Multibody Dynamics B - Assignment 7   **Rick Staa**

ME41055   #4511328

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Head TA: Simon vd. Helm   Due Date: 03/05/2018

Statement of integrity

my homework is completely in accordance with
the Academic Integrity

Figure 0.1: My handwritten statement of integrity

Acknoledgements

I used [1] in making this assignment when finished I compared initial values with Prajish
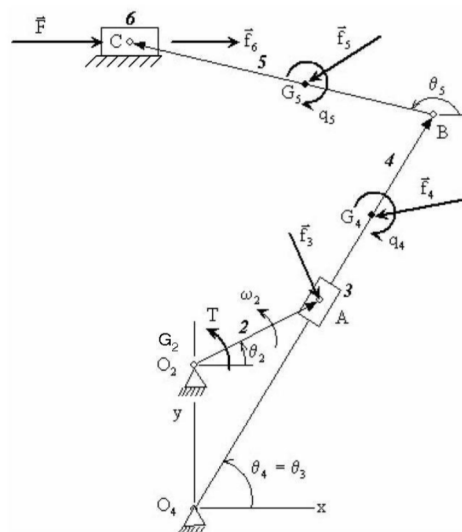Kumar (4743873).

Setup overview

Figure 0.2: Quick return mechanism as depicted in assignment 7

Problem Statement

In this assignment we were asked to determine the motion of the quick return mechanism depicted in homework 7 (see 0.2). This quick return mechanism consisted of 3 bars connected by 2 slider joints and 2 revolute joints. As a result of these joints the quick return mechanism has 1 degree of freedom (3*3 -2 *2 -2*2 = 1 DOF). The qucik retrun mechanism has the following parameters:

$$O_2A = 0.2m \tag{0.1}$$
$$O_4 = 0.7m \tag{0.2}$$
$$O_4O_2 = 0.3m \tag{0.3}$$
$$O_4G_4 = 0.4m \tag{0.4}$$
$$BG_5 = 0.3m \tag{0.5}$$
$$y_c = 0.9m \tag{0.6}$$
$$m_3 = 0.5kg \tag{0.7}$$
$$m_4 = 6kg \tag{0.8}$$
$$m_6 = 2kg \tag{0.9}$$
$$J_4 = 10kgm^2 \tag{0.10}$$
$$J_5 = 6kgm^2 \tag{0.11}$$

On this mechanism the following forces and moments work:

$$F = 1000N \tag{0.12}$$
$$T = 0Nm \tag{0.13}$$

Equations of motion (EOM)

The examine the motion of the mechanism described above we will use the TMT method explained in Chapter 5 of the reader [1] to derive the equations of motion (EOM).

TMT method

The Virtual Power TMT method is like the Lagrange method but differs in the fact that it doesn't go into the energy domain. Instead it stays in the forces domain and uses an incremental approach to obtain the equations of motion. We can derive the method by first looking at the virtual power equation with included D'Alembert forces:

$$\delta P = (F - M\ddot{x})\delta\dot{x} \tag{0.14}$$

Following the TMT method makes use of a transformation matrix T to transform the COM positions to the generalized coordinates. As a result we obtain the following equation:

$$\delta P = \delta\dot{x}_i(F_i - M_{ik}\ddot{x}_k) + \delta\dot{q}_k Q_j \tag{0.15}$$

Following we can derive the virtual accelerations which fulfill the constraints in equation 3 and fill this in in equation 8. After noting that this equation must hold for all virtual velocities and rearranging the equation a bit we obtain:

$$\bar{M}\ddot{q} = \bar{f} \tag{0.16}$$

Where:

$$\bar{M} = T^T M T \, and \, \bar{f} = T^T(F - mG) \tag{0.17}$$

In this T represents the transformation matrix which can be calculated by taking the Jacobian of the states x w.r.t. the general coordinates. In our problem the state is equal to:

$$x_2 = 0 \tag{0.18}$$
$$y_{(2)} = O_4O_2 \tag{0.19}$$
$$x_3 = O_2A cos(\phi_2) \tag{0.20}$$
$$y_3 = O_4O_2 + O_2A sin(\phi_2) \tag{0.21}$$
$$x_4 = O_4G_4 cos(\phi_4) \tag{0.22}$$
$$y_4 = O_4G_4 sin(\phi_4) \tag{0.23}$$
$$x_5 = O_4B cos(\phi_4) + BG_5 cos(\phi_5) \tag{0.24}$$
$$y_5 = O_4B sin(\phi_4) + BG_5 sin(\phi_5) \tag{0.25}$$
$$x_6 = O_4B cos(\phi_4) + BC cos(\phi 5) \tag{0.26}$$

The M is again the mass matrix and the F contains the applied forces and moment at the COM's of the segments.

$$M = \begin{bmatrix} J_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & J_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & J_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & J_5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ m_6 & & & & & & & & & \end{bmatrix} \tag{0.27}$$

$$F = \begin{bmatrix} T \\ F \end{bmatrix} \begin{matrix} 0 & -m_3g & 0 & 0 & -m_4g & 0 & 0 & -m_5g & 0 \end{matrix} \tag{0.28}$$

The new G matrix is a convective term that arises due to deriving the virtual accelerations this term was derived using the symbolic toolbox and will not be displayed here. The MATLAB code implementing the TMT method can be found in Appendix A. The generalised coordinates now become:

$$q = \begin{bmatrix} \phi_2 & \phi_4 & \phi_5 & \dot{\phi}_2 & \dot{\phi}_4 & \dot{\phi}_5 \end{bmatrix} \tag{0.29}$$

