#1

$$f(A) = \sum_{k=0}^{l} \alpha_k A^k = \sum_{k=0}^{l} \alpha_k (V\Lambda V^{-1})^k$$

$$= \sum_{k=0}^{l} \alpha_k V \Lambda^k V^{-1}$$

$$= \sum_{k=0}^{l} V(\alpha_k \Lambda^k) V^{-1}$$

$$= V\left(\sum_{k=0}^{l} \alpha_k \Lambda^k\right) V^{-1}$$

$$\Rightarrow \tilde{\Lambda} = \sum_{k=0}^{l} \alpha_k \Lambda^k$$

$$\Rightarrow \tilde{\lambda}_i = \sum_{k=0}^{l} \alpha_k \lambda_i^k = f(\lambda_i)$$ #

#2

$\because$ A and B is similar matrices

$$\therefore B = P^{-1}AP$$

$$= P^{-1}V\Lambda V^{-1}P$$

$$= \tilde{V} \Lambda \tilde{V}^{-1}$$

The eigenvalue of B will equal to the A

The eigenvector of B will equal to $P^{-1}v_i$ #

#3

#4 Let $x_1 \in S_1 \subseteq \mathbb{R}^n$, $x_2 \in S_2 \subseteq \mathbb{R}^n$

$\because$ S_1 and S_2 are orthogonal

$\therefore$ $\langle x_1, x_2 \rangle = 0$

If $x_1 = x_2$, then

$\langle x_1, x_1 \rangle = 0$

$\Rightarrow x_1 = 0_{\mathbb{R}^n}$ #

#5 $\phi(\lambda) = \det(A - \lambda I) = (\lambda_1 - \lambda)(\lambda_2 - \lambda)\cdots(\lambda_n - \lambda)$

$\Rightarrow \phi(0) = \det(A) = \lambda_1 \lambda_2 \cdots \lambda_n$ #