Linear Systems

HW9

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(a) $\det\left(sI - \begin{bmatrix} 1 & 0 & -4 \\ 0 & 3 & 0 \\ -2 & 0 & -1 \end{bmatrix}\right) = \det\left(\begin{bmatrix} s-1 & 0 & 4 \\ 0 & s-3 & 0 \\ 2 & 0 & s+1 \end{bmatrix}\right) = s^3 - 3s^2 - 9s + 27$

$\Rightarrow s = -3, 3, 3.$ #

For $s = -3$,

$$-3I - \begin{bmatrix} 1 & 0 & -4 \\ 0 & 3 & 0 \\ -2 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -4 & 0 & 4 \\ 0 & -6 & 0 \\ 2 & 0 & -2 \end{bmatrix}$$

$$\begin{bmatrix} -4 & 0 & 4 & | & 0 \\ 0 & -6 & 0 & | & 0 \\ 2 & 0 & -2 & | & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -1 & | & 0 \\ 0 & 1 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix} \Rightarrow \begin{cases} x_1 = x_3 \\ x_2 = 0 \\ x_3 = x_3 \end{cases}$$

eigenvector of $s = -3$ is $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ #

For $s = 3$

$$3I - \begin{bmatrix} 1 & 0 & -4 \\ 0 & 3 & 0 \\ -2 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 4 \\ 0 & 0 & 0 \\ 2 & 0 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 4 & | & 0 \\ 0 & 0 & 0 & | & 0 \\ 2 & 0 & 4 & | & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 2 & | & 0 \\ 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix} \Rightarrow \begin{cases} x_1 = -2x_3 \\ x_2 = x_2 \\ x_3 = x_3 \end{cases}$$

eigenvector of $s = 3$ are $\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} \right\}$ #

$$J = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & -4 \\ 0 & 3 & 0 \\ -2 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$ #

(b) $\det\left(sI - \begin{bmatrix} 2 & 1 & 1 \\ 0 & 3 & 1 \\ 0 & -1 & 1 \end{bmatrix}\right) = s^3 - 6s^2 + 12s - 8$

$\Rightarrow s = 2, 2, 2.$ #

For $S=2$,

$$2I - \begin{bmatrix} 2 & 1 & 1 \\ 0 & 3 & 1 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & -1 \\ 0 & -1 & -1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 & -1 & | & 0 \\ 0 & -1 & -1 & | & 0 \\ 0 & 1 & 1 & | & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 1 & 1 & | & 0 \\ 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix} \Rightarrow \begin{cases} x_1 = x_1 \\ x_2 = -x_3 \\ x_3 = x_3 \end{cases}$$

Select $v_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

$$\begin{bmatrix} 0 & -1 & -1 & | & 1 \\ 0 & -1 & -1 & | & 1 \\ 0 & 1 & 1 & | & -1 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 1 & 1 & | & -1 \\ 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix} \Rightarrow \begin{cases} x_1 = x_1 \\ x_2 = -1-x_3 \\ x_3 = x_3 \end{cases}$$

eigenvector of $S=2$ are $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ #

$$J = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 2 & 1 & 1 \\ 0 & 3 & 1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$ #

(c) $\det\left(sI - \begin{bmatrix} 7 & -11 & -30 \\ -14 & 13 & -43 \\ -8 & 10 & 29 \end{bmatrix}\right) = s^3 - 49s^2 + 707s + 1521$

$\Rightarrow S = -1.8933, 25.4467 \pm 12.4830i$ #

eigenvector of $S = -1.8933$ is $\begin{bmatrix} 0.7534 \\ 0.6573 \\ -0.017n \end{bmatrix}$ #

eigenvector of $S = 25.4467 \pm 12.4830i$ is $\begin{bmatrix} 0.2928 \pm 0.3453i \\ 0.7637 \\ -0.3164 \mp 0.3344i \end{bmatrix}$ #

$$J = \begin{bmatrix} -1.8933 & 0 & 0 \\ 0 & 25.4467+12.483i & 0 \\ 0 & 0 & 25.4467-12.483i \end{bmatrix}$$ #

