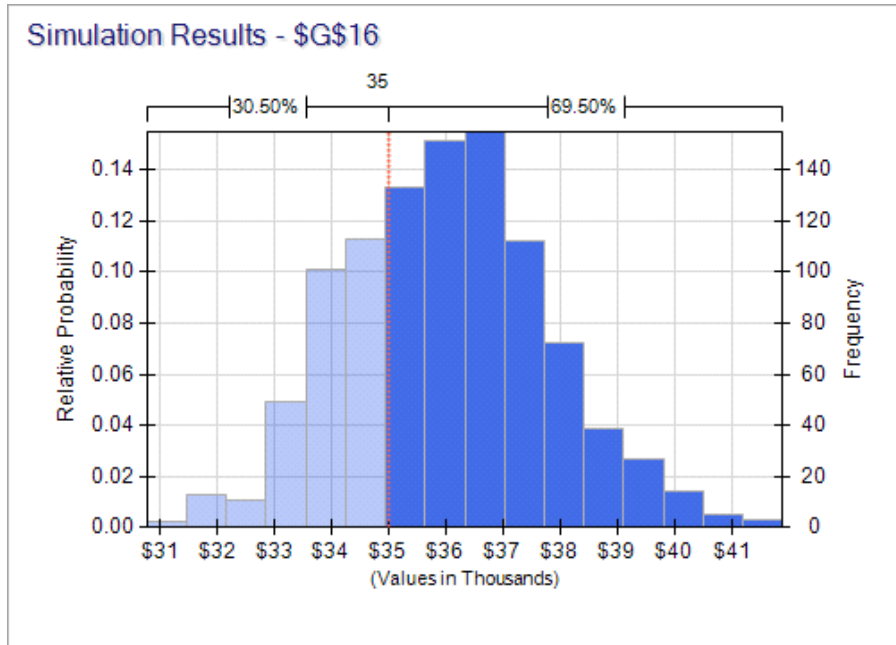
Order Quantity	Mean Profit
50	\$46.45
51	\$46.74
52	\$46.97
53	\$47.13
54	\$47.22
55	\$47.26
56	\$47.22
57	\$47.13
58	\$46.97
59	\$46.74
60	\$46.45

**20.6-7.**

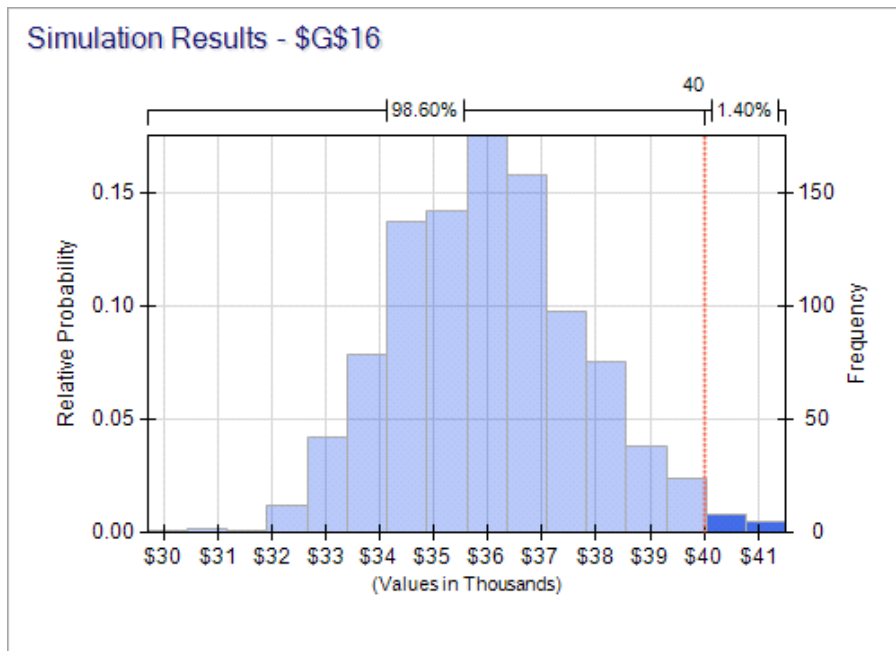
(a) and (b) The expected value of the college fund at year 5 is approximately \$36 thousand. The standard deviation of the college fund at year 5 is just over \$1700.

	Initial	Annual							
Stock Fund	\$3,000	\$2,000							
Bond Fund	\$3,000	\$2,000							
	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5			
Stock Fund Investment	\$5,000	\$2,000	\$2,000	\$2,000	\$2,000	\$2,000			
Stock Fund Start	\$5,000	\$7,519	\$9,962	\$12,804	\$15,794	\$19,738		Mean	St. Dev.
Stock Fund Return (%)	10%	6%	8%	8%	12%		Normal	8%	6%
Stock Fund End	\$5,519	\$7,962	\$10,804	\$13,794	\$17,738				
Bond Fund Investment	\$5,000	\$2,000	\$2,000	\$2,000	\$2,000	\$2,000			
Bond Fund Start	\$5,000	\$7,298	\$9,649	\$11,516	\$13,722	\$16,862			
Bond Fund Return (%)	6%	5%	-1%	2%	8%		Normal	4%	3%
Bond Fund End	\$5,298	\$7,649	\$9,516	\$11,722	\$14,862				
					Total	\$36,600			
					Mean	\$35,993			
					St. Deviation	\$1,729			

(c) The probability that the college fund at year 5 will be at least \$35,000 is approximately 69.5%.



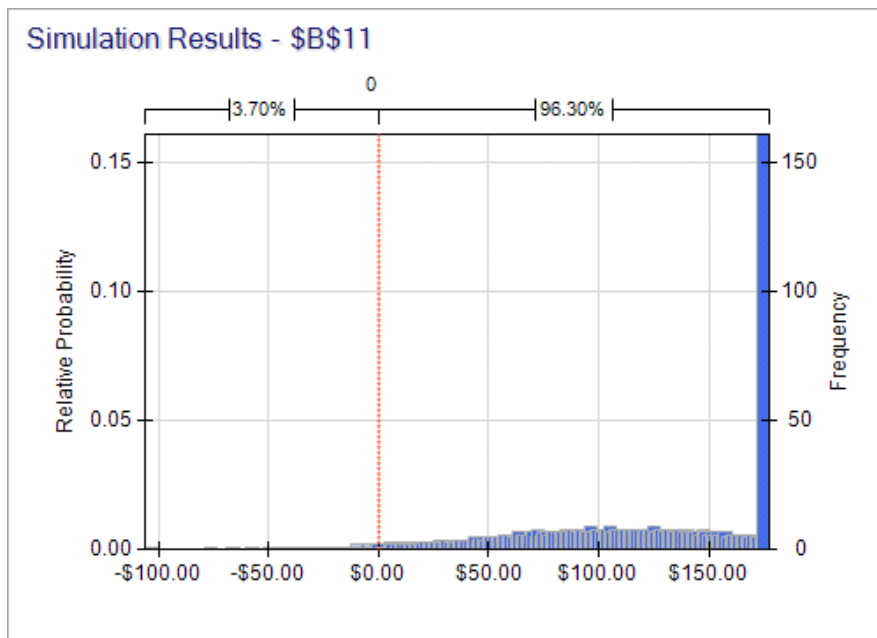
(d) The probability that the college fund at year 5 will be at least \$40,000 is approximately 1.4%.



### 20.6-8.

(a) The mean profit is approximately \$107. There is an approximately 96.3% change of making at least \$0 profit.