The other angles are all dependent on these 3 generalised coordinates. The intial state of the system is:

$$\phi_2 init = 0 \tag{0.30}$$

$$\phi_4 init = \tan^{-1}(\frac{O_4 O_2}{O_2 A}) \tag{0.31}$$

$$\pi - sin^{-1}(\frac{Y_c - O_4 B sin(\phi_4 init)}{BC}) \tag{0.32}$$

$$\dot{\phi_2} init = \frac{(150\pi)}{60} \tag{0.33}$$

$$\dot{\phi_4} init = cos(\phi_4)^2 \dot{\phi_2} \tag{0.34}$$

$$\dot{\phi_5} init = \frac{O_4 B cos(\phi_4) \dot{\phi_4}}{-BC cos(\phi_5)} \tag{0.35}$$

Cut the loop method

Since our mechanism is closed loop we need to make use of the "Cut the loop method" to get these equations of motion. In this method we first make two cuts, one at sliding joint 6 and one at revolute joint 2, as a result we now have a open loop system with 3 degrees of freedom. This system now has 3 generalized coordinates ϕ_2, ϕ_4 and ϕ_5. To get back to the original DOF of the system we need to add 2 extra constraints:

$$C = \begin{bmatrix} O_4 A cos(\phi_4) + O_2 A cos(\phi_2 \\ O_4 B sin(\phi_4 + BC sin(\phi_5) - Y_c \end{bmatrix} \tag{0.36}$$

Full system

If we combine the open loop system with the close loop system we get the following system of equation which can be solved in matlab:

$$\begin{pmatrix} T_{i,l} M_{ij} T_{j,k} & C_{c,l} \\ C_{c,k} & 0_{cc} \end{pmatrix} \begin{pmatrix} \ddot{q}_k \\ \lambda_c \end{pmatrix} = \begin{pmatrix} Q_l + T_{i,l} \left(F_i - M_{ij} q_j \right) \\ -C_{c,kl} \dot{q}_k \dot{q}_l \end{pmatrix}$$

Figure 0.3: the end result

In this the $T_{i,l}$ is the jacobian of state x w.r.t the generalised coordinates, $C_{c,l}$ is the jacobian of the constraints w.r.t. the generalized coordinates and $C_{c,kl}$ is a convective term that comes from taking the second derivative of the constraint. Since all these are calculated with the symbolic toolbox they are not depicted here. To see what their structure is one is refered to the matlab script in appendix A.

Numerical intergration method

To get the movent of the quick release mechanism in time we will use a 4th order Runge-Kuta intergration method combined with a Gauß-Newton correction for position and speed. This correction is done to compensate for intergration drift.

Runge-Kutta 4th order method (RK4)

$$k_1 = f(t_n, y_n) \tag{0.37}$$

$$k_2 = f(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_1) \tag{0.38}$$

$$k_3 = f(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_2) \tag{0.39}$$

$$k_4 = f(t_n + h, y_n + hk_3) \tag{0.40}$$

$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{0.41}$$
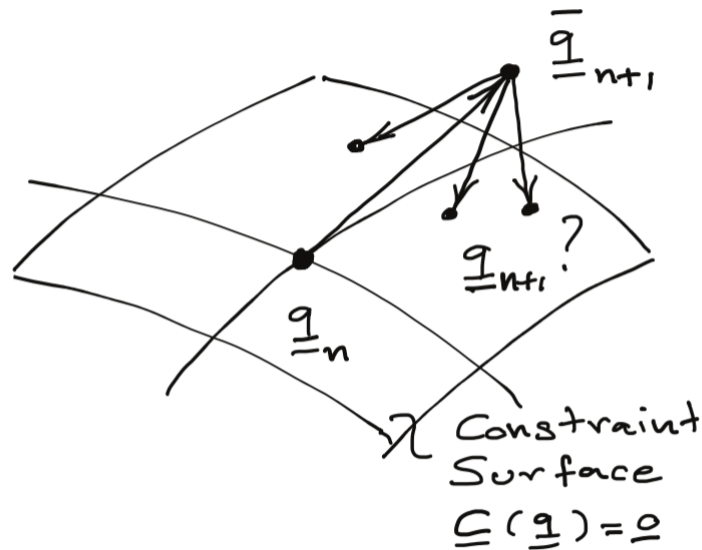
Gauß-Newton corrections

Figure 0.4: A depiction of the constraint surface and Gauß-Newton method as displayed in [1]. This picture was not modified in any sense

The Gauß-Newton we are using here is a non-linear leas-square constraint optimization method. In our problem we the following optimization problem:

$$\left\|\bar{\boldsymbol{q}}_{n+1} - \boldsymbol{q}_{n+1}\right\|_2 = \min_{\boldsymbol{q}_{n+1}}, \quad \forall \quad \{\boldsymbol{q}_{n+1} | C(\boldsymbol{q}_{n+1}) = \boldsymbol{0}\}.$$

In words what your doing with the Gausß method is you look at the solution and see how much it deviates from the constraint surface. You then look for the point on the constraint surface that is closest to our original point. This point searching is what is done by the optimization (see 0.4).Above named non-linear constraint optimization problem is easily solved by an iterative method. The idea of this method is that you look at a small change around the current state q:

$$\boldsymbol{q}_{n+1} = \bar{\boldsymbol{q}}_{n+1} + \Delta\boldsymbol{q}_{n+1}.$$

When you fill this in in the original

$$\Delta\boldsymbol{q}_{n+1} = \boldsymbol{0}, \quad \forall \quad \{\Delta\boldsymbol{q}_{n+1} | \{C(\bar{\boldsymbol{q}}_{n+1}) + C_{,\boldsymbol{q}}\left(\bar{\boldsymbol{q}}_{n+1}\right)\Delta\boldsymbol{q}_{n+1} = \boldsymbol{0}\}.$$

This leads to the following system of equations:

$$\begin{pmatrix} \mathbf{I} & \mathbf{C}^{\mathrm{T}} \\ \mathbf{C} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \boldsymbol{\Delta} \\ \boldsymbol{\mu} \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ e \end{pmatrix}.$$

