Aerodynamics of Lifting Surfaces

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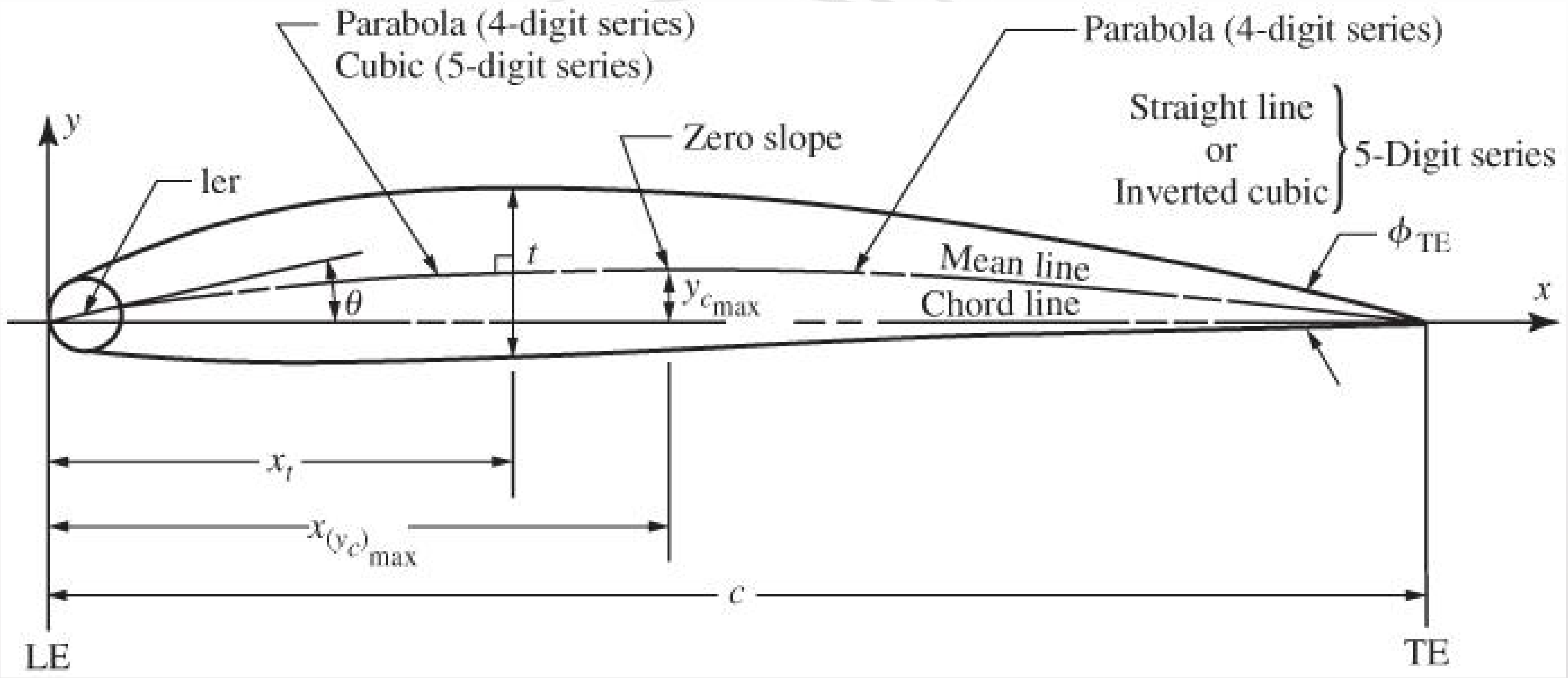
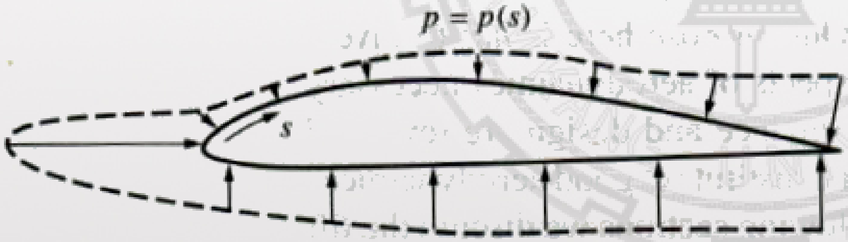
2-D Airfoil cross-section geometry

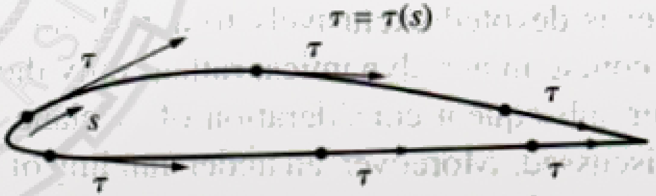


Aerodynamic Lift, Drag, and Moment

The net aerodynamic force on the body is due to the pressure and shear stress distributions integrated over the total exposed surface area.

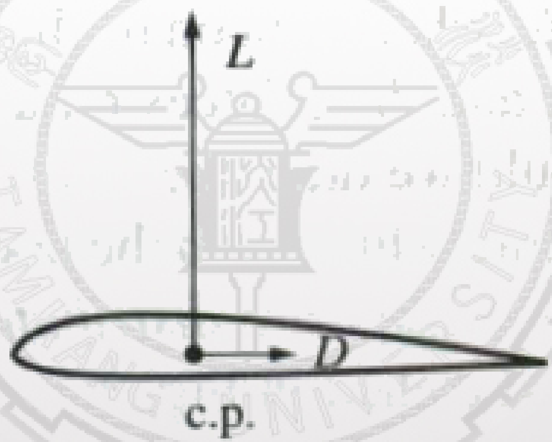


(a) Pressure distribution (schematic only; distorted for clarity)