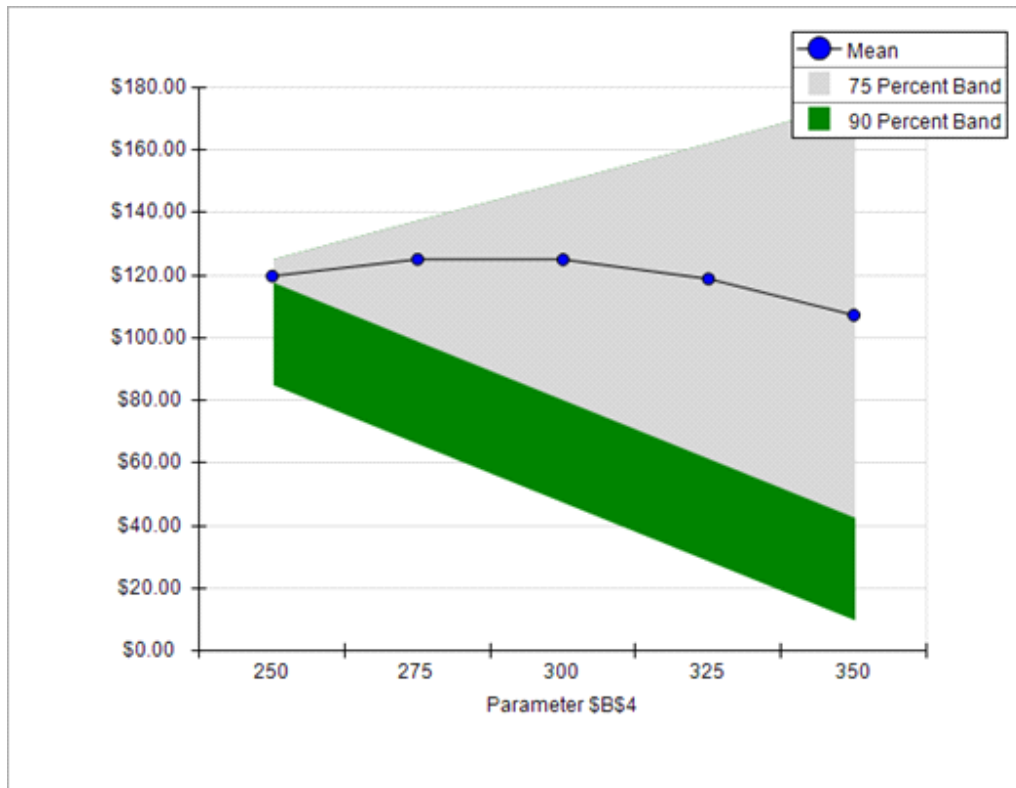
Purchase Price	\$0.75			
Selling Price	\$1.25			
Order Quantity	350			
			Mean	St. Dev.
Demand	298.7126	Normal	300	50
Rounded Demand	299			
Revenue	\$373.75			
Purchase Cost	\$262.50			
Total Profit	\$111.25			
Mean Total Profit	\$107.29			



(b) An order quantity of 275 maximizes the mean profit. An order quantity of 300 is also very close to maximizing the mean profit. The order quantity that actually maximizes the mean profit is probably somewhere between these two quantities.

Order Quantity	Mean Profit
250	\$119.80
275	\$125.14
300	\$125.07
325	\$118.89
350	\$107.30

(c)



(d) An order quantity of approximately 287 maximizes Michael's mean profit.

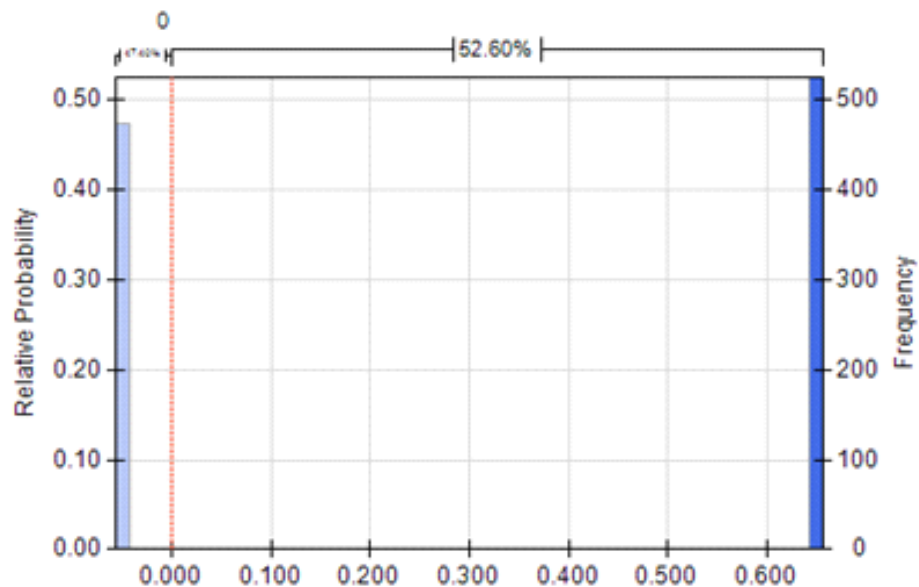
Purchase Price	\$0.75			
Selling Price	\$1.25			
Order Quantity	287			
			Mean	St. Dev.
Demand	348.3337	Normal	300	50
Rounded Demand	348			
Revenue	\$358.75			
Purchase Cost	\$215.25			
Total Profit	\$143.50			
Mean Total Profit	\$125.85			

### 20.6-9.

(a) The mean profit is approximately \$0.3 million. The probability of winning the bid is approximately 52.6%.

<b>Data</b>				
Our Project Cost (\$million)	5.000			
Our Bid Cost (\$million)	0.050			
<b>Competitor Bids</b>				
	Competitor 1	Competitor 2	Competitor 3	Competitor 4
Bid (\$million)	5.824	5.959	6.114	6.136
Distribution	Triangular	Triangular	Triangular	Triangular
<b>Competitor Distribution Parameters (Proportion of Our Project Cost)</b>				
Minimum	105%	105%	105%	105%
Most Likely	120%	120%	120%	120%
Maximum	140%	140%	140%	140%
<b>Competitor Distribution Parameters (\$millions)</b>				
Minimum	5.250	5.250	5.250	5.250
Most Likely	6.000	6.000	6.000	6.000
Maximum	7.000	7.000	7.000	7.000
<b>Minimum Competitor Bid (\$million)</b>				
	5.824			
<b>Our Bid (\$million)</b>				
	5.700			
<b>Win Bid?</b>				
	1	(1=yes, 0=no)		
<b>Profit (\$million)</b>				
	0.650			
<b>Mean Profit (\$million)</b>				
	0.303			

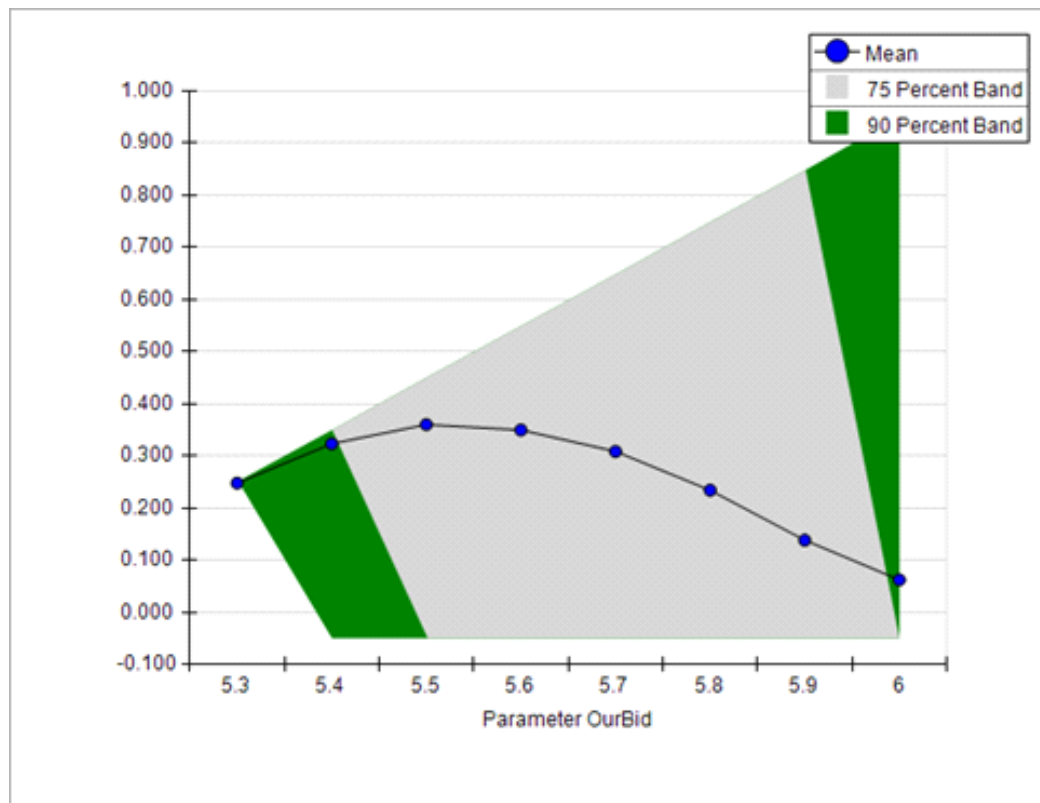
Simulation Results - Profit



(b) A bid of approximately \$5.5 million maximizes RPI's mean profit.