In which:

$$-\mathbf{C}\mathbf{C}^{\mathrm{T}}\boldsymbol{\mu} = e.$$

$$\boldsymbol{\mu} = -\left(\mathbf{C}\mathbf{C}^{\mathrm{T}}\right)^{-1} e,$$
$$\boldsymbol{\Delta} = \mathbf{C}^{\mathrm{T}}\left(\mathbf{C}\mathbf{C}^{\mathrm{T}}\right)^{-1} e.$$

In the end you obtain:

$$\Delta = \mathbf{C}^{+} e.$$

With this you can calculate a new q that is closer to the constraint surface as:

$$q_new = q_old + \Delta \tag{0.42}$$

Following you can recalcaulte the C and $\dot{C}$ and start the process over again. In our example we repeat this process till or 10 function iterations are done or the constraints are smaller than 10^-12. This procedure is applied to both the position and velocity of the quick return mechanism.

position
The matlab code doing this operation is shown below:

```matlab
172      % Solve non-linear constraint least-square problem
173   while (max(abs(C)) > parms.tol)&& (n_iter < parms.nmax)
174         q_tmp        = q(1:3);
175         n_iter = n_iter + 1;
176         q_del   = Cd*inv(Cd.'*Cd)*-C.';
177         q(1:3) = q_tmp+ q_del.';
178
179         % Recalculate constraint
180         [C,Cd]       = constraint_calc(q,parms);
181   end
```

Figure 0.5: Matlab code doing the position correction

velocity
The matlab code doing this operation is shown below:

```matlab
183      % Calculate the corresponding speeds
184   q_tmp_vel         = q(4:6);
185   Dqd_n1            = -Cd*inv(Cd.'*Cd)*Cd.'*q_tmp_vel.';
186   q(4:6)           = q_tmp_vel + Dqd_n1.';
187
```

Figure 0.6: Matlab code doing the velocity correction

Results

Below the results of the simulation are discussed.

Angular speed of crank 2, rocker 4 and connecting bar 5

From figure 0.7 we can see that both the crank, rocker and connecting bar oscilate around their axis of roation. The rocker has the bigest amplitude while the crank the smallest this is what whe would expect of the lengths of the segments.
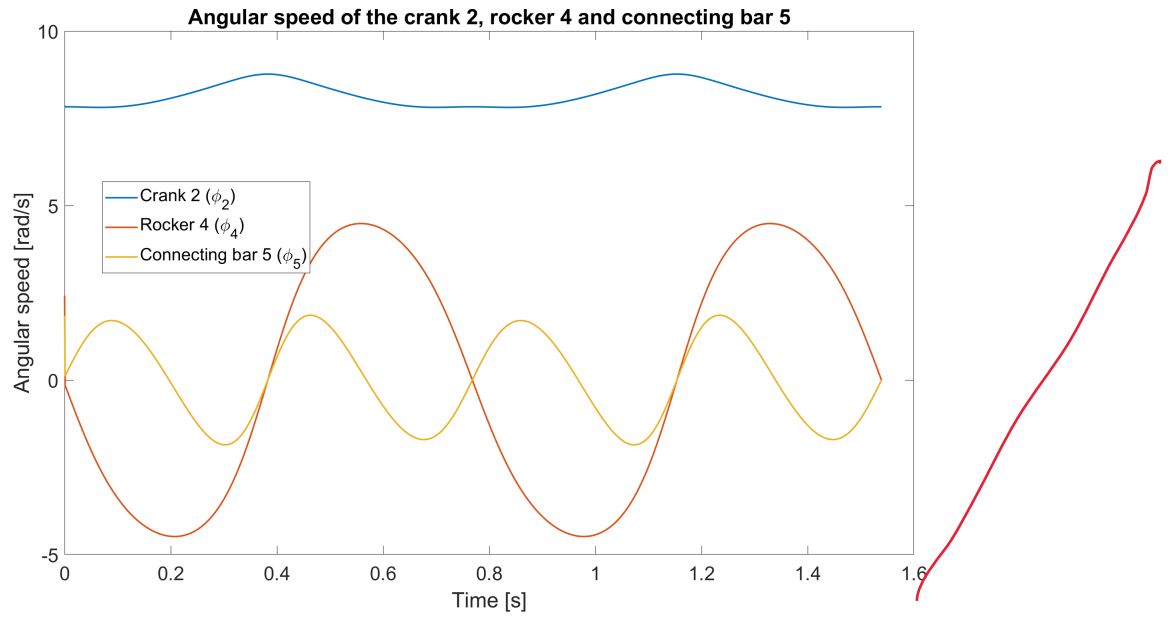
Figure 0.7: Plot of the angular velocity of crank 2, rocker 4 and connecting bar 5

