SUPPLEMENT **8** TO CHAPTER

Linear Goal Programming and Its Solution Procedures

All the algorithms presented in Chap. 8 are for problems that fit the format of linear programming as introduced in Chap. 3. We now turn to an important extension of linear programming and consider how it can be reformulated so that the algorithms of linear programming can again be applied.

We have assumed throughout Chaps. 3–8 that the objectives of the organization conducting the linear programming study can be encompassed within a single overriding objective, such as maximizing total profit or minimizing total cost. However, this assumption is not always realistic. In fact, as we discussed in Sec. 2.1, studies have found that the management of U.S. corporations frequently focuses on a variety of other objectives, e.g., to maintain stable profits, increase (or maintain) market share, diversify products, maintain stable prices, improve worker morale, maintain family control of the business, and increase company prestige. *Goal programming* provides a way of striving toward several such objectives *simultaneously*.

The basic approach of **goal programming** is to establish a specific numeric goal for each of the objectives, formulate an objective function for each objective, and then seek a solution that minimizes the (weighted) sum of deviations of these objective functions from their respective goals. There are three possible types of goals:

1. A **lower, one-sided goal** sets a *lower limit* that we do not want to fall under (but exceeding the limit is fine).
2. An **upper, one-sided goal** sets an *upper limit* that we do not want to exceed (but falling under the limit is fine).
3. A **two-sided goal** sets a *specific target* that we do not want to miss on either side.

Goal programming problems can be categorized according to the type of mathematical programming model (linear programming, integer programming, nonlinear programming, etc.) that it fits except for having multiple goals instead of a single objective. In this book, we only consider *linear* goal programming—those goal programming problems that fit linear programming otherwise (each objective function is linear, etc.) and so we will drop the adjective *linear* from now on.

Another categorization is according to how the goals compare in importance. In one case, called **nonpreemptive goal programming,** all the goals are of *roughly comparable*

importance. In another case, called **preemptive goal programming,** there is a *hierarchy of priority levels* for the goals, so that the goals of primary importance receive first-priority attention, those of secondary importance receive second-priority attention, and so forth (if there are more than two priority levels).

We begin with an example that illustrates the basic features of nonpreemptive goal programming and then discuss the preemptive case.

Prototype Example for Nonpreemptive Goal Programming

The DEWRIGHT COMPANY is considering three new products to replace current models that are being discontinued, so their OR department has been assigned the task of determining which mix of these products should be produced. Management wants primary consideration given to three factors: long-run profit, stability in the workforce, and the level of capital investment that would be required now for new equipment. In particular, management has established the goals of (1) achieving a long-run profit (net present value) of at least $125 million from these products, (2) maintaining the current employment level of 4,000 employees, and (3) holding the capital investment to less than $55 million. However, management realizes that it probably will not be possible to attain all these goals simultaneously, so it has discussed priorities with the OR department. This discussion has led to setting *penalty weights* of 5 for missing the profit goal (per $1 million under), 2 for going over the employment goal (per 100 employees), 4 for going under this same goal, and 3 for exceeding the capital investment goal (per $1 million over). Each new product's contribution to profit, employment level, and capital investment level is *proportional* to the rate of production. These contributions per unit rate of production are shown in Table 1, along with the goals and penalty weights.

Formulation. The Dewright Company problem includes all three possible types of goals: a lower, one-sided goal (long-run profit); a two-sided goal (employment level); and an upper, one-sided goal (capital investment). Letting the decision variables x_1, x_2, x_3 be the production rates of products 1, 2, and 3, respectively, we see that these goals can be stated as

$$12x_1 + 9x_2 + 15x_3 \geq 125 \qquad \text{profit goal}$$
$$5x_1 + 3x_2 + 4x_3 = 40 \qquad \text{employment goal}$$
$$5x_1 + 7x_2 + 8x_3 \leq 55 \qquad \text{investment goal.}$$

More precisely, given the penalty weights in the rightmost column of Table 1, let Z be the *number of penalty points* incurred by missing these goals. The overall objective then is to choose the values of x_1, x_2, and x_3 so as to

Minimize $Z = 5$(amount under the long-run profit goal)
$\qquad\qquad + 2$(amount over the employment level goal)
$\qquad\qquad + 4$(amount under the employment level goal)
$\qquad\qquad + 3$(amount over the capital investment goal),

■ **TABLE 1** Data for the Dewright Co. nonpreemptive goal programming problem

| | Unit Contribution | | | | |
| | Product: | | | | Penalty |
Factor	1	2	3	Goal (Units)	Weight
Long-run profit	12	9	15	$\geq$ 125 (millions of dollars)	5
Employment level	5	3	4	= 40 (hundreds of employees)	2(+), 4(−)
Capital investment	5	7	8	$\leq$ 55 (millions of dollars)	3

where no penalty points are incurred for being over the long-run profit goal or for being under the capital investment goal. To express this overall objective mathematically, we introduce some *auxiliary variables* (extra variables that are helpful for formulating the model) y_1, y_2, and y_3, defined as follows:

$$y_1 = 12x_1 + 9x_2 + 15x_3 - 125 \qquad \text{(long-run profit minus the target).}$$
$$y_2 = 5x_1 + 3x_2 + 4x_3 - 40 \qquad \text{(employment level minus the target).}$$
$$y_3 = 5x_1 + 7x_2 + 8x_3 - 55 \qquad \text{(capital investment minus the target).}$$