OurBid	Mean Profit (\$million)
5.3	0.248
5.4	0.323
5.5	0.364
5.6	0.356
5.7	0.313
5.8	0.234
5.9	0.140
6.0	0.061

(c)





(d) The optimal bid is approximately \$5.57 million, as found by Solver.

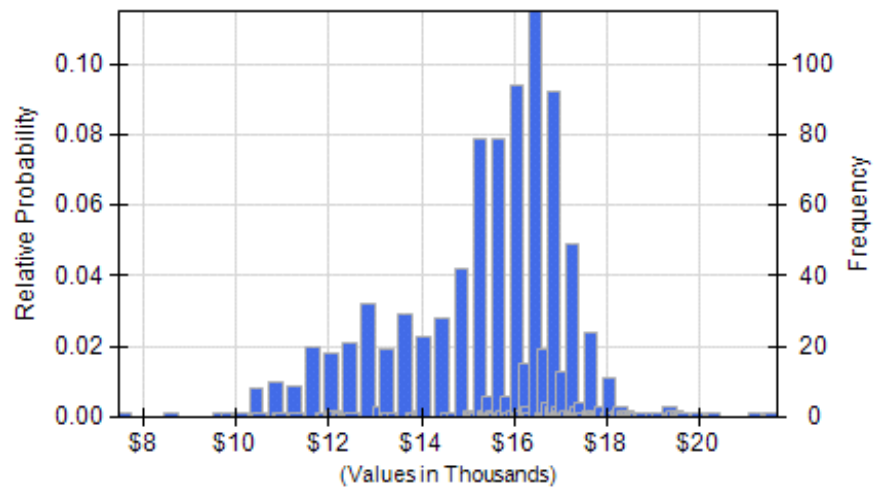
<b>Data</b>				
Our Project Cost (\$million)	5.000			
Our Bid Cost (\$million)	0.050			
<b>Competitor Bids</b>	Competitor 1	Competitor 2	Competitor 3	Competitor 4
Bid (\$million)	6.133	5.864	6.312	6.341
Distribution	<i>Triangular</i>	<i>Triangular</i>	<i>Triangular</i>	<i>Triangular</i>
<b>Competitor Distribution Parameters (Proportion of Our Project Cost)</b>				
Minimum	105%	105%	105%	105%
Most Likely	120%	120%	120%	120%
Maximum	140%	140%	140%	140%
<b>Competitor Distribution Parameters (\$millions)</b>				
Minimum	5.250	5.250	5.250	5.250
Most Likely	6.000	6.000	6.000	6.000
Maximum	7.000	7.000	7.000	7.000
<b>Minimum Competitor Bid (\$million)</b>	5.864			
<b>Our Bid (\$million)</b>	5.569			
<b>Win Bid?</b>	1	(1=yes, 0=no)		
<b>Profit (\$million)</b>	0.519			
<b>Mean Profit (\$million)</b>	0.366			

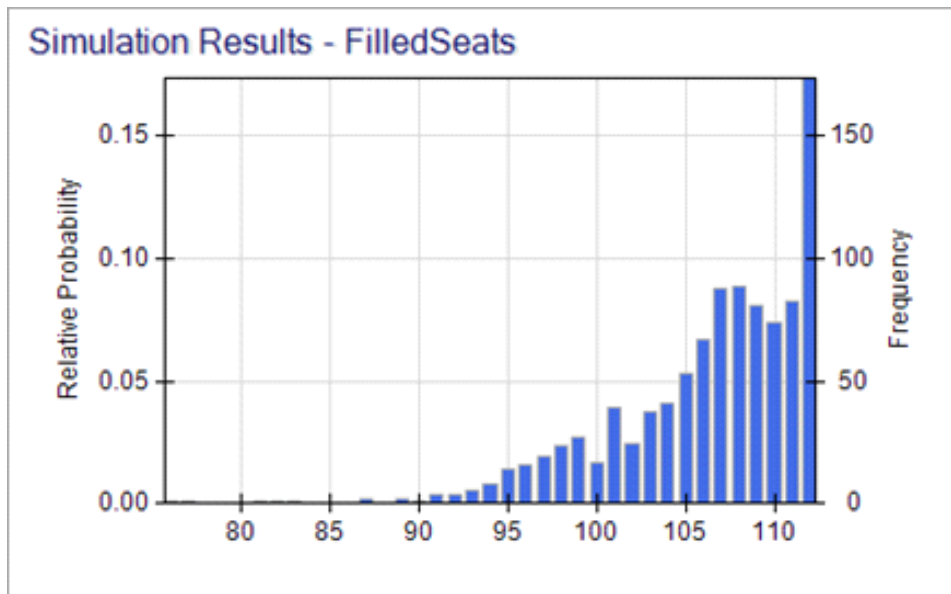
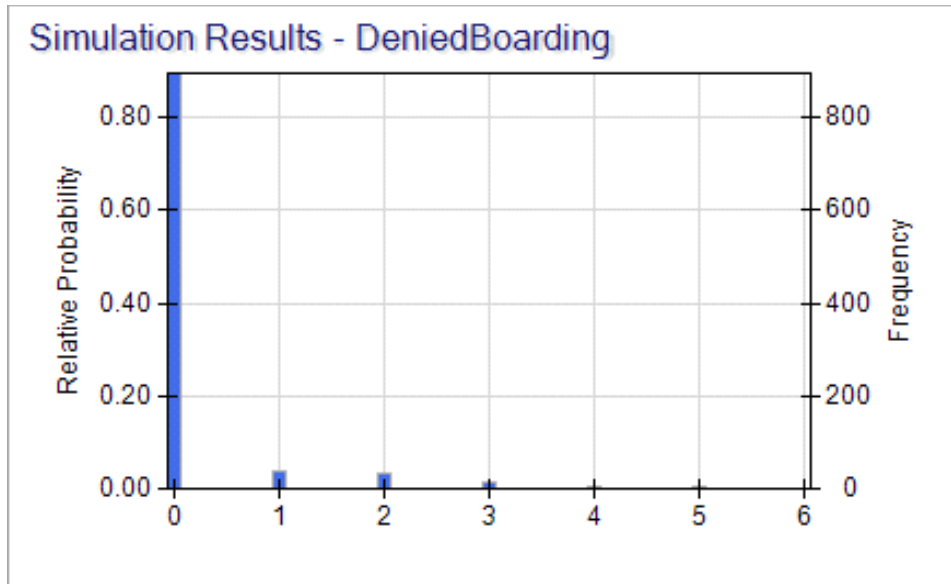
20.6-10.

(a)

Airline Overbooking			
<b>Data</b>		<b>Discount Reservations to Accept</b>	
Seats Available	112		50
Fixed Cost	\$10,000		
Discount Fare	\$150	<b>Total Reservations to Accept</b>	
Full Coach Fare	\$400		112
Cost of Bumping	\$600		
<b>Discount Ticket Demand (Triangular)</b>			
Minimum	50	Discount-Fare Demand	77.01
Most Likely	90	Rounded	77
Maximum	150	Tickets Purchased	50
Probability to Show Up	95%	Number that Show	47
<b>Full-Coach Ticket Demand (Uniform)</b>			
Minimum	30	Full Coach Demand	52.32
Maximum	70	Rounded	52
Probability to Show Up	85%	Tickets Purchased	52
		Number that Show	47
			Mean
		Number Denied Boarding	0
		Number of Filled Seats	94
			Mean
		Revenue (Discount Fare)	\$7,500
		Revenue (Full Coach)	\$18,800
		Bumping Cost	\$0
		Fixed Cost	\$10,000
		<b>Profit</b>	<b>\$16,300</b>

Simulation Results - Profit





(b)

DiscountReservationsToAccept	TotalReservationsToAccept				
	112	117	122	127	132
50	\$14,220	\$14,453	\$14,492	\$14,492	\$14,492
60	\$14,617	\$15,271	\$15,657	\$15,722	\$15,713
70	\$14,183	\$15,232	\$15,925	\$16,024	\$15,860
80	\$13,100	\$14,479	\$15,289	\$15,220	\$14,879
90	\$11,830	\$13,347	\$14,132	\$13,912	\$13,406

(c) They should accept approximately 68 discount reservations and up to approximately 125 total in order to maximize mean profit, as found by Solver.

