

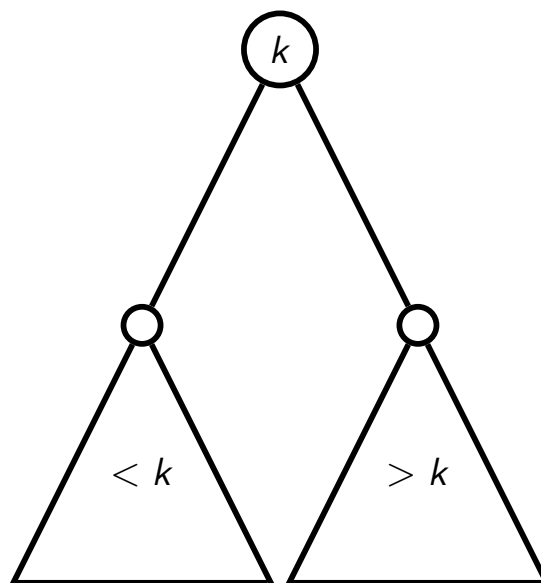
# Binary Search Trees

CS 237

Discrete Mathematics I

## Binary Search Tree Basics

A binary tree with a key at each vertex so that at each vertex, all keys in left subtree are less and all keys in right subtree are greater than key at the vertex is called a **binary search tree (BST)**:



## BST Characteristics

- ▶ Searching—straightforward  $O(\log n)$  algorithm.
- ▶ Insertion—search for key, insert at leaf where search terminated.
- ▶  $n$  is number of nodes (vertexes),  $h$  is height of tree.
- ▶ All operations: worst case number of key comparisons =  $h + 1$
- ▶  $\lfloor \lg n \rfloor \leq h \leq n - 1$
- ▶ With average (random) data:  $h < 1.44 \lg n$
- ▶ Thus all operations have efficiency:
  - ▶ worst case:  $\Theta(n)$
  - ▶ average case:  $\Theta(\lg n)$
- ▶ Bonus: inorder traversal produces sorted list (treesort).

## Building a BST

Build a binary search tree for the following words (using alphabetical order):

*mathematics, physics, geography, zoology, meteorology, geology, psychology, and chemistry*

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*mathematics, physics, geography, zoology, meteorology, geology, psychology, and chemistry*

*physics > mathematics*

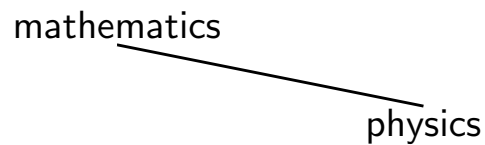
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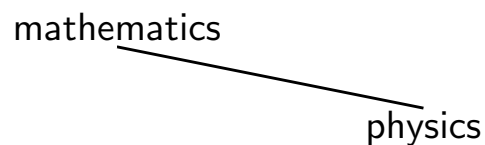


## Building a BST

Build a binary search tree for the following words (using alphabetical order):

*mathematics, physics, geography, zoology, meteorology, geology, psychology, and chemistry*

*geography < mathematics*

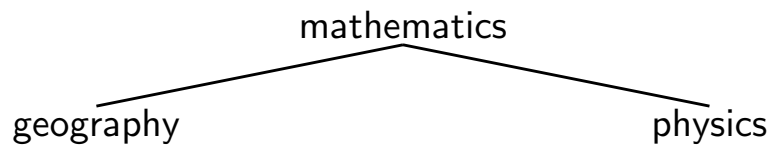


## Building a BST

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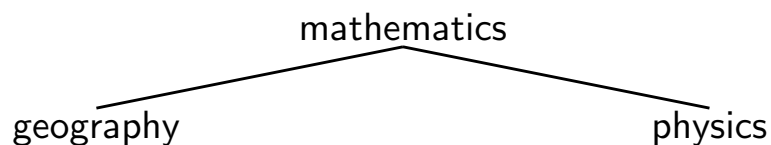


## Building a BST

Build a binary search tree for the following words (using alphabetical order):

*mathematics, physics, geography, zoology, meteorology, geology, psychology, and chemistry*