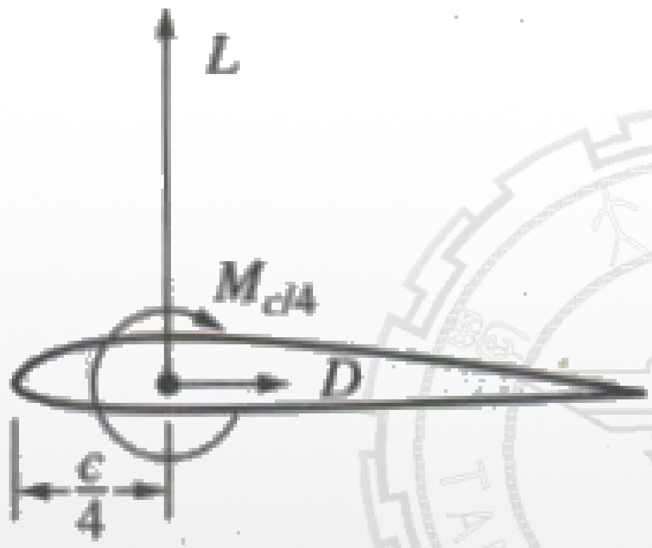


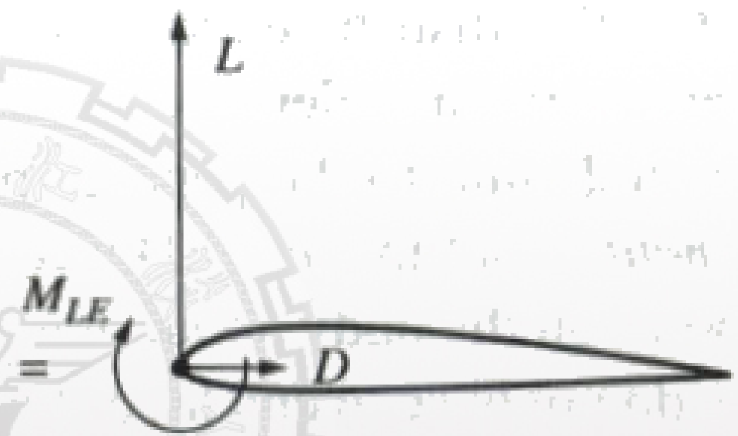
(b) Shear stress distribution

There are three ways of representing the actual distributed load exerted by the pressure and shear stress on the surface of the airfoil by a concentrated force at a point and the moment at that point.



a. Concentrated force acting through the *center of pressure.*





b. Concentrated force acting through the quarter-chord point, plus the moment about the quarter point.

c. Concentrated force acting through the leading edge, plus the moment about the leading edge.

Aerodynamic Coefficients

Intuition, the aerodynamic force on a body depends on:

- the *velocity* of the body through the air,
- the *density* of the ambient air,
- the *size* of the body,
- the *orientation* of the body relative to the free-stream direction,
- the ambient *coefficient of viscosity*,
- the *compressibility* of the air.

Hence, we can state the following relations for lift, drag and moments of a body of given shape:

$$L = L(\rho_\infty, V_\infty, S, \alpha, \mu_\infty, a_\infty)$$
$$D = D(\rho_\infty, V_\infty, S, \alpha, \mu_\infty, a_\infty)$$
$$M = M(\rho_\infty, V_\infty, S, \alpha, \mu_\infty, a_\infty)$$

Defining the lift, drag and moment coefficients for a given body denoted by C_L, C_D and C_M respectively:

$$C_L = \frac{L}{q_\infty S}, \quad C_D = \frac{D}{q_\infty S}, \quad C_M = \frac{M}{q_\infty Sc},$$

where q_∞ is the dynamic pressure, defined as

$$q_\infty = \frac{1}{2}\rho V_\infty^2$$

and c is a *characteristic length* of a body (for an airfoil, the usual choice for c is the *chord length*).

Also defining the *similarity parameters, Reynolds number* (based on chord length) and *Mach number*, respectively:

$$\mathrm{Re} = \frac{\rho_\infty V_\infty c}{\mu_\infty}, \quad M_\infty = \frac{V_\infty}{a_\infty}$$

The *method of dimensional analysis* leads to the following result. For the given body shape, we have

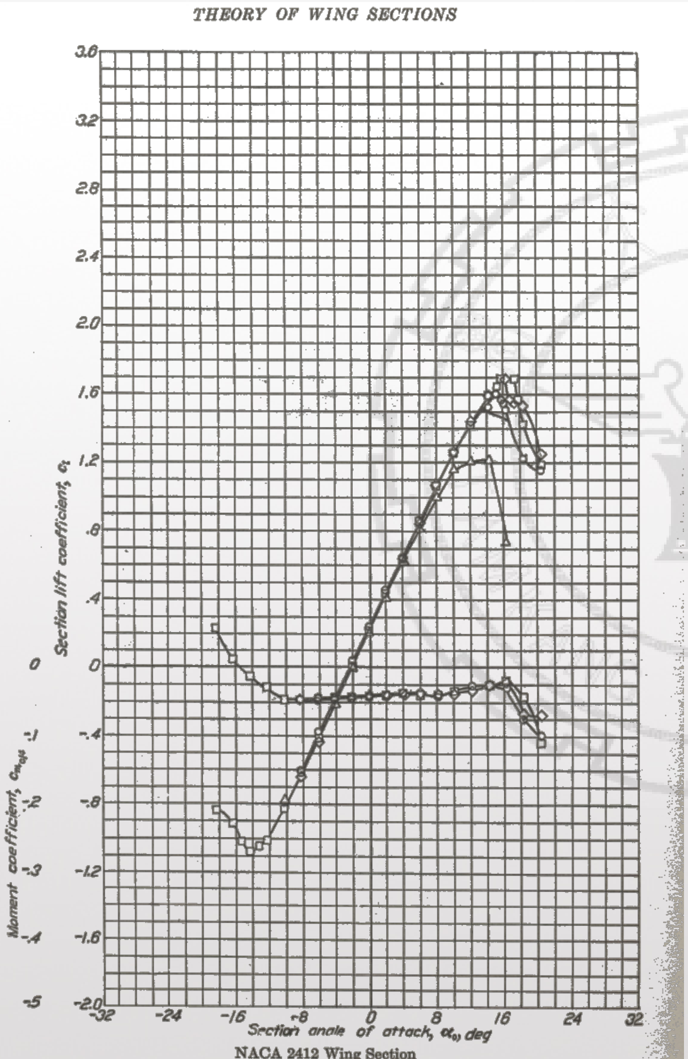
$$C_L = C_L(\alpha, \mathrm{Re}, M_\infty),$$
$$C_D = C_D(\alpha, \mathrm{Re}, M_\infty),$$
$$C_M = C_M(\alpha, \mathrm{Re}, M_\infty).$$

Lift, Drag, and Moment Coefficients: How They Vary

Example: Data for the NACA 2412 airfoil.

