I did not offer assistances to nor receive assistances from others in this exam.

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#1 (a)

$$z = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 3 \cdot 3}}{2 \cdot 3} = \frac{-1 \pm \sqrt{-35}}{6} = \frac{-1}{6} \pm i \frac{\sqrt{35}}{6}$$ #

(b)

$$3(a+bi)^2 + (a+bi) + 3 = 0$$

$$\Rightarrow 3(a^2 - b^2 + 2abi) + (a+bi) + 3 = 0$$

$$\Rightarrow \begin{cases} 3a^2 - 3b^2 + a + 3 = 0 \\ 6ab + b = 0 \end{cases}$$ #

(c)

$$f = \begin{bmatrix} 3a^2 - 3b^2 + a + 3 \\ 6ab + b \end{bmatrix}, \quad J = \begin{bmatrix} 6a+1 & -6b \\ 6b & 6a+1 \end{bmatrix}$$

$$x_0 = \begin{bmatrix} r \\ 0 \end{bmatrix}$$

$$x_1 = \begin{bmatrix} r \\ 0 \end{bmatrix} - \begin{bmatrix} 6r+1 & 0 \\ 0 & 6r+1 \end{bmatrix}^{-1} \begin{bmatrix} r \\ 0 \end{bmatrix} = \begin{bmatrix} r - \frac{r}{(6r+1)^2} \\ 0 \end{bmatrix}$$

After the iteration, the image term will not be change. So that, the start point can NOT converge to the solution. #

(d) The result by Newton method is same to the #1(a) #

#2 (a)

$$\min \; -V\sin\gamma$$

s.t.

$$T\cos\alpha - D - mg\sin\gamma = 0$$

$$T\sin\alpha + L - mg\cos\gamma = 0$$

#

(b) Let $x = \begin{bmatrix} V \\ \alpha \\ \gamma \end{bmatrix}$

$$H = -V\sin\gamma + \lambda_1\left(T\cos\alpha - D - mg\sin\gamma\right) + \lambda_2\left(T\sin\alpha + L - mg\cos\gamma\right)$$

$$H_x = \begin{bmatrix} -\sin\gamma + \lambda_1\left[-\rho V S\left(C_{D_0} + C_{D_\alpha}\alpha^2\right)\right] + \lambda_2\left[\rho V S\left(C_{L_0} + C_{L_\alpha}\alpha\right)\right] \\ \lambda_1\left(-T\sin\alpha - \rho V^2 S C_{D_\alpha}\alpha\right) + \lambda_2\left(T\cos\alpha + \frac{1}{2}\rho V^2 S C_{L_\alpha}\right) \\ -V\cos\gamma + \lambda_1\left(-mg\cos\gamma\right) + \lambda_2\left(mg\sin\gamma\right) \end{bmatrix} = 0$$

$$H_{xx} = \begin{bmatrix} -\lambda_1 \rho S\left(C_{D_0} + C_{D_\alpha}\alpha^2\right) + \lambda_2 \rho S\left(C_{L_0} + C_{L_\alpha}\alpha\right) & \lambda_1\left(-2\rho V S C_{D_\alpha}\alpha\right) + \lambda_2\left(\rho V S C_{L_\alpha}\right) & -\cos\gamma \\ \lambda_1\left(-2\rho V S C_{D_\alpha}\alpha\right) + \lambda_2\left(\rho V S C_{L_\alpha}\right) & \lambda_1\left(-T\cos\alpha - \rho V^2 S C_{D_\alpha}\right) + \lambda_2\left(-T\sin\alpha\right) & 0 \\ -\cos\gamma & 0 & V\sin\gamma + \lambda_1\left(mg\sin\gamma\right) + \lambda_2\left(mg\cos\gamma\right) \end{bmatrix} \geq 0$$

#

#3 (a) $\min \quad J=\int_0^\infty x^2+\rho(kx)^2\,dt=\int_0^\infty(1+\rho k^2)x^2\,dt$

s.t.

$$\dot{x}=ax+b(-kx)=(a-bk)x \quad , \quad x(0)=x_0$$

$$\dot{x}-(a-bk)x=0 \Rightarrow sX-x_0-(a-bk)X=0$$

$$\Rightarrow X=\frac{x_0}{s-(a-bk)}$$

$$\Rightarrow x(t)=x_0e^{-(a-bk)t}$$

$$\int_0^\infty(1+\rho k^2)\left[x_0e^{-(a-bk)t}\right]^2dt=(1+\rho k^2)x_0^2\int_0^\infty e^{-2(a-bk)t}\,dt$$

$$=(1+\rho k^2)x_0^2\left(\frac{1}{-2(a-bk)}\ e^{-2(a-bk)t}\Big|_0^\infty\right)$$

$$=\frac{(1+\rho k^2)x_0^2}{-2(a-bk)}\left(e^{-\infty}-e^0\right)=\frac{(1+\rho k^2)x_0^2}{2(a-bk)} \quad \#$$

(b) $$\frac{\partial J}{\partial k}=\frac{(2\rho k)x_0^2}{2(a-bk)}+\frac{(1+\rho k^2)x_0^2}{[2(a-bk)]^2}(2b)$$

$$=\frac{x_0^2\left[4\rho k(a-bk)+(1+\rho k^2)(2b)\right]}{[2(a-bk)]^2}$$

$$=\frac{x_0^2\left[-2b\rho k^2+4a\rho k+2b\right]}{4(a-bk)^2}=0$$

$$\Rightarrow x_0^2\left(-b\rho k^2+2a\rho k+b\right)=0$$

$$\Rightarrow b\rho k^2-2a\rho k+b=0$$

$$\Rightarrow k=\frac{-2a\rho\pm\sqrt{(2a\rho)^2-4b^2\rho}}{2b\rho}=\frac{a\rho\pm\sqrt{a^2\rho^2-b^2\rho}}{b\rho} \quad \#$$