Stability guarantees in variable impedance control for rigid robotics manipulators in contact with (semi)-rigid environments

HOW TO GUARANTEE STABLE VARIABLE IMPEDANCE CONTROL AT ALL TIMES

ME51010: ME-BMD Literature Report

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Project Duration: March, 2022 - april, 2023
Faculty: Cognitive Robotics, Delft

Cover: Picture of Franka Research 3 robot from https://www.franka.de/
Style: TU Delft Report Style, with modifications by Daan Zwaneveld

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Abstract

This literature provides an extensive overview of the stability/passivity considerations and methods used in the current variable impedance literature for stable control of rigid robotics manipulators in contact with (semi)-rigid environments. Stability is essential for the safe operation of robotic systems in the real world, as unstable systems can exhibit unpredictable and dangerous behaviour. Impedance control, which models robot-environment interaction as a spring-damper system, is commonly used for safe interactions, but variable impedance parameters can cause instability. To solve this, researchers have used Lyapunov's theory and its extension passivity to ensure variable impedance control stays stable in free motion and is bounded during an interaction. Two passivity-based approaches are found within the literature for ensuring the stability of interaction tasks: pacifying control algorithms and stability constraints. Pacifying control algorithms, such as energy tanks and potential fields, rely on an energy-storing element to filter non-passive actions and protect the total system stability. They are easy to implement, offer clear physical insights and can be used with any controller. However, they require task-specific tuning and can only be used online, making them unsuitable for ensuring predetermined trajectories and impedance profiles. Stability constraints, conversely, can be used to ensure the passivity of a given controller or impedance profile offline, but deriving them is challenging. Additionally, their effectiveness depends on the choice of Lyapunov candidates or contractive metrics used during their derivation, giving rise to an accuracy-stability trade-off. Nonetheless, state-independent stability constraints have been derived for the general-impedance controller and have been utilized to ensure the stability of adaptive or optimal control architectures. Similar stability constraints have also been derived and applied to imitation-based impedance controllers. Reinforcement learning-based variable impedance control has limited research on passivity and stability. However, recent papers have used constraint-based methods to impose stability constraints on either the sampling policy or NN-based policies to ensure passivity and stability. As these approaches rely on deterministic policies, ensuring passivity and stability in stochastic policy-based VIC remains an open question. Although the stability methods discussed in this literature review are adequate for ensuring stability in interaction tasks, this review has several limitations. It does not address interactions with active or compliant environments, and it primarily focuses on passivity-based stability methods while other methods are available. Furthermore, safety constraints such as position, velocity, and acceleration limits must be considered to control robotic manipulators alongside humans or other robots safely. Future research can investigate how these safety constraints can be integrated into learned impedance control policies while maintaining passivity and stability.

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Nomenclature

This chapter contains abbreviations and symbols used throughout this report. It only includes abbreviations and symbols specific to this report or ones with ambiguous definitions. Please check out [122] for a list of standardized math symbols.

Abbreviations

Abbreviation	Definition
AS	Asymptotically stable
CBF	Control barrier function
CLF	Control Lyapunov function
DMP	Dynamic movement primitive
DOF	Degree of freedom
DS	Dynamic system
ES	Exponential stable
GAS	Globally asymptotically stable
GES	Globally exponentially stable
IL	Imitation learning
IOS	Input-Output stability
ISS	Input-to-State stability
LfD	Learning from demonstration
RL	reinforcement learning
SISL	Stable in the sense of Lyapunov
UB	Uniformly bounded
UUB	Ultimately uniformly bounded
VIC	Variable impedance control
ZSD	Zero-state detectable
ZSO	Zero-state observable

Symbols

Symbol	Definition	Unit
V	Lyapunov function	
S	Storage function	
$\mathcal{C}$	Task space	
$\mathcal{J}$	Joint space	
x	Position	$[m]$
$\bar{x}$	Equilibrium point position	$[m]$
x_c	Impedance model reference position	$[m]$
x_d	Desired position	$[m]$
x_e	Cartesian end effector position	$[m]$
q	Generalized coordinates	$[rad]$
$M(.)$	Manipulator inertia matrix	$[kg/m^2]$
$C(.,.)$	Manipulator Coriolis/centrifugal matrix	$[N]$
$f(.)$	Manipulator friction force vector	$[N]$
$g(.)$	Manipulator gravity force vector	$[N]$
$k(.)$	Manipulator forward kinematics	

Symbol	Definition	Unit
M_d	Desired inertia matrix	$[kg/m^2]$
B_d	Desired damping matrix	$[Ns/m]$
K_d	Desired stiffness matrix	$[N \cdot m]$
J	Geometric jacobian	
J_A	Analytic jacobian	

1 Introduction

As more and more robots move out of the factory into the real world, there is a growing need for robots that can safely work alongside humans or other robots [178, 231, 189]. In order to do so, these robots must not only detect and avoid collisions [61, 231] but also safely interact with humans and the environment when a collision or a desired interaction occurs. Several researchers have proposed compliant or soft robots to prevent hard collisions and thus ensure safe interactions [75, 35]. Even though these robots are, without doubt, safer than their rigid counterparts, they are more mechanically complex and unsuitable for tasks requiring high position accuracy, repeatability and effort [35]. As a result, researchers mainly resort to using rigid manipulators (sometimes with soft grippers) for robot manipulation tasks.

Although traditional position, velocity, and acceleration control algorithms, often used with rigid manipulators, achieve high precision and repeatability, they are stiff and not sensitive to force interaction. As a result, they are not suited for interacting with the environment because they may lead to large contact forces, resulting in unstable behaviour, damage or even accidents. Effort control algorithms, on the other hand, can precisely regulate contact force but cannot accurately track a desired path without contact. To perform interaction tasks like manipulation, which contain both a free and contact phase, several researchers have used hybrid force/motion controllers [147, 138]. These controllers divide the task space into a position-controlled (i.e., free) and force-controlled (constraint) subspace and switch between these spaces during the task. However, designing the switching behaviour of these hybrid controllers requires prior knowledge of the structure and geometry of the environment, which might not be available and is task-dependent also. Because of this, hybrid control algorithms cannot perform well in unstructured or dynamically changing environments.

Impedance control, in which the robot-environment interaction is modelled as a spring-damper system, is a better candidate. It can track a free-space trajectory while limiting the force applied to the environment when in contact. Traditionally, impedance controllers with constant parameters have been used [70, 69, 17, 185, 21, 113, 68, 118, 79, 84, 140]. These controllers are passive and guarantee that the control never becomes unstable when controlling a passive robot in a passive environment. However, these controllers require accurate knowledge about the stiffness and location of the environment and can only attain a constant desired force. They can, therefore, not be used for dynamic force tracking or in uncertain or changing environments.

Variable impedance control (VIC) with time-varying impedance parameters provides a solution to this problem. By allowing the parameters to change over time, more complex tasks can be executed while improving robustness against uncertainties and changing environments. These variable impedance profiles can be implemented using adaptive or optimal controllers [185, 43, 42, 108, 78, 129, 229] or learned from human demonstrations or through reinforcement learning (RL) [16, 157, 132, 23, 216, 71, 1, 19, 100]. Although promising results have been obtained using these controllers in surgery [47, 46], human-robot interaction [221, 162, 176] and welding and grinding tasks [43, 42, 233, 213], it introduced one big problem. By varying the impedance parameters, the passivity property of the system no longer holds as energy can now be injected into the system, possibly making it unstable [46]. Because unstable systems exhibit unpredictable and uncontrollable behaviour, they can not be deployed safely alongside humans or other robots.

Several solutions have been proposed in the relevant literature for keeping the control passive with varying impedance parameters. Although in recent years, multiple reviews have been conducted on the current state of the literature regarding VIC algorithms that robotic manipulators use for interaction tasks, they only briefly touch on this passivity issue [189, 185, 1, 180]. Therefore, this literature review aims to fill this gap by providing an extensive overview of the stability/passivity considerations and methods used in the current variable impedance literature for stable control of rigid robotics manipulators in contact with **(semi)-rigid environments**. Even though most of the techniques discussed in this literature review can also be applied to VIC in human-robot interaction (HRI) tasks, it is not the primary focus of this review. Readers interested in that subject can read the review done by [177]. This review also does not consider stability problems caused by time delays in the control loop; such is often encountered during teleoperation. An extensive review on this subject can be found in [45].

This literature review is structured as follows: chapter 2 depicts the methodology used for finding the relevant literature used in this review. After that, in chapter 3, essential concepts like VIC, Lyapunov stability and passivity are reviewed. The currently used techniques for ensuring closed-loop stability of classical variable impedance controllers are presented in chapter 4, while chapter 5 describes the stability techniques used in learning-based controllers. Finally, chapter 6 discusses the current research trends and future challenges.

2 Method

An extensive search of Web Of knowledge, Scopus, Semantic Scholar and Google Scholar was performed. During this search, keywords related to ”variable impedance control”, ”stability”, ”passivity”, ”safety”, and ”robot manipulators” were used. In this search, articles before 2010 or related to ”Human-Robot interaction”, ”teleoperation”, ”soft-robotics”, ”hydraulic joints”, and ”cable joints” were filtered out. A complete list of the keywords used can be found in appendix A. In addition, the citation graphs of influential articles found in the first search [46, 6, 102, 12, 77, 94, 133, 136, 153, 15, 101, 96, 88] were inspected to check for missing articles. After these searches, the papers were manually inspected to check for relevance. The most important selection criteria were the significance of the contribution to stable variable impedance control and the technical quality of the work. If multiple studies presented a similar idea, the one with the highest technical quality was selected. If the quality was equal, the one published in a more prominent journal or with a higher citation count was selected.

3 Background

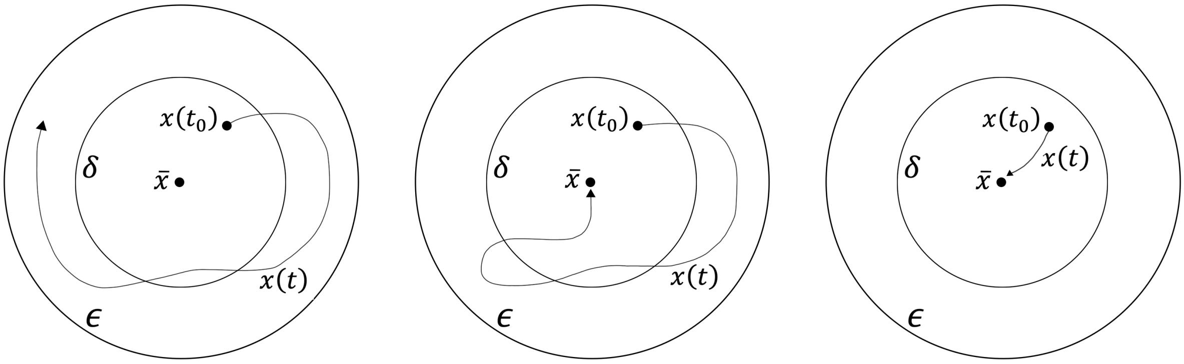
This chapter reviews some general concepts needed to understand the rest of the paper. First, the concept of stability is explained, and several methods used to prove stability are presented. This section is followed by a brief description of the dynamical model of the manipulator. Lastly, several indirect force control methods are introduced, such as impedance and admittance control.

3.1. Stability

The concept of stability is often explained using the example of a ball in a hilly landscape. A stable system can be represented by a ball placed in a valley. When this ball is disturbed (i.e., carried up the hill), it will always return to the lowest point, also called the equilibrium point, when released. When we now put this ball on top of a hill and disturb it (i.e., give it a push), we will see an unstable system instead. The ball will roll off the hill and never return without us carrying it up again. This stability can be local, only valid in a region around the equilibrium point or global and valid for all system states. In control theory, stability means that the control is guaranteed to always converges to a given setpoint while staying bounded in state space. This setpoint can be any arbitrary quantity like position, angle, velocity, effort, rotational speed, or voltage. In the case of a regularisation task, this setpoint can be static, but it can also move in time for trajectory tracking or manipulation tasks.

Multiple general methods have been designed to investigate the behaviour and stability of linear systems. Since a general closed-loop solution exists for a linear system, a simple eigenvalue analysis of this solution can be used to determine stability. The stability type and boundaries can be further investigated using frequency domain mathematical techniques like the Laplace transform, Fourier transform, z-transform, Bode plots, root locus, and Nyquist stability criterion [8]. Because a linear system only has one equilibrium point, it is always globally stable or unstable. On the other hand, no general closed-loop solution is available for nonlinear systems and determining stability is not trivial. Furthermore, nonlinear systems can also have multiple equilibrium points, meaning that stability can only be investigated locally around an equilibrium point and only under some conditions is global.