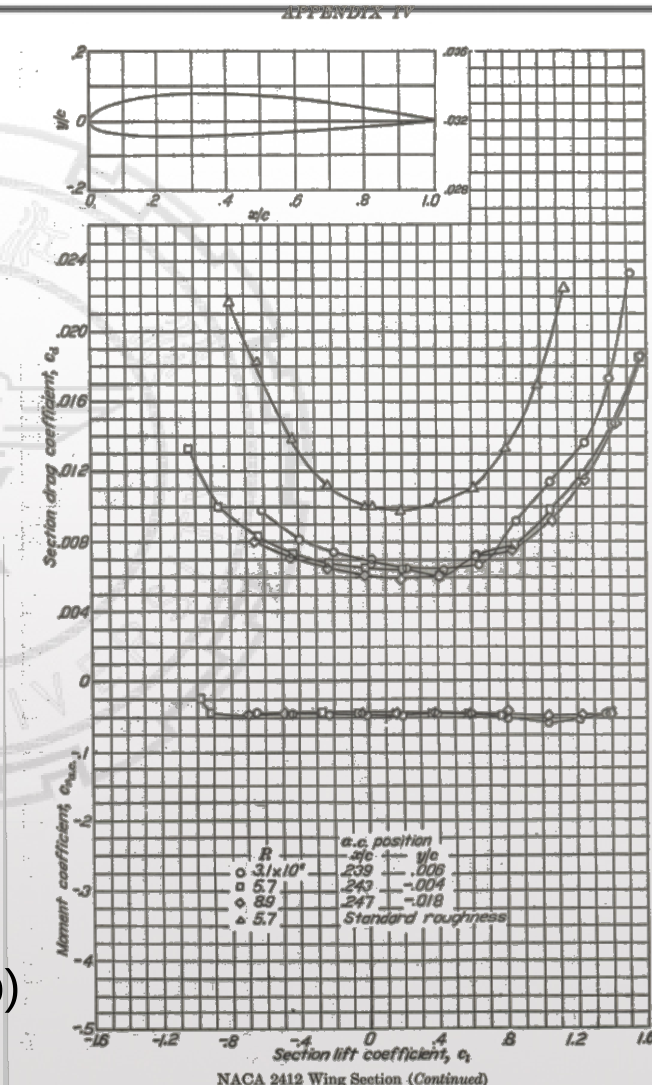
(a) Lift coefficient and moment coefficient about the quarter-chord versus angle of attack.
(b) Drag coefficient and moment coefficient about the *aerodynamic center* as a function of the lift coefficient.

(a)

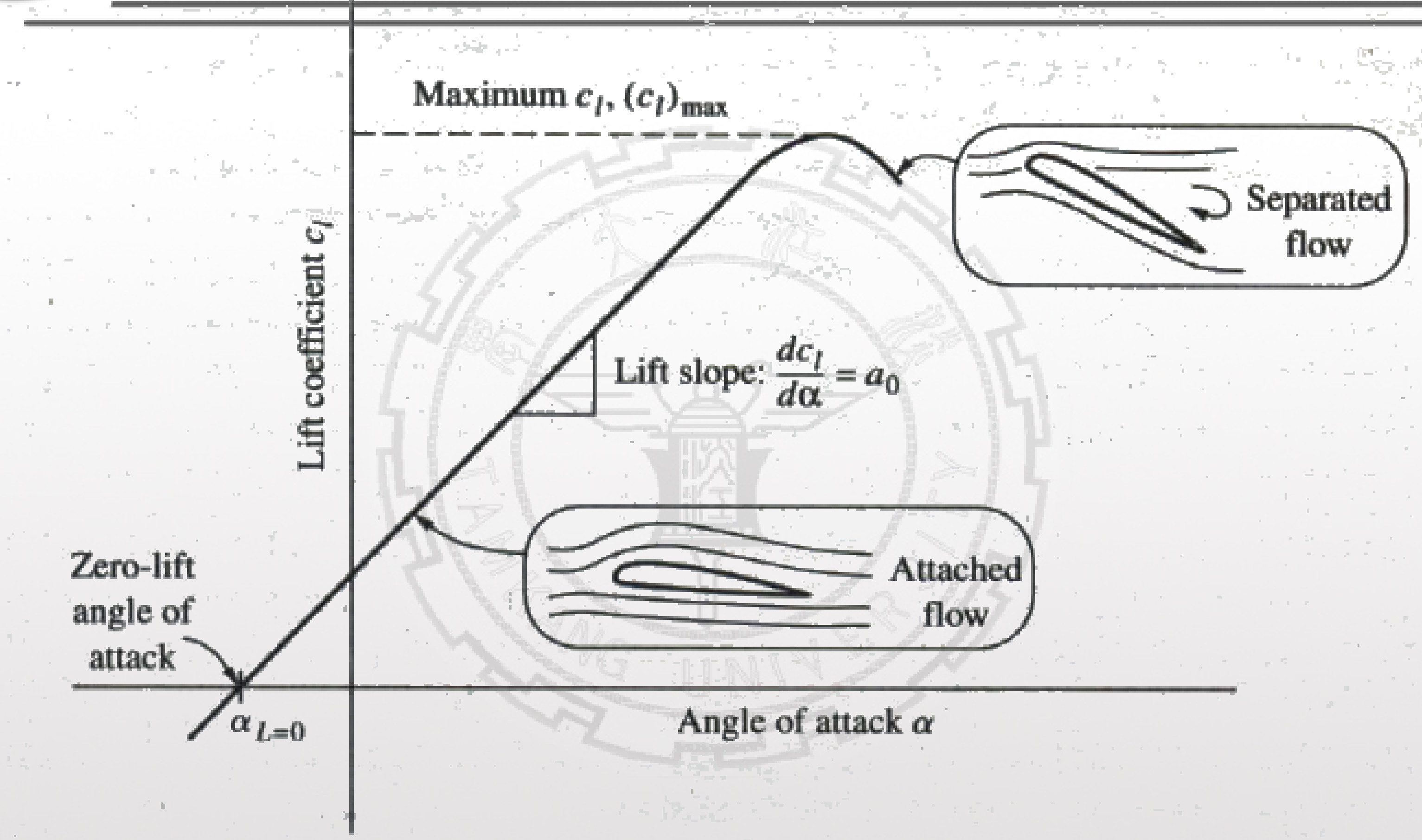


THEORY OF WING SECTIONS
Section lift coefficient, c_l
Moment coefficient, c_m c/4
Section angle of attack, α_0, deg
NACA 2412 Wing Section