*zoology > mathematics   zoology > physics*

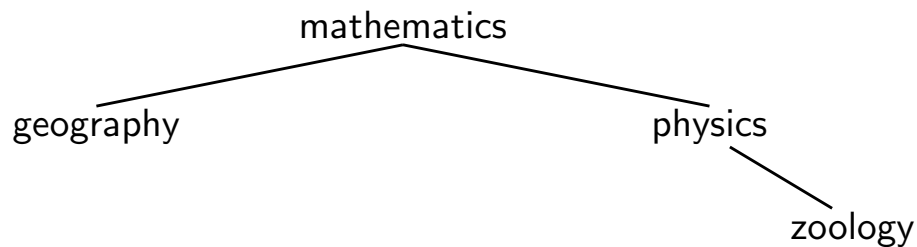


## Building a BST

Build a binary search tree for the following words (using alphabetical order):

*mathematics, physics, geography, zoology, meteorology, geology, psychology, and chemistry*

*zoology > mathematics   zoology > physics*

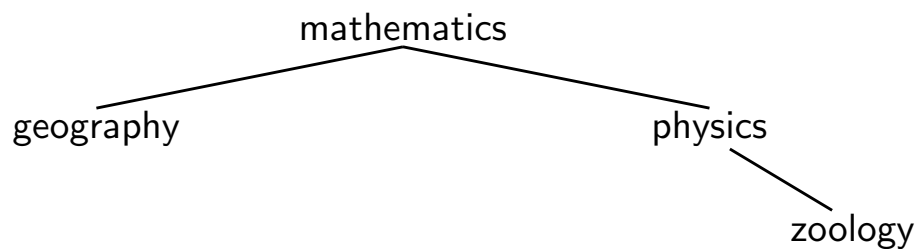


## Building a BST

Build a binary search tree for the following words (using alphabetical order):

*mathematics, physics, geography, zoology, meteorology, geology, psychology, and chemistry*

*meteorology > mathematics   meteorology < physics*

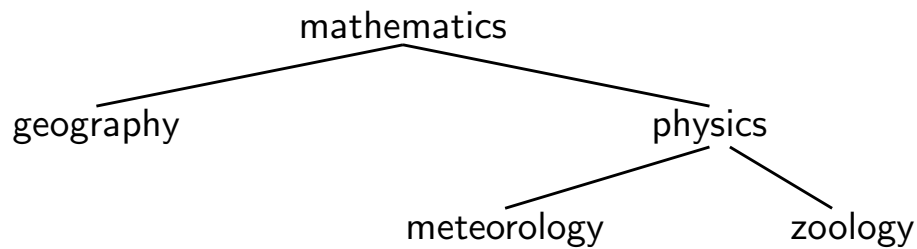


## Building a BST

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*mathematics, physics, geography, zoology, meteorology, geology, psychology, and chemistry*

*meteorology > mathematics   meteorology < physics*

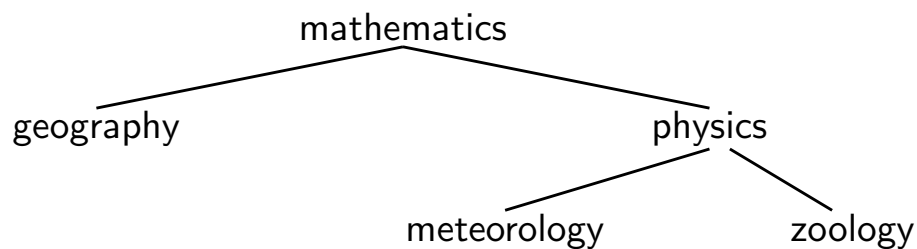


## Building a BST

Build a binary search tree for the following words (using alphabetical order):

*mathematics, physics, geography, zoology, meteorology, geology, psychology, and chemistry*

*geology < mathematics   geology > geography*

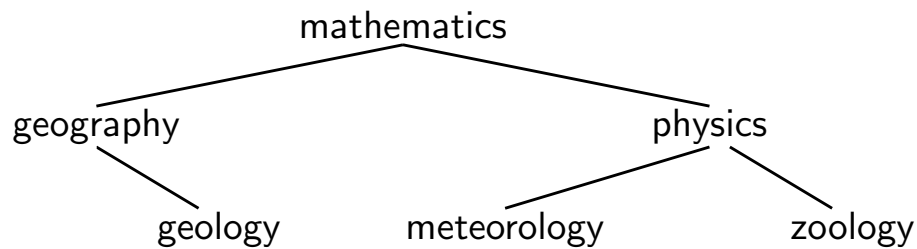


## Building a BST

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*mathematics, physics, geography, zoology, meteorology, geology, psychology, and chemistry*