Since each y_i can be either positive or negative, we next use the technique described at the end of Sec. 4.6 for dealing with such variables; namely, we replace each one by the difference of two nonnegative variables:

$$y_1 = y_1^+ - y_1^-, \qquad \text{where } y_1^+ \geq 0,\, y_1^- \geq 0,$$
$$y_2 = y_2^+ - y_2^-, \qquad \text{where } y_2^+ \geq 0,\, y_2^- \geq 0,$$
$$y_3 = y_3^+ - y_3^-, \qquad \text{where } y_3^+ \geq 0,\, y_3^- \geq 0.$$

As discussed in Sec. 4.6, for any BF solution, these new auxiliary variables have the interpretation

$$y_j^+ = \begin{cases} y_j & \text{if } y_j \geq 0, \\ 0 & \text{otherwise;} \end{cases}$$
$$y_j^- = \begin{cases} |y_j| & \text{if } y_j \leq 0, \\ 0 & \text{otherwise;} \end{cases}$$

so that y_j^+ represents the positive part of the variable y_j and y_j^- its negative part (as suggested by the superscripts).

Given these new auxiliary variables, the overall objective can be expressed mathematically as

$$\text{Minimize} \qquad Z = 5y_1^- + 2y_2^+ + 4y_2^- + 3y_3^+,$$

which now is a legitimate objective function for a linear programming model. (Because there is no penalty for exceeding the profit goal of 125 or being under the investment goal of 55, neither y_1^+ nor y_3^- should appear in this objective function representing the total penalty for deviations from the goals.)

To complete the conversion of this goal programming problem to a linear programming model, we must incorporate the above definitions of the y_j^+ and y_j^- directly into the model. (It is not enough to simply record the definitions, as we just did, because the simplex method considers only the objective function and constraints that constitute the model.) For example, since $y_1^+ - y_1^- = y_1$, the above expression for y_1 gives

$$12x_1 + 9x_2 + 15x_3 - 125 = y_1^+ - y_1^-.$$

After we move the variables $(y_1^+ - y_1^-)$ to the left-hand side and the constant (125) to the right-hand side,

$$12x_1 + 9x_2 + 15x_3 - (y_1^+ - y_1^-) = 125$$

becomes a legitimate equality constraint for a linear programming model. Furthermore, this constraint forces the auxiliary variables $(y_1^+ - y_1^-)$ to satisfy their definition in terms of the decision variables (x_1, x_2, x_3).

Proceeding in the same way for $y_2^+ - y_2^-$ and $y_3^+ - y_3^-$, we obtain the following linear programming formulation of this goal programming problem:

$$\text{Minimize} \qquad Z = 5y_1^- + 2y_2^+ + 4y_2^- + 3y_3^+,$$

subject to

$$12x_1 + 9x_2 + 15x_3 - (y_1^+ - y_1^-) = 125$$
$$5x_1 + 3x_2 + 4x_3 - (y_2^+ - y_2^-) = 40$$
$$5x_1 + 7x_2 + 8x_3 - (y_3^+ - y_3^-) = 55$$

and

$$x_j \geq 0, \qquad y_k^+ \geq 0, \qquad y_k^- \geq 0 \qquad (j = 1, 2, 3; k = 1, 2, 3).$$

(If the original problem had any actual linear programming constraints, such as constraints on fixed amounts of certain resources being available, these would be included in the model.)

Applying the simplex method to this formulation yields an optimal solution $x_1 = \frac{25}{3}$, $x_2 = 0$, $x_3 = \frac{5}{3}$, with $y_1^+ = 0$, $y_1^- = 0$, $y_2^+ = \frac{25}{3}$, $y_2^- = 0$, $y_3^+ = 0$, and $y_3^- = 0$. Therefore, $y_1 = 0$, $y_2 = \frac{25}{3}$, and $y_3 = 0$, so the first and third goals are fully satisfied, but the employment level goal of 40 is exceeded by $8\frac{1}{3}$ (833 employees). The resulting penalty for deviating from the goals is $Z = 16\frac{2}{3}$.

Preemptive Goal Programming

In the preceding example we assume that all the goals are of roughly comparable importance. Now consider the case of *preemptive* goal programming, where there is a hierarchy of priority levels for the goals. Such a case arises when one or more of the goals clearly are far more important than the others. Thus, the initial focus should be on achieving as closely as possible these *first-priority* goals. The other goals also might naturally divide further into second-priority goals, third-priority goals, and so on. After we find an optimal solution with respect to the first-priority goals, we can break any ties for the optimal solution by considering the second-priority goals. Any ties that remain after this reoptimization can be broken by considering the third-priority goals, and so on.