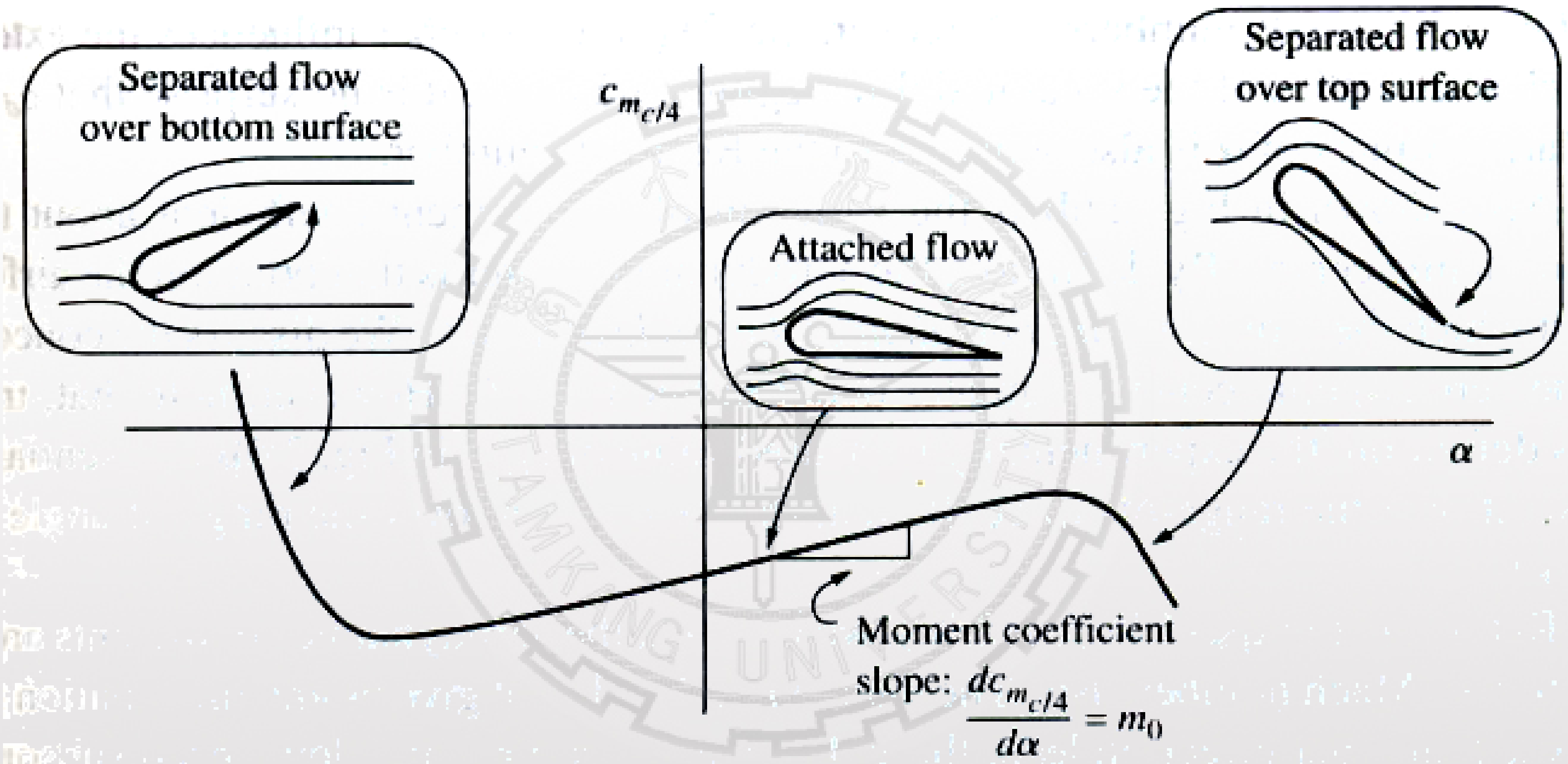
(b)



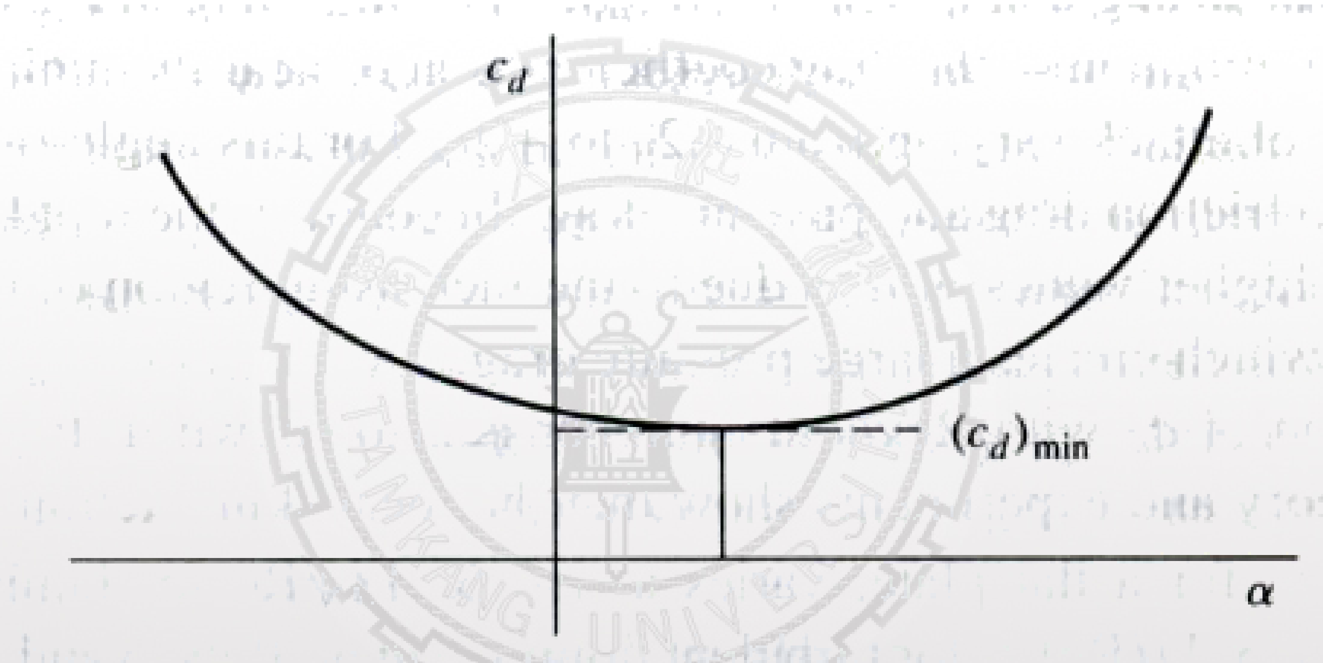
APPENDIX IV
y/c
x/c
Section drag coefficient, c_d
Moment coefficient, c_m a.c.
a.c. position
R x/c y/c
○ 3.1×10^6 .239 .006
□ 5.7 .243 -.004
◇ 8.9 .247 -.018
△ 5.7 Standard roughness
Section lift coefficient, c_l
NACA 2412 Wing Section (Continued)



