# Digital Control

## HW2

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$$\ddot{y} + a_1\dot{y} + a_0 y = b_1\dot{u} + b_0 u \Rightarrow \ddot{y} = -a_1\dot{y} - a_0 y + b_1\dot{u} + b_0 u$$

Define :

$$x_1 = y \qquad\qquad \dot{x}_1 = \dot{y}$$

$$x_2 = \dot{y} - b_1 u \quad \Rightarrow \quad \dot{x}_2 = \ddot{y} - b_1\dot{u} = -a_1\dot{y} - a_0 y \cancel{+ b_1\dot{u}} + b_0 u \cancel{- b_1\dot{u}}$$

$$= -a_1\dot{y} - a_0 y + b_0 u$$

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \dot{y} \\ -a_1\dot{y} - a_0 y + b_0 u \end{bmatrix} = \begin{bmatrix} x_2 \\ -a_1 x_2 - a_0 x_1 + b_0 u \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ b_0 \end{bmatrix} u \quad \#$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \#$$

#1

Chosing the gain of $p$ and $K_p$ on Matlab. And the resule is

$P = 2.88$ , $K_p = 5.32$

$$G_{p_f} = 5.32 \frac{S+1}{S+2.88} \quad \#$$

Poles : $-1.99$ , $-0.694 \pm 0.924j$

$$T = \frac{1}{0.694} = 1.4409 \text{ sec}$$

$$\zeta = \frac{0.694}{\sqrt{0.694^2 + 0.924^2}} = 0.6006$$

# 2

Calculate the phase margin of original system.

$$\left|\frac{0.5}{-\omega_m^2+j0.5\omega_m}\right| = 1 \Rightarrow \frac{0.5}{\sqrt{\omega_m^4+\frac{1}{4}\omega_m^2}} = 1 \Rightarrow \omega_m = 0.6248$$

$$PM = 180° - \measuredangle G(j\omega_m) = 180° + \measuredangle\left(\frac{0.5}{-0.3904+j0.3124}\right)$$

$$= 180° + 0° - \tan^{-1}\left(\frac{0.3124}{-0.3904}\right) = 38.667°$$

The required phase compensation is

$$\phi_m = 70° - 38.667° + 5° = 36.333°$$

$$\sin\phi_m = \frac{\alpha-1}{\alpha+1} \Rightarrow \alpha = 3.9077$$

$$\begin{cases} \dfrac{\omega_2}{\omega_1} = 3.9077 \\ \sqrt{\omega_1\omega_2} = \omega_m = 0.6248 \end{cases} \Rightarrow \omega_2 = 3.9077\,\omega_1$$