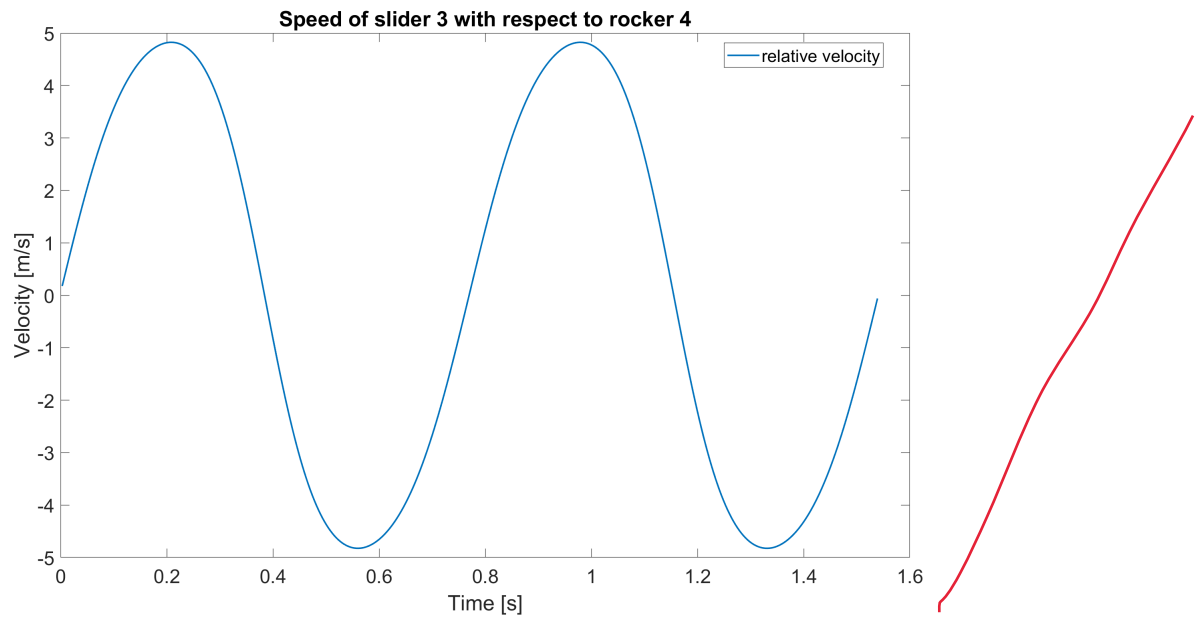## Sliding speed of 3 relative to rocker 4

Figure 0.8: Speed slider 3 relative to rocker 3

Position slider 6

Now we look at the velocity position and acceleration of slider 6:
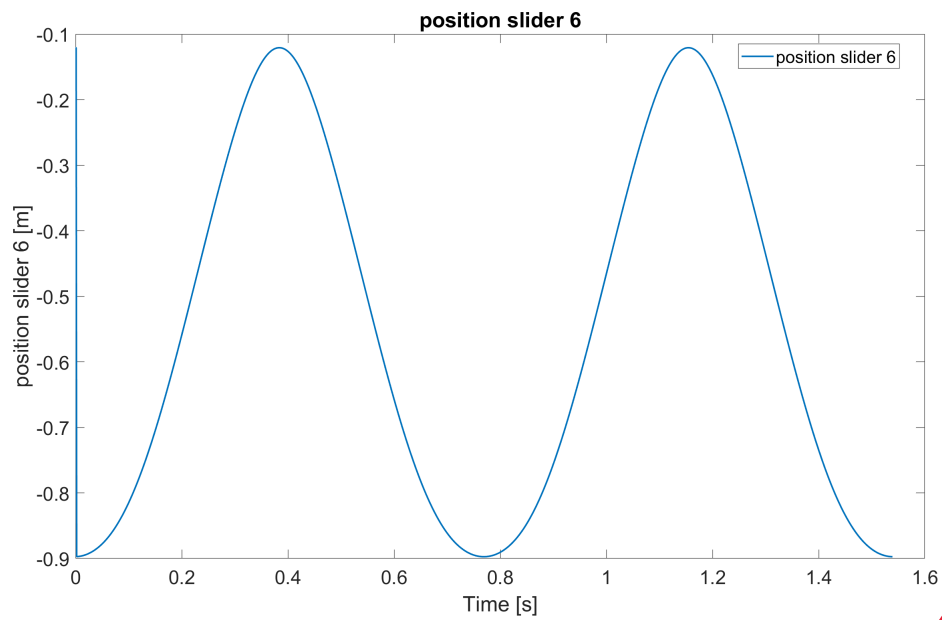
Figure 0.9: position slider 6

Velocity slider 6

Velocity of slider 6 can be found in figure 0.10.

Acceleration slider 6

The acceleration of slider 6 can be found in figure 0.11

Figure 0.10: velocity slider 6

Figure 0.11: acceleration slider 6

Reaction forces in slider 6 and 3

In this section you will find the reaction forces experienced in slider 6 and 3. These reaction forces are shown in figure 0.12. From this figure we can see two things. First we also clearly

