UNIVERSITY OF WISCONSIN
DATA SCIENCE

DS 775

Prescriptive Analytics

Linear Programming: Model and Assumptions

How does Linear Programming fit under the Prescriptive Analytics Umbrella?

The most common use of linear programming is summarized as "...allocating limited resources among competing activities in a best possible (i.e., *optimal*) way."

From p.25 of the textbook

The word programming in linear programming is not used in the sense of programming a computer but rather in the sense of planning. Prescriptive analytics is all about planning. It's about making decisions regarding future courses of action informed by data.

Presumably, we'd like the decisions we make to result in the best outcome. That is, in many situations, we are interested in maximizing or minimizing an important variable, such as maximizing profit or minimizing cost, for two common examples. Furthermore, real world situations are often complicated by having limited resources and competing activities.

Model Formulation

The standard form for a linear programming model contains:

- The Objective Function
- Functional Constraints
- Non-negativity Constraints

While it is possible to solve linear programming problems where the goal is to minimize the objective function or when the functional constraints are either greater than or equal to or simply equality constraints or even when the design variables are not restricted to being non-negative, the standard form is such that the objective function is to be maximized, the functional constraints are all of the less than or equal to variety, and the design variables are all non-negative.

More Important Definitions

- **Feasible Solution** – All constraints are satisfied
- **Infeasible Solution** – At least one constraint is violated
- **Feasible Region** – The collection of all feasible solutions

The optimal solution must be one that's feasible, that is, one that doesn't violate any of the constraints. The collection of all such possibilities is called a feasible /

Optimal Solutions

- **Optimal Solution** – A feasible solution that has the most favorable value (max or min) of the objective function
- **Multiple Optimal Solutions** – An infinite number of points on a line produce the same optimal value
- **No Optimal Solutions** – If no feasible solutions exist or if the feasible region is unbounded

Corner-Point Feasible Solutions

Corner-point Feasible (CPF) Solution – lies at a corner of the feasible region

In *any* LP problem with feasible solutions and a bounded feasible region, the best CPF solution *must* be an optimal solution (for multiple optimal solutions, at least two must be CPF solutions).

Assumptions of Linear Programming

- **Proportionality & Additivity** - Objective function & constraints are linear combinations of the design variables
- **Divisibility** - Design variables are not restricted to integer values, but all real numbers in feasible reason are possible
- **Certainty** - All parameters are known constants

The proportionality and additivity assumptions mean that the objective function and the constraints will be linear combinations of the design variables. We would have nonlinear programming without the proportionality assumption which we won't cover in this class, but you can read about it in chapter 13 if you'd like to learn more.

The divisibility assumption means that the design variables are not restricted to integer values, but that all real numbers in the feasible region are possible. So fractional values of the design variables can be realistically assigned and implemented.

Design variables that are restricted to integers require a special approach. These are covered in chapter 12 of this textbook, and options for solving this type are available in the Excel and ASB solvers.

The certainty assumption is that all of the parameters in a linear programming model are known. This isn't particularly realistic, because the nature of prescriptive analytics is that a future course of action is being considered. The parameters can be estimated through descriptive and predictive analytics, and as such, they would be variables, which adds a layer of complexity to the situation.

Sensitivity analysis is the follow-up procedure to see how the optimal solution is affected by changes in the parameters that were assumed to be known constants.

Question 1

Linear Programming: Model and Assumptions

Question for Self Assessment: Multiple Choice

In a linear programming model, the proportionality assumption

- ○ rules out any exponent other than 1 for any variable in any term of any function.
- ○ says that every function is the sum of the individual contributions of the respective activities.
- ○ restricts the design variables to integer values.
- ○ says that the value assigned to each parameter of a linear programming model is assumed to be a known constant.
- ○ says that the constrains must be proportional to each other.

SUBMIT

Answer is at the end of this transcript

Question 2

Linear Programming: Model and Assumptions

Question for Self Assessment: Multiple Choice

Linear programming problems with only two decision variables can be solved graphically.

