

淡江大學

航空太空工程學系研究所

高等工程數學

作業 3

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座號：6

#1(a)

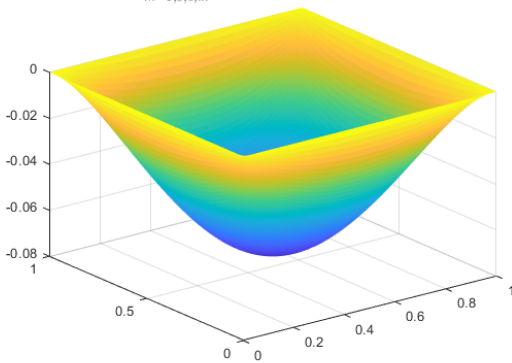
```
clear;clc;close all
[x, y] = meshgrid(0:0.01:1,0:0.01:1);
len = size(x);
for i = 1:len(1)
    for j = 1:len(2)
        f = @(m) (sinh((2*m-1)*pi*(1-y(i,j)))+sinh((2*m-1)*pi*y(i,j)))/sinh((2*m-1)*pi) ...
            *sin((2*m-1)*pi*x(i,j))/(2*m-1)^3;
        T(i,j) = (x(i,j)^2-x(i,j))/2 + 4/pi^3*limsum(f);
    end
end

figure()
surf(x,y,T, 'edgecolor', 'none')
title("\frac{q_0}{2}(x^2-x)+\frac{4q_0}{\pi^3}\sum\limits^{\infty}_{m=1,3,5,...} ...
\frac{1}{m^3}[\frac{\sinh\{m\pi(1-y)\}+\sinh\{m\pi y\}}{\sinh\{m\pi\}}]\sin\{m\pi x\}$ 3D Plot", ...
"fontsize", 14, 'interpreter', 'latex')

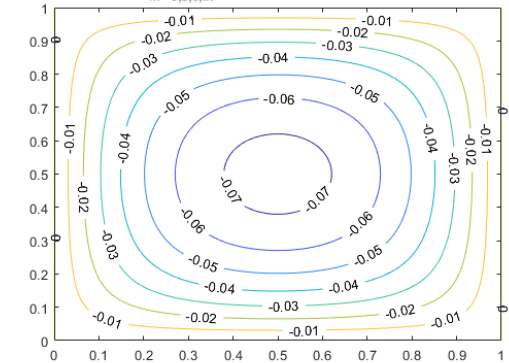
figure()
contour(x,y,T,'ShowText','on')
title("\frac{q_0}{2}(x^2-x)+\frac{4q_0}{\pi^3}\sum\limits^{\infty}_{m=1,3,5,...} ...
\frac{1}{m^3}[\frac{\sinh\{m\pi(1-y)\}+\sinh\{m\pi y\}}{\sinh\{m\pi\}}]\sin\{m\pi x\}$ 2D Plot", ...
"fontsize", 14, 'interpreter', 'latex')
```

.....

$\frac{q_0}{2}(x^2-x) + \frac{4q_0}{\pi^3} \sum_{m=1,3,5,\dots}^{\infty} \frac{1}{m^3} \left[\frac{\sinh m\pi(1-y) + \sinh m\pi y}{\sinh m\pi} \right] \sin m\pi x$ 3D Plot



$\frac{q_0}{2}(x^2-x) + \frac{4q_0}{\pi^3} \sum_{m=1,3,5,\dots}^{\infty} \frac{1}{m^3} \left[\frac{\sinh m\pi(1-y) + \sinh m\pi y}{\sinh m\pi} \right] \sin m\pi x$ 2D Plot