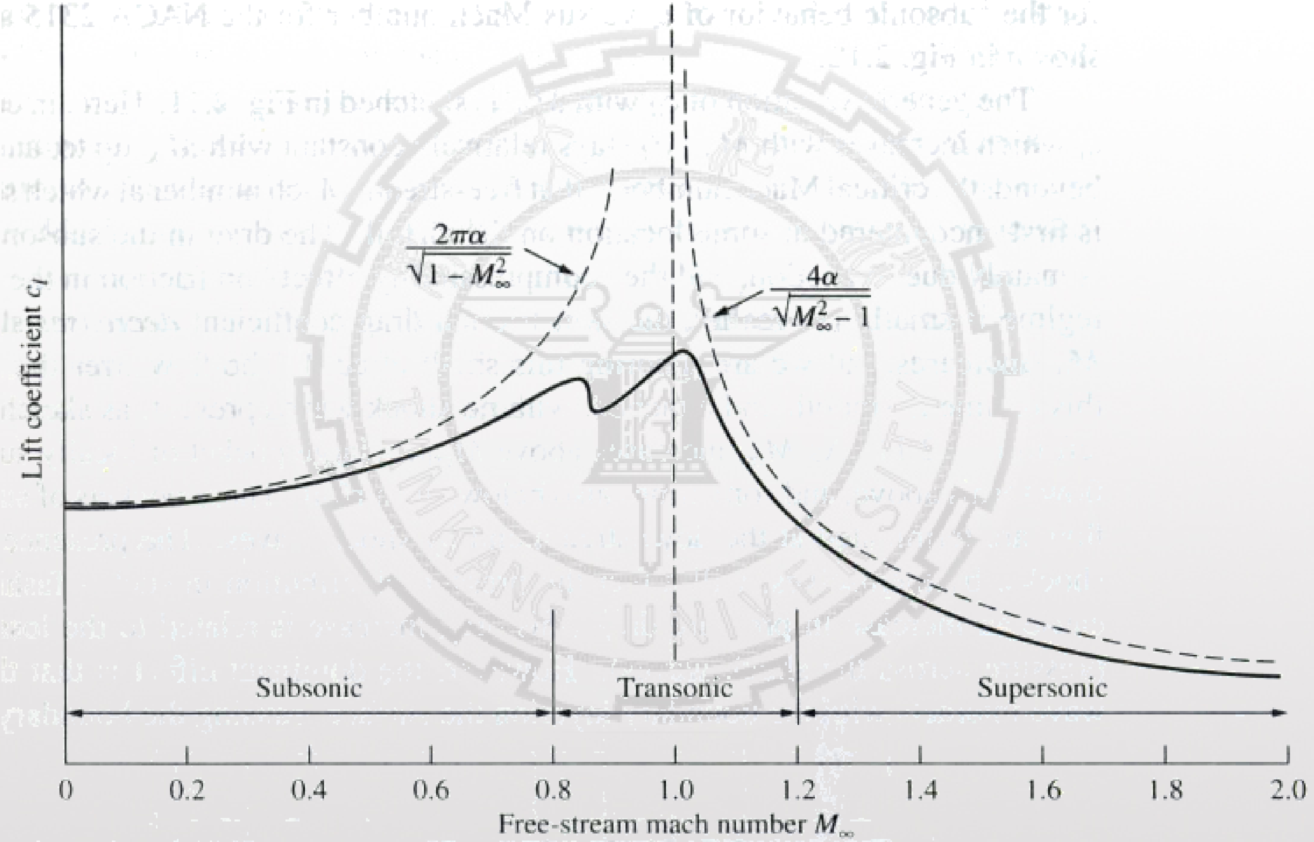
Sketch of a generic lift curve



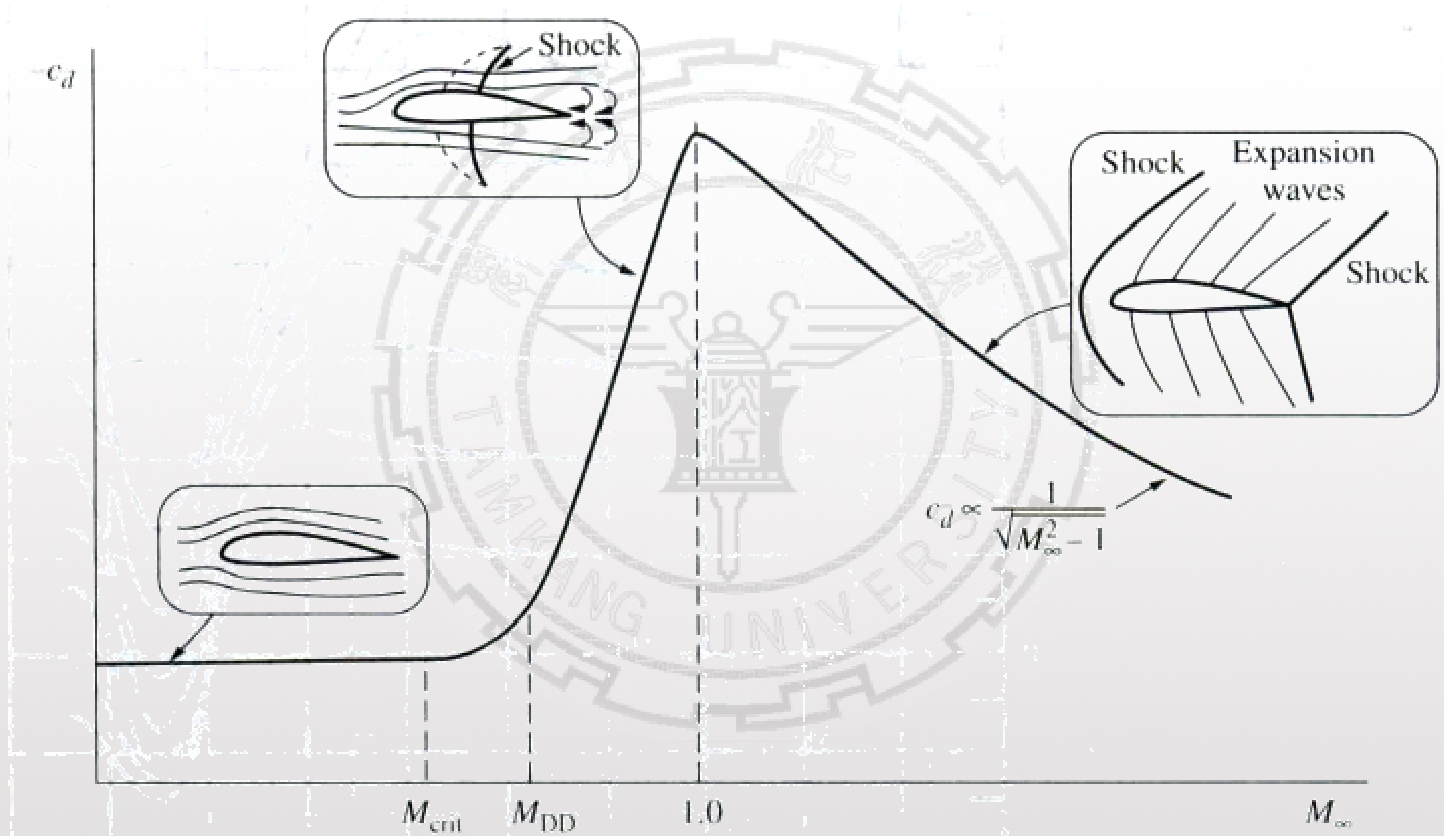
Sketch of a generic moment curve.



