Advanced Dynamics

HW7

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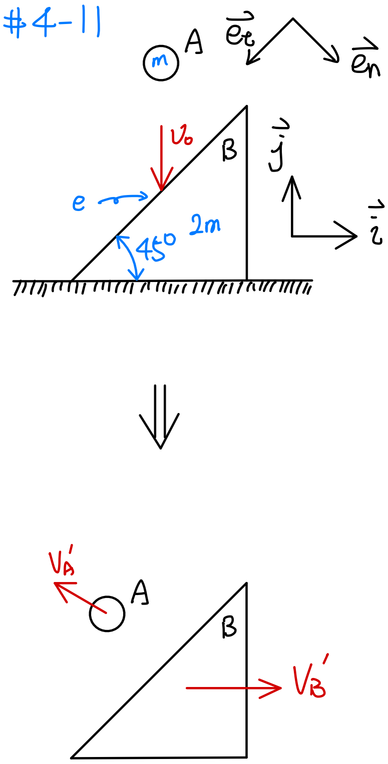

∵ Triangular block is smooth.

∴ $V_{At} = V_{At}' = v_0 \cos 45° = \frac{v_0}{\sqrt{2}}$ — ①

Define restitution e

$$V_{Bn}' - V_{An}' = e\left(V_{An} - \cancel{V_{Bn}}^{\,0}\right)$$

$$\Rightarrow V_B' \cos 45° - V_{An}' = e\, v_0 \cos 45°$$

$$\Rightarrow \frac{v_0}{\sqrt{2}} e = \frac{V_B'}{\sqrt{2}} - V_{An}' \quad \Rightarrow \quad V_{An}' = \frac{V_B'}{\sqrt{2}} - \frac{v_0}{\sqrt{2}} e \quad \text{— ②}$$

Linear momentum:

$$m\vec{V}_A + 2m\vec{V}_B = m\vec{V}_A' + 2m\vec{V}_B'$$

$$\Rightarrow -v_0 \vec{j} = \vec{V}_A' + 2\vec{V}_B'$$

$$= V_{An}'\left(\frac{1}{\sqrt{2}}\vec{i} + \frac{-1}{\sqrt{2}}\vec{j}\right) + V_{At}'\left(\frac{-1}{\sqrt{2}}\vec{i} + \frac{1}{\sqrt{2}}\vec{j}\right) + 2V_B'\vec{i}$$

$$= \left(\frac{V_{An}'}{\sqrt{2}} - \frac{V_{At}'}{\sqrt{2}} + 2V_B'\right)\vec{i} + \left(-\frac{V_{An}'}{\sqrt{2}} + \frac{V_{At}'}{\sqrt{2}}\right)\vec{j}$$

$$\Rightarrow \begin{cases} \dfrac{V_{An}'}{\sqrt{2}} - \dfrac{V_{At}'}{\sqrt{2}} + 2V_B' = 0 & \text{— ③} \\ -\dfrac{V_{An}'}{\sqrt{2}} + \dfrac{V_{At}'}{\sqrt{2}} = -v_0 & \text{— ④} \end{cases}$$

Substitute ① and ② into ③

$$\frac{V_B'}{2} - \frac{v_0}{2} e - \frac{v_0}{2} + 2V_B' = 0$$

$$\Rightarrow \frac{5}{2} V_B' = \left(\frac{1+e}{2}\right) v_0$$

$$\Rightarrow V_B' = \left(\frac{1+e}{5}\right) v_0 \quad \text{— ⑤}$$

$$\Rightarrow \vec{V}_B' = \underline{\left(\frac{1+e}{5}\right) v_0 \vec{i}} \ \#$$

$-5V_0$

Substitute ⑤ into ②

$$V_{An}' = \left(\frac{1+e}{5\sqrt{2}}\right) v_0 - \frac{v_0}{\sqrt{2}} e = \left(\frac{1-4e}{5\sqrt{2}}\right) v_0$$