Lyapunov's stability theory, first introduced in 1892, provides a way to reason about the stability of equilibrium points of linear and nonlinear systems without solving the complete system behaviour. It consists of several notions of stability, fundamental theorems and methods that can be used to investigate the system's stability. Lyapunov's stability theory can be applied to both autonomous and non-autonomous systems. Below, several notions of stability for autonomous systems will be discussed, followed by methods that can be used to prove the stability of equilibrium points. After that, these notions and methods will be extended to unforced non-autonomous time-dependent systems. In doing so, the concept of boundedness, which is used for systems with no clear equilibrium point, and methods for dealing with perturbed systems are presented. Lastly, input-state stability and passivity will be introduced to investigate the stability of forced non-autonomous systems.



(a) Stability in the sense of Lyapunov **(b)** Asymptotic stability **(c)** Exponential stability

Figure 3.1: Several notions of stability. In these figures, δ, ϵ represent the start and end stability bounds, $\bar{x}$ the equilibrium point, and $x\left(t_0\right)$ and $x\left(t\right)$ the initial point and trajectory, respectively.

3.1.1. Stability of autonomous systems

3.1.1.1. Stability notions

The introduction of Lyapunov's stability theory resulted in several new stability notions. The most used in the impedance literature are stability (and its absence, instability), asymptotic stability, and exponential stability.

3.1.1.1.1 Stability in the sense of Lyapunov

In Lyapunov's stability theory, stability is concerned with the trajectories' behaviour rather than the equilibrium point's stability. This notion of stability is handy since a noise or disturbance can always perturb a physical system from its equilibrium. Although, as shown in the following chapters, Lyapunov's theory can also be extended to work with specific non-autonomous systems, let us, for simplicity, consider the following autonomous nonlinear system:

$$\dot{x} = f\left(x\right), \tag{3.1}$$

where $f : D \to \mathbb{R}^n$ is a locally Lipschitz map from a domain $D \subset \mathbb{R}^n$ into $\mathbb{R}^n$. An equilibrium point of this system $\bar{x}$ is **stable in the sense of Lyapunov (SISL)** if, for each $\epsilon > 0$, there exists some $\delta > 0$ such that:

$$\left\|x\left(t\right) - \bar{x}\right\| < \epsilon, \quad \forall\, t \geq t_0, \tag{3.2}$$

whenever $\left\|x\left(t_0\right) - \bar{x}\right\| < \delta$. Intuitively this means that any trajectory starting inside circle δ will never leave circle ϵ (see figure 3.1a). Since SISL is not a global condition, δ and ϵ should be chosen as small as possible to provide the most robust bounds. Logically, every system that does not adhere to these conditions is **unstable**.

3.1.1.1.2 Asymptotic stability

A stricter notion of stability called asymptotic stability can be created by imposing some additional conditions. An equilibrium point $\bar{x}$ is **asymptotically stable (AS)** if:

- It is SISL.
- Some $\delta > 0$ exists such that when $\left\|x\left(t_0\right) - \bar{x}\right\| < \delta, \quad \lim_{t\to\infty} \left\|x\left(t\right) - \bar{x}\right\| = 0$.

Intuitively this means that the trajectory will always converge to $\bar{x}$ if started inside circle δ (see figure 3.1b). If $\delta = \infty$, the system is called **globally asymptotically stable (GAS)**.

3.1.1.1.3 Exponential stability

Lastly, an equilibrium point $\bar{x}$ is **exponentially stable (ES)** with rate of convergence α if:

- It is SISL.
- There exists a $M, \lambda > 0$ such that $\left\|x\left(t\right)\right\| \leq M e^{-\lambda\left(t-t_0\right)} \cdot \left\|x\left(t_0\right)\right\|$.

The intuition for this type of stability is like asymptotic stability, but now the trajectory exponentially converges to the equilibrium point (see figure 3.1c). Since this results in a stricter condition, exponential stability implies asymptotic stability, but the converse does not always hold. If $\delta = \infty$, the system is called **globally exponentially stable (GES)**.

3.1.1.2. Stability methods

Based on these new stability notions, two general stability methods were designed to investigate the stability of (non-)linear systems: **Lyapunov's indirect** and **Lyapunov's direct method**.

3.1.1.2.1 Lyapunov's indirect method

Lyapunov's indirect method linearises the system using a first-order Taylor series around an equilibrium point. After this linearisation, the techniques developed for linear systems can be used to conclude the stability of the nonlinear system at the equilibrium point [210]. Although this method provides an easy way to investigate the local stability of an equilibrium point of both autonomous and non-autonomous time-dependent systems, it is only valid in a small region around this point. Therefore, it cannot give any conclusions about the global stability of the nonlinear system or the region of attraction of an equilibrium point (i.e., the set of states for which the system converges to the equilibrium point as $t \to \infty$).

3.1.1.2.2 Lyapnov's direct method

Lyapunov's direct method takes a different approach by investigating the stability of a system using an energy-based method. Doing this provides sufficient conditions for the local stability of an equilibrium point without investigating the full system dynamics. In some cases, it also allows for conclusions about the global system stability and can estimate the region of attraction [93]. Let us first introduce the intuition behind this method using the example of a simple damped pendulum (see example 3.1).

Example 3.1: Simple damped pendulum energy investigation.

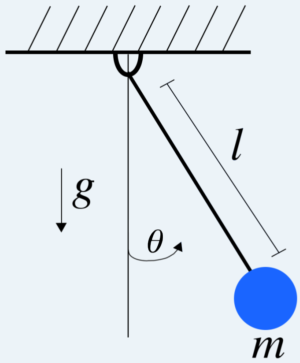


Figure 3.2: Simple damped pendulum. In this figure, m, l, and θ are the pendulum's mass, length, and angle, and g and b are the gravity constant and damping coefficient, respectively.

Let us investigate the stability of the pendulum found in figure 3.2 above. The equations of motion of this simple damped pendulum are given by

$$ml^2\ddot{\theta} + mgl\sin\theta = -b\dot{\theta}, \tag{3.3}$$

where m, l, and θ are the mass, length, and angle of the pendulum and g and b[a] are the gravity constant and damping coefficient, respectively. Although a closed-form solution exists for this relatively simple example, it consists of integrating several elliptical integrals and provides us with relatively little intuition. More importantly, a closed-form solution is not guaranteed to exist for a more complicated system. However, from experience, we know that this damped pendulum (with $b > 0$) when released at an angle θ_{t_0} will lose energy due to air friction and, after some oscillations, will eventually rest at the lowest point, with minimal energy (i.e., $\theta = 2\pi k$). We can use this intuition to prove the stability of the system by looking at its total energy (kinetic + potential), which can be written down as[b]

$$E\left(\theta, \dot{\theta}\right) = \frac{1}{2}ml^2\dot{\theta}^2 + mgl(1 - \cos\theta). \tag{3.4}$$

Evaluating the time derivative of this energy function gives us

$$\frac{d}{dt}E = ml^2\dot{\theta}\ddot{\theta} + \dot{\theta}mgl\sin\theta, \tag{3.5}$$

and substituting in the system dynamics from equation (3.3) reveals

$$\frac{d}{dt}E = -b\dot{\theta} \leq 0. \tag{3.6}$$

Since this condition ensures that the system's energy will never increase, it is sufficient for proving stability. Unfortunately, since $\theta = 2\pi k$ is not the only minimum for which $\dot{E} = 0$, it does not prove that the system will converge to this minimum. It is also zero when the pendulum changes direction during the oscillating behaviour. However, as shown later, we can use La Salle's Invariance principle to prove convergence.

[a] Some texts like [92] use kl^2 instead of b, where k is the friction coefficient.
[b] The reference position is chosen so that the potential energy is zero at the lowest point.

The example above showed that a relatively simple energy function (i.e., mechanical energy) could be used to say something about the system stability without computing the complex analytical closed-form solution. Lyapunov's direct method generalizes this idea to systems that might not be stable in mechanical energy. It states that a general (non-)linear system is stable if a **positive definite** (energy) function of the state variables $V(x)$[1], called a **Lyapunov function**, exists that decreases with time, meaning $\dot{V}(x)$ is **negative (semi-)definite**. These Lyapunov functions, when found, can be used to demonstrate several notions of stability when certain conditions are met. The Lyapunov conditions for the earlier defined stability notions (i.e., SISL, AS, EX) are shown without proof[2] in theorem 3.1. In this theorem, the origin (i.e., $x = 0$) is taken as the equilibrium point. However, since any point can be shifted to the origin using a change of variables, it does not lead to a loss of generality.

Theorem 3.1: Sufficient conditions for stability of autonomous systems [93][3]

Let $x = 0$ be an equilibrium point of (3.1) and $D \subset \mathbb{R}^n$ be a domain containing $x = 0$. Let $V : D \to \mathbb{R}$ be a continuously differentiable function such that

$$V(0) = 0 \quad \text{and} \quad V(x) > 0 \quad \text{in} \quad D - \{0\} \tag{3.7}$$

$$\dot{V}(x) = \frac{\partial V}{\partial x} f(x) \leq 0 \quad \text{in} \quad D \tag{3.8}$$

then, $x = 0$ is (locally) **SISL**. Moreover, if

$$\dot{V}(x) = \frac{\partial V}{\partial x} f(x) < 0 \quad \text{in} \quad D - \{0\} \tag{3.9}$$

then $x = 0$ is (locally) **AS**. Furthermore, if we have

$$\dot{V}(x) = \frac{\partial V}{\partial x} f(x) \leq -\alpha V(x) \quad \text{in} \quad D - \{0\} \tag{3.10}$$

then $x = 0$ is (locally) **ES**. Finally, if $D = R^n$, and (3.7) holds for all $x \neq 0$, and x is

$$\|x\| \to \infty \Rightarrow V(x) \to \infty \tag{3.11}$$

then x is said to be **radially unbounded**, meaning that trajectories cannot diverge to infinity even as V decreases. Consequently, $x = 0$ is **GAS** if (3.9) holds and **GES** if (3.10) holds.

Please be aware that the conditions of theorem 3.1 used in Lyapunov's direct method are **sufficient conditions** for stability. Sufficient in that they only prove stability if a Lyapunov function V is found. If they are violated for a given Lyapunov candidate, this does not prove that the equilibrium is unstable but simply that it is not a proper Lyapunov function. Lyapunov's theory contains several other theorems that can be used to prove the instability of an equilibrium point [93].

[1] A small positive constant can be added to the energy function in 3.4 to make it strictly positive definite.
[2] The proofs can be found in chapter 4 of [93] and section 3.3 of [92].
[3] Theorem 4.1 and 4.10 of [93] and theorem 3.3 [92] were slightly adjusted and combined to improve clarity.

As shown in the pendulum example, for some systems, the derivative of the Lyapunov function $\dot{V}(x)$ ends up being **negative semi-definite**. As a result, for these systems, we can only prove SISL. However, it turns out that we can still show that an equilibrium point or set of equilibrium points is AS in some cases. For the damped pendulum, we, again from experience, know that for all the other points for which $\dot{E} = 0$ the system will not stay there because $\ddot{\theta} \neq 0$. Consequently, the system will eventually converge to $\theta = 2\pi k$, making it a stable equilibrium point. In mathematical terms, it means that $\theta = 2\pi k$ is the largest positively invariant set and that, as $t \to \infty$, the system will eventually come to rest at this set. This relationship, called **LaSalle's invariance principle**, is mathematically expressed in theorem 3.2.

Theorem 3.2: LaSalle's invariance principle [93][4]

Given (3.1) with f being continuous. If a scalar function $V(x)$ exists with a continuous derivative such that

$$V(x) > 0, \quad \dot{V}(x) \leq 0 \tag{3.12}$$

and $V(x) \to \infty$ as $x \to \infty$, then x will converge to the largest invariant set where $\dot{V}(x) = 0$.

As seen above, Lyapunov's direct method allows us to conclude the stability of (non-)linear systems when a valid Lyapunov function is found. From this, we can directly see the main weakness of this method, namely that finding these functions is not trivial. Since no general method exists for finding these Lyapunov functions in nonlinear systems, they must be found by trial and error. In practice, however, the situation is not as bad as it seems due to the tremendous amount of research on this topic. As a result, when trying to find a Lyapunov function for a new system, Lyapunov functions of prior research that consider similar systems can be used as candidates [60]. Moreover, in recent years several optimisation methods have been designed for specific types of systems that can iteratively find Lyapunov functions [93, 56, 149].