Airline Overbooking				
Data			Discount	
	Seats Available	112	Reservations to Accept	68
	Fixed Cost	\$10,000		
	Discount Fare	\$150	Total	
	Full Coach Fare	\$400	Reservations to Accept	125
	Cost of Bumping	\$600		
Discount Ticket Demand (Triangular)				
	Minimum	50	Discount-Fare Demand	73.05
	Most Likely	90	Rounded	73
	Maximum	150	Tickets Purchased	68
	Probability to Show Up	95%	Number that Show	63
Full-Coach Ticket Demand (Uniform)				
	Minimum	30	Full Coach Demand	31.78
	Maximum	70	Rounded	32
	Probability to Show Up	85%	Tickets Purchased	32
			Number that Show	30
				Mean
			Number Denied Boarding	0
			Number of Filled Seats	93
				0.69
				104.28
			Revenue (Discount Fare)	\$10,200
			Revenue (Full Coach)	\$12,000
			Bumping Cost	\$0
			Fixed Cost	\$10,000
				Mean
			Profit	\$12,200
				\$16,045

20.7-1.

Answers will vary.

20.7-2.

Answers will vary.

## Case 20.1 Reducing In-Process Inventory (Revisited)

a) Status quo at the presses – 7.5 sheets of in-process inventory.

	A	B	C	D	E	F	G	H
1		<b>Template for Queueing Simulation</b>						
2								
3			<b>Data</b>			<b>Results</b>		
4		Number of Servers =	10			Point Estimate	95% Confidence Interval	
5							Low	High
6		<b>Interarrival Times</b>			L =	7.48596004	7.122474949	7.849445126
7		Distribution =	Exponential		L <sub>q</sub> =	0.55020043	0.347368991	0.753031867
8		Mean =	0.142857143		W =	1.0770836	1.036422591	1.117744603
9					W <sub>q</sub> =	0.07916311	0.050901621	0.107424593
10								
11		<b>Service Times</b>			P <sub>0</sub> =	0.00110924	0.000312762	0.001905718
12		Distribution =	Exponential		P <sub>1</sub> =	0.00582387	0.003292739	0.008355008
13		Mean =	1		P <sub>2</sub> =	0.02306409	0.018701971	0.027426208
14					P <sub>3</sub> =	0.05166684	0.043052172	0.060281501
15					P <sub>4</sub> =	0.0866959	0.077527167	0.09586463
16		<b>Length of Simulation Run</b>			P <sub>5</sub> =	0.12118604	0.112124348	0.130247735
17		Number of Arrivals =	10,000		P <sub>6</sub> =	0.14062225	0.13442836	0.14681614
18					P <sub>7</sub> =	0.14294653	0.134902634	0.150990419
19					P <sub>8</sub> =	0.12452751	0.11900339	0.130051626
20					P <sub>9</sub> =	0.08806336	0.084082813	0.092043901
21		<b>Run Simulation</b>			P <sub>10</sub> =	0.06192446	0.055935883	0.067913035

Status quo at the inspection station – 3.6 wing sections of in-process inventory.

	A	B	C	D	E	F	G	H
1		<b>Template for Queueing Simulation</b>						
2								
3			<b>Data</b>			<b>Results</b>		
4		Number of Servers =	1			Point Estimate	95% Confidence Interval	
5							Low	High
6		<b>Interarrival Times</b>			L =	3.57765981	3.096884037	4.058435589
7		Distribution =	Exponential		L <sub>q</sub> =	2.71234549	2.244158962	3.180532014
8		Mean =	0.142857143		W =	0.51681506	0.454627294	0.57900283
9					W <sub>q</sub> =	0.39181506	0.329627294	0.45400283
10								
11		<b>Service Times</b>			P <sub>0</sub> =	0.13468567	0.118076335	0.151295015
12		Distribution =	Constant		P <sub>1</sub> =	0.18444199	0.164766618	0.204117359
13		Value =	0.125		P <sub>2</sub> =	0.16054199	0.145653686	0.175430299
14					P <sub>3</sub> =	0.12577666	0.114607169	0.136946159
15					P <sub>4</sub> =	0.09279878	0.083029162	0.102568391
16		<b>Length of Simulation Run</b>			P <sub>5</sub> =	0.07546784	0.065828646	0.085107034
17		Number of Arrivals =	10,000		P <sub>6</sub> =	0.0548405	0.045754492	0.063926513
18					P <sub>7</sub> =	0.04326737	0.033313657	0.053221074
19					P <sub>8</sub> =	0.03643173	0.026094365	0.046769093
20					P <sub>9</sub> =	0.02983638	0.020206033	0.039466733
21		<b>Run Simulation</b>			P <sub>10</sub> =	0.02245891	0.014710033	0.030207788

Inventory cost = (7.5 + 3.6)(\$8/hour) = \$88.80 / hour

Machine cost = (10)(\$7/hour) = \$70 / hour

Inspector cost = \$17 / hour

Total cost = \$175.80 / hour

- b) Proposal 1 will increase the in-process inventory at the presses to 10.6 sheets since the mean service rate has decreased.

	A	B	C	D	E	F	G	H
1	<b>Template for Queueing Simulation</b>							
2								
3			<b>Data</b>				<b>Results</b>	
4		Number of Servers =	10			Point Estimate	95% Confidence Interval	
5							Low	High
6		<b>Interarrival Times</b>			L =	10.6124208	10.07045277	11.15438883
7		Distribution =	Exponential		L <sub>q</sub> =	2.34034351	1.812410733	2.868276277
8		Mean =	0.142857143		W =	1.5192496	1.422904809	1.615594383
9					W <sub>q</sub> =	0.33503816	0.255248897	0.41482742
10								
11		<b>Service Times</b>			P <sub>0</sub> =	0.00034416	-0.000135983	0.000824295
12		Distribution =	Exponential		P <sub>1</sub> =	0.00330079	0.002146705	0.004454878
13		Mean =	1.2		P <sub>2</sub> =	0.00683624	0.005191338	0.008481139
14					P <sub>3</sub> =	0.0225304	0.017788623	0.027272181
15					P <sub>4</sub> =	0.0437143	0.041059108	0.0463695
16		<b>Length of Simulation Run</b>			P <sub>5</sub> =	0.06530488	0.058747044	0.071862716
17		Number of Arrivals =	10,000		P <sub>6</sub> =	0.08305601	0.074729232	0.091382794
18					P <sub>7</sub> =	0.09066307	0.081970997	0.099355138
19					P <sub>8</sub> =	0.09495054	0.09393376	0.095967318
20					P <sub>9</sub> =	0.09944674	0.090813615	0.108079863
21		<b>Run Simulation</b>			P <sub>10</sub> =	0.08672109	0.077951648	0.095490525

The in-process inventory at the inspection station will not change.

Inventory cost =  $(10.6 + 3.6)(\$8/\text{hour}) = \$113.60 / \text{hour}$

Machine cost =  $(10)(\$6.50) = \$65 / \text{hour}$

Inspector cost =  $\$17 / \text{hour}$

Total cost =  $\$195.60 / \text{hour}$

This total cost is higher than for the status quo so should not be adopted. The main reason for the higher cost is that slowing down the machines won't change in-process inventory for the inspection station.

- c) Proposal 2 will increase the in-process inventory at the inspection station to 4.2 wing sections since the variability of the service rate has increased.

	A	B	C	D	E	F	G	H
1		<b>Template for Queueing Simulation</b>						
2								
3			<b>Data</b>				<b>Results</b>	
4		Number of Servers =	1			Point Estimate	95% Confidence Interval	
5							Low	High
6		<b>Interarrival Times</b>			L =	4.15349196	3.51945922	4.787524705
7		Distribution =	Exponential		L <sub>q</sub> =	3.31066782	2.691612222	3.929723426
8		Mean =	0.142857143		W =	0.58953022	0.506614288	0.672446148
9					W <sub>q</sub> =	0.46990309	0.387653637	0.552152552
10								
11		<b>Service Times</b>			P <sub>0</sub> =	0.15717586	0.13797724	0.176374483
12		Distribution =	Erlang		P <sub>1</sub> =	0.16164362	0.143938659	0.179348578
13		Mean =	0.12		P <sub>2</sub> =	0.1417251	0.127306603	0.156143599
14		k =	2		P <sub>3</sub> =	0.11157869	0.100074725	0.123082653
15					P <sub>4</sub> =	0.08340382	0.074497166	0.092310469
16		<b>Length of Simulation Run</b>			P <sub>5</sub> =	0.0729656	0.064546969	0.081384232
17		Number of Arrivals =	10,000		P <sub>6</sub> =	0.05422094	0.04655526	0.061886616
18					P <sub>7</sub> =	0.04033746	0.033104015	0.047570898
19					P <sub>8</sub> =	0.03068653	0.023437928	0.037935133
20					P <sub>9</sub> =	0.02468793	0.018553583	0.030822285
21		<b>Run Simulation</b>			P <sub>10</sub> =	0.02288346	0.016278465	0.029488445

The in-process inventory at the presses will not change.