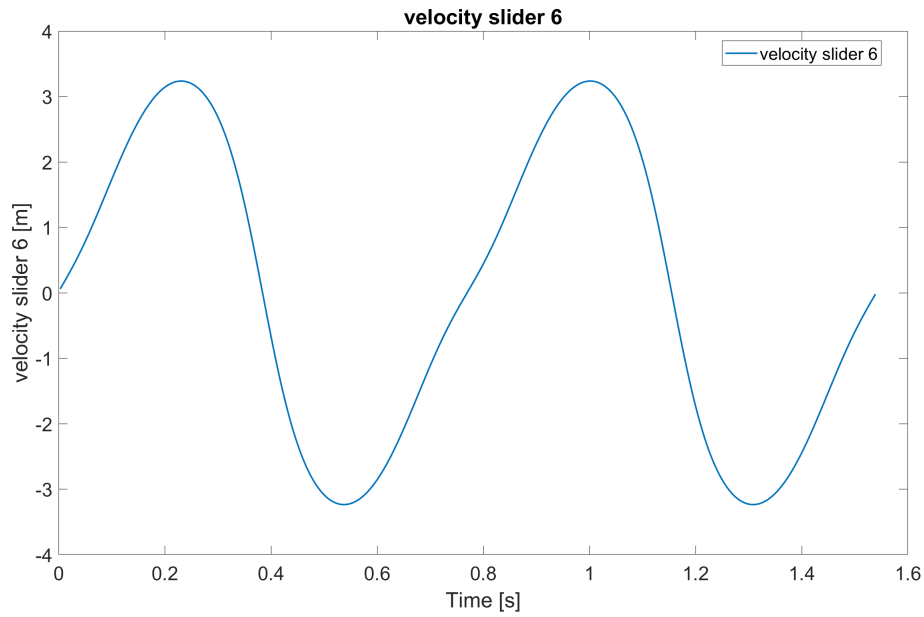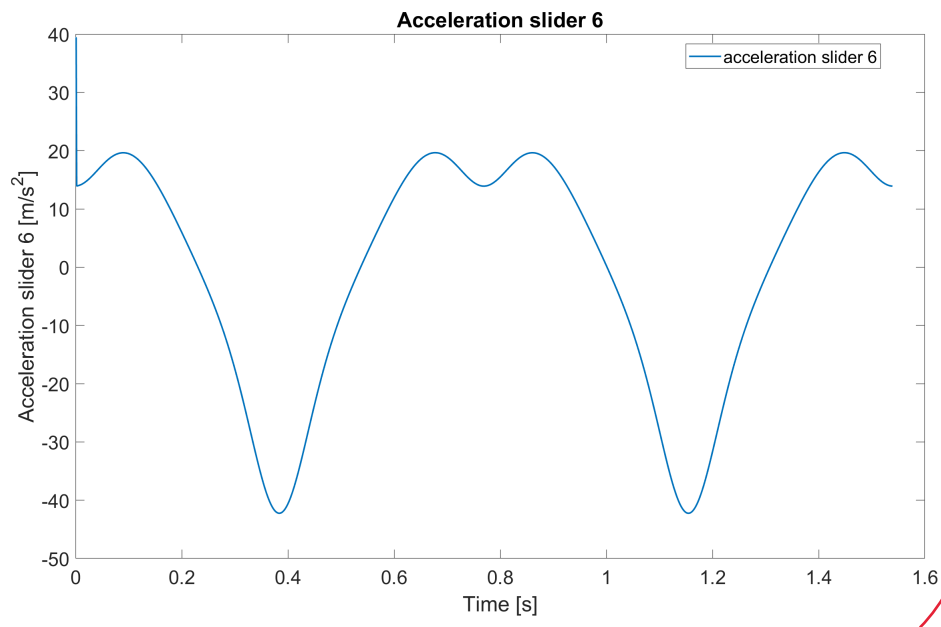see that the quick release mechanism in our simulation experience a oscillatory motion. Second the reaction force of the slider on the ground is way bigger than the reaction force of slider 3 on rocker 4. Lastly we see that also the amplitude of the reaction force of slider 6 on the ground is bigger. These results can be explained by the lower relative impact angle between slider 3 and rocker 4 compared to the impact angle between slider 6 and the ground.
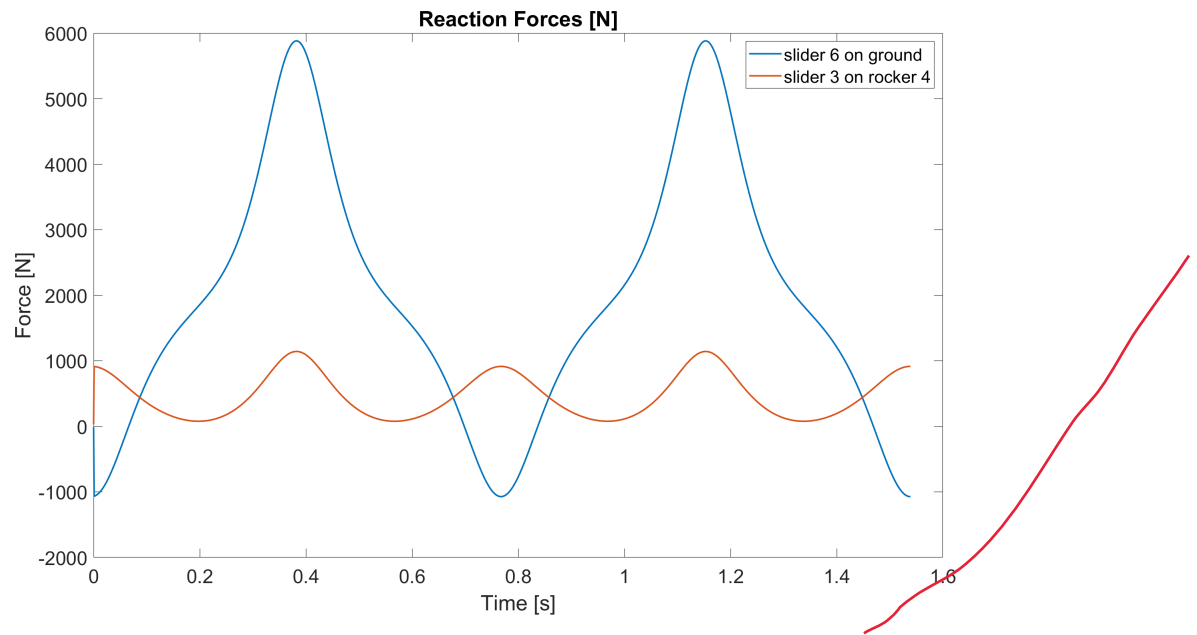
Figure 0.12: Reaction forces experienced in the quick release mechanism during the simulated motion

Validation checks

First of all I checked if the motion was cyclic since this is what I would expect based on intuition. That this is the case can be clearly seen from the plots. Secondly I used a open-source four bar mechanism plotter by "Mohammad Saadeh", which can be found on the MATLAB file exchange server, to check if the motion looked reasonable. Other possible checks would be calculating the kinetic and potential energy of the mechanism to see if energy is lost during the simulation. Lastly one can also calculate the static forces and torques which cause equilibrium in the mechanism and apply these to the model to see If our quick release mechanism is modelled the right way. The last two checks I unfortunatelly couldn't perform due to a recent bug in the MATLAB symbolic toolbox.