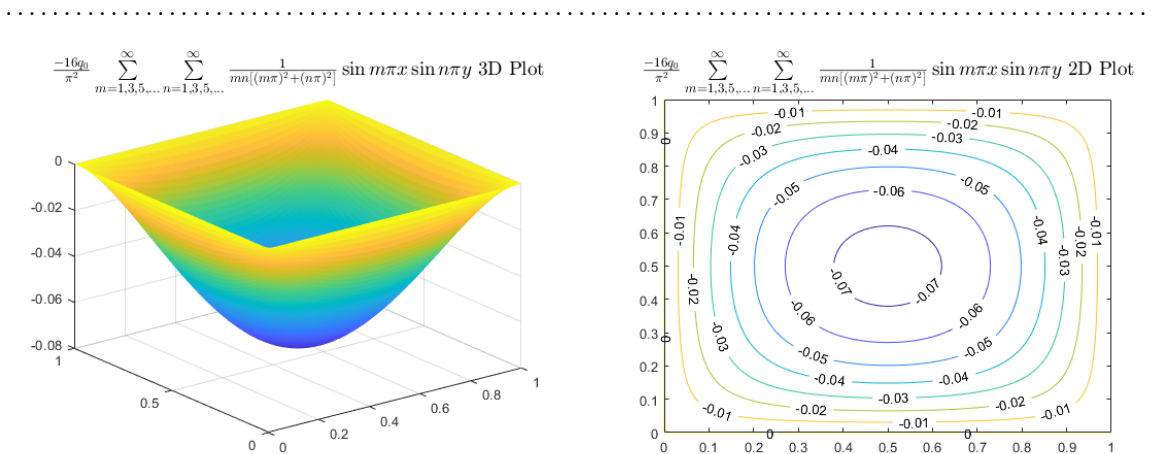


#1(b)

```
clear;clc;close all
[x, y] = meshgrid(0:0.01:1,0:0.01:1);
len = size(x);
for i = 1:len(1)
    for j = 1:len(2)
        tamp = 0;
        f = @(n,m) sin((2*m-1)*pi*x(i,j))*sin((2*n-1)*pi*y(i,j))/((2*m-1)*(2*n-1) ...
            *(((2*m-1)*pi)^2+((2*n-1)*pi)^2));
        for n = 1:100
            for m = 1:100
                tamp = tamp + f(n,m);
            end
        end
        T(i,j) = -16/pi^2*tamp;
    end
end

figure()
surf(x,y,T, 'edgecolor', 'none')
title("$\frac{-16q_0}{\pi^2}\sum\limits_{m=1,3,5,\dots}\sum\limits_{n=1,3,5,\dots}\frac{1}{mn[(m\pi)^2+(n\pi)^2]}\sin\{m\pi x\}\sin\{n\pi y\}$ 3D Plot" ...
, "fontsize", 14, "interpreter", "latex")

figure()
contour(x,y,T,'ShowText','on')
title("$\frac{-16q_0}{\pi^2}\sum\limits_{m=1,3,5,\dots}\sum\limits_{n=1,3,5,\dots}\frac{1}{mn[(m\pi)^2+(n\pi)^2]}\sin\{m\pi x\}\sin\{n\pi y\}$ 2D Plot" ...
, "fontsize", 14, "interpreter", "latex")
```