$$\Rightarrow \sqrt{3.9077\,\omega_1^2} = 0.6248$$

$$\Rightarrow \omega_1 = 0.3161$$

$$\omega_2 = 1.2352$$

$$G_C = 3.9077\frac{s+0.3161}{s+1.2352}$$

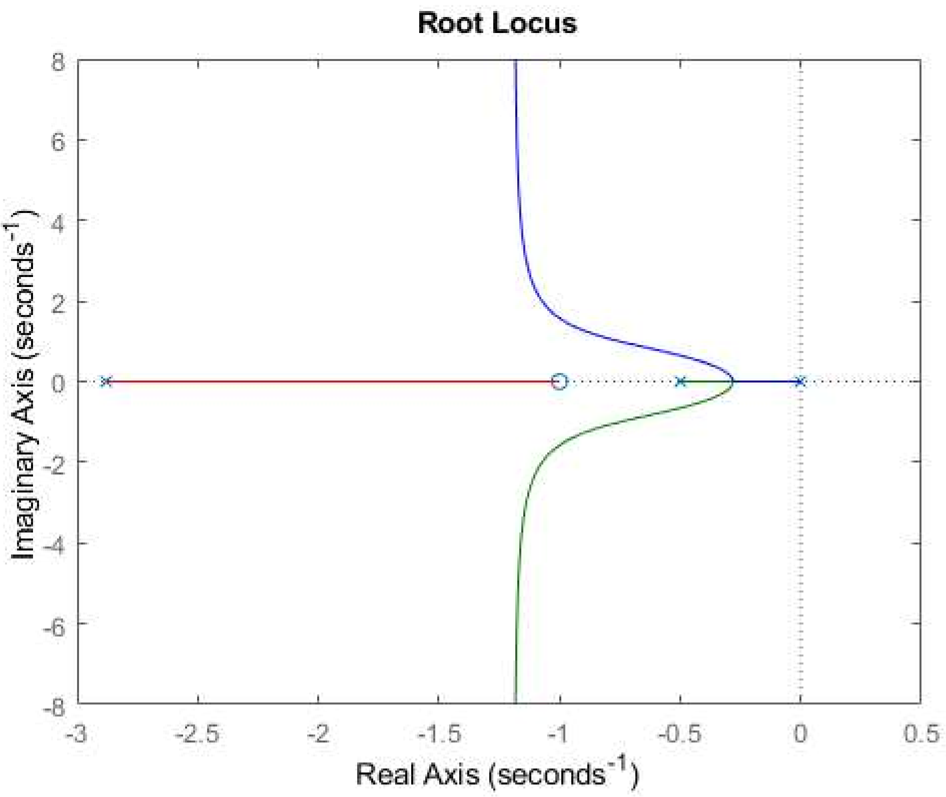
#

## 1 Plot the root locus and calculate the result

```
clear;clc;close all
p = 2.88;
Kp = 5.32;
G = tf(0.5, [1 0.5 0]);
Gp = tf([1 1], [1 p]);
rlocus(G*Gp)
damp(feedback(Kp*Gp*G,1))
```

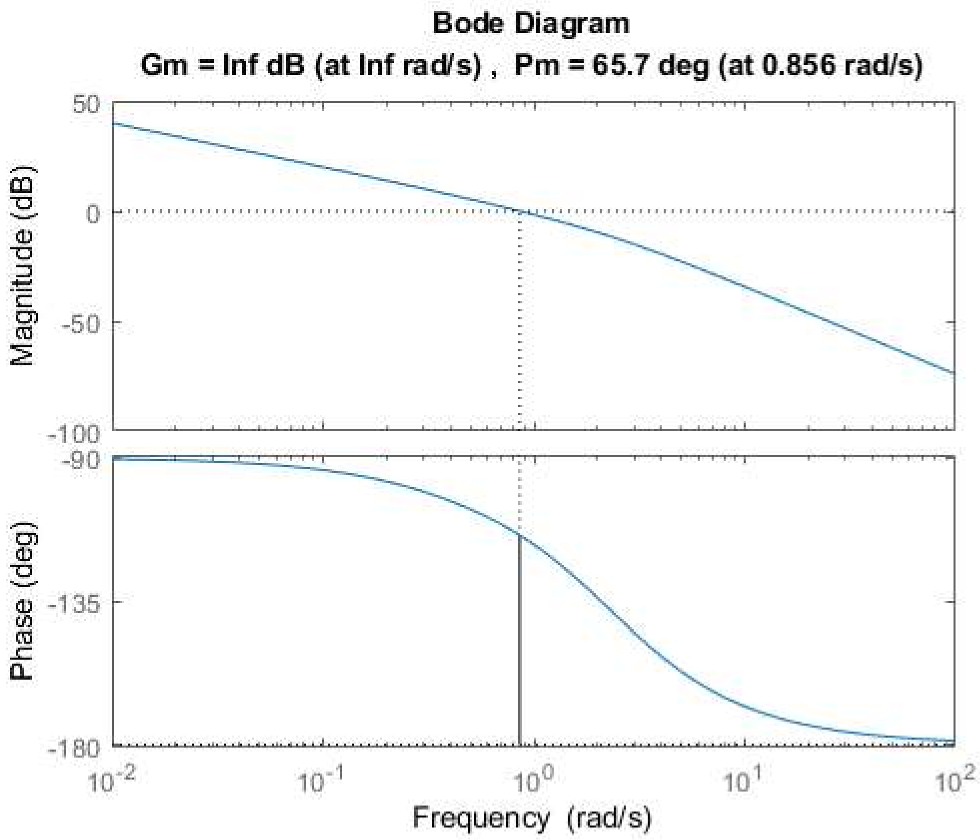
```
        Pole              Damping       Frequency      Time Constant
                                      (rad/seconds)      (seconds)

 -6.94e-01 + 9.24e-01i    6.01e-01       1.16e+00         1.44e+00
 -6.94e-01 - 9.24e-01i    6.01e-01       1.16e+00         1.44e+00
 -1.99e+00                1.00e+00       1.99e+00         5.02e-01
```




## 2 Calculate the phase margin with phase lead controller

```
clear;clc;close all
G = tf(0.5, [1 0.5 0]);
Gc = 3.9077*tf([1 0.5622],[1 2.1969]);
margin(G*Gc)
```


Bode Diagram
Gm = Inf dB (at Inf rad/s) , Pm = 65.7 deg (at 0.856 rad/s)
Magnitude (dB)
50
0
-50
-100
Phase (deg)
-90
-135
-180
$10^{-2}$
$10^{-1}$
$10^{0}$
$10^{1}$
$10^{2}$
Frequency (rad/s)