Appendix A

The main MATLAB script

```matlab
%% MBD_B: Assignment 7 - Quick return mechanism
%   Rick Staa (4511328)
%   Last edit: 09/05/2018
clear all; % close all; clc;
fprintf('--- A7 ---\n');

%% Set up needed symbolic parameters
% Create needed symbolic variables
syms phi2 phi4 phi5 phi2d phi4d phi5d

% Put in parms struct for easy function handling
parms.syms.phi2         = phi2;
parms.syms.phi4         = phi4;
parms.syms.phi5         = phi5;
parms.syms.phi2d        = phi2d;
parms.syms.phi4d        = phi4d;
parms.syms.phi5d        = phi5d;

%% Intergration parameters
time                    = 3;
                                    % Intergration time
parms.h                 = 1e-3;
                                    % Intergration step size
parms.tol               = 1e-12;
                                    % Intergration
    constraint error tolerance
parms.nmax              = 10;
                                    % Maximum number of
    Gauss-Newton drift correction iterations

%% Model Parameters
% Lengths and distances
parms.O2A               = 0.2;
                                    % Length segment 2 [m]
parms.O4B               = 0.7;
                                    % Length segment 4 [m]
parms.BC                = 0.6;
                                    % Length segment 5 [m]
```

```matlab
parms.O4O2                  = 0.3;                  % Distance between
    joint 4 and joint 2 [m]
parms.O4G4                  = 0.4;                  % Distance bewteen
    COM4 and joint 4 [m]
parms.BG5                   = 0.3;                  % Distance joint 5 and
     COM 5 [m]
parms.Yc                    = 0.9;                  % Height joint C (COM
    body 6) [m]
parms.O4A                   = sqrt(parms.O2A^2+parms.O4O2^2);
                % Distance between joint 4 and joint 3 [m]

% Masses and inertias
parms.m3                    = 0.5;                  % Body 3 weight [kg]
parms.m4                    = 6;                    % Body 4 weight [kg]
parms.m5                    = 4;                    % Body 5 weight [kg]
parms.m6                    = 2;                    % Body 6 weight [kg]
parms.J2                    = 100;                  % Moment of inertia
    body 2 [kgm^2]
parms.J3                    = 0;                    % Moment of inertia
    body 3 [kgm^2] - Put on 0 because no moment possible
parms.J4                    = 10;                   % Moment of inertia
    body 4 [kgm^2]
parms.J5                    = 6;                    % Moment of inertia
    body 5 [kgm^2]

%% World parameters
% Gravity
parms.g                     = 9.81;                 % [parms.m/s^2]

% Forces
```

```matlab
51  parms.F6_x                 = 1000;
                                            % x force on body 6 [N]
52  parms.T2                   = 0;
                                            % Torque around
        joint 6 [Nm]
53
54  %% Calculate Initial states
55  phi2_init                  = 0;
56  phi4_init                  = atan2(parms.O4O2,parms.O2A);
57  phi5_init                  = pi-asin((parms.Yc-parms.O4B*sin
        (phi4_init))/parms.BC);
58  phi2d_init                 = (150*pi)/60;
59  phi4d_init                 = cos(phi4_init)^2*phi2d_init; %
        Not real value but failed to calculate
60  phi5d_init                 = (parms.O4B*cos(phi4_init)*
        phi4d_init)/(-parms.BC*cos(phi5_init));  % Not real value
        but failed to calculate
61  q0                         = [phi2_init phi4_init phi5_init
        phi2d_init phi4d_init phi5d_init];
62
63  %% Derive equation of motion
64  [EOM_qdp,C_handle,Cd_handle,X_handle,Xp_handle] = EOM_calc(
        parms);                              % Calculate symbolic
        equations of motion and put in parms struct
65  parms.C_handle             = C_handle;
66  parms.Cd_handle            = Cd_handle;
67  parms.X_handle             = X_handle;
68  parms.Xp_handle            = Xp_handle;
69
70  %% Calculate movement by mean sof a Runge-Kuta 4th order
        intergration method
71  tic
72  [t_RK4,q_RK4,x_RK4,xdp_RK4]                    =
        RK4_custom(EOM_qdp,q0,parms);
73  toc
74
75  %% Calculate com velocities
76  xp = diff(x_RK4)/parms.h;
77
78  %% Create plots
79
80  %% Plot Angular speed crank as a function of time
81  figure;
```

```matlab
82  plot(t_RK4,q_RK4(:,4:6),'linewidth',1.5);
83  set(gca,'fontsize',18);
84  title('Angular speed of the crank 2, rocker 4 and connecting
        bar 5');
85  xlabel('Time [s]');
86  ylabel('Angular speed [rad/s]');
87  legend('Crank 2 (\phi_2)','Rocker 4 (\phi_4)','Connecting bar
        5 (\phi_5)','Location', 'Best');
88
89  %% Plot the sliding speedof slider 3 with respect to rocker 4
90  v_slider_rel = xp(2:end,4).*cos(q_RK4(3:end,3)) + xp(2:end,5)
        .*sin(q_RK4(3:end,3));
91
92  figure;
93  plot(t_RK4,xdp_RK4(:,end),'linewidth',1.5);
94  set(gca,'fontsize',18);
95  xlabel('Time [s]');
96  ylabel('Acceleration slider 6 [m/s^2]');
97  title('Acceleration slider 6');
98  legend('acceleration slider 6','Location', 'Best');
99
100 figure;
101 plot(t_RK4,x_RK4(:,end),'linewidth',1.5);
102 set(gca,'fontsize',18);
103 xlabel('Time [s]');
104 ylabel('position slider 6 [m]');
105 title('position slider 6');