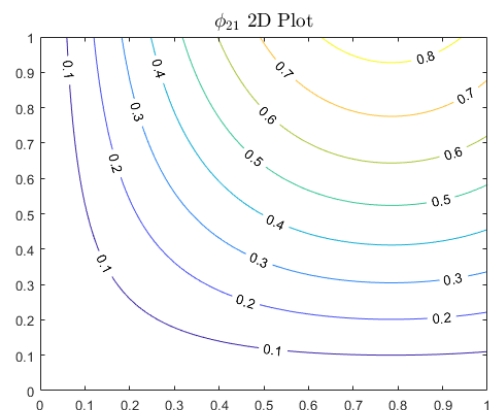
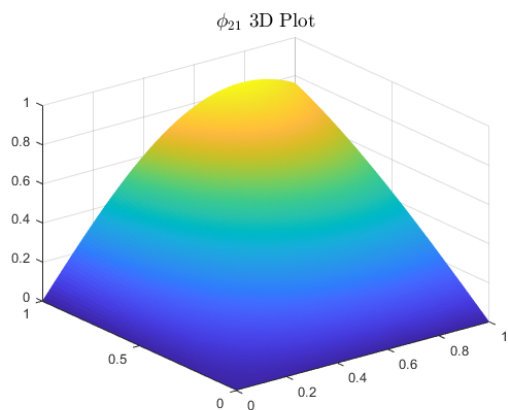
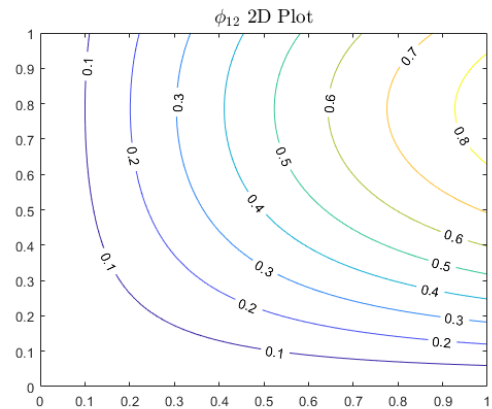
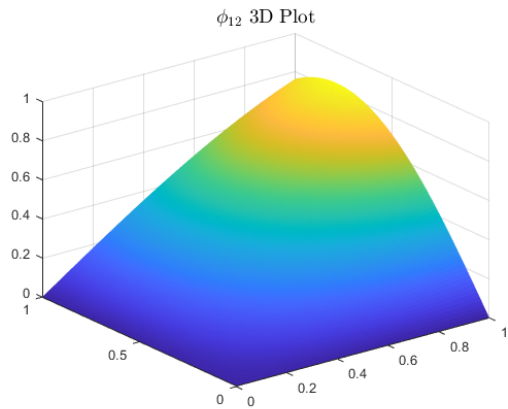
#2(a)

```
clear;clc;close all
[x, y] = meshgrid(0:0.01:1,0:0.01:1);
phi_12 = sin(1*x).*sin(2*y);
phi_21 = sin(2*x).*sin(1*y);
phi_13 = sin(1*x).*sin(3*y);
phi_31 = sin(3*x).*sin(1*y);

figure()
surf(x,y,phi_12, 'edgecolor', 'none')
title("\phi_{12}$ 3D Plot", "fontsize", 14, "interpreter", "latex")
figure()
surf(x,y,phi_21, 'edgecolor', 'none')
title("\phi_{21}$ 3D Plot", "fontsize", 14, "interpreter", "latex")

figure()
contour(x,y,phi_12,'ShowText','on')
title("\phi_{12}$ 2D Plot", "fontsize", 14, "interpreter", "latex")
figure()
contour(x,y,phi_21,'ShowText','on')
title("\phi_{21}$ 2D Plot", "fontsize", 14, "interpreter", "latex")
```

.....