Sketch of a generic drag curve.



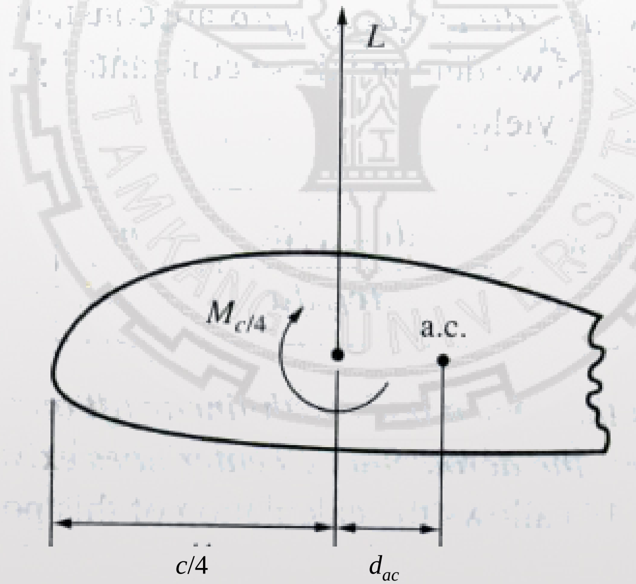
Sketch of a generic lift coefficient variation with Mach number.



Sketch of a generic drag coefficient variation with Mach number.

The Aerodynamic Center

The aerodynamic center is the point on a body about which the moments are independent of the angle of attack.



$$x_{ac} = c/4 + d_{ac}$$

Assuming the point is located a distance $d_{a.c.}$ from the quarter-chord. Taking moments about the point a.c, we have

$$M_{a.c.} = Ld_{a.c.} + M_{c/4}$$

Dividing the last equation by $q_\infty Sc$, we have

$$\frac{M_{a.c.}}{q_\infty Sc} = \frac{L}{q_\infty S}\left(\frac{d_{a.c.}}{c}\right) + \frac{M_{c/4}}{q_\infty Sc}$$

or

$$c_{m_{a.c.}} = c_l\left(\frac{d_{a.c.}}{c}\right) + c_{m_{c/4}}$$

Differentiating the last equation with respect to angle of attack α gives

$$\frac{dc_{m_{a.c.}}}{d\alpha} = \frac{dc_l}{d\alpha}\left(\frac{d_{a.c.}}{c}\right) + \frac{dc_{m_{c/4}}}{d\alpha}$$

By the definition of aerodynamic center, if the point exist, we have

$$0 = \frac{dc_l}{d\alpha}\left(\frac{d_{a.c.}}{c}\right) + \frac{dc_{m_{c/4}}}{d\alpha}$$

This results

$$\frac{d_{a.c.}}{c} = -\frac{dc_{m_{c/4}} / d\alpha}{dc_l / d\alpha} = -\frac{m_0}{a_0}$$

Example – For the NACA 2412 airfoil, calculate the location of the aerodynamic center and $c_{m_{ac}}$

From the airfoil data of NACA 2412 (for example on page 12), we can find a_0 and m_0 as follows. First examining the lift coefficient curve, we can read off the following data:

$$\text{At } \alpha = -8^o,\ c_l = -0.6; \quad \text{at } \alpha = 8^o,\ c_l = 1.08$$

Hence,

$$a_0 = \frac{dc_l}{d\alpha} = \frac{1.08 - (-0.6)}{8^o - (-8^o)} = 0.105$$

Examining the moment coefficient curve, we can read off the following data:

At $\alpha = -8^o$, $c_{m_{c/4}} = -0.045$; at $\alpha = 10^o$, $c_{m_{c/4}} = -0.035$

Hence,

$$m_0 = \frac{dc_{m_{c/4}}}{d\alpha} = \frac{-0.035-(-0.045)}{10^o-(-8^o)} = 5.56\times10^{-4}$$

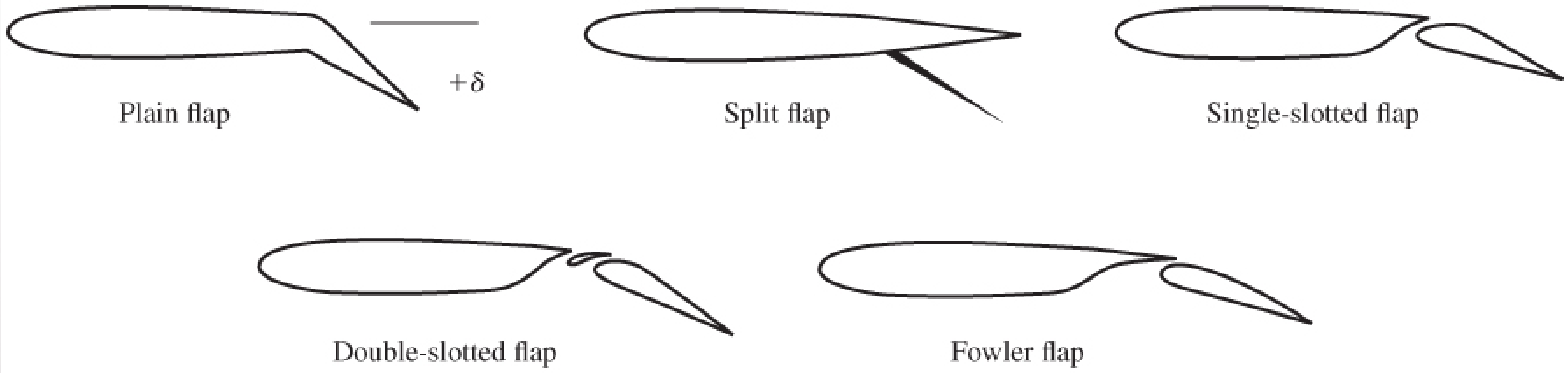
Thus, from the equation of calculating the position of aerodynamic center

$$\frac{d_{a.c.}}{c} = -\frac{dc_{m_{c/4}}/d\alpha}{dc_l/d\alpha} = -\frac{m_0}{a_0} = -\frac{5.56\times10^{-4}}{0.105} = -0.0053$$