When we deal with goals on the *same* priority level, our approach is just like the one described for nonpreemptive goal programming. Any of the same three types of goals (lower one-sided, two-sided, upper one-sided) can arise. Different penalty weights for deviations from different goals still can be included, if desired. The same formulation technique of introducing auxiliary variables again is used to reformulate this portion of the problem to fit the linear programming format.

There are two basic methods based on linear programming for solving preemptive goal programming problems. One is called the *sequential procedure,* and the other is the *streamlined procedure.* We shall illustrate these procedures in turn by solving the following example.

Example.   Faced with the unpleasant recommendation to increase the company's workforce by more than 20 percent, the management of the Dewright Company has reconsidered the original formulation of the problem that was summarized in Table 1. This increase in workforce probably would be a rather temporary one, so the very high cost of training 833 new employees would be largely wasted, and the large (undoubtedly wellpublicized) layoffs would make it more difficult for the company to attract high-quality employees in the future. Consequently, management has concluded that a very high priority should be placed on avoiding an increase in the workforce. Furthermore, management has learned that raising *more than* $55 million for capital investment for the new products would be extremely difficult, so a very high priority also should be placed on avoiding capital investment above this level.

Based on these considerations, management has concluded that a *preemptive goal programming* approach now should be used, where the two goals just discussed should

be the first-priority goals, and the other two original goals (exceeding \$125 million in long-run profit and avoiding a decrease in the employment level) should be the second-priority goals. Within the two priority levels, management feels that the relative penalty weights still should be the same as those given in the rightmost column of Table 1. This reformulation is summarized in Table 2, where a factor of *M* (representing a huge positive number) has been included in the penalty weights for the first-priority goals to emphasize that these goals preempt the second-priority goals. (The portions of Table 1 that are not included in Table 2 are *unchanged.*)

The Sequential Procedure for Preemptive Goal Programming

The *sequential procedure* solves a preemptive goal programming problem by solving a *sequence* of linear programming models.

At the first stage of the sequential procedure, the only goals included in the linear programming model are the first-priority goals, and the simplex method is applied in the usual way. If the resulting optimal solution is *unique,* we adopt it immediately without considering any additional goals.

However, if there are *multiple* optimal solutions with the same optimal value of Z (call it Z^*), we prepare to break the tie among these solutions by moving to the second stage and adding the second-priority goals to the model. If $Z^* = 0$, all the auxiliary variables representing the *deviations from first-priority goals* must equal zero (full achievement of these goals) for the solutions remaining under consideration. Thus, in this case, all these auxiliary variables now can be completely deleted from the model, where the equality constraints that contain these variables are replaced by the mathematical expressions (inequalities or equations) for these first-priority goals, to ensure that they continue to be fully achieved. On the other hand, if $Z^* > 0$, the second-stage model simply adds the second-priority goals to the first-stage model (as if these additional goals actually were first-priority goals), but then it also adds the constraint that the *first-stage objective function* equals Z^* (which enables us again to delete the terms involving first-priority goals from the second-stage objective function). After we apply the simplex method again, if there still are multiple optimal solutions, we repeat the same process for any lower-priority goals.

Example.   We now illustrate this procedure by applying it to the example summarized in Table 2.

At the first stage, only the two *first-priority* goals are included in the linear programming model. Therefore, we can drop the common factor *M* for their penalty weights, shown in Table 2. By proceeding just as for the nonpreemptive model if these were the only goals, the resulting linear programming model is

Minimize $Z = 2y_2^+ + 3y_3^+,$

■ **TABLE 2** Revised formulation for the Dewright Co. preemptive goal programming problem

Priority Level	Factor	Goal	Penalty Weight
First priority	Employment level	≤40	2*M*
	Capital investment	≤55	3*M*
Second priority	Long-run profit	≥125	5
	Employment level	≥40	4

subject to

$$5x_1 + 3x_2 + 4x_3 - (y_2^+ - y_2^-) = 40$$
$$5x_1 + 7x_2 + 8x_3 - (y_3^+ - y_3^-) = 55$$

and

$$x_j \geq 0, \qquad y_k^+ \geq 0, \qquad y_k^- \geq 0 \qquad (j = 1, 2, 3; k = 2, 3).$$

(For ease of comparison with the nonpreemptive model with all four goals, we have kept the same subscripts on the auxiliary variables.)

By using the simplex method (or inspection), an optimal solution for this linear programming model has $y_2^+ = 0$ and $y_3^+ = 0$, with $Z = 0$ (so $Z^* = 0$), because there are innumerable solutions for (x_1, x_2, x_3) that satisfy the relationships

$$5x_1 + 3x_2 + 4x_3 \leq 40$$
$$5x_1 + 7x_2 + 8x_3 \leq 55$$

as well as the nonnegativity constraints. Therefore, these two first-priority goals should be used as *constraints* hereafter. Using them as constraints will force y_2^+ and y_3^+ to remain zero and thereby disappear from the model automatically.