Inventory cost =  $(7.5 + 4.2)(\$8/\text{hour}) = \$93.60 / \text{hour}$

Machine cost =  $(10)(\$7/\text{hour}) = \$70 / \text{hour}$

Inspector cost =  $\$17 / \text{hour}$

Total cost =  $\$180.60 / \text{hour}$

This total cost is higher than for the status quo so should not be adopted. The main reason for the higher cost is the increase in the service rate variability (Erlang rather than constant) and the resulting increase in the in-process inventory.

- d) They should consider *increasing* power to the presses (increasing their cost to \$7.50 per hour but reducing their average time to form a wing section to 0.8 hours). This would decrease the in-process inventory at the presses to 5.7.

	A	B	C	D	E	F	G	H
1		<b>Template for Queueing Simulation</b>						
2								
3			<b>Data</b>				<b>Results</b>	
4		Number of Servers =	10			Point Estimate	95% Confidence Interval	
5							Low	High
6		<b>Interarrival Times</b>			L =	5.74237458	5.581608211	5.903140941
7		Distribution =	Exponential		L <sub>q</sub> =	0.11624317	0.076181593	0.156304743
8		Mean =	0.142857143		W =	0.81487258	0.801697429	0.828047729
9					W <sub>q</sub> =	0.01649551	0.011001805	0.021989206
10								
11		<b>Service Times</b>			P <sub>0</sub> =	0.00445475	0.002433487	0.00647602
12		Distribution =	Exponential		P <sub>1</sub> =	0.0241519	0.019051394	0.0292524
13		Mean =	0.8		P <sub>2</sub> =	0.06075455	0.0522877	0.069221409
14					P <sub>3</sub> =	0.10828334	0.096234	0.120332681
15					P <sub>4</sub> =	0.14577459	0.138731319	0.152817867
16		<b>Length of Simulation Run</b>			P <sub>5</sub> =	0.1580859	0.148929657	0.167242144
17		Number of Arrivals =	10,000		P <sub>6</sub> =	0.14882682	0.137378613	0.160275035
18					P <sub>7</sub> =	0.12347465	0.116102784	0.13084652
19					P <sub>8</sub> =	0.0909915	0.084900257	0.097082738
20					P <sub>9</sub> =	0.05514285	0.050413495	0.059872212
21		<b>Run Simulation</b>			P <sub>10</sub> =	0.03360049	0.029185971	0.038015016

Inventory cost =  $(5.7 + 3.6)(\$8/\text{hour}) = \$74.40 / \text{hour}$

Machine cost =  $(10)(\$7.50/\text{hour}) = \$75 / \text{hour}$

Inspector cost =  $\$17 / \text{hour}$

Total cost =  $\$166.40 / \text{hour}$

This total cost is lower than the status quo and both proposals.



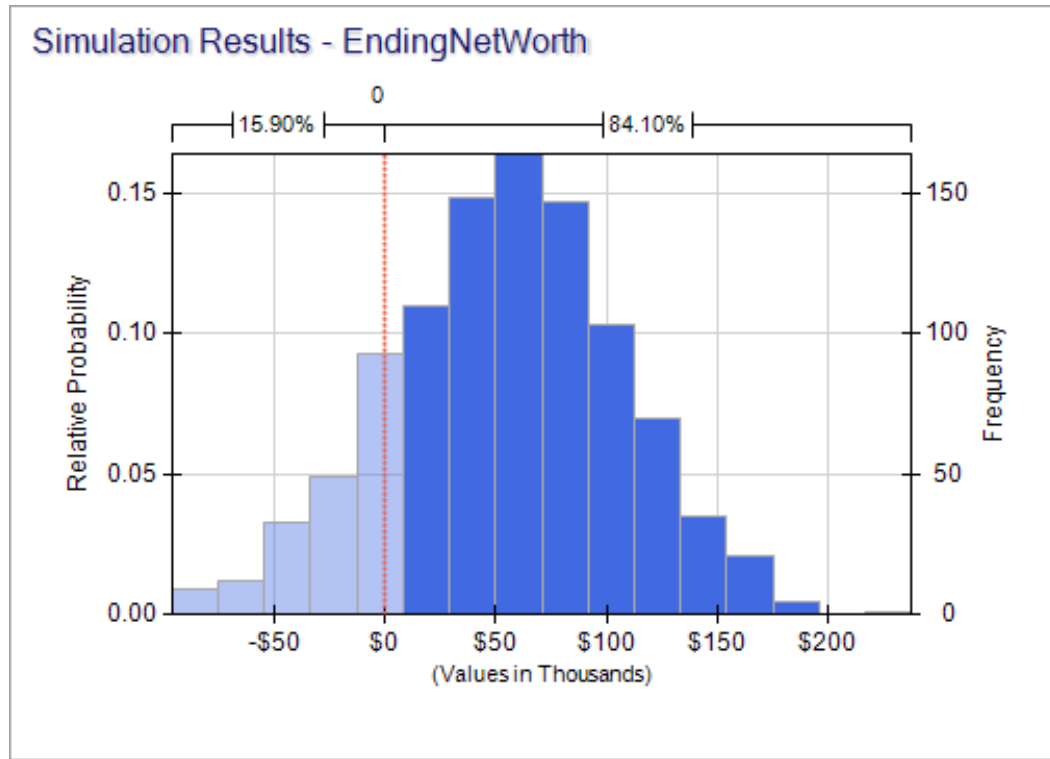
## Case 20.2 Action Adventures

a) The spreadsheet model is spread over the next two pages:

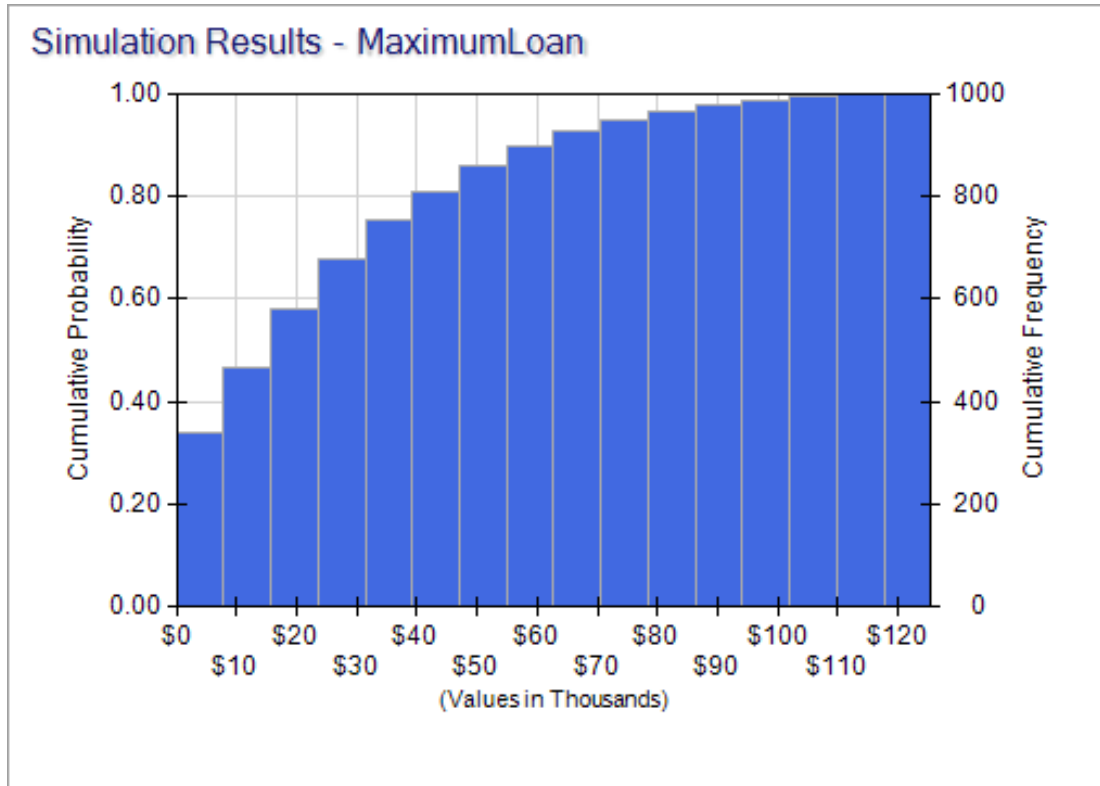
	A	B	C	D	E	F	G	H	I
1	<b>Cost &amp; Revenue Data</b>				<b>Interest Rate Data</b>				
2	Selling Price	\$10			Initial Prime Rate	5%			
3	Replacement Part Cost	\$5,000			Loan Rate Prime Gap	2%			
4	Monthly Fixed Cost	\$15,000			Loan Rate Maximum	9%			
5	Minimum Balance	\$20,000			Savings Rate Prime Gap	-2%			
6	Starting Balance	\$25,000			Savings Rate Minimum	2%			
7									
8	<b>Sales</b>	Dec	Jan	Feb	Mar	Apr	May	June	July
9	Seasonality Index	1.18	0.79	0.88	0.95	1.05	1.09	0.84	0.74
10	Base Sales	6,000	5,250	5,534	4,937	5,562	5,706	5,647	5,137
11	Actual Sales	7,080	4,148	4,870	4,690	5,840	6,219	4,743	3,801
12	Fraction Cash Customers	42%	42%	40%	45%	41%	32%	38%	39%
13									
14	<b>Interest Rates</b>								
15	Prime Rate Change		0.00%	-0.25%	0.50%	0.25%	0.00%	0.00%	0.00%
16	Prime Rate	5.00%	5.00%	4.75%	5.25%	5.50%	5.50%	5.50%	5.50%
17	Loan Interest Rate	7.00%	7.00%	6.75%	7.25%	7.50%	7.50%	7.50%	7.50%
18	Savings Interest Rate	3.00%	3.00%	2.75%	3.25%	3.50%	3.50%	3.50%	3.50%
19									
20	<b>Manufacturing Costs</b>								
21	Replacement Parts Needed		3	0	0	1	0	0	0
22									
23	Variable Cost		\$6.25	\$7.96	\$6.87	\$6.85	\$7.24	\$6.39	\$7.63
24									
25	<b>Cash Flows</b>								
26	Beginning Balance		\$25,000	\$28,415	\$20,000	\$22,627	\$20,000	\$20,000	\$22,377
27	Cash Receipts		\$17,517	\$19,344	\$21,251	\$24,195	\$19,934	\$18,177	\$15,007
28	30-Day Credit Receipts		\$41,064	\$23,960	\$29,351	\$25,653	\$34,203	\$42,257	\$29,254
29	Fixed Cost		-\$15,000	-\$15,000	-\$15,000	-\$15,000	-\$15,000	-\$15,000	-\$15,000
30	Total Variable Cost		-\$25,916	-\$38,784	-\$32,231	-\$39,993	-\$45,055	-\$30,309	-\$28,994
31	Repair Cost		-\$15,000	\$0	\$0	-\$5,000	\$0	\$0	\$0
32	Loan Payoff		\$0	\$0	-\$1,212	\$0	-\$6,783	-\$12,509	\$0
33	Loan Interest		\$0	\$0	-\$82	\$0	-\$509	-\$938	\$0
34	Savings Interest		\$750	\$852	\$550	\$735	\$700	\$700	\$783
35	Balance Before Loan		\$28,415	\$18,788	\$22,627	\$13,217	\$7,491	\$22,377	\$23,427
36	New Loan		\$0	\$1,212	\$0	\$6,783	\$12,509	\$0	\$0
37	Ending Balance	\$25,000	\$28,415	\$20,000	\$22,627	\$20,000	\$20,000	\$22,377	\$23,427
38			>=	>=	>=	>=	>=	>=	>=
39	Minimum Balance		\$20,000	\$20,000	\$20,000	\$20,000	\$20,000	\$20,000	\$20,000
40									
41		Simulated							
42		Value	Mean						
43	Ending Net Worth	\$14,935	\$54,298						
44									
45	Maximum Loan	\$36,775	\$25,990						

	A	J	K	L	M	N	O	P	Q	R
1										
2	Selling Price									
3	Replacement Part Cost									
4	Monthly Fixed Cost									
5	Minimum Balance									
6	Starting Balance									
7										
8	<b>Sales</b>	August	Sept	October	November	December	January			
9	Seasonality Index	0.98	1.06	1.1	1.16	1.18				
10	Base Sales	5,614	5,652	5,354	5,729	5,549	Normal	prev mo.	500	
11	Actual Sales	5,501	5,991	5,889	6,645	6,548				
12	Fraction Cash Customers	37%	33%	38%	43%	39%	Triangular	28%	40%	48%
13										
14	<b>Interest Rates</b>									
15	Prime Rate Change	0.00%	0.00%	0.00%	0.00%	0.00%	Custom	-0.50%	0.05	
16	Prime Rate	4.75%	4.75%	4.75%	4.75%	4.75%	Discrete	-0.25%	0.1	
17	Loan Interest Rate	6.75%	6.75%	6.75%	6.75%	6.75%		0%	0.7	
18	Savings Interest Rate	2.75%	2.75%	2.75%	2.75%	2.75%		0.25%	0.1	
19								0.50%	0.05	
20	<b>Manufacturing Costs</b>									
21	Replacement Parts Needed	1	1	1	0	1	Binomial	10%	8	
22										
23	Variable Cost	\$7.18	\$7.49	\$7.05	\$7.15	\$6.63	Uniform	\$6	\$8	
24										
25	<b>Cash Flows</b>									
26	Beginning Balance	\$27,676	\$20,000	\$20,000	\$20,000	\$20,000	\$20,000			
27	Cash Receipts	\$20,491	\$19,687	\$22,385	\$28,906	\$25,395				
28	30-Day Credit Receipts	\$25,782	\$34,523	\$40,226	\$36,509	\$37,548	\$40,081			
29	Fixed Cost	-\$15,000	-\$15,000	-\$15,000	-\$15,000	-\$15,000				
30	Total Variable Cost	-\$39,486	-\$44,866	-\$41,494	-\$47,529	-\$43,428				
31	Repair Cost	-\$5,000	-\$5,000	-\$5,000	\$0	-\$5,000				
32	Loan Payoff	\$0	-\$4,776	-\$15,204	-\$14,563	-\$12,111	-\$12,863			
33	Loan Interest	\$0	-\$322	-\$1,026	-\$983	-\$818	-\$868			
34	Savings Interest	\$761	\$550	\$550	\$550	\$550	\$550			
35	Balance Before Loan	\$15,224	\$4,796	\$5,437	\$7,889	\$7,137	\$46,899			
36	New Loan	\$4,776	\$15,204	\$14,563	\$12,111	\$12,863				
37	Ending Balance	\$20,000	\$20,000	\$20,000	\$20,000	\$20,000				
38		>=	>=	>=	>=	>=				
39	Minimum Balance	\$20,000	\$20,000	\$20,000	\$20,000	\$20,000				

- b) As seen on the spreadsheet in part a, the mean ending net worth is approximately \$54.4 thousand. The probability that it will be greater than \$0 is approximately 84.1%.



- c) The maximum short-term loan is shown in row 45 of the spreadsheet. The maximum short-term loan averages just over \$25 thousand. However, to be fairly sure that the credit limit is high enough, it should probably be set quite a bit higher. The cumulative chart shows the probability that any given credit limit will be large enough. For example, a \$70 thousand credit limit has about a 95% chance of being sufficient.



### Case 20.3 Planning Planers

Current Situation: A simulation run (shown below) indicates that the average number of jobs in the system is 2.0. Of these, half will be platen castings (1) and half will be housing castings (1). The waiting cost is therefore  $(\$200)(1) + (\$100)(1) = \$300$  / hour.