#2(b)(c)(d)(e)

```
clear;clc;close all
[x, y] = meshgrid(0:0.01:1,0:0.01:1);
phi_12 = sin(1*x).*sin(2*y);
phi_21 = sin(2*x).*sin(1*y);
phi_13 = sin(1*x).*sin(3*y);
phi_31 = sin(3*x).*sin(1*y);

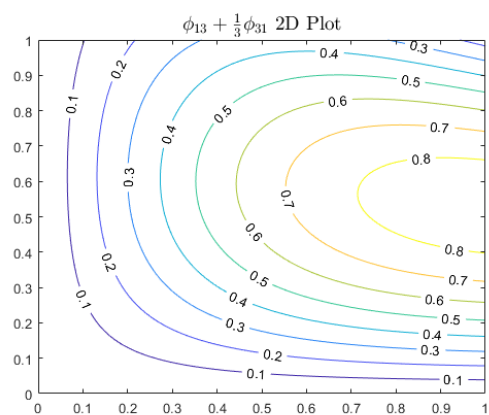
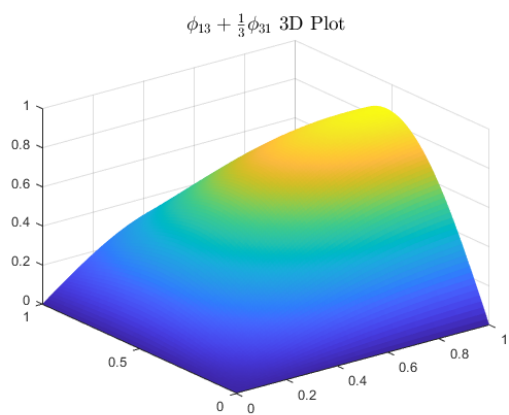
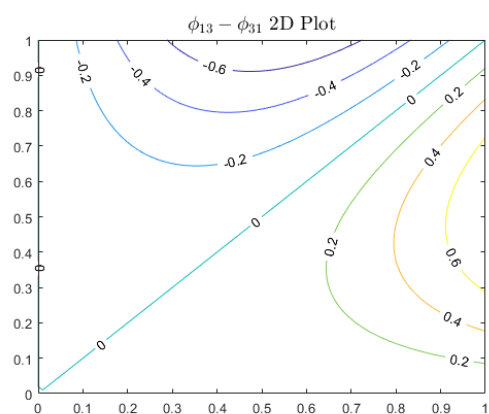
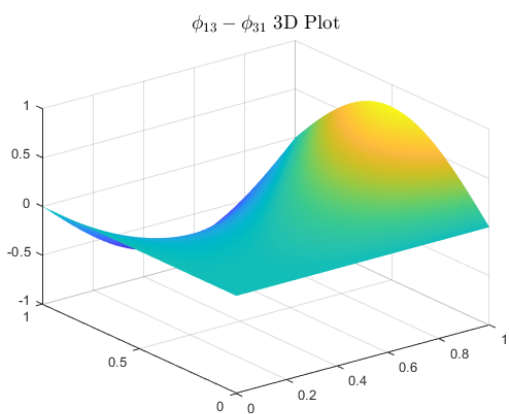
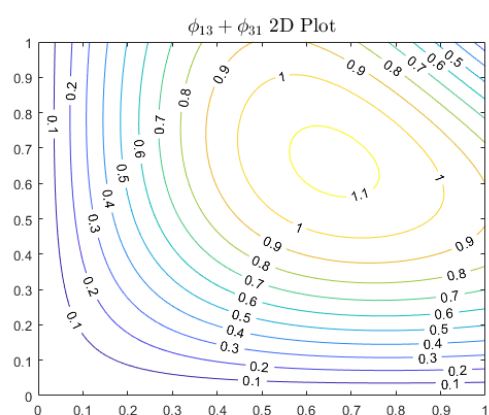
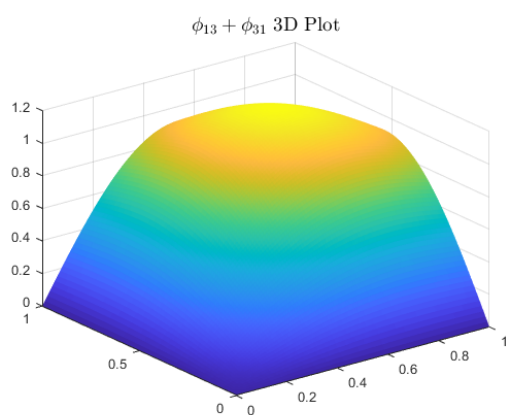
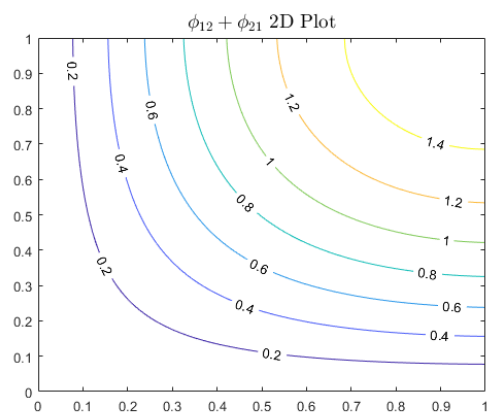
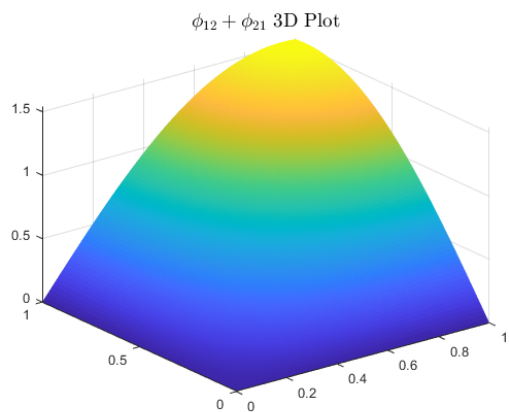
figure()
surf(x,y,phi_12+phi_21, 'edgecolor', 'none')
title("\phi_{12}+\phi_{21}$ 3D Plot", "fontsize", 14, "interpreter", "latex")
figure()
surf(x,y,phi_13+phi_31, 'edgecolor', 'none')
title("\phi_{13}+\phi_{31}$ 3D Plot", "fontsize", 14, "interpreter", "latex")
```

```

figure()
surf(x,y,phi_13-phi_31, 'edgecolor', 'none')
title("\phi_{13}-\phi_{31}$ 3D Plot", "fontsize", 14, "interpreter", "latex")
figure()
surf(x,y,phi_13+phi_31/3, 'edgecolor', 'none')
title("\phi_{13}+\frac{1}{3}\phi_{31}$ 3D Plot", "fontsize", 14, "interpreter", "latex")

figure()
contour(x,y,phi_12+phi_21,'ShowText','on')
title("\phi_{12}+\phi_{21}$ 2D Plot", "fontsize", 14, "interpreter", "latex")
figure()
contour(x,y,phi_13+phi_31,'ShowText','on')
title("\phi_{13}+\phi_{31}$ 2D Plot", "fontsize", 14, "interpreter", "latex")
figure()
contour(x,y,phi_13-phi_31,'ShowText','on')
title("\phi_{13}-\phi_{31}$ 2D Plot", "fontsize", 14, "interpreter", "latex")
figure()
contour(x,y,phi_13+phi_31/3,'ShowText','on')
title("\phi_{13}+\frac{1}{3}\phi_{31}$ 2D Plot", "fontsize", 14, "interpreter", "latex")
.....