106 legend('position slider 6','Location', 'Best');
107
108 figure;
109 plot(t_RK4(2:end),xp(:,end),'linewidth',1.5);
110 set(gca,'fontsize',18);
111 xlabel('Time [s]');
112 ylabel('velocity slider 6 [m]');
113 title('velocity slider 6');
114 legend('velocity slider 6','Location', 'Best');
115
116 plot(t_RK4(4:end),v_slider_rel(2:end),'linewidth',1.5);
117 set(gca,'fontsize',18);
118 xlabel('Time [s]');
119 ylabel('Velocity [m/s]');
120 title('Speed of slider 3 with respect to rocker 4');
121 legend('relative velocity','Location', 'Best');
```

```matlab
122
123 %% Normal forces
124 figure;
125 plot(t_RK4,q_RK4(:,end-1:end),'linewidth',1.5);
126 set(gca,'fontsize',18);
127 xlabel('Time [s]');
128 ylabel('Force [N]');
129 title('Reaction Forces [N]');
130 legend('slider 6 on ground','slider 3 on rocker 4','Location'
        , 'Best')
131
132 %% FUNCTIONS
133
134 %% Runge-Kuta numerical intergration function
135 % This function calculates the motion of the system by means
        of a
136 % Runge-Kuta numerical intergration. This function takes as
        inputs the
137 % parameters of the system (parms), the EOM of the system (
        parms.EOM)
138 % and the initial state.
139 function [t,q,x,xdp] = RK4_custom(EOM,q0,parms)
140
141 % Calculate x0
142 q_new_tmp       = num2cell(q0,1);
143 x0   = feval(parms.X_handle,q_new_tmp{1:3}).';
144 xdp0 = feval(parms.Xp_handle,q_new_tmp{:}).';
145
146 % Initialise variables
147 t                      = 0;
                                                     % Initiate
        time
148 q                      = [q0 0 0 0 0 0];
                                           % Put initial state in
        array
149 x                      = x0;
150 xdp                    = xdp0;
151 % Caculate the motion for the full simulation time by means
        of a
152 % Runge-Kutta4 method
153
154 % Perform intergration till two full rotations of the crank
```

```matlab
155  ii = 1;

         % Create counter
156  while abs(q(ii,1)) < (4*pi)
157
158      % Calculate the next state by means of a RK4 method
159      q_now_tmp       = num2cell(q(ii,1:end-5),1);
                                                     % Create
             cell for feval function
160      K1              = [cell2mat(q_now_tmp(1,end-2:end)),
             feval(EOM,q_now_tmp{:}).'];            % Calculate the
             second derivative at the start of the step
161      q1_tmp          = num2cell(cell2mat(q_now_tmp) + (parms
             .h*0.5)*K1(1,1:end-2));                % Create cell for
             feval function
162      K2              = [cell2mat(q1_tmp(1,end-2:end)),feval(
             EOM,q1_tmp{:}).'];                     % Calculate the
             second derivative halfway the step
163      q2_tmp          = num2cell(cell2mat(q_now_tmp) + (parms
             .h*0.5)*K2(1:end-2));                  % Refine value
             calculation with new found derivative
164      K3              = [cell2mat(q2_tmp(1,end-2:end)),feval(
             EOM,q2_tmp{:}).'];                     % Calculate new
             derivative at the new refined location
165      q3_tmp          = num2cell(cell2mat(q_now_tmp) + (parms
             .h)*K3(1:end-2));                      % Calculate state
             at end step with refined derivative
166      K4              = [cell2mat(q3_tmp(1,end-2:end)),feval(
             EOM,q3_tmp{:}).'];                     % Calculate last
             second derivative                      % Take
             weighted sum of K1, K2, K3
167      q_now_p         = (1/6)*(K1(end-4:end)+2*K2(end-4:end)
             +2*K3(end-4:end)+K4(end-4:end));       % Estimated
             current derivative
168      q_next          = cell2mat(q_now_tmp) + (parms.h/6)*(K1
             (1:6)+2*K2(1:6)+2*K3(1:6)+K4(1:6));  % Perform euler
             intergration step
169
170      % Save reaction forces and current derivative in state
171      q(ii,end-4:end)   = q_now_p;
172
173      % Save full state back in q array
174      q              = [q;[q_next 0 0 0 0 0]];
```

```matlab
175
176      % Correct for intergration drift
177      q_now_tmp = q(ii+1,:);
178      [q_new,error] = gauss_newton(q_now_tmp,parms);
179
180      % Update the second derivative and the constraint forces
181      q_new_tmp        = num2cell(q(ii,1:end-5),1);
182      q_update         = feval(EOM,q_new_tmp{:}).';
183
184      % Overwrite position coordinates
185      q(ii+1,:)        = [q_new(1:6) q_update];
186
187      % Create time array
188 t                    = [t;t(ii)+parms.h];          % Perform
    Gauss-Newton drift correction
189 ii                   = ii + 1;
                                                      % Append
    counter
190 t(ii)
191 q(ii,1)
192
193 % Calculate COM coordinates
194 % Calculate COM coordinates
195 x_tmp   = feval(parms.X_handle,q_new_tmp{1:3}).';
196 xdp_tmp = feval(parms.Xp_handle,q_new_tmp{:}).';
197
198 % Save x in state
199 x       = [x;x_tmp];
200 xdp     = [xdp;xdp_tmp];
201
202 end
203 end
204
205 %% Constraint calculation function
206 function [C,Cd] = constraint_calc(q,parms)
207
208 % Get needed angles out
209 q_now_tmp        = num2cell(q,1);
210
211 % Calculate the two needed constraints
212 C                = [parms.O4A*cos(q(2))+parms.O2A*cos(q(1))
