Optimal Control

Midterm

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I did not offer assistances to nor receive assistances from others in this exam.

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#1 (a)

$$Z = \frac{-1 \pm \sqrt{1^2 - 4\cdot 3\cdot 3}}{2\cdot 3} = \frac{-1 \pm \sqrt{-35}}{6} = \frac{-1}{6} \pm i\frac{\sqrt{35}}{6}$$ #

(b)

$$3(a+bi)^2 + (a+bi) + 3 = 0$$

$$\Rightarrow 3(a^2 - b^2 + 2abi) + (a+bi) + 3 = 0$$

$$\Rightarrow \begin{cases} 3a^2 - 3b^2 + a + 3 = 0 \\ 6ab + b = 0 \end{cases}$$ #

(c)

$$f = \begin{bmatrix} 3a^2 - 3b^2 + a + 3 \\ 6ab + b \end{bmatrix}, \quad J = \begin{bmatrix} 6a+1 & -6b \\ 6b & 6a+1 \end{bmatrix}$$

$$X_0 = \begin{bmatrix} r \\ 0 \end{bmatrix}$$

$$X_1 = \begin{bmatrix} r \\ 0 \end{bmatrix} - \begin{bmatrix} 6r+1 & 0 \\ 0 & 6r+1 \end{bmatrix}^{-1} \begin{bmatrix} r \\ 0 \end{bmatrix} = \begin{bmatrix} r - \frac{r}{(6r+1)^2} \\ 0 \end{bmatrix}$$

After the iteration, the image term will not be change. So that, the start point can NOT converge to the solution. #

(d) The result by Newton method is same to the #1(a) #

#1(d)

```
clear;clc;close all
syms a b
f = [3*a^2-3*b^2+a+3; 6*a*b+b];
df = [diff(f(1), a), diff(f(1), b)
      diff(f(2), a), diff(f(2), b)];

x = [1; 1];
stop = false;
iter_no = 0;

while ~stop
    delta = -inv(double(subs(df, [a, b], x')))*double(subs(f, [a, b], x'));

    x = x + delta;
    iter_no = iter_no + 1;

    if iter_no >= 100
        stop = true;
    end
end
```

```
x =

   -0.1667
    0.9860
```

#2 (a)

$$\min \ -V\sin\gamma$$

s.t.

$$T\cos\alpha - D - mg\sin\gamma = 0$$

$$T\sin\alpha + L - mg\cos\gamma = 0$$

#

(b) Let $x = \begin{bmatrix} V \\ \alpha \\ \gamma \end{bmatrix}$

$$H = -V\sin\gamma + \lambda_1(T\cos\alpha - D - mg\sin\gamma) + \lambda_2(T\sin\alpha + L - mg\cos\gamma)$$

$$\begin{aligned} H_x = \Big[& -\sin\gamma + \lambda_1\left[-\rho V S(C_{D_0} + C_{D\alpha}\alpha^2)\right] + \lambda_2\left[\rho V S(C_{L_0} + C_{L\alpha}\alpha)\right] \\ & \lambda_1(-T\sin\alpha - \rho V^2 S C_{D\alpha}\alpha) + \lambda_2\left(T\cos\alpha + \tfrac{1}{2}\rho V^2 S C_{L\alpha}\right) \\ & -V\cos\gamma + \lambda_1(-mg\cos\gamma) + \lambda_2(mg\sin\gamma) \Big] = 0 \end{aligned}$$

$$H_{xx} = \begin{bmatrix} -\lambda_1 \rho S(C_{D_0} + C_{D\alpha}\alpha^2) + \lambda_2 \rho S(C_{L_0} + C_{L\alpha}\alpha) & \lambda_1(-2\rho V S C_{D\alpha}\alpha) + \lambda_2(\rho V S C_{L\alpha}) & -\cos\gamma \\ \lambda_1(-2\rho V S C_{D\alpha}\alpha) + \lambda_2(\rho V S C_{L\alpha}) & \lambda_1(-T\cos\alpha - \rho V^2 S C_{D\alpha}) + \lambda_2(-T\sin\alpha) & 0 \\ -\cos\gamma & 0 & V\sin\gamma + \lambda_1(mg\sin\gamma) + \lambda_2(mg\cos\gamma) \end{bmatrix} \geq 0$$