If we drop y_2^+ and y_3^+ but add the second-priority goals, the second-stage linear programming model becomes

Minimize $Z = 5y_1^- + 4y_2^-$,

subject to

$$12x_1 + 9x_2 + 15x_3 - (y_1^+ - y_1^-) \qquad\qquad = 125$$
$$5x_1 + 3x_2 + 4x_3 \qquad\qquad + y_2^- \qquad = 40$$
$$5x_1 + 7x_2 + 8x_3 \qquad\qquad\qquad + y_3^- = 55$$

and

$$x_j \geq 0, \qquad y_1^+ \geq 0, \qquad y_k^- \geq 0 \qquad (j = 1, 2, 3; k = 1, 2, 3).$$

Applying the simplex method to this model yields the unique optimal solution $x_1 = 5$, $x_2 = 0$, $x_3 = 3\frac{3}{4}$, $y_1^+ = 0$, $y_1^- = 8\frac{3}{4}$, $y_2^- = 0$, and $y_3^- = 0$, with $Z = 43\frac{3}{4}$.

Because this solution is unique (*or* because there are no more priority levels), the procedure can now stop, with $(x_1, x_2, x_3) = (5, 0, 3\frac{3}{4})$ as the optimal solution for the *overall* problem. This solution fully achieves both first-priority goals as well as one of the second-priority goals (no decrease in employment level), and it falls short of the other second-priority goal (long-run profit $\geq$ 125) by just $8\frac{3}{4}$.

The Streamlined Procedure for Preemptive Goal Programming

Instead of solving a sequence of linear programming models, like the sequential procedure, the *streamlined procedure* finds an optimal solution for a preemptive goal programming problem by solving just *one* linear programming model. Thus, the streamlined procedure is able to duplicate the work of the sequential procedure with just *one run* of the simplex method. This one run *simultaneously* finds optimal solutions based just on first-priority goals and breaks ties among these solutions by considering lower-priority goals. However, this does require a slight modification of the simplex method.

If there are just *two* priority levels, the modification of the simplex method is one you already have seen, namely, the form of the *Big M method* illustrated throughout Sec. 4.6. In this form, instead of replacing M throughout the model by some huge positive number before running the simplex method, we retain the *symbolic* quantity M in the sequence of

simplex tableaux. Each coefficient in row 0 (for each iteration) is some linear function $aM + b$, where a is the current *multiplicative factor* and b is the current *additive term*. The usual decisions based on these coefficients (entering basic variable and optimality test) now are based solely on the *multiplicative* factors, except that any ties would be broken by using the *additive* terms. This is how the IOR Tutorial operates when solving interactively by the simplex method (and choosing the Big M method).

The linear programming formulation for the streamlined procedure with two priority levels would include *all* the goals in the model in the usual manner, but with basic penalty weights of M and 1 assigned to deviations from first-priority and second-priority goals, respectively. If different penalty weights are desired within the same priority level, these basic penalty weights then are multiplied by the individual penalty weights assigned within the level. This approach is illustrated by the following example.

Example. For the Dewright Co. preemptive goal programming problem summarized in Table 2, note that (1) different penalty weights are assigned within each of the two priority levels and (2) the individual penalty weights (2 and 3) for the first-priority goals have been multiplied by M. These penalty weights yield the following single linear programming model that incorporates all the goals.

Minimize $\qquad Z = 5y_1^- + 2My_2^+ + 4y_2^- + 3My_3^+,$

subject to

$$12x_1 + 9x_2 + 15x_3 - (y_1^+ - y_1^-) = 125$$
$$5x_1 + 3x_2 + 4x_3 - (y_2^+ - y_2^-) = 40$$
$$5x_1 + 7x_2 + 8x_3 - (y_3^+ - y_3^-) = 55$$

and

$$x_j \geq 0, \qquad y_k^+ \geq 0, \qquad y_k^- \geq 0 \qquad (j = 1, 2, 3; k = 1, 2, 3).$$

Because this model uses M to symbolize a huge positive number, the simplex method can be applied as described and illustrated throughout Sec. 4.6. Alternatively, a very large positive number can be substituted for M in the model and then any software package based on the simplex method can be applied. Doing either naturally yields the same unique optimal solution obtained by the sequential procedure.

More than Two Priority Levels. When there are more than two priority levels (say, p of them), the streamlined procedure generalizes in a straightforward way. The basic penalty weights for the respective levels now are $M_1, M_2, \ldots, M_{p-1}, 1$, where M_1 represents a number that is vastly larger than M_2, M_2 is vastly larger than $M_3, \ldots,$ and M_{p-1} is vastly larger than 1. Each coefficient in row 0 of each simplex tableau is now a linear function of all of these quantities, where the multiplicative factor of M_1 is used to make the necessary decisions, with tie breakers beginning with the multiplicative factor of M_2 and ending with the additive term.

■ PROBLEMS

8S-1. One of management's goals in a goal programming problem is expressed algebraically as

$$3x_1 + 4x_2 + 2x_3 = 60,$$

where 60 is the specific numeric goal and the left-hand side gives the level achieved toward meeting this goal.
(a) Letting y^+ be the amount by which the level achieved exceeds this goal (if any) and y^- the amount under the goal (if any),

show how this goal would be expressed as an equality constraint when reformulating the problem as a linear programming model.

