Digital Control

HW9

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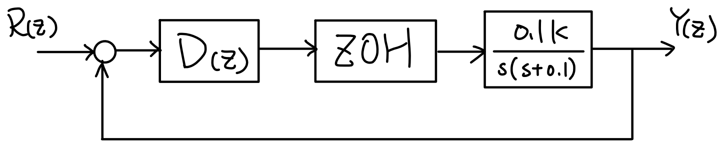


$$\begin{cases} P.O. < 17\% \\ t_s \le 10 \end{cases} \qquad T = 0.1\ \text{sec}$$

From fig. 7.4, we can know required of P.O. < 17% could be meet the required of $\zeta \ge 0.5$

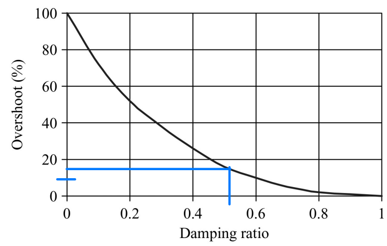


Figure 7.4 Variation of overshoot with damping ratio

For required of settling time $t_s \le 10$ sec.

$$t_s = \frac{4.6}{\zeta\omega_n} \le 10\ \text{sec}$$

The requirement to meet the goal can be determined by following equation.

$$\begin{cases} \zeta \ge 0.7 \\ \frac{4.6}{\zeta\omega_n} \le 10 \end{cases} \Rightarrow \begin{cases} \zeta \ge 0.7 \\ \zeta\omega_n \ge 0.46 \end{cases}$$

Aside:

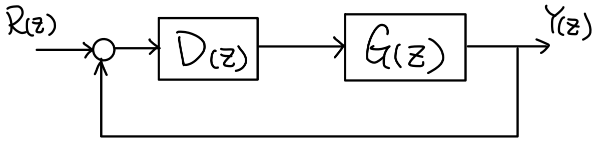
$\zeta\omega_n \ge 0.46 \Rightarrow \omega_n \ge 0.575$

Choose $\zeta = 0.8$, $\omega_n = 20$

$$\begin{aligned} s &= -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} \\ &= -16 \pm j12 \end{aligned}$$

$$\begin{aligned} \Rightarrow z &= e^{sT} \\ &= e^{(-16 \pm j12)\times 0.1} \\ &= e^{-1.6}(\cos 1.2 \pm j\sin 1.2) \\ &= 0.0732 \pm j0.1882 \end{aligned}$$

$\longleftarrow$ Desired poles



$$G(s) = \frac{1-e^{-sT}}{s}\,\frac{0.1K}{s(s+0.1)} = (1-e^{-sT})\frac{0.1K}{s^2(s+0.1)}$$

$$G(z) = \mathcal{Z}\{G(s)\} = (1-z^{-1})\,\mathcal{Z}\left\{\frac{0.1K}{s^2(s+0.1)}\right\}$$

$$= K(1-z^{-1})\,\mathcal{Z}\left\{\frac{0.1}{s^2(s+0.1)}\right\}$$

$$= K(1-z^{-1})\,\mathcal{Z}\left\{\frac{A}{s^2}+\frac{B}{s}+\frac{C}{s+0.1}\right\}$$

$$= K(1-z^{-1})\,\mathcal{Z}\left\{\frac{1}{s^2}+\frac{-10}{s}+\frac{10}{s+0.1}\right\}$$

$$= K(1-z^{-1})\left[\frac{Tz}{(z-1)^2}+\frac{-10z}{z-1}+\frac{10z}{z-e^{-aT}}\right],\quad \begin{cases} a=0.1 \\ T=0.1 \end{cases}$$

$$= \frac{K\cancel{(z-1)}}{\cancel{z}}\,\frac{\cancel{z}\left[(T+10e^{-aT}-10)z+10-(T+10)e^{-aT}\right]}{(z-1)^{\cancel{2}}(z-e^{-aT})}$$

$$= \frac{K(0.0004983z+0.0004967)}{(z-1)(z-0.99)}$$

Aside:

$$\frac{A}{s^2}+\frac{B}{s}+\frac{C}{s+0.1} = \frac{0.1}{s^2(s+0.1)}$$

A:

$$\xRightarrow{\times s^2} A+Bs+\frac{C}{s+0.1}s^2 = \frac{0.1}{s+0.1}$$

$$\xRightarrow{\lim_{s\to 0}(\cdot)} A+0+0 = \frac{0.1}{0.1} \Rightarrow A=1$$

B:

$$\xRightarrow{\times s^2} A+Bs+\frac{C}{s+0.1}s^2 = \frac{0.1}{s+0.1}$$

$$\xRightarrow{\frac{d}{ds}(\cdot)} 0+B+\frac{-Cs^2}{(s+0.1)^2}+\frac{2Cs}{s+0.1} = \frac{-0.1}{(s+0.1)^2}\times 1$$

$$\xRightarrow{\lim_{s\to 0}(\cdot)} B+0+0 = \frac{-0.1}{0.1^2} \Rightarrow B=-10$$