#

#2(b)

```
clear;clc;close all
syms V alpha gamma real
syms T mg lambda1 lambda2 real

ddiff = @(f, x, y) diff(diff(f, x), y);

syms rho S CL_0 CD_0 CL_alpha CD_alpha real
L = 0.5*rho*V^2*S*(CL_0+CL_alpha*alpha);
D = 0.5*rho*V^2*S*(CD_0+CD_alpha*alpha^2);

H = - V*sin(gamma) + lambda1*(T*cos(alpha)-D-mg*sin(gamma)) ...
    + lambda2*(T*sin(alpha)+L-mg*cos(gamma));
Hx = [diff(H, V) diff(H, alpha) diff(H, gamma)];
Hxx = [ddiff(H, V, V)     ddiff(H, V, alpha)     ddiff(H, V, gamma)
       ddiff(H, alpha, V) ddiff(H, alpha, alpha) ddiff(H, alpha, gamma)
       ddiff(H, gamma, V) ddiff(H, gamma, alpha) ddiff(H, gamma, gamma)];
```

#2(c)

```
clc;close all
mg_ = 95000*9.81;   S_ = 153;
rho_ = 0.7782;      T_ = 200000;
CL_0_ = 0.3;        CL_alpha_ = 0.1;
CD_0_ = 0.07351;    CD_alpha_ = 0.01;

Hx = subs(Hx, [mg S rho CL_0 CD_0 CL_alpha CD_alpha T], ...
             [mg_ S_ rho_ CL_0_ CD_0_ CL_alpha_ CD_alpha_ T_]);
Hxx = subs(Hxx, [mg S rho CL_0 CD_0 CL_alpha CD_alpha T], ...
               [mg_ S_ rho_ CL_0_ CD_0_ CL_alpha_ CD_alpha_ T_]);

eqn = @(x) double(subs(Hx, [V alpha gamma lambda1 lambda2], x));

opts = optimoptions(@fsolve,'Algorithm', 'levenberg-marquardt', 'Display', 'off');
x = fsolve(eqn, [100 10 20 1 0], opts)

x =

   99.9864   10.4153   19.7082    0.3536    0.3053
```

#3 (a)

$$\min \quad J=\int_0^\infty x^2+\rho(kx)^2\,dt=\int_0^\infty (1+\rho k^2)x^2\,dt$$

s.t.

$$\dot{x}=ax+b(-kx)=(a-bk)x \quad , \quad x(0)=x_0$$

$$\dot{x}-(a-bk)x=0 \Rightarrow sX-x_0-(a-bk)X=0$$

$$\Rightarrow X=\frac{x_0}{s-(a-bk)}$$

$$\Rightarrow x(t)=x_0 e^{-(a-bk)t}$$

$$\int_0^\infty (1+\rho k^2)\left[x_0 e^{-(a-bk)t}\right]^2 dt=(1+\rho k^2)x_0^2\int_0^\infty e^{-2(a-bk)t}\,dt$$

$$=(1+\rho k^2)x_0^2\left(\frac{1}{-2(a-bk)}\,e^{-2(a-bk)t}\Big|_0^\infty\right)$$

$$=\frac{(1+\rho k^2)x_0^2}{-2(a-bk)}\left(e^{-\infty}-e^0\right)=\frac{(1+\rho k^2)x_0^2}{2(a-bk)}$$ #

(b)

$$\frac{\partial J}{\partial k}=\frac{(2\rho k)x_0^2}{2(a-bk)}+\frac{(1+\rho k^2)x_0^2}{[2(a-bk)]^2}(2b)$$

$$=\frac{x_0^2\left[4\rho k(a-bk)+(1+\rho k^2)(2b)\right]}{[2(a-bk)]^2}$$

$$=\frac{x_0^2\left[-2b\rho k^2+4a\rho k+2b\right]}{4(a-bk)^2}=0$$

$$\Rightarrow x_0^2\left(-b\rho k^2+2a\rho k+b\right)=0$$

$$\Rightarrow b\rho k^2-2a\rho k+b=0$$

$$\Rightarrow k=\frac{-2a\rho\pm\sqrt{(2a\rho)^2-4b^2\rho}}{2b\rho}=\frac{a\rho\pm\sqrt{a^2\rho^2-b^2\rho}}{b\rho}$$ #