Optimal Control

HW5

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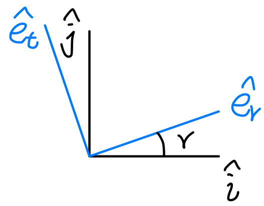

$$V = \dot{x}\hat{i} + \dot{y}\hat{j} = V\hat{e}_r$$

$$a = \dot{V} = \ddot{x}\hat{i} + \ddot{y}\hat{j} = \dot{V}\hat{e}_r + \frac{V^2}{\rho}\hat{e}_t$$

$$\begin{bmatrix}\hat{i}\\ \hat{j}\end{bmatrix} = \begin{bmatrix}\cos\gamma & -\sin\gamma\\ \sin\gamma & \cos\gamma\end{bmatrix}\begin{bmatrix}\hat{e}_r\\ \hat{e}_t\end{bmatrix}$$

$$\Rightarrow m\ddot{x}\hat{i} + m\ddot{y}\hat{j} = m\dot{V}\hat{e}_r + m\frac{V^2}{\rho}\hat{e}_t$$

$$\Rightarrow -L\sin\gamma\,\hat{i} + (L\cos\gamma - mg)\hat{j}$$

$$= -L\sin\gamma(\cos\gamma\,\hat{e}_r - \sin\gamma\,\hat{e}_t) + (L\cos\gamma - mg)(\sin\gamma\,\hat{e}_r + \cos\gamma\,\hat{e}_t)$$

$$= (\cancel{-L\sin\gamma\cos\gamma} + \cancel{L\sin\gamma\cos\gamma} - mg\sin\gamma)\hat{e}_r + (\underbrace{L\sin^2\gamma + L\cos^2\gamma}_{=L} - mg\cos\gamma)\hat{e}_t$$

$$= -mg\sin\gamma\,\hat{e}_r + (L - mg\cos\gamma)\hat{e}_t = m\dot{V}\hat{e}_r + m\frac{V^2}{\rho}\hat{e}_t$$

$$\Rightarrow \begin{cases}\dot{V} = -g\sin\gamma\\ \dfrac{V^2}{\rho} = \dfrac{L}{m} - g\cos\gamma\end{cases}$$

#

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state : V

control : γ #

3.

Lagrange : $\min\ -\int_{t_0}^{t_f} V\cos\gamma\, dt$

s.t.

$\dot{V} = -g\sin\gamma$, $V(t_0) = V_0$ #

Mayer : $\min\ -x(t_f) + x(0)$

s.t.

$\dot{V} = -g\sin\gamma$, $V(t_0) = V_0$ #

#4. Lagrange: $H = -V\cos\gamma + \lambda(-g\sin\gamma)$

$$\dot{V} = -g\sin\gamma, \quad V(t_0) = V_0 \qquad \text{— ①}$$

$$\dot{\lambda} = -H_V = \cos\gamma \qquad \text{— ②}$$

$$H_\gamma = V\sin\gamma - \lambda g\cos\gamma = 0 \qquad \text{— ③}$$

$$\lambda(t_f) = 0 \qquad \text{— ④}$$

#

Mayer: $H = \lambda(-g\sin\gamma)$

$$\dot{V} = -g\sin\gamma, \quad V(t_0) = V_0$$

$$\dot{\lambda} = -H_V = 0$$

$$H_\gamma = -\lambda g\cos\gamma = 0$$

$$\lambda(t_f) - \phi_V(V(t_f)) = 0$$

#

#5 ③ $\Rightarrow \lambda = \frac{V}{g}\tan\gamma$

$$\overset{\frac{d}{dt}(\cdot)}{\Rightarrow} \dot{\lambda} = \frac{\dot{V}}{g}\tan\gamma + \frac{V}{g}\sec^2\gamma \cdot \dot{\gamma}$$

$$= \frac{-g\sin\gamma}{g}\,\frac{\sin\gamma}{\cos\gamma} + \frac{V}{g}\,\frac{\dot{\gamma}}{\cos^2\gamma} = \cos\gamma$$

$$\Rightarrow \cos^2\gamma + \sin^2\gamma = \frac{V}{g}\,\frac{\dot{\gamma}}{\cos\gamma} = 1$$

$$\Rightarrow \dot{\gamma} = \frac{g\cos\gamma}{V}$$

$$\frac{dV}{d\gamma} = \frac{\dot{V}}{\dot{\gamma}} = \frac{-g\sin\gamma}{\frac{g\cos\gamma}{V}} \Rightarrow \frac{1}{V}dV = \frac{-\sin\gamma}{\cos\gamma}d\gamma$$

$$\Rightarrow \ln V = \ln\cos\gamma + \ln k = \ln k\cos\gamma$$

$$\Rightarrow V = k\cos\gamma$$

#6

```
clear;clc;close all
[time, state] = ode45(@ode, [0 10], [0; 0; 10; 0]);

figure(); hold on
plot(state(:,1), state(:,2))
grid on
xlabel("x"); ylabel("y")

function dstate = ode(~, state)
% Parameter define:
%   - state: [x; y; V; gamma]

g = 9.81;
dx = state(3)*cosd(state(4));
dy = state(3)*sind(state(4));

dstate = [dx; dy; -g*sind(state(4)); g*cosd(state(4))/state(3)];
end
```