(b) If each unit over the goal is considered twice as serious as each unit under the goal, what is the relationship between the coefficients of y^+ and y^- in the objective function being minimized in this linear programming model.

8S-2. Management of the Albert Franko Co. has established goals for the market share it wants each of the company's two new products to capture in their respective markets. Specifically, management wants Product 1 to capture at least 15 percent of its market and Product 2 to capture at least 10 percent of its market. Three advertising campaigns are being planned to try to achieve these market shares. One is targeted directly on the first product. The second targets the second product. The third is intended to enhance the general reputation of the company and its products. Letting x_1, x_2, and x_3 be the amount of money allocated (in millions of dollars) to these respective campaigns, the resulting market share (expressed as a percentage) for the two products are estimated to be

Market share for Product 1 = $0.5x_1 + 0.2x_3$,
Market share for Product 2 = $0.3x_2 + 0.2x_3$.

A total of $55 million is available for the three advertising campaigns, but management wants at least $10 million devoted to the third campaign. If both market share goals cannot be achieved, management considers each 1 percent decrease in the market share from the goal to be equally serious for the two products. In this light, management wants to know how to most effectively allocate the available money to the three campaigns.

(a) Formulate a goal programming model for this problem.
(b) Reformulate this model as a linear programming model.
(c) Use the simplex method to solve this model.

8S-3. The Research and Development Division of the Emax Corporation has developed three new products. A decision now needs to be made on which mix of these products should be produced. Management wants primary consideration given to three factors: total profit, stability in the workforce, and achieving an increase in the company's earnings next year from the $75 million achieved this year. In particular, using the units given in the following table, they want to

Maximize $Z = P - 6C - 3D$,

where P = total (discounted) profit over the life of the new products,

C = change (in either direction) in the current level of employment,

D = decrease (if any) in next year's earnings from the current year's level.

The amount of any increase in earnings does not enter into Z, because management is concerned primarily with just achieving some increase to keep the stockholders happy. (It has mixed feelings about a large increase that then would be difficult to surpass in subsequent years.)

The impact of each of the new products (per unit rate of production) on each of these factors is shown in the following table:

Factor	Unit Contribution Product: 1	2	3	Goal	Units
Total profit	20	15	25	Maximize	Millions of dollars
Employment level	6	4	5	= 50	Hundreds of employees
Earnings next year	8	7	5	≥ 75	Millions of dollars

(a) Define y_1^+ and y_1^-, respectively, as the amount over (if any) and the amount under (if any) the employment level goal. Define y_2^+ and y_2^- in the same way for the goal regarding earnings next year. Define x_1, x_2, and x_3 as the production rates of Products 1, 2, and 3, respectively. With these definitions, use the goal programming technique to express y_1^+, y_1^-, y_2^+, and y_2^- algebraically in terms of x_1, x_2, and x_3. Also express P in terms of x_1, x_2, and x_3.
(b) Express management's objective function in terms of x_1, x_2, x_3, y_1^+, y_1^-, y_2^+, and y_2^-.
(c) Formulate a linear programming model for this problem.
(d) Use the simplex method to solve this model.

8S-4. Reconsider the original version of the Dewright Co. problem summarized in Table 1. After further reflection about the solution obtained by the simplex method, management now is asking some what-if questions.
(a) Management wonders what would happen if the penalty weights in the rightmost column of Table 1 were to be changed to 7, 4, 1, and 3, respectively. Would you expect the optimal solution to change? Why?
(b) Management is wondering what would happen if the total profit goal were to be increased to wanting at least $140 million (without any change in the original penalty weights). Solve the revised model with this change.
(c) Solve the revised model if both changes are made.

8S-5. Montega is a developing country which has 15,000,000 acres of publicly controlled agricultural land in active use. Its government currently is planning a way to divide this land among three basic crops (labeled 1, 2, and 3) next year. A certain percentage of each of these crops is exported to obtain badly needed foreign capital (dollars), and the rest of each of these crops is used to feed the populace. Raising these crops also provides employment for a significant proportion of the population. Therefore, the main factors to be considered in allocating the land to these crops are (1) the amount of foreign capital generated, (2) the number of citizens fed, and (3) the number of citizens employed in raising these crops. The following table shows how much each 1,000 acres of each crop contributes toward these factors, and the last column gives the goal established by the government for each of these factors.

Factor	Contribution per 1,000 Acres Crop: 1	2	3	Goal
Foreign capital	$3,000	$5,000	$4,000	≥ $70,000,000
Citizens fed	150	75	100	≥ 1,750,000
Citizens employed	10	15	12	= 200,000

In evaluating the relative seriousness of *not* achieving these goals, the government has concluded that the following deviations from the goals should be considered *equally undesirable:* (1) each $100 under the foreign-capital goal, (2) each person under the citizens-fed goal, and (3) each deviation of one (in either direction) from the citizens-employed goal.

