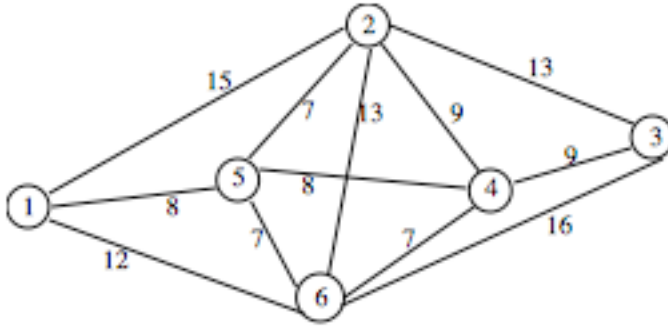# Solved Examples for Chapter 14

Example For Section 14.3

Consider the traveling salesman problem shown below, where city 1 is the home city.

Using 1-2-3-4-6-5-1 as the initial trial solution, perform one iteration of the basic simulated annealing algorithm presented in Sec. 14.3 by hand. Follow the instructions given at the beginning of the Problems section for Chapter 14 to obtain the needed random numbers.

Initial trial solution: 1-2-3-4-6-5-1 $Z_c = 59$ $T_1 = 0.2 Z_c = 11.8$

      0.00000-0.24999:   Sub-tour begins in slot 2.
      0.25000-0.49999:   Sub-tour begins in slot 3.
      0.50000-0.74999:   Sub-tour begins in slot 4.
      0.75000-0.99999:   Sub-tour begins in slot 5.

The random number is 0.09656, so we choose a sub-tour that begins in slot 2.

By beginning in slot 2, the sub-tour needs to end somewhere between slots 3 and 5.

      0.00000-0.33332:   Sub-tour begins in slot 3.
      0.33333-0.66666:   Sub-tour begins in slot 4.
      0.66667-0.99999:   Sub-tour begins in slot 5.

The random number is 0.96657, so we choose a sub-tour that ends in slot 5.

Reverse 2-3-4-6: new solution: 1-6-4-3-2-5-1 $Z_n = 56$

Since $Z_n < Z_c$, we accept 1-6-4-3-2-5-1 as the new trial solution.

Example For Section 14.4

Reconsider the 8-city traveling salesman problem introduced in Problem 14.2-6. The links for this problem have the associated distances shown in the following table (where a dash indicates the absence of a link).

City	2	3	4	5	6	7	8
1	14	15	—	—	—	—	17
2		13	14	20	—	—	21
3			11	21	17	9	9
4				11	10	8	20
5					15	18	—
6						9	—
7							13

(a) When applying the basic genetic algorithm presented in Sec. 14.4, suppose that members 2 - 10 of the initial population are the following.

Member	Initial Population	Distance
1		
2	1-8-7-6-5-2-4-3-1	114
3	1-3-6-5-7-4-8-2-1	128
4	1-2-5-6-4-7-3-8-1	102
5	1-2-4-5-6-7-3-8-1	97
6	1-2-5-6-7-3-4-8-1	115
7	1-3-5-6-7-8-4-2-1	121
8	1-3-4-7-6-5-2-8-1	116
9	1-3-6-5-7-8-4-3-1	126
10	1-3-2-5-4-6-7-8-1	108

Use the procedure described in the fifth paragraph of the subsection of Sec. 14.4 entitled "The Traveling Salesman Problem Example" to generate member 1 of this population by hand. Follow the instructions given at the beginning of the Problems section for Chapter 14 to obtain the needed random numbers.

Start from city 1.
Possible links: 1-2, 1-3, 1-8
Random Number: 0.09656   Choose 1-2.

Start from city 2. Current tour 1-2
Possible links: 2-3, 2-4, 2-5, 2-8
Random Number: 0.96657   Choose 2-8.

Start from city 8. Current tour 1-2-8
Possible links: 8-3, 8-4, 8-7
Random Number: 0.64842   Choose 8-4.

Start from city 4. Current tour 1-2-8-4
Possible links: 4-3, 4-5, 4-6, 4-7
Random Number: 0.49222   Choose 4-5.

Start from city 5. Current tour 1-2-8-4-5
Possible links: 5-3, 5-6, 5-7
Random Number: 0.49506   Choose 5-6.

Start from city 6. Current tour 1-2-8-4-5-6
Possible links: 6-3, 6-7
Random Number: 0.10145   Choose 6-3.

Start from city 3. Current tour 1-2-8-4-5-6-3
Possible links: 3-7
Random Number: 0.48455   Choose 3-7.

Start from city 7. Current tour 1-2-8-4-5-6-3-7
Dead End. Repeat this process again.

Start from city 1.
Possible links: 1-2, 1-3, 1-8
Random Number: 0.23505   Choose 1-2.

Start from city 2. Current tour 1-2
Possible links: 2-3, 2-4, 2-5, 2-8
Random Number: 0.90430   Choose 2-8.

Start from city 8. Current tour 1-2-8
Possible links: 8-3, 8-4, 8-7
Random Number: 0.04180   Choose 8-3.

Since we now have chosen all the cities that have links to city 1, we will reach a dead end
eventually. We begin the process of generating parent 1 again.

Start from city 1.
Possible links: 1-2, 1-3, 1-8
Random Number: 0.24712   Choose 1-2.

