(e)

	1	2		6 Optimal Value
Variables	2	2		
				RHS
Constraints	1	3	8 <=	8
	1	1	4 <=	4

Adjustable Cells

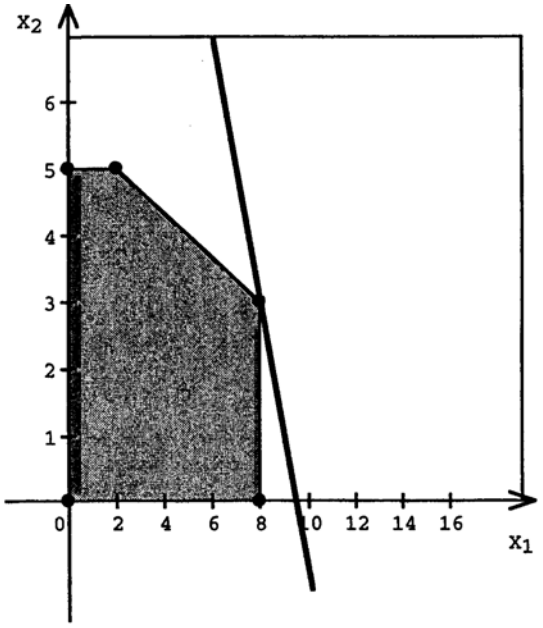
Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B2		2	0	1	1	0.333333
C2		2	0	2	1	1

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
E4		8	0.5	8	4	4
E5		4	0.5	4	4	1.333333

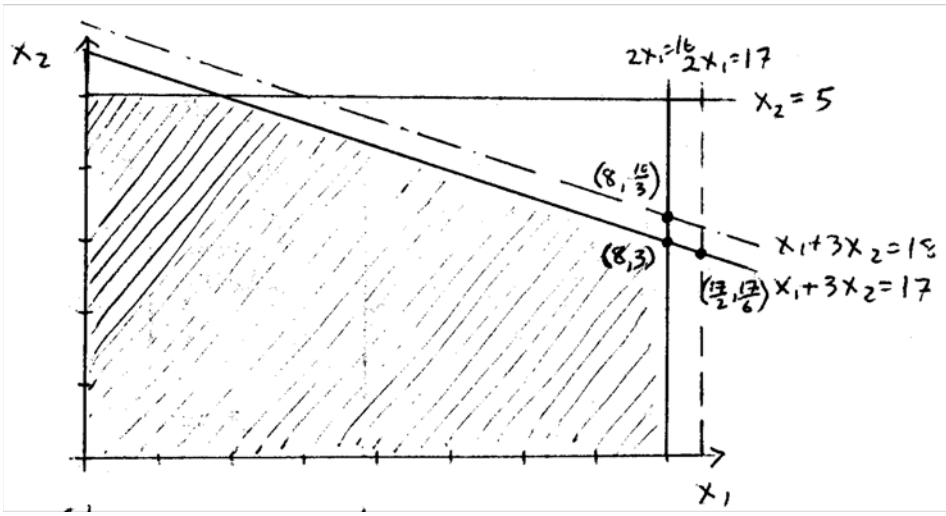
4.7-3.

(a) Optimal Solution: $(x_1^*, x_2^*) = (8, 3)$ and $Z^* = 38$



Corner Point	Z
$(8, 3)$	$38*$
$(8, 0)$	32
$(2, 5)$	18
$(0, 5)$	10
$(0, 0)$	0

(b)



Increasing resource 1 to 17 units increases Z to $4(8.5)+2(2.83)=39.67$, so $\Delta Z = y_1^* = 1.67$.
Increasing resource 2 to 18 units increases Z to $4(8)+2(3.33)=38.33$, so $\Delta Z = y_2^* = 0.67$.
The third constraint is not binding, so $y_3^* = 0$.

(c) To increase Z by 15, resource 1 should be increased by $\frac{15}{y_1^*} = \frac{15}{1.67} \approx 9$. Solving the LP problem with resource 1 set to $16+9=25$ returns the result $Z=53$.

4.7-4.

(a) Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (0.5, 0, 4.5)$ and $Z^* = 14$

Bas Var	Eq No	Coefficient of Z	X_1	X_2	X_3	X_4	X_5	X_6	Right Side
Z	0	1	-1	7	-3	0	0	0	0
X_4	1	0	2	1	-1	1	0	0	4
X_5	2	0	4	-3	0	0	1	0	2
X_6	3	0	-3	2	1	0	0	1	3

Bas Var	Eq No	Coefficient of Z	X_1	X_2	X_3	X_4	X_5	X_6	Right Side
Z	0	1	-10	13	0	0	0	3	9
X_4	1	0	-1	3	0	1	0	1	7
X_5	2	0	4	-3	0	0	1	0	2
X_3	3	0	-3	2	1	0	0	1	3

Bas Var	Eq No	Coefficient of Z	X_1	X_2	X_3	X_4	X_5	X_6	Right Side
Z	0	1	0	5.5	0	0	2.5	3	14
X_4	1	0	0	2.25	0	1	0.25	1	7.5
X_1	2	0	1	-0.75	0	0	0.25	0	0.5
X_3	3	0	0	-0.25	1	0	0.75	1	4.5

(b) The shadow prices for the three resources are given by the reduced costs (in the objective function) for the corresponding slack variables. These values are circled in the table above. The shadow prices for resources 1, 2 and 3 are 0, 2.5 and 3 respectively. They represent the rate at which the objective function value z increases as the corresponding resource is increased. For instance, increasing resource 3 by one unit increases Z by 3, provided that no other constraints cause any trouble.

