for $n=3$,

$$f^{(3)}(\rho) = \frac{2\sin\theta\left[-5\rho^4-6\rho^2+2\cos(2\theta)+1\right]}{1-2\rho^2+\rho^4+2\rho^2\sin^2\theta} - \frac{2\sin\theta\left[-\rho^5-2\rho^3+2\rho\cos(2\theta)+\rho\right]}{(1-2\rho^2+\rho^4+2\rho^2\sin^2\theta)^2} \times (-4\rho+4\rho^3+4\rho\sin^2\theta)$$

$$f^{(3)}(0) = \frac{2\sin\theta\left[2\cos(2\theta)+1\right]}{1} - 0$$

$$= 2\left[2\sin\theta\cos(2\theta)+\sin\theta\right] = 2\left[2\sin(\theta)\cos(2\theta) + \sin(2\theta-\theta)\right]$$

$$= 2\left[\cancel{2}\sin\theta\cos(2\theta) + \sin(2\theta)\cos\theta - \cancel{\cos\theta\sin(2\theta)}\right]$$

$$= 2\sin(\theta+2\theta) = \underline{2\sin(3\theta)}$$

$$f(\rho) = \frac{1}{2}\tan^{-1}\left(\frac{2\rho\sin\theta}{1-\rho^2}\right) = \cancelto{0}{\frac{0}{0!}\cdot 1} + \frac{\sin\theta}{1!}\rho + \cancelto{0}{\frac{0}{2!}\rho^2} + \frac{2\sin(3\theta)}{3!}\rho^3 + \cdots$$

$$\underline{= \rho\sin\theta + \frac{1}{3}\rho\sin(3\theta) + \cdots \quad \#}$$

(b)

$$\sum_{n=0}^{\infty} \frac{e^{-(2n+1)x}}{2n+1}\sin(2n+1)\frac{\pi y}{H} = \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n}e^{-nx}\sin\frac{n\pi}{H}y$$

Let $\rho = e^{-x}$, $\theta = \frac{\pi y}{H}$, then

$$\sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n}\rho^n\sin n\theta = \frac{1}{2}\tan^{-1}\left(\frac{2\rho\sin\theta}{1-\rho^2}\right)$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{2e^{-x}\sin\frac{\pi}{H}y}{1-e^{-2x}}\right)$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{\sin\frac{\pi}{H}y}{\frac{e^{x}-e^{-x}}{2}}\right)$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{\sin\frac{\pi}{H}y}{\sinh x}\right)$$