(d) $\det\left(sI - \begin{bmatrix} \alpha & \omega \\ -\omega & \alpha \end{bmatrix}\right) = \begin{bmatrix} s-\alpha & -\omega \\ \omega & s-\alpha \end{bmatrix} = (s-\alpha)^2 + \omega^2 = 0$

$\Rightarrow S = \alpha \pm \omega i$ #

For $s = \alpha + \omega i$

$\begin{bmatrix} \alpha+\omega i & 0 \\ 0 & \alpha+\omega i \end{bmatrix} - \begin{bmatrix} \alpha & \omega \\ -\omega & \alpha \end{bmatrix} = \begin{bmatrix} \omega i & -\omega \\ \omega & \omega i \end{bmatrix}$

$\left[\begin{array}{cc|c} \omega i & -\omega & 0 \\ \omega & \omega i & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} -1 & -i & 0 \\ 1 & i & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & i & 0 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{cases} x_1 = -i\,x_2 \\ x_2 = x_2 \end{cases}$

eigenvector of $S = \alpha + \omega i$ is $\begin{bmatrix} -i \\ 1 \end{bmatrix}$ #

For $s = \alpha - \omega i$

$\begin{bmatrix} \alpha-\omega i & 0 \\ 0 & \alpha-\omega i \end{bmatrix} - \begin{bmatrix} \alpha & \omega \\ -\omega & \alpha \end{bmatrix} = \begin{bmatrix} -\omega i & -\omega \\ \omega & -\omega i \end{bmatrix}$

$\left[\begin{array}{cc|c} -\omega i & -\omega & 0 \\ \omega & -\omega i & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & -i & 0 \\ 1 & -i & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & -i & 0 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{cases} x_1 = i\,x_2 \\ x_2 = x_2 \end{cases}$

eigenvector of $S = \alpha - \omega i$ is $\begin{bmatrix} i \\ 1 \end{bmatrix}$ #

$J = \begin{bmatrix} -i & i \\ 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \alpha & \omega \\ -\omega & \alpha \end{bmatrix} \begin{bmatrix} -i & i \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} a+bi & 0 \\ 0 & a-bi \end{bmatrix}$ #

#2.

(a) $\int_0^{2\pi} \sin(t-x) f(x)\, dx = \int_0^{2\pi} [\sin(t)\cos(-x) + \cos(t)\sin(-x)] f(x)\, dx$

$= \sin t \int_0^{2\pi} \cos x\, f(x)\, dx - \cos t \int_0^{2\pi} \sin x\, f(x)\, dx$

$T = \begin{bmatrix} -\int_0^{2\pi} \sin x\, f(x)\,dx & 0 \\ 0 & \int_0^{2\pi} \cos x\, f(x)\, dx \end{bmatrix}$ #

(b)

$$\det\left(sI-T\right) = \det\left(\begin{bmatrix} s+\int_0^{2\pi} \sin x\, f(x)dx & 0 \\ 0 & s-\int_0^{2\pi} \cos x\, f(x)dx \end{bmatrix}\right)$$

$$= \left(s+\int_0^{2\pi} \sin x\, f(x)\, dx\right)\left(s-\int_0^{2\pi} \cos x\, f(x)\, dx\right)$$

$$\Rightarrow s = -\int_0^{2\pi} \sin x\, f(x)\, dx\, ,\ \int_0^{2\pi} \cos x\, f(x)\, dx \quad \#$$

For $s = -\int_0^{2\pi} \sin x\, f(x)\, dx$

$$\left(sI-T\right)v = 0$$

$$\left[\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & -\int_0^{2\pi} \sin x\, f(x)\, dx - \int_0^{2\pi} \cos x\, f(x)dx & 0 \end{array}\right] \Rightarrow \begin{cases} x_1 = x_1 \\ x_2 = 0 \end{cases}$$

eigenvector of $s = -\int_0^{2\pi} \sin x\, f(x)\, dx$ is $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\#$

For $s = \int_0^{2\pi} \cos x\, f(x)\, dx$

$$\left[\begin{array}{cc|c} \int_0^{2\pi} \sin x\, f(x)\, dx + \int_0^{2\pi} \cos x\, f(x)\, dx & 0 & 0 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{cases} x_1 = 0 \\ x_2 = x_2 \end{cases}$$

eigenvector of $s = \int_0^{2\pi} \cos x\, f(x)\, dx$ is $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\#$