$\frac{V_0}{\sqrt{2}}$

$$\vec{V}_A' = V_{An}'\left(\frac{1}{\sqrt{2}}\vec{i}-\frac{1}{\sqrt{2}}\vec{j}\right) + V_{At}'\left(\frac{-1}{\sqrt{2}}\vec{i}-\frac{1}{\sqrt{2}}\vec{j}\right)$$

$$= \frac{1}{\sqrt{2}}\left(V_{An}'-V_{At}'\right)\vec{i} + \frac{-1}{\sqrt{2}}\left(V_{An}'+V_{At}'\right)\vec{j}$$

$$= \left(\frac{1-4e}{10}v_0 - \frac{1}{2}v_0\right)\vec{i} - \left(\frac{1-4e}{10}v_0 + \frac{1}{2}v_0\right)\vec{j}$$

$$= \frac{-4-4e}{10}v_0\vec{i} - \frac{6-4e}{10}v_0\vec{j}$$

$$= v_0\left[-\frac{2}{5}(1+e)\vec{i} - \frac{3-2e}{5}\vec{j}\right] \#$$

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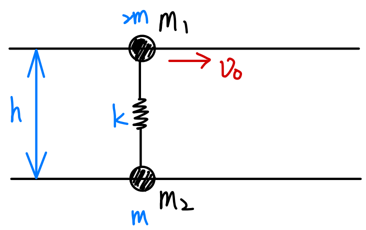

From conservation of energy,

$$\frac{1}{2}(2m)v_0^2 = \frac{1}{2}(2m)v_1^2 + \frac{1}{2}mv_2^2 + \frac{1}{2}k\delta^2$$

$$\Rightarrow 2v_0^2 = 2v_1^2 + v_2^2 + \frac{k}{m}\delta^2 \quad — ①$$

From conservation of linear momentum,

$$2mv_0 = 2mv_1 + mv_2$$

$$\Rightarrow 2v_0 = 2v_1 + v_2 \quad — ②$$

(a) The maximum velocity of v_2 will happen at $\delta = 0$, then

$$2v_0^2 = 2v_1^2 + v_2^2 + \frac{k}{m}\cancel{\delta^2}^{\,0}$$

$$\Rightarrow 2v_0^2 = 2v_1^2 + v_2^2$$

Substitute "v_1" by using ①

$$2v_0^2 = 2\left(\frac{2v_0-v_2}{2}\right)^2 + v_2^2$$

$$\Rightarrow \cancel{4v_0^2} = (2v_0-v_2)^2 + 2v_2^2$$

$$= \cancel{4v_0^2} - 4v_0v_2 + v_2^2 + 2v_2^2$$

$$\Rightarrow 3v_2^2 = 4v_0v_2$$

for $v_2 \neq 0$

$$\Rightarrow v_2 = \frac{4}{3}v_0 \#$$

Aside:

$$2v_0 = 2v_1 + v_2$$

$$\Rightarrow v_1 = \frac{2v_0 - v_2}{2}$$

(b)

Substitude ① into ②

$$2v_0^2 = 2v_1^2 + (2v_0 - 2v_1)^2 + \frac{k}{m}\delta^2$$

$$\Rightarrow \frac{k}{m}\delta^2 = 2v_0^2 - 2v_1^2 - 4v_0^2 + 8v_0v_1 - 4v_1^2$$

$$= -6v_1^2 + 8v_0v_1 - \underbrace{2v_0^2}_{\text{const.}} = f(v_1) \quad \text{— ③}$$

Find maximum of ③

$$f' = -12v_1 + 8v_0 = 0$$

$$\Rightarrow v_1 = \frac{8}{12}v_0 = \frac{2}{3}v_0$$

$$f''(v_1) = -12$$

$\because$ $f''(\frac{2}{3}v_0)$ is negative

$\therefore$ The maximum will happen at $v_1 = \frac{2}{3}v_0$

Substitute the result into ③

$$\frac{k}{m}\delta_{max}^2 = f(\tfrac{2}{3}v_0)$$

$$= -6\left(\tfrac{2}{3}v_0\right)^2 + 8v_0\left(\tfrac{2}{3}v_0\right) - 2v_0^2$$

$$= \frac{-8}{3}v_0^2 + \frac{16}{3}v_0^2 - 2v_0^2$$

$$= \frac{2}{3}v_0^2$$

$$\Rightarrow \delta_{max}^2 = \frac{2m}{3k}v_0^2$$

$$\Rightarrow \delta_{max} = \sqrt{\frac{2m}{3k}}\,v_0 \quad \#$$