3.1.2. Stability of unforced non-autonomous systems

In the previous section, we considered autonomous time-invariant systems. Doing this provided us with a good intuition of the workings of Lyapunov's direct method. In these systems, the solutions and thus the trajectories are dependent only on $(t - t_0)$. Therefore, a specific initial condition $x(t_0)$ will invariably lead to the same trajectory independent of the starting time t_0. Because of this, a system is always stable when a circle δ exists, for which all trajectories starting there will stay inside circle ϵ. However, in most applications, the system solutions may depend on both t and t_0. As a result, the circle δ is now not only dependent on ϵ but also on t_0. Because of this, there is no guarantee that a δ exists, dependent only on ϵ, that is valid for all t_0. Therefore, the stability notions defined for autonomous systems no longer hold and must be extended such that they hold **uniformly** over time (i.e., holds for all t_0). For this purpose, let us consider the following **forced** non-autonomous system:

$$\dot{x} = f(t, x, u). \tag{3.13}$$

In this $f : [0, \infty) \times \mathbb{R}^m \times \mathbb{R}^n \to \mathbb{R}^n$ is piecewise continuous in t and locally Lipschitz in x and u on $[0, \infty) \times D$, and $D \subset \mathbb{R}^n$ is a domain that contains the origin $x = 0$. Further, let us assume for now that the system is **unforced** and thus does not have additional inputs other than time (i.e., $u = 0$). For the origin of this system, $x = 0$ to be **uniformly SISL**, for each $\epsilon > 0$, and any $t_0 \geq 0$, there must be a $\delta = \delta(\epsilon, t_0) > 0$, dependent on initial time t_0, such that

$$\|x(t_0)\| < \delta \Rightarrow \|x(t)\| < \epsilon, \quad \forall\, t \geq t_0. \tag{3.14}$$

Extending this definition to the earlier mentioned stability notions results in definition 3.1.

Definition 3.1: Time-dependent stability notions [92][5]

The equilibrium point $x = 0$ of the **unforced** (i.e., $u = 0$) system given in (3.13) is

1. **Uniformly SISL**, if there exists a class $\mathcal{K}$ function α and a positive constant c, independent

[4] Theorem 4.4 of [93] was reworded to improve readability.

of t_o, such that

$$\|x(t)\| < \alpha(\|x(t_0)\|), \quad \forall\, t \geq t_0 \geq 0, \quad \forall\, \|x(t_0)\| < c. \tag{3.15}$$

2. **Uniformly AS** if there exists a class $\mathcal{KL}$ function β and a positive constant c, independent of t_0, such that

$$\|x(t)\| < \beta(\|x(t_0)\|, t - t_0), \quad \forall\, t \geq t_0 \geq 0, \quad \forall\, \|x(t_0)\| < c. \tag{3.16}$$

3. **Globally uniformly AS** if the previous inequality holds $\forall\, x(t_0)$.
4. **Uniformly ES** if there exist positive constants c, M, and λ such that

$$\|x(t)\| \leq Me^{-\lambda(t-t_0)} \cdot \|x(t_0)\|, \quad \forall\, \|x(t_0)\| < c. \tag{3.17}$$

5. **Globally uniformly ES** if the previous inequality holds $\forall\, x(t_0)$.

In these notions, so-called class $\mathcal{K}$ and class $\mathcal{KL}$ functions (see definition 3.2) were used for transparency [93]. These functions are used to generalise ϵ of the previous stability notions, while c can be seen as a generalisation of δ.

Definition 3.2: Class $\mathcal{K}$ and class $\mathcal{KL}$ functions [92]

1. A scalar continuous function $\alpha(r)$, defined for $r \in [0, a)$, belongs to class $\mathcal{K}$ if it is strictly increasing and function $\alpha(0) \equiv 0$. It belongs to class $\mathcal{K}_\infty$ if it is defined for all $r \geq 0$ and $\alpha(r) \to \infty$ as $r \to \infty$.
2. A scalar continuous function $\beta(r, s)$, defined for $r \in [0, a)$, and $s \in [0, \infty)$, belongs to class $\mathcal{KL}$ if, for each fixed s, the mapping $\beta(r, s)$ belongs to class $\mathcal{K}$ with respect to r and, for each fixed r, the mapping $\beta(r, s)$ is decreasing with respect to s and $\beta(r, s) \to 0$ as $s \to \infty$.

Now that we have defined the stability notions for unforced non-autonomous systems, we can determine the accompanying Lyapunov stability conditions. These Lyapunov stability conditions are presented without proof[6] in theorem 3.3 below. The big difference with theorem 3.1 is that now f and V are dependent on time. Consequently, these conditions now also contain time derivates and several time-independent bounds (i.e., W_1, W_2, W_3, $k_{\{n\,\in\,\mathbb{R}:\,1-3\}}\,\|x^a\|$).

Theorem 3.3: Sufficient conditions for stability of unforced non-autonomous systems [92][7]

Uniformly SISL
Let the origin $x = 0$ be an equilibrium point of the **unforced** (i.e., $u = 0$) system (3.13) and $D \subset \mathbb{R}^n$ be a domain containing $x = 0$. Suppose (3.13) is piecewise continuous in t and locally Lipschitz in x for all $t \geq 0$ and $x \in D$. Let $V(t, x)$ be a continuously differentiable function such that

$$W_1(x) \leq V(t, x) \leq W_2(x) \tag{3.18}$$

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(t, x) \leq 0 \tag{3.19}$$

for all $t \geq 0$ and $x \in D$, where $W_1(x)$ and $W_2(x)$ are continuous positive definite class $\mathcal{K}$ functions on D. Then the origin is **uniformly SISL**.

(Globally) Uniformly AS
Suppose the assumptions above for uniformly SISL are satisfied with inequality (3.19) strength-

[5] Definition 4.2 of [92] was slightly changed for consistency with earlier stability notions.
[6] The proofs can be found in section 4.5 of [93].

ened to

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(t, x) \leq W_3(x) \tag{3.20}$$

for all $t \geq 0$ and $x \in D$, where $W_3(x)$ is a continuous positive definite function on D. Then, the origin is **uniformly AS**. Moreover, if some constants r and c are chosen such that $B_r = \{\|x\| \leq r\} \subset D$ and $c < \min_{\|x\|=r} W_1(x)$, then every trajectory starting in $\{x \in B_r \mid W_2(x) \leq c\}$ satisfies

$$\|x(t)\| \leq \beta(\|x(t_0)\|, t - t_0), \quad \forall\, t \geq t_0 \geq 0 \tag{3.21}$$

for some class $\mathcal{KL}$ function β. Finally, if $D = \mathbb{R}^n$ and $W_1(x)$ is radially unbounded, then the origin is **globally uniformly AS**.

(Globally) Uniformly ES
Suppose the assumptions above for uniformly SISL are satisfied with inequality (3.18) and (3.19) strengthened to

$$k_1 \|x\|^a \leq V(t, x) \leq k_2 \|x\|^a \tag{3.22}$$

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(t, x) \leq -k_3 \|x\|^a \tag{3.23}$$

for all $t \geq 0$ and $x \in D$, where k_1, k_2, k_3 and a are positive constants. Then the origin is **ES**. If the assumptions hold globally, the origin will be **globally ES**.

Similar to the autonomous case, there are situations where the Lyapunov analysis can only prove uniform SISL because $W_3(x)$ ends up being positive semi-definite. In these cases, an extension of LaSalle's invariant principle for non-autonomous systems, called **Barbalat's lemma**, can be used to prove AS [93].

3.1.2.1. Boundedness and ultimate boundedness

In the previous sections, the nonlinear systems (3.1) and (3.13) were assumed to have distinct equilibrium points. However, Lyapunov's stability theory can also show the **boundedness** of the nonlinear system solution when there is no equilibrium point at the origin. Let us, for example, consider a simple nonlinear system in which the following scalar equation governs the system behaviour:

$$\dot{x} = -x + \delta \sin(t), \quad x(t_0) = a, \quad a > \delta > 0. \tag{3.24}$$

The solution to this system can be expressed as

$$x(t) = e^{-(t-t_0)} a + \delta \int_{t_0}^{t} e^{-(t-\tau)} \sin \tau d\tau. \tag{3.25}$$

This system has no clear equilibrium point due to the presence of the trigonometric term $\sin t$. It, however, is not unstable since it satisfies the following bound:

$$|x(t)| \leq e^{-(t-t_0)} a + \delta \int_{t_0}^{t} e^{-(t-\tau)} d\tau = e^{-(t-t_0)} a + \delta \left[1 - e^{-(t-t_0)}\right] \leq a, \quad \forall\, t \geq t_0. \tag{3.26}$$

As a result, the system is shown to be **uniformly bounded (UB)** by a for all $t \geq t_0$, that is, with a bound independent of t_0. Although this bound is valid for all times $t \geq t_0$, since it does not consider the exponentially decaying term, it becomes a conservative estimate of the solution as time progresses. We can, therefore, instead pick any number b, such that $\delta < b < a$, to get the following bound:

$$|x(t)| \leq b, \quad \forall\, t \geq t_0 + ln\left(\frac{a-\delta}{b-\delta}\right) = t_o + T. \tag{3.27}$$

This new bound b, which is again independent of t_0, provides us with a less conservative estimate of the solution after a transient period T has passed (see figure 3.3a). In this case, the system is said to be uniformly **ultimately bounded (UUB)**, with bound b being the ultimate bound. These notions are summarized in definition 3.3 below. The prefix "uniformly" can be dropped for autonomous systems since they only depend on $t - t_0$.

[7] This theorem combines theorem 4.1-4.3 of [92]. These theorems were slightly adjusted for clarity.

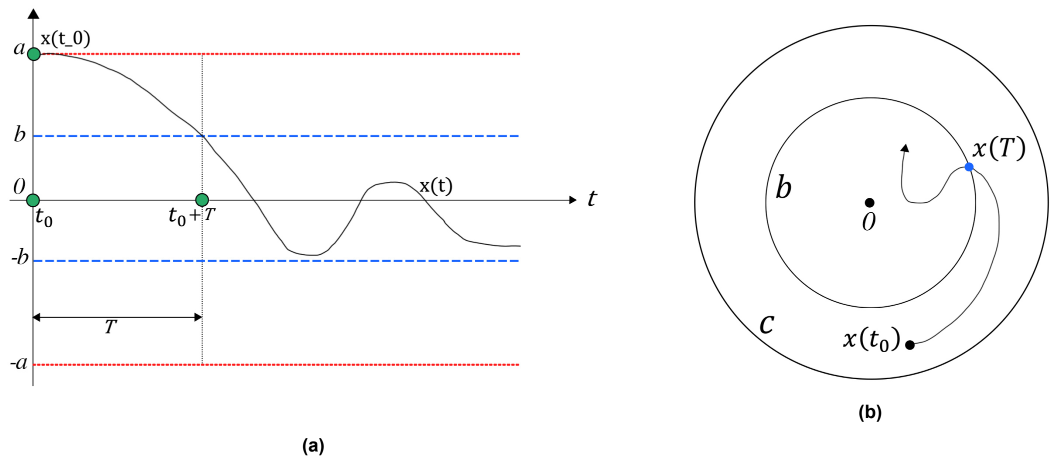


Figure 3.3: Uniform and ultimate boundedness visualized in 1D (figure 3.3a) and 2D (figure 3.3b). In these figures, t_0 is the initial time, T the time after the transient period, x the trajectory and a and b are the uniform and ultimate bound, respectively.

Definition 3.3: Uniform and ultimate boundedness [92]

The solutions of the **unforced** (i.e., $u = 0$) system (3.13) are

- **UB** if there exists $c > 0$, independent of t_0, and for every $a \in (0, c)$, there is $\beta > 0$, dependent on a but independent of t_0, such that

$$\|x(t_0)\| \leq a \Rightarrow \|x(t)\| \leq \beta, \quad \forall\, t \geq t_0. \tag{3.28}$$

- **Globally UB** if (3.28) holds for arbitrarily large a.
- **UUB** with ultimate bound b if there exists a positive constant c, independent of t_0, and for every $a \in (0, c)$, there is a $T \geq 0$, dependent on a and b but independent of t_0, such that

$$\|x(t_0)\| \leq a \Rightarrow \|x(t)\| \leq b, \quad \forall\, t \geq t_0 + T. \tag{3.29}$$

- **Globally UUB** if (3.29) holds for arbitrarily large a.

It is important to note that in contrast to ϵ in SISL, here, the (ultimate) bound b can not be made arbitrarily small by starting closer to the equilibrium point of origin. This bound instead depends on the system dynamics, uncertainties and disturbances. As a result, these stability notions can be seen as "milder" forms of SISL. As they are not defined around specific equilibrium points, they only require that trajectories converge to a given bound in finite time and stay therein for all future times (see figure 3.3b). Therefore, they are easier to achieve in practical dynamical systems subject to uncertainties and disturbances.

Like the other stability notions, a Lyapunov analysis can be used to investigate the UB and UUB of nonlinear systems. Since the Lyapunov conditions used in this analysis are very similar to those in theorem 3.3, they will not be displayed here. Instead, they can be found in [92] with their proof and several examples.

