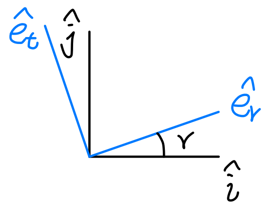
\# 1



$$V = \dot x \hat i + \dot y \hat j = V \hat e_r$$

$$a = \dot V = \ddot x \hat i + \ddot y \hat j = \dot V \hat e_r + \frac{V^2}{\rho} \hat e_t$$

$$\begin{bmatrix} \hat i \\ \hat j \end{bmatrix} = \begin{bmatrix} \cos\gamma & -\sin\gamma \\ \sin\gamma & \cos\gamma \end{bmatrix} \begin{bmatrix} \hat e_r \\ \hat e_t \end{bmatrix}$$

$$\Rightarrow m\ddot x \hat i + m\ddot y \hat j = m\dot V \hat e_r + m\frac{V^2}{\rho}\hat e_t$$

$$\Rightarrow -L\sin\gamma\, \hat i + (L\cos\gamma - mg)\hat j$$

$$= -L\sin\gamma(\cos\gamma\, \hat e_r - \sin\gamma\, \hat e_t) + (L\cos\gamma - mg)(\sin\gamma\, \hat e_r + \cos\gamma\, \hat e_t)$$

$$= (\cancel{-L\sin\gamma\cos\gamma} + \cancel{L\sin\gamma\cos\gamma} - mg\sin\gamma)\hat e_r + (\underbrace{L\sin^2\gamma + L\cos^2\gamma}_{=L} - mg\cos\gamma)\hat e_t$$

$$= -mg\sin\gamma\, \hat e_r + (L - mg\cos\gamma)\hat e_t = m\dot V \hat e_r + m\frac{V^2}{\rho}\hat e_t$$

$$\Rightarrow \begin{cases} \dot V = -g\sin\gamma \\ \dfrac{V^2}{\rho} = \dfrac{L}{m} - g\cos\gamma \end{cases}$$

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\# 2

state : V

control : γ

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\# 3.

Lagrange : $\min \; -\int_{t_0}^{t_f} V\cos\gamma \, dt$

s.t.

$\dot V = -g\sin\gamma \;,\; V(t_0) = V_0$

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Mayer : $\min \; -x(t_f) + x(0)$

s.t.

$\dot V = -g\sin\gamma \;,\; V(t_0) = V_0$

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#4. Lagrange: $H = -V\cos\gamma + \lambda(-g\sin\gamma)$

$$\dot{V} = -g\sin\gamma, \quad V(t_0) = V_0 \qquad — ①$$

$$\dot{\lambda} = -H_V = \cos\gamma \qquad — ②$$

$$H_\gamma = V\sin\gamma - \lambda g\cos\gamma = 0 \qquad — ③$$

$$\lambda(t_f) = 0 \qquad — ④$$

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Mayer: $H = \lambda(-g\sin\gamma)$

$$\dot{V} = -g\sin\gamma, \quad V(t_0) = V_0$$

$$\dot{\lambda} = -H_V = 0$$

$$H_\gamma = -\lambda g\cos\gamma = 0$$

$$\lambda(t_f) - \phi_V(V(t_f)) = 0$$

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#5 ③ $\Rightarrow \lambda = \frac{V}{g}\tan\gamma$

$$\overset{\frac{d}{dt}(\cdot)}{\Rightarrow} \dot{\lambda} = \frac{\dot{V}}{g}\tan\gamma + \frac{V}{g}\sec^2\gamma \cdot \dot{\gamma}$$

$$= \frac{-g\sin\gamma}{g}\frac{\sin\gamma}{\cos\gamma} + \frac{V}{g}\frac{\dot{\gamma}}{\cos^2\gamma} = \cos\gamma$$

$$\Rightarrow \cos^2\gamma + \sin^2\gamma = \frac{V}{g}\frac{\dot{\gamma}}{\cos\gamma} = 1$$

$$\Rightarrow \dot{\gamma} = \frac{g\cos\gamma}{V}$$

$$\frac{dV}{d\gamma} = \frac{\dot{V}}{\dot{\gamma}} = \frac{-g\sin\gamma}{\frac{g\cos\gamma}{V}} \Rightarrow \frac{1}{V}dV = \frac{-\sin\gamma}{\cos\gamma}d\gamma$$

$$\Rightarrow \ln V = \ln\cos\gamma + \ln k = \ln k\cos\gamma$$

$$\Rightarrow V = k\cos\gamma$$