(a) Formulate a goal programming model for this problem.
(b) Reformulate this model as a linear programming model.
(c) Use the simplex method to solve this model.
(d) Now suppose that the government concludes that the importance of the various goals differs greatly so that a preemptive goal programming approach should be used. In particular, the first-priority goal is citizens fed $\geq 1,750,000$, the second-priority goal is foreign capital $\geq \$70,000,000$, and the third-priority goal is citizens employed $= 200,000$. Use the goal programming technique to formulate one complete linear programming model for this problem.
(e) Use the streamlined procedure to solve the problem as formulated in part (*d*).
(f) Use the sequential procedure to solve the problem as presented in part (*d*).

8S-6. Consider a *preemptive goal programming* problem with three priority levels, just one goal for each priority level, and just two activities to contribute toward these goals, as summarized in the following table:

Priority Level	Unit Contribution Activity: 1	2	Goal
First priority	1	2	≤ 20
Second priority	1	1	= 15
Third priority	2	1	≥ 40

(a) Use the *goal programming technique* to formulate one complete linear programming model for this problem.
(b) Construct the initial simplex tableau for applying the *streamlined procedure.* Identify the *initial BF solution* and the *initial entering basic variable,* but do not proceed further.
(c) Starting from (*b*), use the *streamlined procedure* to solve the problem.
(d) Use the logic of preemptive goal programming to solve the problem graphically by focusing on just the two decision variables. Explain the logic used.
(e) Use the *sequential procedure* to solve this problem. After using the *goal programming technique* to formulate the linear programming model (including auxiliary variables) at each stage, solve the model *graphically* by focusing on just the two decision variables. Identify *all* optimal solutions obtained for each stage.

8S-7. Redo Prob. 8.S-6 with the following revised table:

Priority Level	Unit Contribution Activity: 1	2	Goal
First priority	1	1	≤20
Second priority	1	1	≥30
Third priority	1	2	≥50

8S-8. One of the most important problems in the field of *statistics* is the *linear regression problem*. Roughly speaking, this problem involves fitting a straight line to statistical data represented by points—$(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$—on a graph. If we denote the line by $y = a + bx$, the objective is to choose the constants a and b to provide the "best" fit according to some criterion. The criterion usually used is the *method of least squares,* but there are other interesting criteria where linear programming can be used to solve for the optimal values of a and b.

Formulate a linear programming model for this problem under the following criterion:

Minimize the sum of the absolute deviations of the data from the line; that is,

$$\text{Minimize} \quad \sum_{i=1}^{n} \left| y_i - (a + bx_i) \right|.$$

(*Hint:* Note that this problem can be viewed as a nonpreemptive goal programming problem where each data point represents a "goal" for the regression line.)

■ CASES

CASE 8S-1   A Cure for Cuba

Fulgencio Batista led Cuba with a cold heart and iron fist—greedily stealing from poor citizens, capriciously ruling the Cuban population that looked to him for guidance, and violently murdering the innocent critics of his politics. In 1958, tired of watching his fellow Cubans suffer from corruption and tyranny, Fidel Castro led a guerilla attack against the Batista regime and wrested power from Batista in January 1959.

Cubans, along with members of the international community, believed that political and economic freedom had finally triumphed on the island. The next two years showed, however, that Castro was leading a Communist dictatorship—killing his political opponents and nationalizing all privately held assets. The United States responded to Castro's leadership in 1961 by invoking a trade embargo against Cuba. The embargo forbade any country from selling Cuban products in the United States and forbade businesses from selling American products to Cuba. Cubans did not feel the true impact of the embargo until 1989 when the Soviet economy collapsed. Prior to the disintegration of the Soviet Union, Cuba had received an average of $5 billion in annual economic assistance from the Soviet Union. With the disappearance of the economy that Cuba had almost exclusively depended upon for trade, Cubans had few avenues from which to purchase food, clothes, and medicine. The avenues narrowed even further when the United States passed the Torricelli Act in 1992 that forbade American subsidiaries in third countries from doing business with Cuba that had been worth a total of $700 million annually.

Since 1989, the Cuban economy has certainly felt the impace from decades of frozen trade. Today poverty continues to be a serious problem on the island of Cuba. Many families do not have sufficient money to purchase bare necessities, such as food, milk, and clothing. Children die from malnutrition or exposure. Disease infects the island because medicine is not sufficiently available. Optical neuritis, tuberculosis, pneumonia, and influenza run rampant among the population.

Few Americans hold great sympathy for Cuba, but Robert Baker, director of Helping Hand, leads a handful of tender souls on Capitol Hill who cannot bear to see politics destroy so many human lives. His organization distributes humanitarian aid annually to needy countries around the world. Mr. Baker recognizes the dire situation in Cuba, and he wants to allocate aid to Cuba for the coming year.

Mr. Baker wants to send numerous aid packages to Cuban citizens. Three different types of packages are available. The basic package contains only food, such as grain and powdered milk. Each basic package costs $300, weighs 120 pounds, and aids 30 people. The advanced package contains food and clothing, such as blankets and fabrics. Each advanced package costs $350, weighs 180 pounds, and aids 35 people. The supreme package contains food, clothing, and medicine. Each supreme package costs $720, weighs 220 pounds, and aids 54 people.