3.1.2.2. Stability of perturbed systems

Besides being time-dependent and possibly lacking distinct equilibrium points, real systems can also be subject to perturbations caused by modelling errors, ageing, uncertainties, and disturbances. It turns out that for stable unperturbed systems, Lyapunov's stability theory can be extended to include perturbations when information about these perturbations, like an upper bound, is available. To demonstrate this, let us extend the system of (3.13) with a perturbation $g(t, x)$:

$$\dot{x} = f(x, t, u) + g(t, x), \quad u = 0. \tag{3.30}$$

Like f, $g : [0, \infty) \times D \to \mathbb{R}^n$ is also piecewise continuous in t and locally Lipschitz in x and u. Further g can be a vanishing (i.e., $g(t, 0) = 0$) or non-vanishing (i.e., $g(t, 0) \neq 0$). Now suppose that the nominal system (3.13) has an ES equilibrium point $x = 0$, and $V(t, x)$ is a Lyapunov function that satisfies

$$c_1 \|x\|^2 \leq V(t, x) \leq c_2 \|x\|^2 , \tag{3.31}$$

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(t, x) \leq -k_3 \|x\|^2 , \tag{3.32}$$

$$\left\| \frac{\partial V}{\partial x} \right\| \leq c_4 \|x\| , \tag{3.33}$$

for all $(t, x) \in [0, \infty) \times D$ for some positive constants c_1, c_2, c_3 and c_4. These conditions, which can be seen as the inverse or converse[8] of the Lyapunov conditions in theorem 3.3, prove that some $V(t, x)$ exists, given an ES equilibrium exists.

Now let us assume that we know g is a vanishing perturbation (i.e., $g(t, 0) = 0$) which is linearly bounded by

$$\|g(t, 0)\| \leq \gamma \|x\| , \quad \forall\, t \geq 0, \quad \forall\, x \in D, \tag{3.34}$$

in which γ is a nonnegative constant. Given this information and the fact that the nominal system has an ES equilibrium point, a natural approach would be to use the Lyapunov function of the nominal system as a candidate for investigating the stability of the perturbed system. Doing this results in the following derivative of V along the trajectories of (3.30):

$$\dot{V}(t, x) = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(t, x) + \frac{\partial V}{\partial x} g(t, x) . \tag{3.35}$$

The first two terms on the right-hand side of (3.35) represent the contribution of the nominal system. Because these terms are always negative definite, the resulting $\dot{V}(t, x)$ is also negative definite, and condition (3.32) is satisfied. The third term, $\left[\frac{\partial V}{\partial x}\right] g(t, x)$, represents the effect of the perturbation. Since no exact knowledge is available about g the effect of this term on the definiteness of $\dot{V}(t, x)$ is unknown. Therefore, we can at best use the linear bound on g, which was given in (3.34), to investigate the worst-case scenario. Using this information with (3.32) and (3.33) results in the following inequality:

$$\dot{V}(t, x) \leq -c_3 \|x\|^2 + \left\| \frac{\partial V}{\partial x} \right\| \|g(t, x)\| \leq -c_3 \|x\|^2 + c_4 \|x\|^2 . \tag{3.36}$$

From this, we can determine that if γ is small enough to satisfy the bound

$$\gamma < \frac{c_3}{c_4}, \tag{3.37}$$

then

$$\dot{V}(t, x) \leq -(c_3 - \gamma c_4) \|x\|^2, \quad (c_3 - \gamma c_4) > 0. \tag{3.38}$$

Using Theorem 3.3 it can therefore be concluded that the origin of the nominal system is also a (globally) ES equilibrium point of the perturbed system. As a result, although the perturbation can disturb the system trajectories away from the equilibrium point, the system will return to this equilibrium point when it is gone. A similar relationship can be derived when the nominal system has an AS equilibrium point [92].

In the case that g is a non-vanishing perturbation, the equilibrium point $x = 0$ of the nominal system (3.13) may no longer be the equilibrium point of the perturbed system (3.30). As a result, the reasoning above is no longer valid. For these systems, we can, at best, prove that $x(t)$ is bounded by a small bound (i.e., UB or UUB) if the perturbation is relatively small. More information about this extension of UB and UUB to non-vanishing perturbed systems can be found in Chapter 9 of [93].

[8] More information on the inverse Lyapunov theorems can be found in section 4.7 of [93].

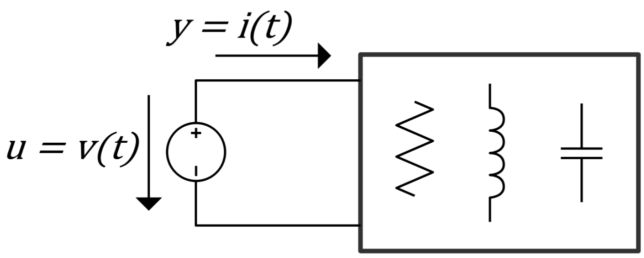


Figure 3.4: Schematic diagram of a general RLC circuit containing a combination of (passive) resistances, inductors and capacitors. In this, y and x represent the input and output while v and i represent the voltage and current, respectively.

3.1.3. Stability of forced non-autonomous systems

The previous chapter extended Lyapunov's stability theory to work for unforced, non-autonomous and perturbed systems. In most cases, however, we are not interested in the stability of free or perturbed systems but that of the system when it is being controlled (i.e., $u \neq 0$). Two distinct approaches can be taken for investigating the stability of controlled systems: the state-space approach and the input-output approach. In the state-space approach, a detailed model of the inner structure of the system is used to investigate the behaviour of the state variables. On the other hand, the input-output approach treats the system as a black box and relates the system's output directly to the input. These two approaches seed two distinct stability notions: **input-to-state stability (ISS)** and **input-output stability (IOS)**. IOS is used for systems with no exact state model, for example, systems which can only be approximated due to time delays. Since a dynamics model is readily available for robot manipulators, the rest of the text will focus on ISS. However, concepts like dissipativity and passivity, explained below, can also be applied to IOS. Interested readers can check out [92] for more information.

3.1.3.1. Input-to-state stability

Roughly speaking, a system is said to be ISS if it is GAS or GES in the absence of external inputs and the trajectories $x(t)$ stay bounded for any bounded input $u(t)$. The mathematical definition naturally follows the previous sections on UB and perturbed systems if we consider the new input u as a bounded disturbance to the unforced system $\dot{x} = f(x, t, 0)$. For this perturbed system, since u is bounded, it might be possible to show that $\dot{V}$ is negative (semi-)definite outside a ball of radius μ, where μ depends on the upper bound of u (i.e., sup $\|u\|$). As a result, we can use the concept of UB to arrive at the formal definition of ISS shown in definition 3.4.

Definition 3.4: Input-to-state stability [92][9]

The **forced** (i.e., $u \geq 0$) system (3.13) is **input-to-state stable** if there exists a class $\mathcal{KL}$ function β and a class $\mathcal{K}$ function γ such that for any $t_0 \geq 0$, any initial state $x(t_0)$, and any bounded input $u(t)$, the solution $x(t)$ exists for all $t \geq t_0$ and satisfies

$$\|x(t)\| \leq \max\left\{\beta\left(\|x(t_0)\|, t - t_0\right),\ \gamma\left(\sup_{t_0 \leq \tau \leq t} \|u(\tau)\|\right)\right\}, \quad \forall\, t \geq t_0. \tag{3.39}$$

Several Lyapunov-based approaches exist in the literature to prove the ISS of systems: dissipativity, robustness margins and classical Lyapunov-like conditions [93, 186]. Of these approaches, the notion of dissipativity and its special case passivity is especially convenient when working with the interconnected systems often encountered in control.

3.1.3.2. Passivity

The concept of a passive system first emerged in the circuit theory field from the energy dissipation in passive component [145]. In electrical networks, an electronic component is called passive if it does not require electrical energy to operate. It can only receive energy from other components and either dissipate, absorb, or store it in an electric or magnetic field. Because of this, networks that only contain passive elements will not generate energy and are, therefore, stable [4]. An example of such a passive

[9] Definition 4.4 of [92] was slightly changed for consistency with the rest of the text.

network is the general RLC circuit depicted in figure 3.4. Since it consists of only passive elements (e.g., inductors, resistors and capacitors), it cannot supply more energy to its environment than it receives. As the power supplied to this network equals the product of voltage and current (i.e., $p(t) = v(t)\,i(t)$) this results in the following inequality:

$$E(t_1) - E(t_0) \leq \int_{t_0}^{t_1} v(t)\,i(t)\,dt, \quad t_0 \leq t_1. \tag{3.40}$$

This inequality which relates the total energy stored in the network $E(t)$ to the energy supplied to the network, holds for all passive electrical systems.

This notion of passivity was later generalized to other dissipative systems through the introduction of a **storage function** (energy stored in the system) and a **supply rate** (externally supplied energy) [217, 218]. A system is said to be dissipative if its energy (storage function) increases no more than the energy provided (via the supply rate) during a given time interval. This relationship is mathematically expressed in definition 3.5 below.

Definition 3.5: Dissipative system [11]

System H with **supply rate** $w(t)$ is said to be **dissipative** if there exists a nonnegative real function $S(x) : X \to \mathbb{R}^+$ called the **storage function**, such that, for all $t_1 \geq t_0 \geq 0$, $x(t_0) \in X$ and $u \in U$,

$$S(x_1) - S(x(t_0)) \leq \int_{t_0}^{t_1} w(t)\,dt \tag{3.41}$$

where $x_1 = \phi(t_1, t_0, x(t_0), u)$ and $\mathbb{R}^+$ is a set of nonnegative real numbers.

In this definition, H can be any general (non-)linear system, while the supply rate $w(t)$, which determines the type of dissipativity, can be any bounded function defined on the input and output space (i.e., $\int_{t_0}^{t_1} w(t)\,dt < \infty$). If the storage function S is continuously differentiable, this relationship simplifies to

$$\frac{dS(x(t))}{dt} \leq w(t), \tag{3.42}$$

which emphasizes the storage function's positive semi-definiteness. When a bilinear supply rate is used, we receive the general definition of a passive system given in definition 3.6. This definition spans a broad spectrum of passive systems, with **lossless** and **state strictly passive** systems being the most extreme cases. Where lossless systems store all energy supplied to them, strictly passive systems lose additional energy due to, for example, heat or friction.

Definition 3.6: Passive system [92, 11][10]

A system is said to be **passive** if it is dissipative with respect to the following supply rate:

$$w(u(t), y(t)) = u(t)^T y(t) \tag{3.43}$$

and the storage function $S(x)$ satisfies $S(0) = 0$. Moreover, it is

1. **Lossless** if $\dot{S} = u(t)^T y(t)$.
2. **Input strictly passive** if $\dot{S} + u(t)^T \varphi(u(t)) \leq u(t)^T y(t)$ *and* $u(t)^T \varphi(u(t)) > 0, \quad \forall\, u \neq 0$.
3. **Output strictly passive** if $\dot{S} + y(t)^T \rho(y(t)) \leq u(t)^T y(t)$ *and* $y(t)^T \varphi(y(t)) > 0, \quad \forall\, u \neq 0$.
4. **Strictly passive** if $\dot{S} + \psi(x) \leq u(t)^T y(t)$ for some positive definite function ψ.

In all cases, the inequality should hold for all (x, u). A system that does not adhere to these conditions is said to be **active**.

From the definitions above, it is easy to see that the inequality reduces to the aforementioned Lyapunov inequality $\dot{V}(x(t)) \leq 0$ when $u = 0$. Because of this, Lyapunov functions, called **Control**

[10] Definition 2.7 of [11] was combined with a slightly modified version of definition 5.3 of [92].

Lyapunov functions (CLF) for controlled systems, can be used as candidate energy functions in dissipative and passive systems. To better understand the relationship between passivity and stability, let us apply the concept of passivity to the earlier used simple damped pendulum (see example 3.2).

Example 3.2: Passivity analysis of simple damped pendulum

Consider the simple damped pendulum of example 3.1. Let us add an input torque u and rewrite the equations of motion in (3.3) as two coupled first-order differential equations:

$$\dot{x}_1 = \dot{x}_2, \quad \dot{x}_2 = -\frac{g}{l}\sin(x_1) - \frac{b}{ml^2}x_2 + \frac{1}{ml^2}u, \tag{3.44}$$

where x_1 is the pendulum angle θ. Now let us take $y = x_2$ as the output and use the expression for the total energy we found in example 3.1 as a candidate energy function S:

$$S\left(x_1, x_2\right) = \frac{1}{2}ml^2x_2^2 + mgl(1-\cos x_1). \tag{3.45}$$

Note that this function is positive semi-definite but not positive definite since it is zero at points other than the origin. Evaluating the time derivative of this energy function gives us

$$\dot{S}\left(x_1, x_2\right) = mlx_2\left(l\dot{x}_2 + g\sin x_1\right), \tag{3.46}$$

and substituting in the system dynamics from equation (3.44) reveals

$$\dot{S}\left(x_1, x_2\right) = -bx_2 + x_2u. \tag{3.47}$$

Using the passivity relationship in definition 3.6 and definition 3.5 results in the following inequality:

$$\dot{S} - u^Ty = -bx_2 + x_2u - ux_2 = -bx_2^2 \leq x_2u - ux_2 \leq 0. \tag{3.48}$$

Hence, the pendulum is **passive** (i.e lossless) when $b = 0$ and is **strictly (output) passive** when $b > 0$.