                 ...
```

```matlab
213                             parms.O4B*sin(q(2))+parms.BC*sin(q(3))-
                                    parms.Yc];

214
215  C_test              = feval(parms.C_handle,q_now_tmp{1:3}).';
216
217  % Calculate constraint derivative
218  Cd                  = feval(parms.Cd_handle,q_now_tmp{1:3}).';
219
220  end
221
222  %% Speed correct function
223  function [q,error] = gauss_newton(q,parms)
224
225  % Get rid of the drift by solving a non-linear least square
         problem by
226  % means of the Gaus-Newton method
227  % Calculate the two needed constraints
228  [C,Cd] = constraint_calc(q,parms);
229
230  %% Guass-Newton position correction
231  n_iter          = 0;

         % Set iteration counter

         % Get position data out
232
233  % Solve non-linear constraint least-square problem
234  while (max(abs(C)) > parms.tol)&& (n_iter < parms.nmax)
235      q_tmp           = q(1:3);
236      n_iter = n_iter + 1;
237      q_del   = Cd*inv(Cd.'*Cd)*-C.';
238      q(1:3) = q_tmp+ q_del.';
239
240      % Recalculate constraint
241      [C,Cd]       = constraint_calc(q,parms);
242  end
243
244  % Calculate the corresponding speeds
245  q_tmp_vel           = q(4:6);
246  Dqd_n1              = -Cd*inv(Cd.'*Cd)*Cd.'*q_tmp_vel.';
247  q(4:6)              = q_tmp_vel + Dqd_n1.';
248
249  error = C;
```

```matlab
250  end
251
252  %% Calculate (symbolic) Equations of Motion four our setup
253  function [qdp_handle,C_handle,Cd_handle,X_handle,Xd_handle] =
         EOM_calc(parms)
254
255  % Unpack symbolic variables from varargin
256  phi2             = parms.syms.phi2;
257  phi4             = parms.syms.phi4;
258  phi5             = parms.syms.phi5;
259  phi2d            = parms.syms.phi2d;
260  phi4d            = parms.syms.phi4d;
261  phi5d            = parms.syms.phi5d;
262
263  % Create generalized coordinate vectors
264  q                = [phi2;phi4;phi5];
265  qp               = [phi2d;phi4d;phi5d];
266
267  % COM of the bodies expressed in generalised coordinates
268  % x2             = 0;
269  % y2             = parms.O4O2;
270  x3               = parms.O2A*cos(phi2);
271  y3               = parms.O4O2+parms.O2A*sin(phi2);
272  x4               = parms.O4G4*cos(phi4);
273  y4               = parms.O4G4*sin(phi4);
274  x5               = parms.O4B*cos(phi4)+parms.BG5*cos(phi5);
275  y5               = parms.O4B*sin(phi4)+parms.BG5*sin(phi5);
276  x6               = parms.O4B*cos(phi4)+parms.BC*cos(phi5);
277  % y6              = parms.O4B*sin(phi4)+parms.BC*sin(phi5);
278
279  % Create mass matrix
280  % x2 = 0, y2 = 0 and y5 =0 also no moments around slider 3
         and 6
281  parms.M          = diag([parms.J2,parms.m3,parms.m3,parms.J3,
         parms.m4,parms.m4,parms.J4,parms.m5,parms.m5,parms.J5,parms
         .m6]);
282
283  % Put in one state vector
284  x                = [phi2;x3;y3;phi4;x4;y4;phi4;x5;y5;phi5;x6];
285
286  % Calculate the two needed constraints
287  C                = [parms.O4A*cos(phi4)+parms.O2A*cos(phi2)
                     ...
```

```matlab
288                          parms.O4B*sin(phi4)+parms.BC*sin(phi5)-
                                parms.Yc];
289
290   % Compute the jacobian of state and constraints
291   Jx_q            = simplify(jacobian(x,q.'));
292   JC_q            = simplify(jacobian(C,q.'));
293
294   %% Calculate convective component
295   Jx_dq           = jacobian(Jx_q*qp,q);
296   JC_dq           = jacobian(JC_q*qp,q);
297
298   % Solve with virtual power
299   M_bar           = Jx_q.'*parms.M*Jx_q;
300
301   % Add forces F=[M2,F3_x,F3_y,M3,F4_x,F4_y,M4,F5_x,F5_y,M5,
         F6_x];
302   F               = [parms.T2, 0, -parms.m3*parms.g, 0, 0, -
         parms.m4*parms.g, 0, 0, -parms.m5*parms.g, 0, parms.F6_x];
303
304   % Create system of DAE
305   A = [M_bar JC_q.'; JC_q zeros(size(JC_q,1))];
306   B = [Jx_q.'*(F.'-parms.M*Jx_dq*qp); ...
307       -JC_dq*qp];
308
309   % Calculate result expressed in generalized coordinates
310   qdp             = A\B;
311
312   % % Get result back in COM coordinates
313   % xdp     = simplify(jacobian(xp,qp.'))*qdp + simplify(
         jacobian(xp,q.'))*qp;
314
315   %% Convert to function handles
316   % xdp_handle       = matlabFunction(xdp);
                                          % Create function handle of
            EOM in terms of COM positions
317   qdp_handle         = matlabFunction(simplify(qdp));
                                        % Create function handle of EOM in
            terms of generalised coordinates
318   % matlabFunction(qdp,'file',qdp_cal')
319
320   % Constraint function handle
321   C_handle          = matlabFunction(simplify(C));
322
```

```matlab
% Constraint derivative function handle
Cd               = JC_q;
Cd_handle        = matlabFunction(simplify(Cd));

% Get back to COM coordinates
X_handle         = matlabFunction(simplify(x));
xp               = Jx_q*qp;
xdp              = simplify(jacobian(xp,qp))*qdp(1:3)+simplify
    (jacobian(xp,q))*qp(1:3);
Xd_handle        = matlabFunction(simplify(xdp));

end
```

References

[1] Arend L. Schwab. Reader: MultiBody Dynamics B. In *Multibody Dynamics*, chapter 3. TU Delft, Delft, The Netherlands, 2018.