- True
- False

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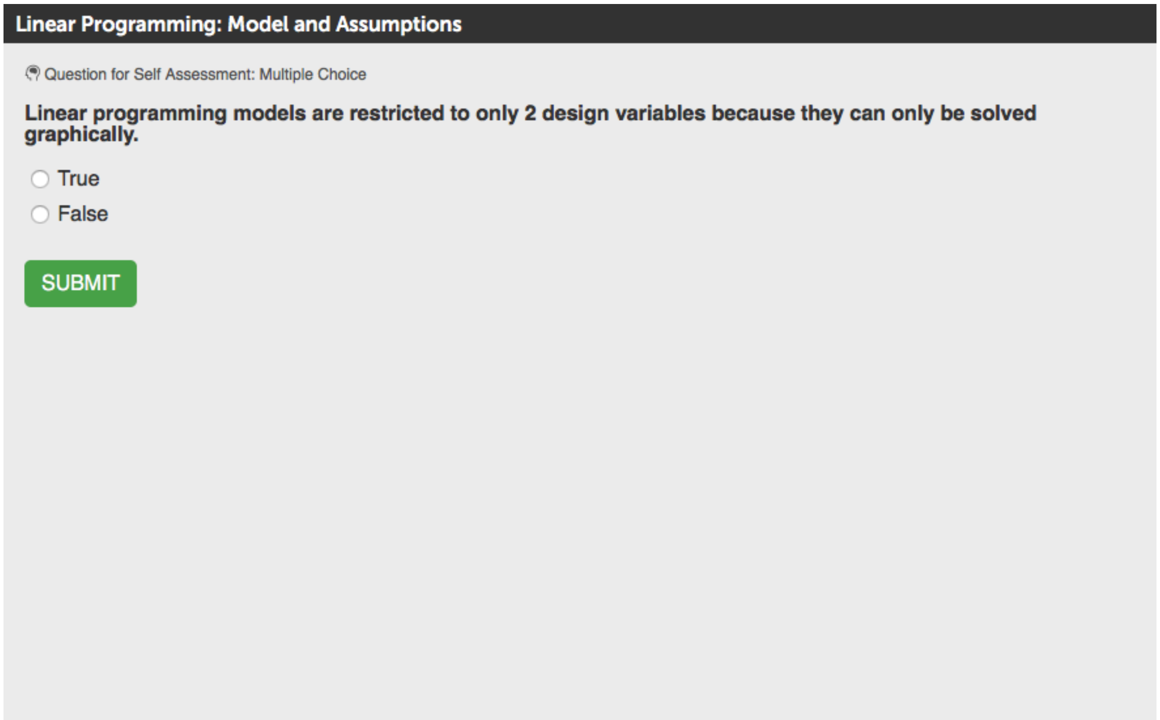
Answer is at the end of this transcript

Question 3



Answer is at the end of this transcript

Question 4

Linear Programming: Model and Assumptions

Question for Self Assessment: Multiple Choice

In sensitivity analysis, parameters whose values cannot be changed without changing the optimal solution are called sensitive parameters.

- True
- False

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Answer is at the end of this transcript

Question 1 Answer

Linear Programming: Model and Assumptions

Feedback for Self Assessment

✓ Correct!

In a linear programming model, the proportionality assumption

Your answer:

rules out any exponent other than 1 for any variable in any term of any function.

Correct answer:

rules out any exponent other than 1 for any variable in any term of any function.

Feedback:

Correct. See page 38 in the textbook.

Question 2 Answer

Feedback for Self Assessment

✓ Correct!

Linear programming problems with only two decision variables can be solved graphically.

Your answer:

True

Correct answer:

True

Feedback:

Absolutely! When a solution exists…

Question 3 Answer

Linear Programming: Model and Assumptions

Feedback for Self Assessment

✓ Correct!

Linear programming models are restricted to only 2 design variables because they can only be solved graphically.

Your answer:

False

Correct answer:

False

Feedback:

Correct. Models with more design variables can be solved mathematically. See the Kibbutz example in sec. 3.4 of the text.

Question 4 Answer

Linear Programming: Model and Assumptions

Feedback for Self Assessment

✓ Correct!

In sensitivity analysis, parameters whose values cannot be changed without changing the optimal solution are called sensitive parameters.

Your answer:

True

Correct answer:

True

Feedback:

Nice work.