The pendulum example above suggests that passivity implies stability if a positive definite storage function is used. However, passivity does not guarantee stability because the storage function in definition 3.5 is only required to be positive semi-definite (i.e., nonnegative). This is because the presence of an unobservable unstable part of the system can still destabilise the equilibrium [11]. Additional conditions like zero-state detectability and observability, which can be seen as extensions of the invariance principles described above, are required to prove stability through passivity to exclude these situations:

Definition 3.7: Zero-state detectability and observability [11]

A system H is **zero-state observable (ZSO)** if for any $x \in X$,

$$y\left(t\right) = h\left(\phi\left(t, t_0, x, u\right)\right) = 0, \quad u = 0, \quad \forall\, t \geq t_0 \geq 0 \quad implies\ x = 0, \tag{3.49}$$

and the system is **locally ZSO** if there exists a neighbourhood X_n of 0, such that for all $x \in X_n$, (3.49) holds. The system is **zero-state detectable (ZSD)** if for any $x \in X$,

$$y\left(t\right) = h\left(\phi\left(t, t_0, x, u\right)\right) = 0, \quad u = 0, \quad \forall\, t \geq t_0 \geq 0\ implies \lim_{t\to\infty}\phi\left(t, t_0, x, 0\right) = 0, \tag{3.50}$$

and the system is **locally ZSD** if there exists a neighbourhood X_n of 0, such that for all $x \in X_n$, (3.50) holds.

With these conditions, several relationships between passivity and stability can be derived. The relationships between passivity, SISL and AS are shown in theorem 3.4. However, relationships for the other stability notions can also be obtained by modifying the storage function [60]. It is important to note that passivity, like Lyapunov stability, is only a sufficient condition for stability, as examples of non-passive systems exist that are stable [137].

Theorem 3.4: Passivity based stability [93][11]

If the **forced** (i.e., $u \geq 0$) system (3.13) is **passive** or **input-strictly passive** with a **positive definite** storage function $S(x)$, then the origin of the undisturbed system $x = f(t, x, 0)$ is **SISL**. Moreover, it is **AS** if the system

- **Strictly passive** or
- **Output strictly passive and ZSO or ZSD**. Furthermore, if the storage function is **radially unbounded**, the origin will be **GAS**.

The discussion above showed that dissipativity and its subclass passivity could be seen as an extension of Lyapunov's stability theory. Where Lyapunov's direct method allows for reasoning about the stability of (unforced) systems, dissipativity also gives information about the input-output relation. In addition, a useful property of passive systems is that passivity is preserved when two passive systems are interconnected in parallel or feedback [60]. This makes passivity especially well-suited when working with compositional control designs like controlled robot manipulators. For example, if a passive controller controls a passive manipulator, the resulting combined system is also passive, and the controlled system is guaranteed to never becomes unstable. As a result, Lyapunov's stability theory and the concept of passivity are valuable tools for designing stable controllers for these systems.

3.2. Manipulator kinematics/dynamics

Several formulations are found in the impedance literature describing the dynamics of an n-degree of freedom (DOF) manipulator: the Newton-Euler, Euler-Lagrange, and Hamiltonian formulations. The Euler-Lagrange and Hamiltonian formulations are often used in stability research because of their close relationship to the system energy [60], while the Newton-Euler formulation is often used in simulations [182]. In this section, the Euler-Lagrange formulation will be depicted to aid the understanding of the control methods in the next section. More information about the Newton-Euler and Hamiltonian formulations can be found in [182] and [11], respectively. By using this formulation, the joint-space dynamics of a general n-DOF robot manipulator operating in an m-dimensional Cartesian space can be formulated as

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + f(\dot{q}) + g(q) = u + J^T(q) F_e, \tag{3.51}$$

where $q \in J \in \mathbb{R}^n$ are the joint positions, $M \in \mathbb{R}^{n \times n}$ represents the inertia matrix, $C \in \mathbb{R}^{n \times n}$ gives the contributions due to coriolis and centripetal forces, while $g \in \mathbb{R}^n$ and $f \in \mathbb{R}^n$ are the torques due to gravity and friction forces, respectively. Further, $F_e \in \mathbb{R}^m$ denotes a vector of forces and moments exerted on the end-effector by the environment, $J \in \mathbb{R}^{m \times n}$ the geometric jacobian[12] and $u \in \mathbb{R}^n$ the joint actuation (i.e., control) torques. Although equation (3.51) is expressed in the joint space $\mathcal{J}$, it can also be converted to the task space $\mathcal{C}$ using the following kinematic relationships:

$$\dot{x}_e = J_A(q)\dot{q}, \tag{3.52}$$

$$J_A(q) = \frac{\partial k(q)}{\partial q}, \tag{3.53}$$

$$\ddot{x}_e = J_A(q)\ddot{q} + \dot{J}_A(q, \dot{q})\dot{q}, \tag{3.54}$$

in which $x_e \in C \in \mathbb{R}^m$ are the Cartesian positions of the end effector, $k : \mathbb{R}^n \rightarrow \mathbb{R}^m$ the forward kinematics and $J_A \in \mathbb{R}^{m \times n}$ the analytical Jacobian[13]. If we assume $n = m$ and we neglect the joint friction torques, the cartesian space dynamic model becomes[14]

$$M_x(q)\ddot{x}_e + C_x(q, \dot{q})\dot{x}_e + g_x(q) = J_A^{-T}(q) u + F_a, \tag{3.55}$$

[11] Lemma 5.5-5.6 of [92] were combined and slightly adjusted for clarity.

[12] The geometric Jacobian relates the joint velocities $\dot{q}$ to the end-effector linear $\dot{x}_e$ and angular ω_e velocities using the geometry of the manipulator. More information can be found in section 3.1 of [182].

[13] The analytical Jacobian relates the joint velocities $\dot{q}$ to the translational $\dot{x}_e$ and rotational ϕ_e velocities of the end-effector frame through direct differentiation of the manipulator kinematics. More information can be found in 3.6 of [182].

[14] The full derivation can be found in section 7.8 of [182].

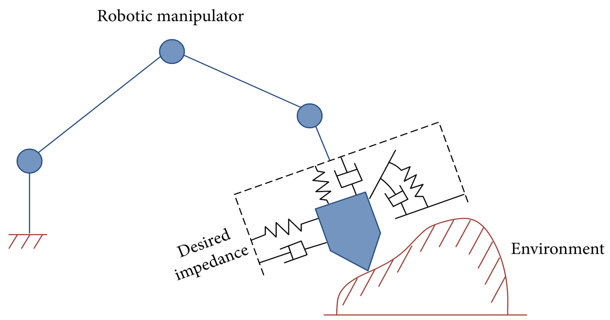


Figure 3.5: Visual representation of impedance control in contact with the external environment [180].

where

$$M_x(q) = J_A^{-T}(q)\, M(q)\, J_A^{-1}(q), \tag{3.56}$$

$$C_x(q,\dot{q}) = J_A^{-T}(q)\, C(q,\dot{q})\, J_A^{-1}(q) - M_x(q)\, \dot{J}_A(q)\, J_A^{-1}(q), \tag{3.57}$$

$$g_x(q) = J_A^{-T}(q)\, g(q), \tag{3.58}$$

$$F_A = T_A^{-T}(x_e)\, F_e, \tag{3.59}$$

and F_A is a vector of equivalent forces and moments in the frame of the analytic Jacobian, while T_A is the transformation matrix between the geometric and analytic Jacobian.

3.3. Indirect force control

Now that we have a clear picture of stability, passivity, and the manipulator model, we are ready to introduce indirect force control methods like impedance and its reciprocal admittance. As the name implies, indirect force control methods do not directly control desired reaction forces like force control does but instead enforce them indirectly via reference models. These reference models produce a desired dynamic interaction between the manipulator and its (unknown) environment.

3.3.1. Impedance control

In impedance control, the desired robot-environment interaction is modelled as a spring-damper system (see figure 3.5) and is generally expressed[15] as

$$M_d(\ddot{x}_e - \ddot{x}_d) + B_d(\dot{x}_e - \dot{x}_d) + K_d(x_e - x_d) = M_d\ddot{\tilde{x}} + B_d\dot{\tilde{x}} + K_d\tilde{x} = F_a, \tag{3.60}$$

in which x_e, $\dot{x}_e$, and $\ddot{x}_e \in \mathbb{R}^n$ are the current position, velocity and acceleration of the manipulator's end effector in the cartesian coordinates, respectively, while the subscript d denotes their desired values. Further, F_a denotes the vector of external forces and torques affecting the system as expressed in the frame of the analytic Jacobian, and M_d, B_d and $K_d \in \mathbb{R}^{n\times n}$ are the inertia, damping and stiffness matrices of the desired impedance relationship, respectively, all of which are symmetric and nonnegative definite. This general impedance relationship, called the second-order impedance, can also be simplified to its first or zero-order form by setting the $\ddot{x}_d$ and $\dot{x}_d$ to zero, respectively.

This desired impedance relationship can be enforced on the environment interaction through feedback linearization. Feedback linearization, also called dynamic inversion control, is a control technique that transforms a nonlinear system into a linear system by cancelling out (or compensating) the nonlinear terms. The manipulator model in (3.55) can be feedback linearized in the cartesian space using the following control law:

$$u = J_A^T(q)\left[M_x(q)\, y + C_x(q,\dot{q})\,\dot{x}_e + g_x(q) - F_a\right]. \tag{3.61}$$

Applying this control law to (3.55) gives us a linear and decoupled closed-loop system

$$\ddot{x}_e = y, \tag{3.62}$$

[15] This relationship can also be expressed in the joint space [201, 114, 117, 115]. This representation was omitted here since the cartesian version offers more intuition due to its close relationship to the task space.

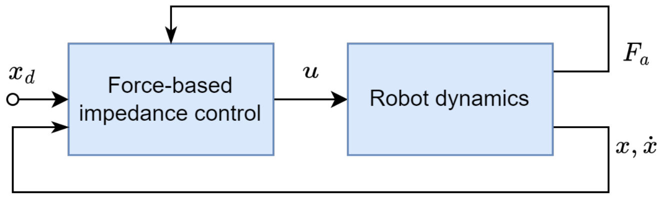


Figure 3.6: Schematic diagram of force-based impedance control. In this figure, x and $\dot{x}$ are the current position and velocity, respectively, x_d is the desired position, u the control command and F_a the external forces and torques [139].

where y represents a new input vector that directly influences, with a double integrator, the independent cartesian coordinates x. This input allows us to enforce any dynamic relationship onto the controlled robot manipulator. By setting y equal to

$$y = \ddot{x}_d + M_d^{-1}\left[-B_d\dot{\widetilde{x}} - K_d\widetilde{x} + F_a\right], \tag{3.63}$$

we end up with the impedance model given in (3.60). Combining and simplifying (3.63) and (3.61) gives us the general impedance control law:

$$\begin{aligned} u = {} & M(q)\, J_A^{-1}(q)\left\{\ddot{x}_d - \dot{J}_A(q)\,\dot{q} + M_d^{-1}\left[-B_d\dot{\widetilde{x}} - K_d\widetilde{x}\right]\right\} \\ & + C(q,\dot{q})\,\dot{q} + g(q) + J_A^T(q)\left[M_x(q)\, M_d^{-1} - I\right] F_A. \end{aligned} \tag{3.64}$$

This control law, sometimes called **torque-based impedance control**, produces forces in response to changes in motion and can, therefore, directly be applied to effort-controlled manipulators (see figure 3.6) or through an optional inner force control loop [180]. When an accurate model of the robot dynamics (i.e., $M(q)$, $C(q,\dot{q})$ and $g(q)$) and a measurement of the external forces and torques F_a are available, it allows precise shaping of the environment interaction using the impedance parameters M_d, B_d, K_d. Of these, M_d influences the control system's response speed, B_D the shape of the transient behaviours and K_d the trade-off between contact forces and free space position accuracy.