	A	B	C	D	E	F	G	H
1		<b>Template for Queueing Simulation</b>						
2								
3			<b>Data</b>				<b>Results</b>	
4		Number of Servers =	2			Point Estimate	95% Confidence Interval	
5							Low	High
6		<b>Interarrival Times</b>			L =	1.98365641	1.870700578	2.096612244
7		Distribution =	Exponential		L <sub>q</sub> =	0.66628639	0.575783306	0.756789465
8		Mean =	15		W =	30.0811805	28.73655618	31.4258049
9					W <sub>q</sub> =	10.1039076	8.845870432	11.36194473
10								
11		<b>Service Times</b>			P <sub>0</sub> =	0.19988054	0.188799674	0.210961405
12		Distribution =	Translated Exponential		P <sub>1</sub> =	0.2828689	0.271597628	0.294140162
13		Minimum Value =	10		P <sub>2</sub> =	0.21948306	0.211376682	0.227589435
14		Mean =	20		P <sub>3</sub> =	0.13257277	0.125756108	0.13938943
15					P <sub>4</sub> =	0.0722497	0.0660523	0.078447105
16		<b>Length of Simulation Run</b>			P <sub>5</sub> =	0.04178641	0.036150923	0.047421901
17		Number of Arrivals =	10,000		P <sub>6</sub> =	0.02261418	0.018147395	0.027080961
18					P <sub>7</sub> =	0.0129863	0.009197547	0.016775062
19					P <sub>8</sub> =	0.00771744	0.004659116	0.010775773
20					P <sub>9</sub> =	0.003861	0.001884639	0.005837354
21		<b>Run Simulation</b>			P <sub>10</sub> =	0.00185903	0.000591296	0.003126755

Proposal 1: A simulation run (shown below) indicates that the average number of jobs in the system with three planers is approximately 1.4. Of these, half will be platen castings (0.7) and half will be housing castings (0.7). The waiting cost is therefore  $(\$200)(0.7) + (\$100)(0.7) = \$210$  / hour. The savings  $(\$90$  / hour) is substantially more than the added cost of the third planer  $(\$30$  / hour), so this looks to be worthwhile. The net savings would be  $\$60$  / hour.

	A	B	C	D	E	F	G	H
1		<b>Template for Queueing Simulation</b>						
2								
3			<b>Data</b>				<b>Results</b>	
4		Number of Servers =	3			Point Estimate	95% Confidence Interval	
5							Low	High
6		<b>Interarrival Times</b>			L =	1.42409865	1.380256124	1.467941167
7		Distribution =	Exponential		L <sub>q</sub> =	0.09771456	0.076609893	0.11881923
8		Mean =	15		W =	21.4712624	21.07385924	21.86866564
9					W <sub>q</sub> =	1.47325117	1.168148546	1.778353785
10								
11		<b>Service Times</b>			P <sub>0</sub> =	0.25534157	0.245577077	0.265106061
12		Distribution =	Translated Exponential		P <sub>1</sub> =	0.33796761	0.329893149	0.346042064
13		Minimum Value =	10		P <sub>2</sub> =	0.231656	0.224819899	0.238492092
14		Mean =	20		P <sub>3</sub> =	0.11158027	0.106409547	0.116750986
15					P <sub>4</sub> =	0.04233244	0.038546771	0.046118111
16		<b>Length of Simulation Run</b>			P <sub>5</sub> =	0.01406273	0.011836939	0.016288531
17		Number of Arrivals =	10,000		P <sub>6</sub> =	0.00409905	0.002819395	0.005378708
18					P <sub>7</sub> =	0.00133435	0.000425396	0.002243309
19					P <sub>8</sub> =	0.00080672	-0.000131529	0.001744969
20					P <sub>9</sub> =	0.00036429	-0.000147459	0.000876045
21		<b>Run Simulation</b>			P <sub>10</sub> =	0.00025729	-0.000188128	0.000702701

Proposal 2: A simulation run (shown below) indicates that the average number of jobs in the system with constant interarrival times is approximately 1.4. Of these, half will be platen castings (0.7) and half will be housing castings (0.7). The waiting cost is therefore  $(\$200)(0.7) + (\$100)(0.7) = \$210$  / hour. The savings (\$90 / hour) is somewhat more than the added cost of changing the preceding production cost (\$60 / hour). The net savings (\$30) is less than for proposal 1, so this option is less worthwhile.

	A	B	C	D	E	F	G	H
1	<b>Template for Queueing Simulation</b>							
2								
3			<b>Data</b>				<b>Results</b>	
4		Number of Servers =	2			Point Estimate	95% Confidence Interval	
5							Low	High
6		<b>Interarrival Times</b>			L =	1.40396168	1.383412164	1.424511205
7		Distribution =	Constant		L <sub>q</sub> =	0.06455913	0.055139154	0.073979097
8		Value =	15		W =	21.0594253	20.75118246	21.36766807
9					W <sub>q</sub> =	0.96838689	0.82708731	1.109686461
10								
11		<b>Service Times</b>			P <sub>0</sub> =	0.05142644	0.049243293	0.053609594
12		Distribution =	Translated Exponential		P <sub>1</sub> =	0.55774455	0.547877528	0.56761158
13		Minimum Value =	10		P <sub>2</sub> =	0.33345643	0.325678768	0.341234095
14		Mean =	20		P <sub>3</sub> =	0.05060063	0.045262513	0.055938746
15					P <sub>4</sub> =	0.00635733	0.004037203	0.008677449
16		<b>Length of Simulation Run</b>			P <sub>5</sub> =	0.00041461	2.30036E-06	0.000826928
17		Number of Arrivals =	10,000		P <sub>6</sub> =	0	0	0
18					P <sub>7</sub> =	0	0	0
19					P <sub>8</sub> =	0	0	0
20					P <sub>9</sub> =	0	0	0
21		<b>Run Simulation</b>			P <sub>10</sub> =	0	0	0

Proposal 1 and 2: A simulation run (shown below) indicates that the average number of jobs in the system with both three planers and constant interarrival times is approximately 1.33. Of these, half will be platen castings (0.665) and half will be housing castings (0.665). The waiting cost is therefore  $(\$200)(0.665) + (\$100)(0.665) = \$200$  / hour. The savings (\$85 / hour) is less than the combined cost of adding a third planer and changing the preceding production cost (\$90 / hour), so this combined option does not appear to be worthwhile.

	A	B	C	D	E	F	G	H
1	<b>Template for Queueing Simulation</b>							
2								
3			<b>Data</b>				<b>Results</b>	
4		Number of Servers =	3			Point Estimate	95% Confidence Interval	
5							Low	High
6		<b>Interarrival Times</b>			L =	1.32985569	1.316690172	1.343021211
7		Distribution =	Constant		L <sub>q</sub> =	0.00052554	0.000184596	0.00086648
8		Value =	15		W =	19.9478354	19.75035259	20.14531816
9					W <sub>q</sub> =	0.00788307	0.002768944	0.012997201
10								
11		<b>Service Times</b>			P <sub>0</sub> =	0.05754771	0.055474824	0.059620587
12		Distribution =	Translated Exponential		P <sub>1</sub> =	0.58946474	0.581401676	0.5975278
13		Minimum Value =	10		P <sub>2</sub> =	0.31909725	0.311531281	0.326663225
14		Mean =	20		P <sub>3</sub> =	0.03336476	0.03022801	0.036501519
15					P <sub>4</sub> =	0.00052554	0.000184596	0.00086648
16		<b>Length of Simulation Run</b>			P <sub>5</sub> =	0	0	0
17		Number of Arrivals =	10,000		P <sub>6</sub> =	0	0	0
18					P <sub>7</sub> =	0	0	0
19					P <sub>8</sub> =	0	0	0
20					P <sub>9</sub> =	0	0	0
21		<b>Run Simulation</b>			P <sub>10</sub> =	0	0	0

Overall recommendation: Proposal 1 appears to be the most worthwhile, with a net savings of about \$36 / hour over the current situation. Other proposals that may be worth looking into should include giving priority to platen castings, because of the higher waiting cost for that type of job.

#### Case 20.4 Pricing Under Pressure

- a) Before we begin the formal problem, we must first calculate the mean  $\mu$  and standard deviation  $\sigma$  of the normally distributed random variable  $N$ . We are told that the annual interest rate will be used to estimate  $\mu$  and the historical annual volatility will be used to estimate  $\sigma$ . Because the case is simulating weekly – not yearly – change, we must convert these yearly values to weekly values.

We first convert the annual interest rate  $r = 8\%$  to a weekly interest rate  $w$  with the following formula:

$$\begin{aligned} w &= (1 + r)^{(1/52)} - 1 \\ &= (1 + 0.08)^{(1/52)} - 1 \\ &= (1.08)^{(1/52)} - 1 \\ &= 0.00148 \end{aligned}$$

We next convert the annual volatility  $V_a = 0.30$  to a weekly volatility  $V_w$  with the following formula:

$$\begin{aligned} V_w &= V_a / \sqrt{52} \\ &= 0.30 / \sqrt{52} \\ &= 0.0416 \end{aligned}$$

Once we have the weekly interest rate and volatility, we can calculate  $\mu$  and  $\sigma$ .

$$\begin{aligned} \mu &= w - 0.5(V_w)^2 \\ &= 0.00148 - 0.5(0.0416)^2 \\ &= 0.0006 \end{aligned}$$

$$\begin{aligned} \sigma &= V_w \\ &= 0.0416 \end{aligned}$$

1. One component appears in this system: the stock price. The stock price in the previous week is used to calculate the stock price in the next week. The relationship between the stock price in the previous week and the stock price in the next week is given by  $s_n = e^N s_c$ .