*geology < mathematics geology > geography*

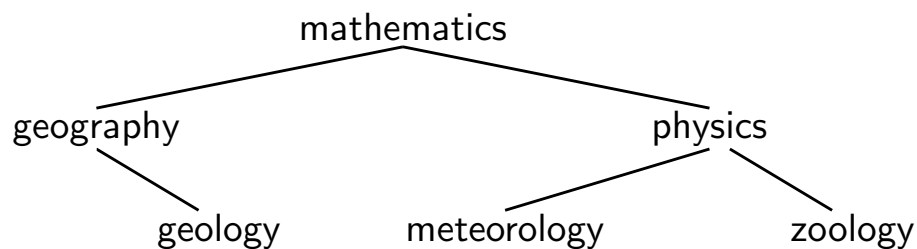


## Building a BST

Build a binary search tree for the following words (using alphabetical order):

*mathematics, physics, geography, zoology, meteorology, geology, psychology, and chemistry*

*psychology > mathematics, physics psychology < zoology*



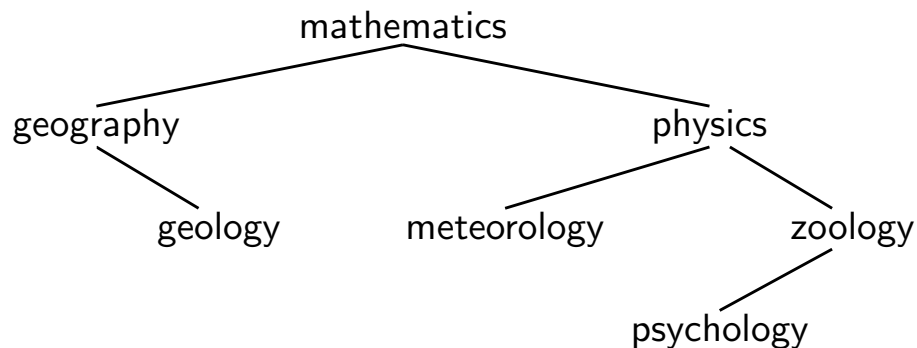


## Building a BST

Build a binary search tree for the following words (using alphabetical order):

*mathematics, physics, geography, zoology, meteorology, geology, psychology, and chemistry*

*psychology > mathematics, physics   psychology < zoology*

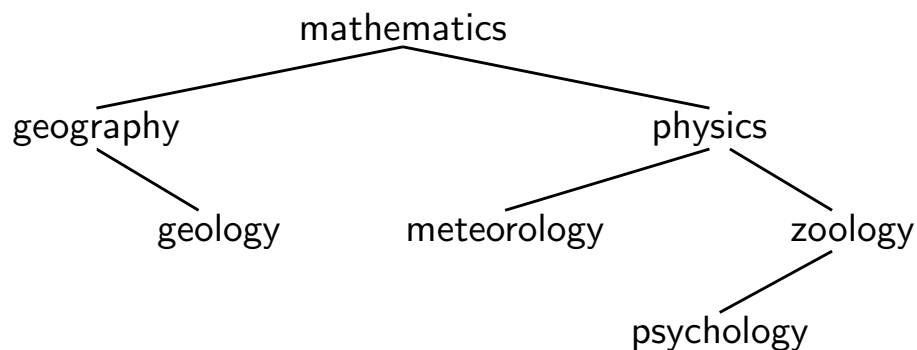


## Building a BST

Build a binary search tree for the following words (using alphabetical order):

*mathematics, physics, geography, zoology, meteorology, geology, psychology, and chemistry*