3.3.1.1. Cartesian stiffness control

The general impedance model above can only be used when a measurement of the external force F_a is available. In some cases like robotic surgery, however, placing a force sensor at the robot end-effector can be difficult [46]. It turns out that impedance control can also be used in these situations if we modify (3.60) so that the desired inertia M_d is equal to the actual inertia of the robot $M_x(q)$:

$$M_x(q)\,\ddot{\widetilde{x}} + (B_d + C(q,\dot{q}))\,\dot{\widetilde{x}} + K_d\widetilde{x} = F_a, \tag{3.65}$$

where $C(q,\dot{q})$ was introduced to preserve the mechanical properties of the configuration-dependent inertia $M_x(q)$. Applying this new impedance model through feedback linearization results in the following control law:

$$u = M(q)\, J_A^{-1}(q)\left\{\ddot{x}_d - \dot{J}_A(q)\, J_A^{-1}(q)\,\dot{x}_d\right\} + C(q,\dot{q})\, J_A^{-1}(q)\,\dot{x}_d + g(q) - J_A^T(q)\left[B_d\dot{\widetilde{x}} + K_d\widetilde{x}\right]. \tag{3.66}$$

This new control law, called **cartesian stiffness control**, is similar to a pure motion control law but designed to keep limited contact forces at the end-effector level. It no longer has a dependency on the contact forces F_a and can therefore be used without a measurement of this force. However, it has two drawbacks: first, we lose the ability to change the system's behaviour using the desired inertia M_d, and second, because of the presence of the extra $C(q,\dot{q})$, it can only be used at low velocities.

3.3.1.2. Admittance control

As mentioned above, a trade-off exists between contact forces and free space position accuracy in general impedance control. In the absence of interaction (i.e., free motion), impedance control becomes equivalent to inverse dynamics position control, and therefore a high K_d is needed for robustness against disturbances or model uncertainties. This high K_d will, however, lead to undesired high impact forces when in contact. **Position-based or velocity-based impedance control** provides a solution

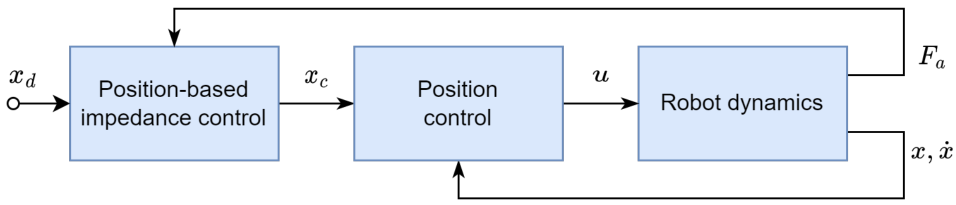


Figure 3.7: Schematic diagram of a position-based impedance controller. In this figure, x and $\dot{x}$ are the current position and velocity, respectively, x_d is the desired position, x_c the reference position, u the control command and F_a the external forces and torques [139].

to this trade-off by splitting the control into two control loops: an inner loop that controls (compliant) position or velocity references and an outer loop that provides these references based on the desired target impedance dynamics (see figure 3.7 for a general description). A simple proportional-integral-derivative (PID) can be used for the inner control loop, while a modified version of the impedance model in (3.60) is used for the outer control loop:

$$M_d\left(\ddot{x}_c - \ddot{x}_d\right) + B_d\left(\dot{x}_c - \dot{x}_d\right) + K_d\left(x_c - x_d\right) = F_a, \tag{3.67}$$

where x_c, $\dot{x}_c$, and $\ddot{x}_c \in \mathbb{R}^n$ now are the reference position, velocity and acceleration, respectively. This type of control, which generates reference motions x_c in response to contact forces F_a, is also known as **admittance control** because of its close resemblance to mechanical admittance. It is often used for interaction tasks dealing with compliant environments or free space, while impedance control is often used for interactions with stiff environments.

3.3.1.3. Variable impedance control

Although the general impedance model in (3.60) allows us to design the interaction with the environment precisely, it is still restrictive since this interaction can not be changed during the motion. It is, therefore, only suited for simple tasks where the environment stiffness is constant and can not be used to track desired contact forces. In contrast, humans change their muscle stiffness during motion to attenuate any disturbances or track desired interaction forces [200]. Doing this allows them to perform complex interaction tasks in changing and uncertain environments while precisely controlling the force applied to these environments. The general impedance model in (3.60) can be extended to match this human behaviour by allowing the impedance parameters M_d, B_d and K_d to vary. This results in the following impedance model:

$$M_d\left(t\right)\ddot{\tilde{x}} + B_d\left(t\right)\dot{\tilde{x}} + K_d\left(t\right)\tilde{x} = F_a, \tag{3.68}$$

in which the impedance parameters $M_d\left(t\right),\ B_d\left(t\right)\ and\ K_d\left(t\right)$ are now a function of time. As stated in the introduction, however, this **variable impedance control (VIC)** model destroys the passivity of the system unless a proper impedance profile is selected. To prove this, let us assume the desired inertia M_d is constant while the desired damping B_d and stiffness K_d are allowed to change with time. Using the following positive definite Lyapunov function:

$$V\left(\tilde{x}, \dot{\tilde{x}}\right) = \frac{1}{2}\dot{\tilde{x}}^T M_d \dot{x} + \frac{1}{2}\tilde{x}^T K\left(t\right)\tilde{x} \tag{3.69}$$

and taking its derivative while substituting $\ddot{x}$ from (3.68) gives

$$\dot{V} = \dot{\tilde{x}}^T F_a + \left[\frac{1}{2}\tilde{x}^T \dot{K}_d\left(t\right)\tilde{x} - \dot{\tilde{x}}^T B_d\left(t\right)\dot{\tilde{x}}\right]. \tag{3.70}$$

From this derivative, since the sign of the term between the brackets is undefined, it is apparent that the passivity condition of definition 3.6 only holds when the stiffness is constant or decreasing $\dot{V} \leq \dot{\tilde{x}}F_a$. As a result, energy might be injected into the system when the stiffness increases, possibly making it unstable. A similar conclusion can be drawn for the case where the inertia can also vary. This loss of passivity, and thus stability, significantly affects the impedance control since a stable environment interaction and accurate trajectory tracking can no longer be guaranteed.

3.3.2. Limitations

While the indirect force control methods above, compared to direct force control methods, are better suited to handle interaction tasks, they also have one major drawback. Due to the use of feedback linearization, an accurate model of the system is required. They are, therefore, not suited in situations where this model is unavailable or significantly impacted by model uncertainties, disturbances or other noises. If so, more advanced indirect control methods like robust or adaptive impedance control can be used [185, 92, 211].

3.4. Further reading

This chapter is intended to be a quick primer on stability, passivity, and impedance control. Readers can check out [8] for more information on stability criteria and control methods used with linear systems. For nonlinear systems, an excellent introduction to the control of nonlinear systems is given by [92], while [93] serves as a deep dive into all the stability proofs and more advanced methods. Further, [11] is an excellent introduction to passivity-based control, while [60] covers more advanced concepts. Lastly, readers are referred to [182] for a detailed explanation of impedance control, while recent reviews of impedance research are found in [185] and [180].

4 Stability methods in classical variable impedance control

As shown in section 3.3.1.3, the beneficial passivity property of constant (or fixed) impedance controllers no longer holds for variable impedance controllers, meaning the system can become unstable due to the control. Since no closed-loop solution exists for most interaction tasks, a classic stability analysis cannot be applied as the environment model is unknown or uncertain. Therefore, most authors have resorted to Lyapunov's stability theory and the concept of passivity to prove that systems controlled by variable impedance stay stable when interacting with these unknown environments. In the literature, two main groups of methods can be found for ensuring the stability of variable impedance controllers: Solutions that try to enforce the passivity online through **pacifying control algorithm** and solutions that use **stability constraints** to ensure the passivity of a given controller or impedance profile offline. The following sections discuss the methods used with classical variable impedance controllers like adaptive and optimal controllers. The methods used in learning-based variable impedance controllers are described in the next chapter. Although this review focuses on stability, some mechanisms for ensuring safety are also briefly mentioned below to aid the understanding of the shortcomings of specific passivity methods.

4.1. Pacifying control algorithms

4.1.1. Energy-tanks

One of the first authors to use a pacifying control algorithm for creating a stable variable impedance controller was Ferraguti et al. [46]. By looking at the passivity condition derived for the constant impedance controller (i.e., equation (3.70)[1]), they realised that the dissipation term serves as a passivity margin. The more power dissipated in the system, the stricter the passivity condition becomes, and the more the desired stiffness can be adjusted without threatening the passivity of the closed-loop system. Therefore, by augmenting the impedance model with an energy-storing element, a tank, they could track the dissipated energy to be later used to implement the VIC. While doing this, a tank upper limit is set to avoid potentially hazardous situations, and a constant stiffness is used to maintain the closed-loop system's passivity when the energy in the tank is almost empty. This strategy was later extended in [47] to work with second-order impedance control algorithms that vary not only the desired stiffness but also the desired damping and inertia.

Several works have used this strategy to ensure passivity while varying impedance through adaptive or optimal control [25, 28, 29, 130, 22, 10, 170, 172, 190, 85, 83]. Although theoretically sound from an energy perspective, this method has three main shortcomings:

1. A singularity exists when the energy tank is empty. As a result, the tank has to be initialised with a certain amount of energy before the control action. This initialisation will make the controller more complex and worsen the performance.

[1] The passivity condition for a constant impedance controller is obtained from equation (3.70) by setting $\dot{K}_d$ equal to zero.

2. The control performance depends on the tank's initial energy and upper energy bound. These values must be correctly tuned to guarantee a particular variable impedance profile. If the initial energy accumulated in the tank is too low, the controller will run out of energy before the variable impedance profile is completed. If the upper bound is too high, the controller might release too much energy at once, leading to fast and unsafe behaviours.
3. It depends on the current robot state and can only be applied online. Consequently, the execution of a desired variable impedance profile cannot be guaranteed beforehand because a constant stiffness is used when the tank is almost empty.

To solve the first shortcoming, Zheng et al. [240] modified the energy tank formula with an exponential so that the singularity does not occur anymore when the tank is empty. However, as pointed out in a recent paper by Califano et al., [18], this only shifts the singularity away from the origin but does not prevent it from occurring elsewhere. More importantly, due to the presence of the exponential, the storage function no longer qualifies as a Lyapunov function, breaking previously derived stability guarantees. Saudrais et al. [163], on the other hand, use a progressive damping injection mechanism to ensure the tank never becomes empty. This mechanism slightly increases the damping parameter to add energy to the tank when the lower limit is reached and decreases it again when the upper bound is reached. Lastly, Secchi et al. [171] reframed the original impedance control problem as a constraint convex optimisation problem, in which the minimum tank energy is constrained to be above a specific limit. They solve this problem online to retrieve the variable impedance behaviour that best approximates the desired impedance behaviour without depleting the tank. Since their method plans the (passive) control behaviour closest to the desired one, they prevent the singularity from occurring while also improving the performance. This method was later generalised by Capelli et al. [20], who encoded the passivity and minimum tank energy constraints of [171] as a Control Barrier Function (CBF)[2]. Using this CBF, their method applies to a broader range of control systems. Additionally, it leads to a more gradual enforcement of the passivity and minimum energy constraints, resulting in better tracking performance.

For the second shortcoming, several solutions have been proposed. Schindlbeck et al. [167], for example, estimate the energy needed to fulfil the desired task using a linear reaction force model and apply it to the tank to ensure enough energy is in the tank to complete this task. While doing so, a configuration-dependent shaping function on the controller output is used to prevent fast and unsafe behaviour at contact loss. Gerlagh et al. [55] take a slightly different approach by recovering the required task energy using an offline optimisation. They do not set this energy at the start but gradually release it throughout the motion to ensure the control behaviour is safe. Finally, to prevent the fast and unsafe behaviours mentioned above, Shahriari et al. 2018, 2020 [173, 175] limit the power exchanged from the tank at any given time. Shahriari et al. [173] accomplish this by adding adjustable valves into the tank design, which can control the power released during the task execution. Later, in Shahriari et al. [175], these valves are replaced by a constraint that directly enforces a specific power exchange profile on the output of the energy tank. This power flow regulation mechanism was used in a later paper by Michel et al. [131] to satisfy a maximum kinetic energy constraint, thereby ensuring that collisions do not lead to injuries.