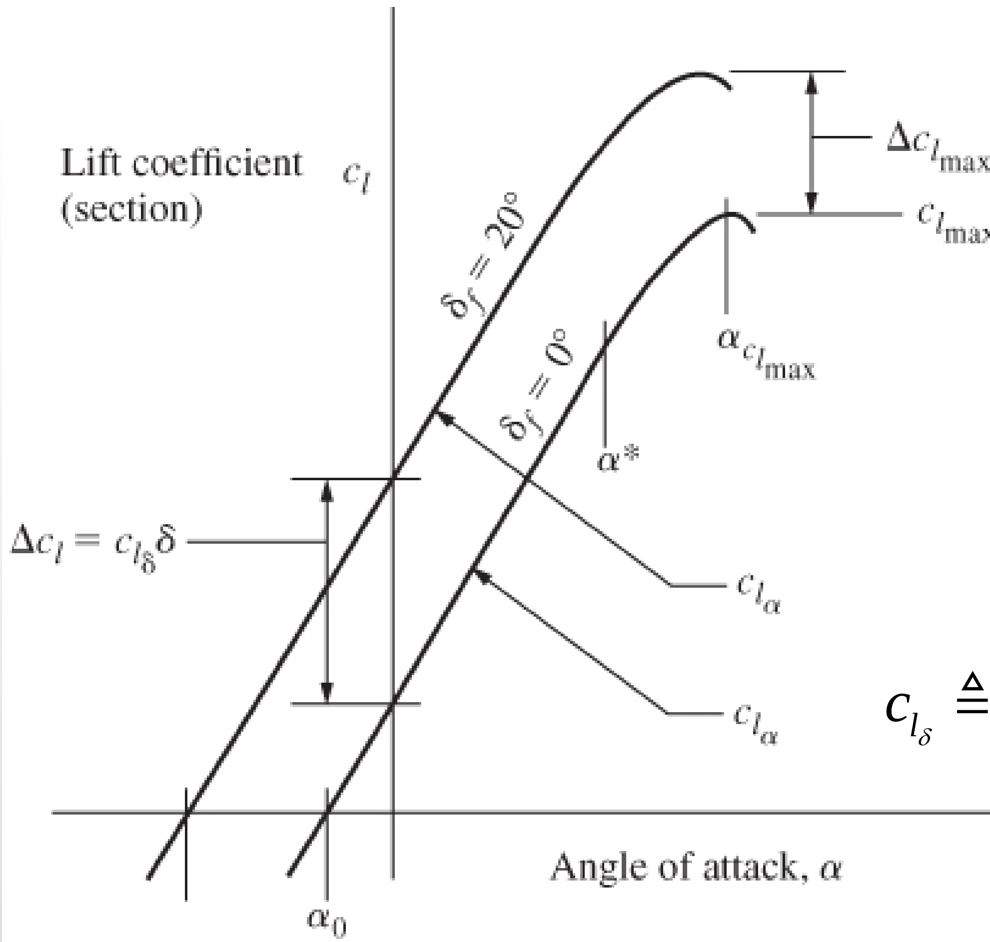
The aerodynamic center is located 0.53% of the chord length ahead of the quarter chord point.

$$c_{m_{a.c.}} = c_l\left(\frac{d_{a.c.}}{c}\right) + c_{m_{c/4}} = (-0.6)(-0.0053) + (-0.045) = -0.04182$$

Effects of Flaps on Subsonic Airfoil Section Characteristics



Examples of common types of flaps



Another parameter is the change of angle of attack per unit flap deflection

$$\alpha_\delta \triangleq \frac{\partial \alpha}{\partial \delta}\Big|_{c_l=\text{constant}}$$

Refer to the figure

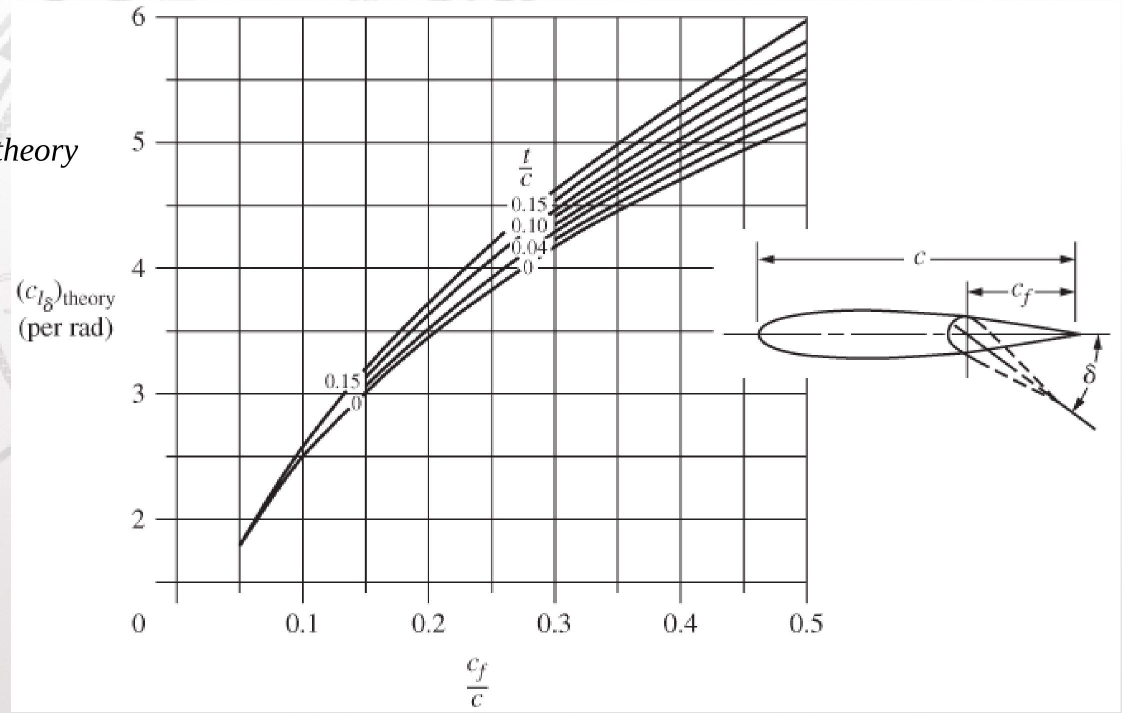
$$\alpha_\delta = \frac{\Delta\alpha_0}{20}$$

$$c_{l_\delta} \triangleq \frac{\partial c_l}{\partial \delta}\Big|_{\alpha=\text{constant}}$$