Start from city 2. Current tour 1-2
Possible links: 2-3, 2-4, 2-5, 2-8
Random Number: 0.55799   Choose 2-5.

Start from city 5. Current tour 1-2-5
Possible links: 5-3, 5-4, 5-6, 5-7
Random Number: 0.60857   Choose 5-6.

Start from city 6. Current tour 1-2-5-6
Possible links: 6-3, 6-4, 6-7
Random Number: 0.73479   Choose 6-7.

Start from city 7. Current tour 1-2-5-6-7
Possible links: 7-3, 7-4, 7-8
Random Number: 0.33581   Choose 7-4.

Start from city 4. Current tour 1-2-5-6-7-4
Possible links: 4-3, 4-8
Random Number: 0.17360   Choose 4-3.

Start from city 3. Current tour 1-2-5-6-7-4-3
Possible links: 3-8
Random Number: 0.30406   Choose 3-8.

Start from city 8. Current tour 1-2-5-6-7-4-3-8

Since all the cities now are in the tour, we automatically add the link 8-1 (which fortunately is available this time) from the last city back to the home city.

Parent 1 is 1-2-5-6-7-4-3-8-1.

(b) Begin the first iteration of the basic genetic algorithm presented in Sec. 14.4 by selecting the parents, pairing up the parents to form couples, and then using the first couple to generate their first child.

Selection of Parents:

We need to select six members from among the following initial population to become parents.

Member	Initial Population	Distance
1	1-2-5-6-7-4-3-8-1	103
2	1-8-7-6-5-2-4-3-1	114
3	1-3-6-5-7-4-8-2-1	128
4	1-2-5-6-4-7-3-8-1	102
5	1-2-4-5-6-7-3-8-1	97
6	1-2-5-6-7-3-4-8-1	115
7	1-3-5-6-7-8-4-2-1	121
8	1-3-4-7-6-5-2-8-1	116
9	1-3-6-5-7-8-4-3-1	126
10	1-3-2-5-4-6-7-8-1	108

We choose 4 members among the 5 having the highest degree of fitness (in order): 5, 4, 1, 10, and 2.
A random number is used to select one member to be rejected.
Random number: 0.09656.
The first member listed (member 5) does not become a parent.

We also select 2 members among the less fit members: 6, 8, 7, 9, and 3.
Random number: 0.96657. We select member 3 to become a parent.
The next random number is used to select a parent among 6, 8, 7, and 9.
Random number: 0.64842. We select member 7 to become a parent.

Pairing up the Parents:

The next step is to pair up parents – members 4, 1, 10, 2, 7, and 3.
We use a random number to determine the mate of the first parent listed (member 4).
Random number: 0.29222. Member 10 is selected to pair up with member 4.
Next, we use a random number to pair up the next member listed (member 1).
Random number: 0.49506. Member 7 is selected to pair up with member 1.
This then leaves member 2 and member 3 to become the last couple.

Generation of a Child from the First Pair of Parents:

Pair 1:   M4:   1-2-5-6-4-7-3-8-1
                 M10: 1-3-2-5-4-6-7-8-1

Start from city 1.
Possible links: 1-2, 1-8, 1-3, 1-8.
Random Number: 0.10145   Choose 1-2.
               0.48455   No mutation

Start from city 2. Current Tour: 1-2.
Possible links: 2-5, 2-3, 2-5.
Random Number: 0.23505   Choose 2-5.
               0.09430   Mutation   Reject 2-5.

Since a mutation has just occurred, we now list all the possible links from city 2 other than the one just rejected and choose one of them randomly.

Possible links: 2-3, 2-4, 2-8
Random number: 0.17953   Choose 2-3.

Start from city 3. Current Tour: 1-2-3
Possible links: 3-7, 3-8, 3-5
Random Number: 0.04180   Choose 3-7.
                  0.24712   No mutation

An interesting feature of the step just completed is that 3-5 is listed as a possible link even though it is not a link used by either parent. The reason it is there is that the second parent does not have a link from city 3 to any of the cities not already in the tour because the child is using a sub-tour reversal (reversing 3-2). Completing this sub-tour reversal requires adding the link 3-5. However, link 3-7 was chosen instead of link 3-5, so we continue with 1-2-3-7 as the current tour.

Start from city 7. Current Tour: 1-2-3-7
Possible links: 7-4, 7-6, 7-8
Random Number: 0.55799   Choose 7-6.
                  0.60857   No mutation

Start from city 6. Current Tour: 1-2-3-7-6
Possible links: 6-5, 6-4, 6-4
Random Number: 0.23479   Choose 6-5.
                  0.33581   No mutation

Start from city 5. Current Tour: 1-2-3-7-6-5
Possible links: 5-4, 5-4
Random Number: 0.17360   Choose 5-4.
                  0.30406   No mutation

The link 5-4 comes from the first parent as well as the second because the child is using a sub-tour reversal (reversing 5-6) of the first parent, which requires adding link 5-4 to complete the sub-tour reversal.

Start from city 4. Current Tour: 1-2-3-7-6-5-4

Since the only city not yet visited on the tour is city 8, link 7-8 is automatically added next, followed by link 8-1 to return to the home city. Therefore, the complete child is

        1-2-3-7-6-5-4-8-1,

which has a distance of 108.