4.1.2. Potential fields

While the methods above solve the first two shortcomings of the original energy tank, they significantly complicate the controller design, rely on additional interaction models, slow down the control frequency or hurt the system performance. More importantly, their effectiveness depends on the ability to track the energy dissipated in the system, and their performance cannot be guaranteed for highly-variable environments that might consume all tank energy. As a result, Babarahmati et al. [6] replaced the energy tank with an asymptotic stable potential field called a fractal attractor (FA). This FA is used to encode the convergence and divergence behaviour of the variable impedance controller. It accumulates the potential energy of the controller when diverging from the desired state so that it can later be used to converge back to this state passively. In their paper, Barbarahmati combines this FA with a cartesian stiffness controller to form a new (passive) variable impedance controller called a Fractal Impedance Controller (FIC). They use this FIC to execute a nonlinear state-dependent variable stiffness

[2]Similar to a CLF, but instead of guaranteeing that a trajectory converges to a particular stable set, it ensures it can never leave a particular safe set (i.e., a desired constraint is always satisfied) [3].

profile. This profile can be changed online to enforce a desired impedance relationship without affecting the system's stability, thus guaranteeing stable interaction with unknown environments. Unlike the original energy tank, this new FIC does not need energy dissipation tracking or an energy initialisation method. It also does not rely on energy damping to dissipate energy, achieves a better control performance, and is intrinsically robust against discretisation, model errors, and noise because it is path-independent. It, however, contains a non-smooth force transition while switching from divergence to convergence, which can hurt control performance. Tiseo et al. [196] solved this problem by replacing the simple stiffness profile used in [6] with a more flexible force profile. This new profile simplifies the controller's tuning for different tasks, and more importantly, it results in a smooth force transition between convergence and divergence. In addition, a force-feedback loop was also added to the FIC, enabling it to be used with force sensing or tracking tasks such as haptic exploration. Since then, this (improved) FIC has been deployed successfully in several manipulation and teleoperation tasks [198, 197, 195, 194, 7, 199].

4.1.3. Stability constraints

Although potential fields solve most of the shortcomings of the original energy tank, they are still state-dependent. As a result, they, like energy tanks, are unsuitable for guaranteeing the execution of a desired variable impedance profile in advance. Additionally, both energy tanks and, to a lesser extent, potential fields need to be tuned for for each task they are applied to. Several authors have therefore used a Lyapunov stability analysis to derive constraints that ensure the stability of their adaptive controllers [108, 121, 125, 126, 188, 229, 212, 116, 62, 63, 53]. These stability constraints are, however, controller specific and can not be used to prove the stability of a general variable impedance profile. Therefore, a paper by Kronander et al. [102] performed a Lyapunov stability analysis on the general variable impedance controller. Using a Lyapunov candidate function, they derived a **state-independent** stability constraint that relates the desired stiffness and its time derivative to the desired damping. This constraint can be used **offline** to verify a system's (asymptotic) stability for a given impedance profile or directly incorporated into an optimization or learning procedure. The resulting profiles can then be executed with any standard impedance control architecture while ensuring the (perturbed) system cannot go unstable, guaranteeing accurate trajectory tracking in free motion. Alternatively, it can also be implemented as a passivity filter that modifies non-passive impedance profiles **online** such that the passivity of the closed-loop system is guaranteed [12].

Several papers have used the constraint of Kronander et al. [102] with adaptive or optimal control-based variable impedance controllers to ensure the stability of the control [51, 123, 119, 67, 31]. Even though the constraint above guarantees the stability of the control in a state-independent way it has several limitations. First, because the constraint is very conservative, some stable impedance profiles may be incorrectly classified as unstable, causing the tracking performance to be significantly impacted. To improve this, Bednarczyk et al. [12] proposed another Lyapunov candidate function that leads to a less conservative stability constraint. Second, the original constraint does not consider external forces in the stability analysis. As a result, the stability of the control system is only guaranteed under the assumption of a free, unconstrained motion. Several papers have, however, derived similar stability constraints on the desired variable impedance dynamics for which stability of the unconstrained motion (i.e. AS and ES) and passivity or ISS of the constraint motion is proven [142, 187, 235, 12]. Lastly, the constraint derived by Kronander et al. [102] assumes that the desired inertia remains constant, which requires correct model knowledge, dynamic decoupling, and external force measurement. As a result, the constraint cannot be used with methods that vary the desired inertia or methods that set the desired inertia equal to the configuration-dependent robot inertia, such as Cartesian stiffness control. To solve this, Dong et al. [33] derived a stability constraint on the desired stiffness, damping, and inertia that can be used to ensure the control system's passivity when all impedance parameters are varied. Further, a paper by Park et al. [142] derived a stability constraint that incorporates the configuration-dependent robot inertia and can therefore be used to guarantee ISS of a Cartesian stiffness controller.

5

Stability methods in learning based variable impedance control

The previous chapter discussed methods for ensuring stable control in classical variable impedance controllers. Although impressive results were achieved using these classical controllers [185], these controllers' variable impedance profiles are manually programmed for a given task. This considerably reduces the usability of these controllers since every time a robot is used for a new task, multiple hours of labour by skilled workers are required for re-programming the controller. In recent years, numerous authors have instead switched to learning-based variable impedance controllers [1]. These controllers allow learning more complex behaviours without requiring the researcher to program this behaviour explicitly. A recent review of these learning-based variable impedance controllers was done by [1]. Whereas [1] focused on the learning-based VIC methods used in the current literature, here, the focus is on the stability and passivity techniques employed to ensure stability while using these methods. To achieve this, in this chapter, the learning-based VIC methods are classified into two main groups: imitation-based VIC, in which the impedance profiles are learned from human demonstrations, and RL-based VIC, in which the impedance control law is directly learned or improved via through trial and error from environmental interactions.

5.1. Imitation-based variable impedance control

5.1.1. Stability of the learned trajectories

As explained above, in imitation-based VIC, the desired trajectory and impedance profiles for a given task are learned from human demonstrations using a supervised learning procedure called Imitation Learning (IM), also called Learning from Demonstrations (LfD) [154]. Two steps generally make up this approach: A step in which kinematic (and dynamic) data is collected from sensors (i.e. data-collection step), followed by a step in which a model is fitted to the collected data using regression (i.e. data-fitting step). Depending on the task, any regression model (e.g. linear models, splines, gaussian mixture models, neural networks, gaussian processes) and accompanying regression technique can be used in this data-fitting step [99, 76]. Traditionally, LfD research has mainly focused on reproducing the demonstrations' kinematics (i.e. trajectories) [181]. At first glance, this trajectory reproduction problem seems to be a simple regression task that can be solved by minimising the squared distance between the sampled trajectories and the regression model. A low-level controller like a PID controller can then be used to reproduce the recorded trajectories. However, in practice, such a naive regression approach has proven to be insufficient because it results in an over-fitted model that does not generalise outside of the demonstrated trajectories [184]. Therefore, this (nonlinear) model is only locally stable and does not converge to the desired trajectories if situations are encountered that were not present in the training data (e.g. different initial states, temporal disturbances and spatial disturbances). One possible solution would be to use linear regression models in which stability outside the demonstrated regions can be proven easily. Such models, however, cannot accurately reproduce desired trajectories that are nonlinear and non-smooth. Therefore, several methods have been proposed in the literature for learning (nonlinear) trajectories while guaranteeing stability outside the demonstrated region.

In LfD research, motions are often modelled as a Dynamic System (DS) [94] to improve generalizability. Compared to classical approaches, which use time-indexed trajectories, in a DS, the trajectories are formulated as a differential equation. Because of this, the DS captures both the trajectories and the essential dynamics that underlie a given task, making it better able to adapt to environmental changes like temporal and spatial disturbances. The most widely used DS-based approach for learning stable nonlinear trajectories is the so-called "Dynamic Movement Primitives" (DMPs) [77, 166, 214, 183, 57, 159, 111]. In DMPs, a globally stable linear DS is coupled with a nonlinear forcing term through a phase variable to create an autonomous, weakly nonlinear DS. This DS can learn complex high-dimensional motions from single [77, 146] or multiple demonstrations [127, 144] while guaranteeing global stability. This global stability is achieved by exponentially or linearly decaying the phase variable during the task, thereby decreasing the effect of the possibly unstable nonlinear forcing term. As these DMPs are time- and scale-invariant, the learnt motions' velocity and amplitude can be scaled without losing stability, making them well-suited for reacting to external perturbations in real-time. Unfortunately, although the phase variable ensures the stability of the motions, it introduces an implicit time dependency in the DMP formulation. Due to this time dependency, the motion of the DMP is very dependent on the phase variable, which is, again, task-dependent. Consequently, DMPs have poor generalisation capabilities outside the demonstrations and are not robust against temporal disturbances [136].

Other authors use a constraint optimisation approach to overcome the abovementioned limitations and encode the demonstrated motions in a state-dependent nonlinear DS while enforcing global stability through Lyapunov constraints. Since these DSs are time-independent, the learned policies are now robust against temporal disturbances. This was first done by Khansari-Zadeh et al. [94], who introduced a stable estimator of a dynamical system (SEDS). By limiting the parameters of a Gaussian mixture regression model (GMM) using a Quadratic Lyapunov constraint, SEDS can learn accurate motions while guaranteeing global stability. Similar Lyapunov constraints were used in literature with other regression models to ensure the global stability of the learned motions [110, 72, 203, 205, 128, 36, 227, 204, 104, 226, 161, 27]. Unfortunately, as these methods' stability criteria are derived based on a simple quadratic Lyapunov function, they can only model trajectories in which the distance to the target decreases monotonically in time, leading to poor accuracy if demonstrated trajectories are not contractive. As a result, several authors have used parametric and non-parametric Lyapunov candidates to learn less conservative Lyapunov constraints directly from data to improve this so-called accuracy-stability dilemma [133, 135, 109, 205, 206, 39, 207, 38, 149, 150, 204, 224, 193, 26, 82]. These less conservative Lyapunov constraints can then be enforced **offline** during learning or **online** through a stabilising control command to ensure the stability of the reproduced trajectories. If applied online [133, 205, 206, 39, 207, 150, 149, 224, 38, 204], any regression method can be used. However, no guarantees can be given about the accuracy of reproduced trajectories since the control command potentially interferes with the dynamic system. If applied offline [135, 109, 193, 26, 82], on the other hand, the shape and accuracy of the demonstrated trajectories are known beforehand, but the implementation is specific to the used regression model. This significantly restricts the applicability of these offline methods since it is unknown beforehand which regression technique would be more effective for a given task.

Although the learned Lyapunov constraints above yield more accurate motions than their fixed counterparts and DMPs, finding them is more computationally intensive and time-consuming. Furthermore, their reproduction accuracy still depends on the Lyapunov candidate function used during learning. While less conservative Lyapunov candidate functions could be used to improve the accuracy of the reproduced trajectories, finding these Lyapunov candidate functions is not easy, time-consuming, and quickly becomes computationally intractable [149, 66, 193]. Additionally, except [82], all of the offline methods of the previous paragraph currently lack constructive mathematical guarantees causing the stability of these methods to be restricted to finite regions in the workspace. Therefore, these methods only apply to applications where the task region is bounded and known a priori. Because of this, recent papers have taken a diffeomorphic approach and transformed the demonstrations into a latent space in which the demonstrations are consistent with a (simpler) predefined or learned Lyapunov function [136, 143, 80, 148, 54, 209, 191, 48, 58, 241, 238, 165, 208, 215, 234]. By enforcing the Lyapunov constraints in this new latent space, the global stability of the learned motions can be guaranteed offline. Since this latent space, which can be of the same or a higher dimension, is topological equivalent to the original space, the stability achieved in this space is retained when the learned demonstrations are eventually transformed back to the original space [107, 106, 105]. Although a transformation that preserves stability has to be found, finding this transformation is relatively easier than finding a suitable Lyapunov function in the untransformed space. As a result, compared to previous methods, these diffeomorphic methods can handle more complex high-dimensional motions with less computational effort. Research has learned that diffeomorphic methods exhibit better reproduction accuracy and generalizability since they can learn a more general group of demonstrations than the abovementioned methods. However, they introduce more tunable parameters, can currently only be used offline and are specific to the used regression technique. Moreover, the expressiveness of the function approximator used for the diffeomorphic transform determines the accuracy and generalizability of these methods.

The methods discussed above were able to achieve high accuracy while guaranteeing the stability of the equilibrium points. They, however, give no guarantee about the convergence of the trajectories. As a result, although learned policies will always reach the target, regardless of the region in the state space where they are perturbed, they will no longer follow the reference trajectories. These techniques are, therefore, inadequate for tasks where trajectory tracking is crucial. Because of this, several authors have used contraction theory to derive stability constraints that guarantee a ”stronger” type of stability called incremental stability. Instead of looking at the stability of the equilibrium points, incremental stability studies the convergence between any two trajectories of a given system [124, 202]. These (incremental) stability constraints have been used **offline** [153, 152, 91, 184, 151] and **online** [15] with different regression models to improve the accuracy of the reproduced trajectories. Contraction-based methods have shown better or similar results to the diffeomorphic methods above. However, since their contraction-based constraints lead to exponential stable DSs, they provide greater robustness against temporal and spatial perturbations [155]. Nonetheless, the robustness and accuracy of these methods are highly dependent on the contraction metrics used to derive these constraints. Furthermore, only a small class of contraction metrics (i.e. positive definite symmetric matrices and a sum of squares polynomials) are currently used in literature and deriving them is non-trivial and dependent on the used regression model. Additionally, due to their inherent contractive nature, these approaches, however, suffer from the risk of over-corrections, making them unsuited for tasks containing global or locally divergent trajectories [50, 49]. Several papers have recently combined contraction theory and diffeomorphisms with global linearization methods inspired by Koopman theory to solve these issues [13]. These Koopman-based approaches work by mapping the system states of the nonlinear system to high (possibly infinite) dimensional spaces of observables in which the dynamics are linear. A globally linear representation of the original nonlinear system can be achieved by learning these Koopman observables and the accompanying mapping from data. Contraction theory can then be used on this representation to guarantee the global stability of the original nonlinear system. These Koopman-based methods, like the diffeomorphic methods above, lead to more challenging system identification but make it easier to guarantee global stability. Even though these methods have shown good accuracy and robustness in simple learning from demonstration datasets [44, 14], they are relatively new and yet to be compared to the methods above on more complex LFD tasks.