2. State of the system:  $P(t)$  = price of the stock at time  $t$ .

3. This simulation requires generating a series of random observations from the normal distribution. Each random observation is a normally distributed random variable that determines the increase or decrease of the stock price at the end of next week. The random variable is substituted for  $N$  in the following equation:

$$s_n = e^N s_c$$

To generate a series of random variables, we define an uncertain variable cell with normal distribution, where  $\mu = 0.0006$  and  $\sigma = 0.0416$ .

4. The formula  $s_n = e^N s_c$  gives us a procedure for changing the price (the state of the system) when an event occurs.

5. In this simulation, the time periods are fixed. We have a twelve-week period, and we need to calculate the change in the stock price each week. We have a formula  $s_n = e^N s_c$  that relates the stock price at the end of the next week to the stock price at the end of the previous week. Thus, we do not have to worry about advancing the clock. We simply have to generate  $N$  for each of the twelve weeks.

6. We need to build a spreadsheet using RSPE. We start with the current stock price of \$42.00. We then use the formula  $s_n = e^N s_c$  to calculate the stock price at the end of each of the twelve weeks. We substitute a RSPE uncertain variable cell with normal distribution (with mean  $\mu = 0.0006$ , and standard deviation  $\sigma = 0.0416$ ) for  $N$ .

We then use the stock price at the end of the twelfth week to calculate the value of the option at the end of the twelfth week. If the stock price at the end of the twelfth week is greater than the exercise price of \$44.00, the value of the option is the difference between the value of the stock at the end of the twelfth week and the exercise price. If the stock price at the end of the twelfth week is less than or equal to the exercise price of \$44.00, the value of the option is \$0.

Finally, we need to discount the value of the option at the end of the twelfth week to the value of the option in today's dollars using the following formula:

$$(\text{Value of the option at the end of the twelfth week}) / (1.00148)^{12}$$

The spreadsheet model is shown below. The uncertain variable cells are the  $N$  values (B8:B19), the result cell is the price of the option today (C22), and the statistic cell is the mean price of the option today (C23).



	A	B	C	D	E	F
1	<b>Simulation Model to Estimate Option Value</b>					
2						
3		Current Stock Price	\$42.00		Annual Interest Rate	8%
4		Exercise Price	\$44.00		Weekly Interest Rate	0.148%
5						
6			Stock Price at		Annual Volatility	30%
7	Week	N	End of Week		Weekly Volatility	4.160%
8	1	0.002944088	\$42.12			
9	2	-0.041470376	\$40.41		$\mu =$	0.0006
10	3	0.051283439	\$42.54		$\sigma =$	0.0416
11	4	0.012944376	\$43.09			
12	5	-0.026906539	\$41.95			
13	6	-0.079994242	\$38.72			
14	7	0.006708864	\$38.99			
15	8	0.00707491	\$39.26			
16	9	-0.003553399	\$39.12			
17	10	0.093240243	\$42.95			
18	11	-0.086581855	\$39.38			
19	12	-0.017522862	\$38.70			
20						
21	Price of Option at end of Week 12		\$0.00			
22	Price of Option Today		\$0.00			
23	Mean(Price of Option Today)		\$1.91			

	A	B	C
3		Current Stock Price	42
4		Exercise Price	44
5			
6			Stock Price at
7	Week	N	End of Week
8	1	=PsiNormal(Mean,StandardDeviation)	=CurrentStockPrice*EXP(B8)
9	2	=PsiNormal(Mean,StandardDeviation)	=EXP(B9)*C8
10	3	=PsiNormal(Mean,StandardDeviation)	=EXP(B10)*C9
11	4	=PsiNormal(Mean,StandardDeviation)	=EXP(B11)*C10
12	5	=PsiNormal(Mean,StandardDeviation)	=EXP(B12)*C11
13	6	=PsiNormal(Mean,StandardDeviation)	=EXP(B13)*C12
14	7	=PsiNormal(Mean,StandardDeviation)	=EXP(B14)*C13
15	8	=PsiNormal(Mean,StandardDeviation)	=EXP(B15)*C14
16	9	=PsiNormal(Mean,StandardDeviation)	=EXP(B16)*C15
17	10	=PsiNormal(Mean,StandardDeviation)	=EXP(B17)*C16
18	11	=PsiNormal(Mean,StandardDeviation)	=EXP(B18)*C17
19	12	=PsiNormal(Mean,StandardDeviation)	=EXP(B19)*C18
20			
21	Price of Option at end of Week 12		=IF(C19>ExercisePrice,C19-ExercisePrice,0)
22	Price of Option Today		=C21/(1+WeeklyInterestRate)^12 + PsiOutput()
23	Mean(Price of Option Today)		=PsiMean(C22)

Range Name	Cells
AnnualInterestRate	F3
AnnualVolatility	F6
CurrentStockPrice	C3
ExercisePrice	C4
Mean	F9
PriceOfOption	C22
StandardDeviation	F10
WeeklyInterestRate	F4
WeeklyVolatility	F7

	E	F
3	Annual Interest Rate	0.08
4	Weekly Interest Rate	$=((1+\text{AnnualInterestRate})^{(1/52)})-1$
5		
6	Annual Volatility	0.3
7	Weekly Volatility	$=\text{AnnualVolatility}/\text{SQRT}(52)$
8		
9	m=	$=\text{WeeklyInterestRate}-0.5*(\text{WeeklyVolatility}^2)$
10	s =	$=\text{WeeklyVolatility}$

The mean of the “Price of Option Today” is the price of the option in today’s dollars. The simulation results after 100, 1,000, and 10,000 trials will vary. Typical mean values might be \$1.70, \$1.91, and \$1.87. The variation is significantly reduced with more trials (the mean standard error drops from \$0.26 to \$0.11 to \$0.03 at 100, 1,000, and 10,000 trials, respectively).

- b) Using the Black-Scholes Formula, the price of the option is \$1.88. The spreadsheet used to calculate the Black-Scholes Formula in Excel follows:

	A	B	C	D	E	F
1	<b>Black-Scholes Calculation of Option Value</b>					
2						
3		Current Stock Price	\$42.00		<b>Black-Scholes</b>	
4					d1 =	-0.127503153
5		Weeks to exercise date	12		d2 =	-0.271618491
6		Exercise Price	\$44.00			
7		Exercise Price Present Value	\$43.23		N[d1] =	0.449271051
8					N[d2] =	0.39295775
9		Annual Interest Rate	8%			
10		Weekly Interest Rate	0.148%		Value =	\$1.88
11						
12		Annual Volatility	30%			
13		Weekly Volatility	4.160%			
14						
15			$\mu =$	0.0006		
16			$\sigma =$	0.0416		

	E	F
3	<b>Black-Scholes</b>	
4	d1 =	=LN(CurrentStockPrice/ExercisePricePV)/(StandardDeviation*SQRT(WeeksToExerciseDate))+StandardDeviation
5	d2 =	=d_1-StandardDeviation*SQRT(WeeksToExerciseDate)
6		
7	N[d1] =	=NORMSDIST(d_1)
8	N[d2] =	=NORMSDIST(d_2)
9		
10	Value =	=Nd1*CurrentStockPrice-Nd2*ExercisePricePV

Range Name	Cells
AnnualInterestRate	C9
AnnualVolatility	C12
CurrentStockPrice	C3
d_1	F4
d_2	F5
ExercisePrice	C6
ExercisePricePV	C7
Mean	C15
Nd1	F7
Nd2	F8
StandardDeviation	C16
Value	F10
WeeklyInterestRate	C10
WeeklyVolatility	C13
WeeksToExerciseDate	C5

The price of the option obtained by simulation and the price of the option obtained by the Black-Scholes formula are fairly close. The 1,000-iteration simulation price is off by just thirteen cents.

- c) No, a random walk does not completely describe the price movement of the stock because the random walk assumes a consistent lognormal increase or decrease in the price of the stock. The price of the stock could change according to a different distribution, however, especially if an event occurs to trigger a dramatic increase or decrease in the stock. In this case, the European Space Agency may award Ellare the International Space Station contract. The award notice would most likely trigger a dramatic movement in the stock. The random walk does not take into account this dramatic event.