# Advanced Dynamics

## HW1

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# Transformation matrices between different frame

From geometry, the unit vector in rotation frame calculated in the fixed frame are

$$\begin{cases} \hat{i}_1 = \cos\alpha\hat{i} + \sin\alpha\hat{j} \\ \hat{j}_1 = -\sin\alpha\hat{i} + \cos\alpha\hat{j} \\ \hat{k}_1 = \hat{k} \end{cases} \tag{1}$$

Rewrite Eq.(1) in matrix form

$$\begin{bmatrix} \hat{i}_1 \\ \hat{j}_1 \\ \hat{k}_1 \end{bmatrix} = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{i} \\ \hat{j} \\ \hat{k} \end{bmatrix} \triangleq \mathbf{T} \begin{bmatrix} \hat{i} \\ \hat{j} \\ \hat{k} \end{bmatrix} \tag{2}$$

The matrix $T$ is the transfer matrix between fix frame to rotation frame. The transfer matrix between rotation frame to fix frame is the inverse of $T$ ($T^{-1}$).

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## Matlab program

```
F2R = [ cos(alpha)  sin(alpha)  0       % from fix frame to rotation frame respect to link 1
       -sin(alpha)  cos(alpha)  0
           0           0        1];
R2F = inv(F2R);                         % from rotation frame respect to link 1 to fix frame
```

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# Position vector of the particle

The position vector of particle 2 in the rotation frame is

$$\mathbf{r} = R\cos\Omega t\hat{i} + R\sin\Omega t\hat{k} \tag{3}$$

Rewrite Eq.(3) in matrix form

$$\mathbf{r} = \begin{bmatrix} R\cos\Omega t \\ 0 \\ R\sin\Omega t \end{bmatrix} \tag{4}$$

Then the position vector in the fix frame is

$$\mathbf{r}_{fix} = \begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix} = \mathbf{T}^{-1}\mathbf{r} = \begin{bmatrix} R\cos\omega t\cos\Omega t \\ R\sin\omega t\cos\Omega t \\ R\sin\Omega t \end{bmatrix} \tag{5}$$

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## Matlab program

```
r_R = [R*cos(beta) 0 R*sin(beta)]';     % in the rotating frame
r_I = simplify(R2F*r_R);                % in the fix frame
```

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## Velocity vector of the particle

The velocity vector of the particle 2 in the fix frame is

$$\mathbf{v}_{fix} = \dot{\mathbf{r}}_{fix} = \begin{bmatrix} -R\omega \sin \omega t \cos \Omega t - R\Omega \cos \omega t \sin \Omega t \\ R\omega \cos \omega t \cos \Omega t - R\Omega \sin \omega t \sin \Omega t \\ R\Omega \cos \Omega t \end{bmatrix} \tag{6}$$

Then the velocity vector in the rotating frame is

$$\mathbf{v} = \begin{bmatrix} v_{x1} \\ v_{y1} \\ v_{z1} \end{bmatrix} = \mathbf{T}\mathbf{v}_{fix} = \begin{bmatrix} -R\Omega \sin \Omega t \\ R\omega \cos \Omega t \\ R\Omega \cos \Omega t \end{bmatrix} \tag{7}$$

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### Matlab program

```
v_I = diff(r_I,t);                      % in the fix frame
v_R = simplify(F2R*v_I);                % in the rotating frame

fprintf('v_x1 = %s \n',char(v_R(1)))
fprintf('v_y1 = %s \n',char(v_R(2)))
fprintf('v_z1 = %s \n',char(v_R(3)))
fprintf('\n')
```

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### Program result

```
v_x1 = -OMEGA*R*sin(OMEGA*t)
v_y1 = R*omega*cos(OMEGA*t)
v_z1 = OMEGA*R*cos(OMEGA*t)
```

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## Acceleration vector of the particle

The acceleration vector of the particle 2 in the fix frame is

$$\mathbf{a}_{fix} = \dot{\mathbf{v}}_{fix} = \begin{bmatrix} -R\omega^2 \cos \omega t \cos \Omega t + 2R\omega\Omega \sin \omega t \sin \Omega t - R\Omega^2 \cos \omega t \cos \Omega t \\ -R\omega^2 \sin \omega t \cos \Omega t - 2R\omega\Omega \cos \omega t \sin \Omega t - R\Omega^2 \sin \omega t \cos \Omega t \\ -R\Omega^2 \sin \Omega t \end{bmatrix} \tag{8}$$

Then the acceleration vector in the rotating frame is

$$\mathbf{a} = \begin{bmatrix} a_{x1} \\ a_{y1} \\ a_{z1} \end{bmatrix} = \mathbf{T}\mathbf{a}_{fix} = \begin{bmatrix} -R(\Omega^2 + \omega^2) \sin \Omega t \\ -2R\omega\Omega \sin \Omega t \\ -R\Omega^2 \sin \Omega t \end{bmatrix} \tag{9}$$

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## Matlab program

```
a_I = diff(v_I,t);                      % in the fix frame
a_R = simplify(F2R*a_I);                % in the rotating frame

fprintf('a_x1 = %s \n',char(a_R(1)))
fprintf('a_y1 = %s \n',char(a_R(2)))
fprintf('a_z1 = %s \n',char(a_R(3)))
fprintf('\n')
```

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## Program result

```
a_x1 = -R*cos(OMEGA*t)*(OMEGA^2 + omega^2)
a_y1 = -2*OMEGA*R*omega*sin(OMEGA*t)
a_z1 = -OMEGA^2*R*sin(OMEGA*t)
```