Lastly, two other noteworthy approaches for guaranteeing stability in kinematic LfD research are found in the current literature. The first one, a paper by Savariano et al. [164], used the energy tank of the previous chapter to ensure the DS's stability **online**. By storing the energy of stable (conservative) actions in this tank, potentially unstable motions can be performed as long as there is enough energy in the tank, thereby preserving the system's overall stability. Their method can achieve comparable accuracy as the aforementioned constrained-based methods, can be used with any regression technique, is less computationally intensive and requires fewer parameters to be tuned. However, it has an implicit time dependency and, like the energy tank-based methods in the previous chapter, requires a tank initialisation procedure which can significantly affect the reproduction accuracy. The second one, a paper of Figueroa et al. [50], used a non-parametric GMM to learn the movement policy and an elliptical Lyapunov function jointly. They introduce a novel similarity metric which leverages locality and directionally of the demonstrations and use this metric as a Bayesian prior in the constraint optimisation. By doing this, they end up with a globally asymptotically stable DS that also converges locally to the demonstrated trajectories. Their algorithm outperforms several methods described earlier in accuracy and generalizability without relying on diffeomorphisms or contraction theory.

5.1.2. Stability of the learned impedance

The previous section depicted several solutions for learning stable kinematic motion from demonstrations. Since robotic manipulators seldom only perform movements in free space, researchers have augmented these methods for interaction tasks. Several papers, for example, have extended the original DMP model of the previous section with an additional forcing or coupling term that encodes the desired impedance behaviour [166, 32, 64, 64, 120, 228, 230, 232, 236]. Since both the kinematic and impedance terms are linked to the same exponentially decaying phase variable, the stability of the reproduced motions and impedance profiles is guaranteed. Other authors use the Lyapunov constraint-based approach from the previous section to ensure the stability of their learned variable impedance controllers. Some directly encode the impedance behaviour on a Riemannian manifold and use a diffeomorphic approach [165, 215] while others use stability constraints similar to those of Kronander et al. [102] [192, 81, 34, 37, 5].

Although the stability of these impedance-based LfD methods in free motion is guaranteed, it does not imply passivity, and thus, the controlled system can become unstable when in contact. Therefore, several recent papers have focused on guaranteeing the passivity of these methods. In Khansari-Zadeh et al. [95], both the motion and impedance control were encoded into a time-invariant DS, which was expressed as a nonlinear combination of a set of linear spring-damper systems. Stability constraints were derived by performing a passivity analysis on this DS to ensure free space stability and passivity in-contact [95]. This idea was further developed in [96], where a non-parametric potential function and dissipative field were learned from demonstrations to encode the desired motion, stiffness and damping behaviour. The passivity and, thus, stability of this potential field-based variable impedance controller were demonstrated using a passivity analysis. Compared to [95], this new method can model a more extensive range of motions, simplifies motion and impedance learning and allows explicit stiffness and damping adjustments without modifying the desired trajectory. Since then, multiple authors have used this method to ensure passivity [219, 158, 134, 9, 73]. Instead, others have modified the energy-tank method given in the previous chapter to be used with LfD-based impedance controllers [101, 174]. Kronander et al. [101] designed a passive DS-based controller that can be used to learn motion and impedance control while ensuring stable interaction with a passive environment. The passivity and, thus, global stability of this controller is ensured through an energy-tank-like mechanism. This method was later extended by [49, 23] to encode locally contractive stiffness-like behaviours. Shahriari et al. [174], on the other hand, augment the DMP approach from the previous section with a passivity observer, which modifies the phase variable when non-passive behaviour is detected. Ferraguti's energy tank is also added since this passivity observer merely reduces non-passive actions but does not guarantee passivity. Multiple authors have used these modified energy tank-based methods to ensure their learned controllers stay passive and thus stable during interaction tasks [87, 86, 30, 141, 98, 2, 41, 132, 223, 222, 239]. Lastly, a paper by Franzese et al. [52] used a non-parametric model to learn stiffness and motion behaviour from human demonstrations. They use the uncertainty measure from the non-parametric model to create a potential field-like stability prior, that steers motion back to regions where the model predictions have high confidence. Additionally, they decrease the stiffness to zero when the uncertainty becomes too large to prevent unstable behaviours. Although intuitively, their controller avoids possible VIC instabilities, no constructive mathematical stability and passivity guarantees were given.

5.2. Reinforcement learning-based variable impedance control

While LfD-based approaches offer an intuitive and user-friendly way to teach robots new contact tasks without explicit programming, they rely on accurate motion and impedance measurements. Unfortunately, obtaining these human impedance parameters directly from sensors can be challenging [237, 1, 83]. Furthermore, the demonstrations collected from humans can contain implicit biases and noise, leading to suboptimal performance [97, 156, 1, 225]. As a result, RL-based methods, sometimes in combination with LfD, [16, 1, 166, 40] have been used to train or improve variable impedance controllers. However, only a few papers have explored passivity or stability guarantees for these RL-based impedance-based methods. The first study on this topic was performed by Rey et al. [156], who used RL to improve a stable DS-based movement policy gathered from human demonstrations. By enforcing the stability constraints of Khansari et al. [94] (i.e. SEDS) on the exploration noise, they ensure only stable samples are encountered during RL, preserving the original policy's stability characteristics. A similar approach was taken in [74]. Although both methods also provide a way to include state-dependent stiffness profiles, they only guarantee free-motion stability. To address this limitation, a recent paper by Khander et al. [88] instead uses the passive VIC model of [95] as the policy parameterisation of their model-free RL algorithm. They prevent unstable policy parameters from being sampled during the policy update by ensuring their sample distribution adheres to the stability constraints of [95]. Using this method, they could jointly learn motion and impedance profiles while ensuring the passivity and stability of the learned policy. However, the simple analytical form used in the policy parameterisation limits the learning of complex motion profiles. Therefore, later studies by the same research group used special NN-based policies of which the structure guarantees passivity and stability [90, 89]. Among these methods, [89] guarantees stability for both policy and exploration but can only take entire trajectories as data points and hence is incompatible with state-action-based RL methods. On the other hand, [90] can be used with these RL methods but only ensures stable final controllers without demonstrating stable exploration. It is worth noting that all of the methods above use deterministic policies, which can limit their robustness and flexibility in situations where the agent's actions are constrained or the environment is noisy or unpredictable [168, 169, 59, 65, 112, 220]. Therefore, ensuring passivity and stability in stochastic policy-based variable impedance controllers remains an open question.

6 Discussions and Conclusion

In the previous chapters, we saw that Lyapunov's stability and passivity are essential for ensuring stable control of rigid robotics manipulators in contact with (semi)-rigid environments. Because unstable systems can exhibit unpredictable and often dangerous behaviours guaranteeing stability is essential if we want robots to transition from their isolated factory workspace to the real world, where they interact with humans and other robots. Impedance control, in which the robot-environment interaction is modelled as a spring-damper system, is frequently used because traditional position or force controllers are unsuited for simultaneously performing free motion trajectory tracking and environment interaction. Although impedance controllers with constant parameters are passive and thus stable by design, this passivity is lost when the impedance parameters vary with time. To solve this, researchers have used Lyapunov's theory and its extension passivity to ensure VIC stays stable in free motion and is bounded during an interaction.

Within the variable impedance literature, two types of passivity-based approaches for ensuring the stability of interaction tasks are found: Solutions that try to enforce passivity online through pacifying control algorithms and solutions that use stability constraints to ensure the passivity of a given controller or impedance profile offline. Under the first approach (i.e. pacifying control algorithms), methods like energy tanks and potential fields are found. First proposed by [46], these methods rely on an energy-storing element (such as a tank or potential field) to track the energy dissipated in the closed-loop system. They act as a passivity filter by only allowing the execution of non-passive actions when enough energy is available in this storage element. When no energy is left, non-passive actions can no longer be performed, thereby protecting the total system passivity. Several authors have used these storage-based methods with optimal, adaptive and LfD-based variable impedance controllers to ensure passivity and stability. They are easy to implement, offer clear physical insights, and can be used with any controller. However, these methods are task-dependent and therefore require tuning of the storage parameters for each task they are applied. More importantly, they are also state-dependent and can only be applied online. As a result, they are not suited for guaranteeing the execution of desired trajectories and impedance profiles beforehand. Therefore, other authors have taken the second approach and derived stability constraints that ensure the stability of their optimal and adaptive controllers. However, as no general method exists for deriving these constraints, this task is challenging. Furthermore, as most papers only prove stability and not passivity, they do not ensure these controllers stay stable when interacting with the environment. As a result, Kronander et al. [102] and follow-up papers, therefore, derived state-independent stability constraints on the general-impedance controller that can be used to check the stability and passivity of a given impedance profile offline or directly incorporated into an optimization or learning procedure. Multiple authors have used these constraints to ensure that impedance profiles from their adaptive or optimal control architectures are stable. Similar stability constraints were derived and used with LfD-based impedance controllers. For RL-based VIC, limited research has been done on passivity and stability. Some recent papers ensured passivity and stability by imposing stability constraints on either the sampling policy or by exploiting special NN-based policies of which the structure guarantees passivity and stability. However, these approaches rely on deterministic policies, while the question of ensuring passivity and stability in stochastic policy-based VIC remains unanswered. These constraint-based methods provide a state-independent way to ensure the passivity and, thus, stability of offline impedance profiles while having less tunable parameters than the storage-based methods above. Nevertheless, except for a recent paper by Bednarcyk et al. [12], these constraint-based methods do not provide a way to modify non-passive impedance profiles online. More importantly, although they generally result in better tracking and impedance accuracy than storage-based methods, this performance highly depends on the Lyapunov candidates or contractive metrics used during the derivation giving rise to a so-called accuracy-stability trade-off. Although, since their initial derivation by [102] less constrictive Lyapunov candidates have been found, this still is an ongoing area of research. Because of this, no clear answer can be given as to which passivity approach is best, as the answer is highly dependent on the exact controller architecture being used and the task that needs to be performed. Where storage-based methods can easily ensure passivity when the exact impedance profile and controller specifics are unknown, constraint-based methods are preferred when the desired impedance profiles are known beforehand.

This literature review tried to fill the gap in the current impedance literature regarding the stability/-passivity considerations and methods used for stable control of rigid robotics manipulators in contact with (semi)-rigid environments. However, it also has several limitations that need to be pointed out. First, it only considers interactions with rigid (passive) environments. It does not look at interactions with active or compliant environments like humans, controlled robots or soft objects. This review's passivity and ISS proofs do not hold for these environments. Therefore, ensuring stability in these conditions is left for future research. Secondly, it mainly focuses on passivity-based stability methods. Even though passivity is well suited for guaranteeing stability in interaction tasks, it is not the only method. Other ISS and IOS methods are also used in the literature but are beyond the scope of this literature review. Lastly, while the controller's stability is trivial for a safe robot interaction, it is not the only quantity that needs to be considered. While a stable controller will not lead to unpredictable free motion and contact behaviour, it can still command high velocities, accelerations and forces, leading to unsafe situations. It should, therefore, also adhere to specific safety constraints like position, velocity, and acceleration limits to safely control robotic manipulators alongside humans or other robots [103, 24, 160, 179]. A possible future area of research could be to investigate how these additional safety constraints can be incorporated into the (learned) impedance control policy while maintaining passivity and stability.

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A
Keywords

Table A.1: Keywords used in the literature review.

AND	NOT
Stability	Human-robot interaction
Stability considerations	HRI
Stability guarantee(s)	Teleoperation
Lyapunov stability	Cable
Passivity	Electrical
Impedance control	Flexible robot
Manipulator(s)	Compliant joints
Environment interaction	Hydraulic
Reinforcement learning	Visual servoing
Imitation learning	
Learning from demonstration(s)	