```



#3

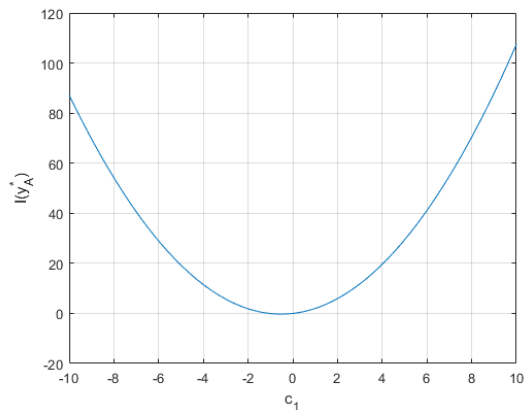
```
clear;clc;close all
syms c1 x y(x)
y = c1*x*(pi/2-x);
I = int(2*x*y-y^2+diff(y,x)^2, x, 0, pi/2);

c1_ = -10:0.01:10;
I_ = double(subs(I, c1, c1_));

plot(c1_, I_)
xlabel("c_1"); ylabel("I(y*_A)")
grid()

[min_I, index] = min(I_);
min_c1 = c1_(index);
fprintf("When c1=%.2f, I(yA) have the minimum value is %.4f\n", min_c1, min_I)
```

.....
When c1=-0.52, I(yA) have the minimum value is -0.2645



#4

#4.

$$\frac{d}{dx} \left[P(x) \frac{dy}{dx} \right] + \lambda \sigma(x) y(x) = 0$$

$$\Rightarrow P(x) \frac{d^2 y}{dx^2} + \frac{dP(x)}{dx} \frac{dy}{dx} + \lambda \sigma(x) y(x) = 0$$

$$\Rightarrow x \frac{d^2 y}{dx^2} + \frac{dy}{dx} + \frac{\lambda}{x} y(x) = 0$$

$$\Rightarrow P(x) = x, \frac{dP}{dx} = 1, \sigma(x) = \frac{1}{x}$$

$$\lambda = \frac{\int_1^2 x \left(\frac{dy}{dx} \right)^2 dx}{\int_1^2 \frac{1}{x} y^2(x) dx}$$

$$y_A(x) = (x-1)(2-x) = -x^2 + 3x - 2$$

$$\frac{dy_A}{dx} = -2x + 3$$

$$\begin{aligned} \tilde{\lambda}_1 &= \frac{\int_1^2 x (-2x+3)^2 dx}{\int_1^2 \frac{1}{x} (-x^2+3x-2)^2 dx} \\ &= \frac{1/2}{\ln 6 - \frac{11}{4}} = 22.1349 \end{aligned}$$