*chemistry < mathematics   chemistry < geography*

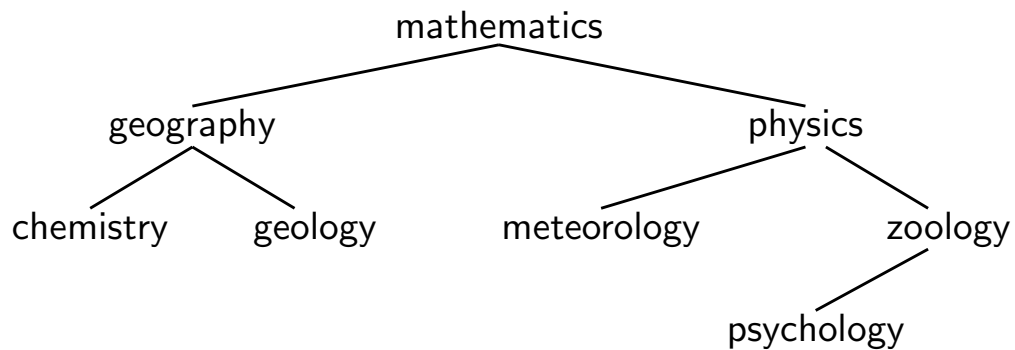


## Building a BST

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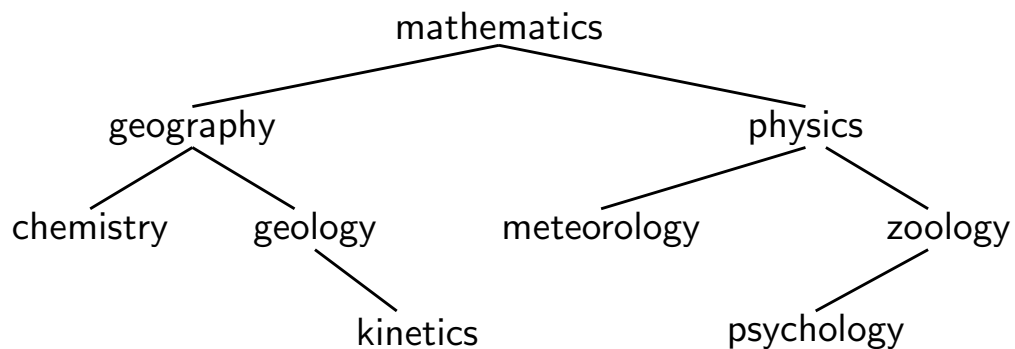
*mathematics, physics, geography, zoology, meteorology, geology, psychology, and chemistry*

*chemistry < mathematics   chemistry < geography*



## Binary Search Tree Insertion

Insert the word *kinetics* into this binary search tree:



## BST Class Exercise

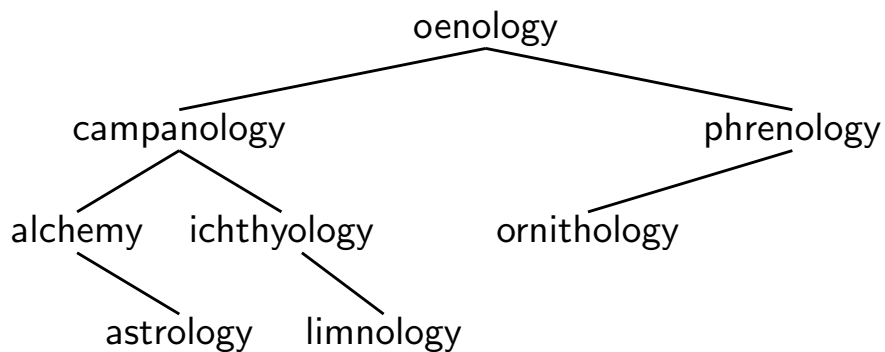
Build a binary search tree for the following words (using alphabetical order):

*oenology, campanology, phrenology, alchemy,*  
*ichthyology, ornithology, astrology, limnology*

## BST Class Exercise

Build a binary search tree for the following words (using alphabetical order):

*oenology, campanology, phrenology, alchemy,*  
*ichthyology, ornithology, astrology, limnology*



## Static and Predictable

From Wikipedia:

“If we don’t plan on modifying a search tree, and we know exactly how often each item will be accessed, we can construct an **optimal binary search tree**, which is a search tree where the average cost of looking up an item (the expected search cost) is minimized.

Even if we only have estimates of the search costs, such a system can considerably speed up lookups on average. For example, if you have a BST of English words used in a **spell checker**, you might balance the tree based on word frequency in text corpuses, placing words like *the* near the root and words like *agerasia* near the leaves.”

An analysis of optimal BSTs will have to wait for CSE 381!