Mr. Baker has several goals he wants to achieve when deciding upon the number and types of aid packages to allocate to Cuba. First, he wants to aid at least 20 percent of Cuba's 11 million citizens. Second, because disease runs rampant among the Cuban population, he wants at least 3,000 of the aid packages sent to Cuba to be the supreme

packages. Third, because he knows many other nations also require humanitarian aid, he wants to keep the cost of aiding Cuba below $20 million.

Mr. Baker places different levels of importance on his three goals. He believes the most important goal is keeping costs down since low costs mean that his organization is able to aid a larger number of needy nations. He decides to penalize his plan by 1 point for every $1 million above his $20 million goal. He believes the second most important goal is ensuring that at least 3,000 of the aid packages sent to Cuba are supreme packages, since he does not want to see an epidemic develop and completely destroy the Cuban population. He decides to penalize his plan by 1 point for every 1,000 packages below his goal of 3,000 packages. Finally, he believes the least important goal is reaching at least 20 percent of the population, since he would rather give a smaller number of individuals all they need to thrive instead of a larger number of individuals only some of what they need to thrive. He therefore decides to penalize his plan by 7 points for every 100,000 people below his 20 percent goal.

Mr. Baker realizes that he has certain limitations on the aid packages that he delivers to Cuba. Each type of package is approximately the same size, and because only a limited number of cargo flights from the United States are allowed into Cuba, he is only able to send a maximum of 40,000 packages. Along with a size limitation, he also encounters a weight restriction. He cannot ship more that 6 million pounds of cargo. Finally, he has a safety restriction. When sending medicine, he needs to ensure that the Cubans know how to use the medicine properly. Therefore, for every 100 supreme packages, Mr. Baker must send one doctor to Cuba at a cost of $33,000 per doctor.

(a) How many basic, advanced, and supreme packages should Mr. Baker send to Cuba?

(b) Mr. Baker reevaluates the levels of importance he places on each of the three goals. To sell his efforts to potential donors, he must show that his program is effective. Donors generally judge the effectiveness of a program on the number of people reached by aid packages. Mr. Baker therefore decides that he must put more importance on the goal of reaching at least 20 percent of the population. He decides to penalize his plan by 10 points for every half a percentage point below his 20 percent goal. The penalties for his other two goals remain the same. Under this scenario, how many basic, advanced, and supreme packages should Mr. Baker send to Cuba? How sensitive is the plan to changes in the penalty weights?

(c) Mr. Baker realizes that sending more doctors along with the supreme packages will improve the proper use and distribution of the packages' contents, which in turn will increase the effectiveness of the program. He therefore decides to send one doctor with every 75 supreme packages. The penalties for the goals remain the same as in part (*b*). Under this scenario, how

many basic, advanced, and supreme packages should Mr. Baker send to Cuba?

(d) The aid budget is cut, and Mr. Baker learns that he definitely cannot allocate more than $20 million in aid to Cuba. Due to the budget cut, Mr. Baker decides to stay with his original policy of sending one doctor with every 100 supreme packages. How many basic, advanced, and supreme packages should Mr. Baker send to Cuba assuming that the penalties for not meeting the other two goals remain the same as in part (*a*)?

(e) Now that the aid budget has been cut, Mr. Baker feels that the levels of importance of his three goals differ so much that it is difficult to assign meaningful penalty weights to deviations from these goals. Therefore, he decides that it would be more appropriate to apply a preemptive goal-programming approach (which will ensure that his budget goal is fully met if possible), while retaining his original policy of sending one doctor with every 100 supreme packages. How many basic, advanced, and supreme packages should Mr. Baker send to Cuba according to this approach?

CASE 8S-2   Airport Security

Shortly after the tragic events of September 11, 2001, the United States Congress enacted emergency legislation to give the Department of Transportation primary responsibility for providing security at over 400 major U.S. airports. The Transportation Security Administration was then created within the Department of Transportation to carry out this responsibility.

A leading OR consultant in the airline industry, Adeline Jonasson, has been hired by the Transportation Security Administration to head up a new task force to further improve airport security. The specific charge to the task force is to investigate what advanced security technology should be developed and used at airport checkpoints to maximize the effectiveness with which passengers can be screened within budget constraints.

Even prior to 2001, airline passengers had become familiar with the two basic types of systems used to check each passenger at a security checkpoint. One is a portal that can detect concealed weapons as the passenger walks through. The other is a screening system that scans the passenger's carry-on luggage. Various proposals have been made for advanced security technology that would improve these two systems. Adeline's task force now needs to make recommendations on which direction to go for the next generation of these systems.

The task force has been told that the functional requirement for the new portal system is that it must be able to detect even one ounce of explosives and hazardous liquids as well as metallic weapons being concealed by a passenger. The technology needed to do this includes quadrupole resonance (closely related to magnetic resonance technology used by the medical industry) and magnetic sensors. There are various ways to design the portal with this technology that would satisfactorily meet the functional requirement. However, the designs would differ greatly in the frequency with which false alarms would occur as well as in the purchase cost and maintenance cost for the portal. The frequency of false alarms is a key consideration since it substantially affects the efficiency with which the passengers can be processed. Even more importantly, a high frequency of false alarms greatly decreases the alertness of the security personnel for detecting the relatively rare terrorists who are actually concealing destructive devices.