Aside:

$$\int_1^2 x (-2x+3)^2 dx = \int_1^2 4x^3 - 12x^2 + 9x dx$$

$$= \left[x^4 - 4x^3 + \frac{9}{2}x^2 \right]_1^2 = (16 - 32 + 18) - (1 - 4 + \frac{9}{2}) = \frac{1}{2}$$

$$\int_1^2 \frac{1}{x} (-x^2+3x-2)^2 dx$$

$$= \int_1^2 x^3 - 6x^2 + 13x - 12 + \frac{4}{x} dx$$

$$= \left[\frac{1}{4}x^4 - 2x^3 + \frac{13}{2}x^2 - 12x + 4\ln x \right]_1^2$$

$$= (4 - 16 + 26 - 24 + \ln 6) - (\frac{1}{4} - 2 + \frac{13}{2} - 12 + 0)$$

$$= \frac{-11}{4} + \ln 6$$

$$\text{Let } z = \ln x, \frac{dz}{dx} = \frac{1}{x}$$

$$x \frac{d^2 y}{dx^2} + \frac{dy}{dx} + \frac{\lambda}{x} y = 0$$

$$\Rightarrow x \left(\frac{1}{x^2} \frac{d^2 y}{dz^2} - \frac{1}{x^2} \frac{dy}{dz} \right) + \frac{1}{x} \frac{dy}{dz} + \frac{\lambda}{x} y = 0$$

$$\Rightarrow \frac{1}{x} \frac{d^2 y}{dz^2} - \frac{1}{x} \frac{dy}{dz} + \frac{1}{x} \frac{dy}{dz} + \frac{\lambda}{x} y = 0$$

$$\Rightarrow \frac{1}{x} \frac{d^2 y}{dz^2} + \frac{\lambda}{x} y = 0$$

$$x \cdot \Rightarrow \frac{d^2 y}{dz^2} + \lambda y = 0$$

$$y(z) = C_1 \cos \sqrt{\lambda} z + C_2 \sin \sqrt{\lambda} z$$

$$\Rightarrow y(x) = C_1 \cos(\sqrt{\lambda} \ln x) + C_2 \sin(\sqrt{\lambda} \ln x) \quad 9$$

Aside:

$$\frac{dy}{dx} = \frac{dy}{dz} \frac{dz}{dx} = \frac{1}{x} \frac{dy}{dz}$$

$$\frac{d^2 y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d}{dx} \left(\frac{1}{x} \frac{dy}{dz} \right)$$

$$= -\frac{1}{x^2} \frac{dy}{dz} + \frac{1}{x} \frac{d}{dx} \left(\frac{dy}{dz} \right)$$

$$= -\frac{1}{x^2} \frac{dy}{dz} - \frac{1}{x^2} \frac{dy}{dz}$$

from B.C.

$$y(1) = C_1 \cos(\sqrt{\lambda} \cdot 0) + C_2 \sin(\sqrt{\lambda} \cdot 0) = 0 \Rightarrow C_1 = 0$$

$$y(2) = C_2 \sin(\sqrt{\lambda} h_2) = 0 \Rightarrow \sin(\sqrt{\lambda} h_2) = 0$$

$$\Rightarrow \sqrt{\lambda} h_2 = n\pi$$

$$\Rightarrow \lambda_n = \left(\frac{n\pi}{h_2}\right)^2, \quad n = 1, 2, 3, \dots$$

$$\lambda_1 = \left(\frac{\pi}{h_2}\right)^2 = \underline{20.5423} \quad \#$$

#5

```
clear;clc;close all
y0 = 0:0.01:1;
x1 = y0.*acosh(1./y0);

figure()
plot(x1,y0)
xlabel("x_1"); ylabel("y_0")
grid()

A = pi*y0.^2.*(sinh(2*x1./y0)+2*x1./y0);

figure()
plot(x1,A)
xlabel("x_1"); ylabel("A_min")
grid()
```

