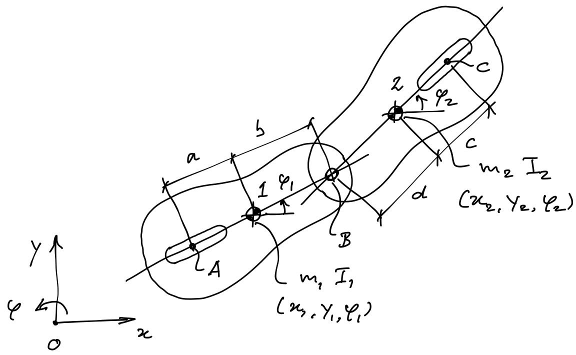
ME41055
Multibody Dynamics B

Spring Term 2018, Tue 13:45-15:30, room 3mE-CZ C, 4 ECTS credits.

Homework Assignment 8 (HW8)



The EzyRoller, shown in the top figure, is the ultimate riding machine for kids. The machine is propelled by an oscillating motion of the steering assembly, which is operated by the feet. For the proper operation watch the video[1]. A mechanical model for this machine is shown in the bottom figure. The model consists of two rigid bodies connected by a hinge in B. Each body has a rolling contact, for body 1 the wheel in A and for body 2 the wheel in C. The centre of mass location is denoted by the numbers 1 and 2. With the help of this model we like to demonstrate the operation of this machine in a number of steps.

The system consists of two rigid bodies and therefore has six coordinates. The system has 2 holonomic constraints, the hinge in B, and two nonholonmic constraints, the rolling contact condition in A and C. Therefore you have two constraints on six coordinates and four constraints on the six velocities. So the number of degrees of freedom in the coordinates space is four, whereas you only have two degrees of freedom in the velocity space. We will solve this problem by setting up the constraint equations of motion in DAE form and stabilise the constraints by means of the Coordinate Projection Method. Note that there is now a difference between the constraints on the coordinates and the constraints on the velocities.

Please address the following questions:

a. Derive the equations of motion for this system in DAE form.

b. Formulate the Coordinate Projection Method for the constraints on the coordinates and for the constraints on the velocities.

c. Implement the DAE together with the constraint stabilisation in a Matlab program. Try not to form the equations of motion in an explicit form but evaluate your equations in a step-by-step manner.

d. Determine the motion of the unpowered system by numerical integration of the equations of motion (use a fixed step RK4 method). For dimensions of the EzyRoller we take: $a = 0.5, b = 0.5, c = 0.125, d = 0.125, m_1 = 1, I_1 = 0.1, m_2 = 0, I_2 = 0$ (SI units). For initial conditions take some non-zero values for the coordinates and the speeds and show that the system is moving properly.

e. Add and action-reaction torque in the hinge such that it resembles an oscillating torque as applied by the rider on the steering assembly. Take for the torque the following function $M = M_0 \cos(\omega t)$, with $M_0 = 0.1$ and $\omega = \pi$ [rad/s]. Start from rest with body 1 and body 2 aligned along the x-axis, $\phi_1 = 0, \phi_2 = \pi$ (point C between A and B). Determine the motion of the system by numerical integration of the constraint equations of motion for 100 seconds.

f. Plot the path of point A and C.

g. Plot the linear and angular velocities of the CM's of body 1 and body 2.

h. Determine the Kinetic energy of the system and plot it as a function of time.

i. Determine the work done by the torque as a function of time and plot this as a function of time in the same figure as the Kinetic Energy of the system. What do you notice? Can you explain this?

[1] https://youtu.be/GMnJ9q1D4hU