The most basic version of the portal system that satisfactorily meets the functional requirement would have an estimated purchase price of $90,000 and, on the average, would incur an annual maintenance cost of $15,000. The drawback of this version is that it would generate a false alarm for approximately 10 percent of the passengers. This false alarm rate can be reduced by using more expensive versions of the system. Each additional $15,000 in the cost of the portal system would lower the false alarm rate 1 percent and also would increase the annual maintenance cost by $1,500. The most expensive version would cost $210,000, so it would have a false alarm rate of only 2 percent of the customers as well as an annual maintenance cost of $27,000.

Regarding the new screening system for carry-on luggage, the functional requirement is that it must clearly reveal suspicious objects as small as the smallest Swiss army knife. The technology needed to do this combines X-ray imaging, a thermal neutron scanner, and computer tomography imaging (which compares the density and other physical properties of any suspicious objects with known high-risk materials). It is estimated that the most basic version that satisfactorily meets this functional requirement would cost $60,000 plus an annual maintenance cost of $9,000. As with the most basic portal system, the drawback of this version is that it isn't sufficiently discriminating between suspicious objects that actually are destructive devices and those that are harmless. Thus, this version would generate false alarms for approximately 6 percent of the customers. In addition to wasting time and delaying passengers, such a high false alarm rate would make it very difficult for the screening operator to pay sufficient attention when the far more unusual true alarms occur. However, more expensive versions of the screening system would be considerably more discriminating. In particular, each additional $30,000 in the cost of the system would enable a reduction of 1 percent in the false alarm rate, while also increasing the annual maintenance cost by $1,200. Thus, the most expensive version, costing $150,000, would decrease the false alarm rate to 3 percent and incur an annual maintenance cost of $12,600.

The task force has been given two budgetary guidelines.

First Budgetary Guideline: Plan on a total expenditure of $250,000 for both the portal system and the screening system for carry-on luggage at each security checkpoint.

Second Budgetary Guideline: Plan on holding down the average total maintenance costs for the two systems at each security checkpoint to no more than $30,000.

These budget guidelines prohibit using the most expensive versions of both the portal system and the screening system for carry-on baggage. Therefore, the task force needs to determine which financially feasible combination of versions for the two systems will

maximize the effectiveness with which passengers can be screened. Doing this requires first obtaining input from the top management of the Transportation Security Administration regarding what the measure of effectiveness should be and then what management's goals and priorities are for achieving substantial effectiveness and meeting the budgetary guidelines.

Fortunately, Adeline already has had extensive discussions with top management to obtain its guidance on these matters. These discussions led to the adoption of a clear policy that was approved all the way up to the Secretary of Transportation (who also informed the chairmen of the Congressional oversight committees of this action). The policy establishes the following order of priorities.

Priority 1: The functional requirement for each of the two new systems *must* be met. (This is satisfied by all the versions under consideration by the task force.)

Priority 2: The total false alarm rate for both systems should not exceed 0.1 per passenger.

Priority 3: Meet the first budgetary guideline.

Priority 4: Meet the second budgetary guideline.

Now that it has obtained all the needed managerial input, the task force is ready to begin its analysis.

(a) Identify the two decisions to be made, and define a decision variable for each one.

(b) Describe why this problem is a preemptive goal programming problem by giving quantitative expressions for each of the goals in terms of the decision variables defined in part (*a*).

(c) Draw a single two-dimensional graph where the two axes correspond to the decision variables defined in part (*a*). Consider each of the goals in order of priority and use the quantitative expression obtained in part (*b*) for this goal to draw a plot on this graph that graphically displays the values of the decision variables that fully satisfy this goal. After completing this for all the goals, use this graph to determine the optimal solution for this preemptive goal programming problem.

(d) Use a linear programming software package (such as Solver, ASPE, MPL/Solvers, LINDO, or LINGO) to formulate and solve this preemptive goal programming problem.

(e) If it is possible to fully satisfy all the goals except the lowest-priority goal, one can quickly solve a preemptive goal programming problem by formulating and solving a linear programming model that includes all the goals except the last one as constraints and then uses the objective function to strive toward the lowest-priority goal. Formulate and solve such a linear programming model for this problem on a spreadsheet. What would be the interpretation for the preemptive goal programming problem if this linear programming model had no feasible solutions?

(f) Perform some postoptimality analysis by determining how far the total false alarm rate per passenger can be reduced (perhaps even below the goal) by ignoring the second budgetary guideline but fully meeting the first one.

(g) What additional postoptimality analysis do you feel should be performed in order to provide top management with the information needed to make a sound judgment decision about the best trade-off between (1) the total false alarm rate per passenger, (2) the total expenditure for the two new security systems per security checkpoint, and (3) the total annual maintenance cost for these two systems per security checkpoint.