C:

$$\xRightarrow{\times(s+0.1)} \frac{A}{s^2}(s+0.1)+\frac{B}{s}(s+0.1)+C = \frac{0.1}{s^2}$$

$$\xRightarrow{\lim_{s\to -0.1}(\cdot)} 0+0+C = \frac{0.1}{(-0.1)^2} \Rightarrow C=10$$

Aside:

$$Tz(z-e^{-aT})-10z(z-1)(z-e^{-aT})+10z(z-1)^2$$

$$\xRightarrow{\div z} T(z-e^{-aT})-10(z-1)(z-e^{-aT})+10(z-1)^2$$

$$\Rightarrow Tz-Te^{-aT}-10\left[z^2-(1+e^{-aT})z+e^{-aT}\right]+10(z^2-2z+1)$$

$$\Rightarrow (-10+10)z^2+\left[T+10(1+e^{-aT})-20\right]z-(T+10)e^{-aT}+10$$

$$\Rightarrow (T+10e^{-aT}-10)z+\left[10-(T+10)e^{-aT}\right]$$

The character equation

$$\Delta(z) = 1+D(z)G(z) = 0$$

$$\Rightarrow 1+\frac{z-n}{z-p}\,\frac{K(0.0004983z+0.0004967)}{(z-1)(z-0.99)} = 0$$

$$\Rightarrow \frac{(z-n)(\overbrace{0.0004983}^{\alpha}z+\overbrace{0.0004967}^{\beta})}{(z-p)(z-1)(z-0.99)} = \frac{-1}{K}$$

Substitude $z = 0.0732+j\,0.1882$ (Desired pole) in to character equation, and then find the phase angle of the system.

$$\Rightarrow \tan^{-1}\left(\frac{0.1882}{0.0732-n}\right)+\tan^{-1}\left(\frac{0.1882\alpha}{0.0732\alpha+\beta}\right)-\tan^{-1}\left(\frac{0.1882}{0.0732-p}\right)-\tan^{-1}\left(\frac{0.1882}{0.0732-1}\right)-\tan^{-1}\left(\frac{0.1882}{0.0732-0.99}\right) = \tan^{-1}\left(\frac{-1}{K}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{0.1882}{0.0732-n}\right)+9.9757^\circ-\tan^{-1}\left(\frac{0.1882}{0.0732-p}\right)-168.5213^\circ-168.3995^\circ = -180^\circ$$

$$\Rightarrow \tan^{-1}\left(\frac{0.1882}{0.0732-n}\right) - \tan^{-1}\left(\frac{0.1882}{0.0732-p}\right) = 146.9451^\circ$$

Choose $n = 0.92$

$$\tan^{-1}\left(\frac{0.1882}{0.0732-0.92}\right) - \tan^{-1}\left(\frac{0.1882}{0.0732-p}\right) = 146.9451^\circ$$

$$\Rightarrow \tan^{-1}\left(\frac{0.1882}{0.0732-p}\right) = \tan^{-1}\left(\frac{0.1882}{0.0732-0.92}\right) - 146.9451^\circ = 20.5249^\circ$$

$$\Rightarrow \frac{0.1882}{0.0732-p} = 0.3744 \Rightarrow p = -0.4295$$

$$\Rightarrow D(z) = \frac{z-0.92}{z+0.4295}$$

Substitude the n, p and desired pole into character equation

$$\Delta(z) = \frac{(z-0.92)(0.0004983z+0.0004967)}{(z+0.4295)(z-1)(z-0.99)} = \frac{-1}{K}$$

$$\|\cdot\| \Rightarrow \left\| -0.0010 + j\,6.56\times10^{-36} \right\| = \left\| \frac{-1}{K} \right\|$$

$$\Rightarrow 0.0010 = \frac{1}{K} \Rightarrow K = 1000$$

Analysis the system by Matlab

$$P.O. = 12.8888\%$$

$$t_s = 2.9 \text{ sec} \quad \#$$