# Optimal Control

HW6

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#1.

$$\min \; -x(t_f)$$

s.t.

$$\dot{x} = x + \sin u, \quad x(0) = 0$$ #

$$H = \lambda(x + \sin u), \quad \phi(x) = -x, \quad \psi(x) = 0$$

$$\begin{cases} \dot{x} = x + \sin u & — ① \\ \dot{\lambda} = -\lambda & — ② \\ H_u = \lambda \cos u = 0 & — ③ \\ \lambda(t_f) + 1 = 0 & — ④ \end{cases}$$

From ③

$$\lambda \cos u = 0 \Rightarrow \cos u(t) = 0$$

$$\Rightarrow \sin u(t) = 1$$ #

From ①

$$\dot{x} = x + \sin u = x + 1$$

$$\frac{dx}{dt} = x + 1 \Rightarrow \int_0^{x(t)} \frac{1}{x+1}\, dx = \int_0^t dt$$

$$\Rightarrow \ln(x+1)\Big|_0^{x(t)} = t\Big|_0^t$$

$$\Rightarrow \ln(x(t)+1) - \ln(1) = t - 0$$

$$\Rightarrow x(t) + 1 = e^t \Rightarrow x(t) = e^t - 1$$ #

## #2

```
clear;clc;close All
% Define u(t) is a constant A
init_A = 1;

[A, Jmin] = fminsearch(@fminfunc, init_A);
u = A
Xmax = -Jmin

function J = fminfunc(A)
    EOM = @(t, x) x+sin(A);
    [~, x] = ode45(EOM, [0 2], 0);
    J = -x(end);

end
```

```
u = 1.5708
```

```
Xmax = 6.3891
```

## #3

```
clear;clc;close all
init_lambda = 1;
options = optimoptions('fsolve','Display','off');

lambda0 = fsolve(@forwardfunc, init_lambda, options)
[t, state] = ode45(@ODE, [0 2], [0; lambda0]);
Xmax = state(end,1)

function F = forwardfunc(lambda0)
    [~, state] = ode45(@ODE, [0 2], [0; lambda0]);
    F = state(end,2) - (-1);

end
```

```
lambda0 = -7.3891
```

```
Xmax = 6.3891
```

## #4

```
clear;clc;close all
init_xf = 7;
options = optimoptions('fsolve','Display','off');

xf = fsolve(@backwardfunc, init_xf, options);
Xmax = xf

function F = backwardfunc(xf)
    [~, state] = ode45(@ODE, [2 0], [xf; -1]);
    F = state(end,1) - 0;

end
```

```
Xmax = 6.3891
```

## functiuon for necessary condition

```
function dstate = ODE(~, state)
    % state: [x; lambda]
    u = acos(0);
    dstate = zeros(2,1);
    dstate(1) = state(1)+sin(u);
    dstate(2) = -1*state(2);

end
```