(c)

	X1	X2	X3			
Maximize	1	-7	3			
						Right-Hand
				Totals		Side
Constraint 1	2	1	-1	-3.5	<=	4
Constraint 2	4	-3	0	2	<=	2
Constraint 3	-3	2	1	3	<=	3
						Objective
Solution	0.5	0	4.5			14

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B10	Solution X1	0.5	0	1	7.33333	10
C10	Solution X2	0	-5.5	-7	5.5	1E+30
D10	Solution X3	4.5	0	3	22	3

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
E5	Constraint 1 Totals	-3.5	0	4	1E+30	7.5
E6	Constraint 2 Totals	2	2.5	2	1E+30	2
E7	Constraint 3 Totals	3	3	3	1E+30	4.5

4.7-5.

(a) Optimal Solution: $(x_1^*, x_2^*, x_3^*) = (0, 1, 3)$ and $Z^* = 7$

Bas Var	Eq No	Z	Coefficient of X_1	X_2	X_3	X_4	X_5	X_6	Right Side
Z	0	1	-2	2	-3	0	0	0	0
X_4	1	0	-1	1	1	1	0	0	4
X_5	2	0	2	-1	1	0	1	0	2
X_6	3	0	1	1	3	0	0	1	12

Bas Var	Eq No	Z	Coefficient of X_1	X_2	X_3	X_4	X_5	X_6	Right Side
Z	0	1	4	-1	0	0	3	0	6
X_4	1	0	-3	2	0	1	-1	0	2
X_3	2	0	2	-1	1	0	1	0	2
X_6	3	0	-5	4	0	0	-3	1	6

Bas Var	Eq No	Z	Coefficient of X_1	X_2	X_3	X_4	X_5	X_6	Right Side
Z	0	1	2.5	0	0	0.5	2.5	0	7
X_2	1	0	-1.5	1	0	0.5	-0.5	0	1
X_3	2	0	0.5	0	1	0.5	0.5	0	3
X_6	3	0	1	0	0	-2	-1	1	2

(b) The shadow prices are $y_1^* = 0.5$, $y_2^* = 2.5$ and $y_3^* = 0$. They are the marginal values of resources 1, 2 and 3 respectively.

(c)

	X1	X2	X3			
Maximize	2	-2	3			
						Right-Hand
				Totals		Side
Constraint 1	-1	1	1	4	<=	4
Constraint 2	2	-1	1	2	<=	2
Constraint 3	1	1	3	10	<=	12
						Objective
Solution	0	1	3			7

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B10	Solution X1	0	-2.5	2	2.5	1E+30
C10	Solution X2	1	0	-2	1.6667	1
D10	Solution X3	3	0	3	1E+30	1

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
E5	Constraint 1 Totals	4	0.5	4	1	2
E6	Constraint 2 Totals	2	2.5	2	2	6
E7	Constraint 3 Totals	10	0	12	1E+30	2

4.7-6.

(a) Optimal Solution: $(x_1^*, x_2^*, x_3^*, x_4^*) = (11, 0, 3, 0)$ and $Z^* = 52$

Bas Var	Eq No	Z	Coefficient of X_1	X_2	X_3	X_4	X_5	X_6	Right Side
Z	0	1	-5	-2	1	-3	0	0	0
X_5	1	0	3	2	-3	1	1	0	24
X_6	2	0	3	3	1	3	0	1	36

Bas Var	Eq No	Z	Coefficient of X_1	X_2	X_3	X_4	X_5	X_6	Right Side
Z	0	1	0	1.3333	-4	-1.333	1.6667	0	40
X_1	1	0	1	0.6667	-1	0.3333	0.3333	0	8
X_6	2	0	0	1	4	2	-1	1	12

Bas Var	Eq No	Z	Coefficient of X_1	X_2	X_3	X_4	X_5	X_6	Right Side
Z	0	1	0	2.3333	0	0.6667	0.6667	1	52
X_1	1	0	1	0.9167	0	0.8333	0.0833	0.25	11
X_3	2	0	0	0.25	1	0.5	-0.25	0.25	3

(b) The shadow prices are $y_1^* = 0.6667$ and $y_2^* = 1$. They are the marginal values of resources 1 and 2 respectively.

(c)

	X1	X2	X3	X4			
Maximize	5	4	-1	3			
							Right-Hand
					Totals		Side
Resource 1	3	2	-3	1	24	<=	24
Resource 2	3	3	1	3	36	<=	36
							Objective
Solution	11	0	3	0			52

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B9	Solution X1	11	0	5	1E+30	0.3636
C9	Solution X2	0	-0.33333	4	0.33333	1E+30
D9	Solution X3	3	0	-1	2.66667	1.33333
E9	Solution X4	0	-0.66667	3	0.66667	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
F5	Resource 1 Totals	24	0.66667	24	12	132
F6	Resource 2 Totals	36	1	36	1E+30	12