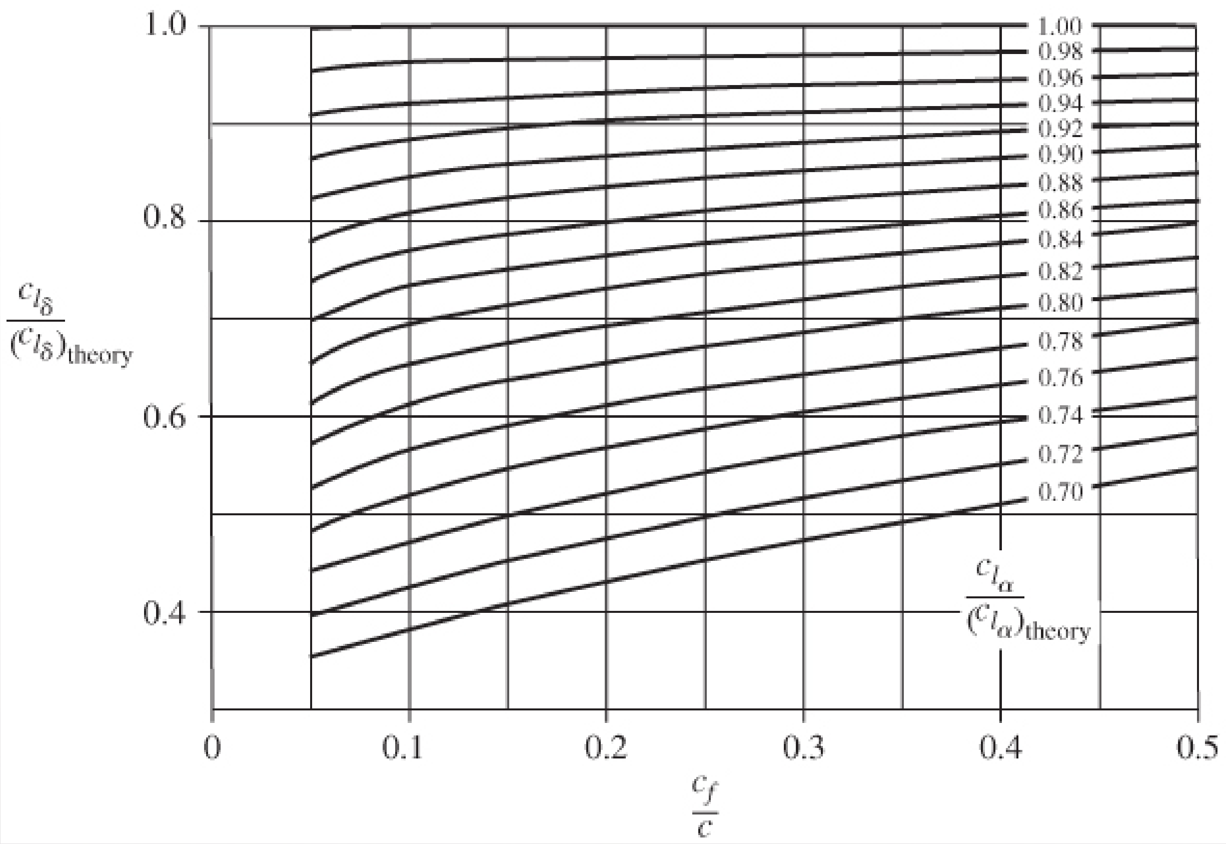
Effect of high-lift devices on section lift

For plain flaps in subsonic flow, the sectional flap effectiveness may be estimated as

$$c_{l_\delta} = \frac{1}{\beta}\left(\frac{c_{l_\delta}}{c_{l_\delta}|_{theory}}\right)c_{l_\delta}|_{theory}$$

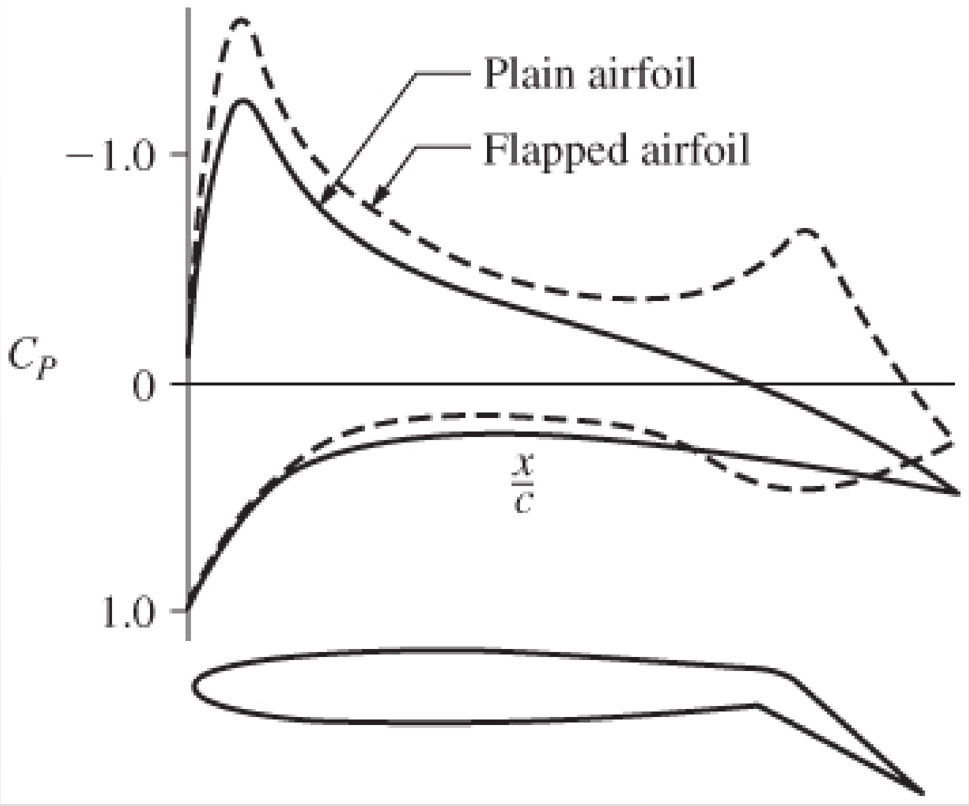


Theoretical section lift effectiveness of plain flaps



Empirical correction for plain flap section lift effectiveness

Flap deflection also affects the pitching moment of airfoil section, due to the change of pressure distribution:



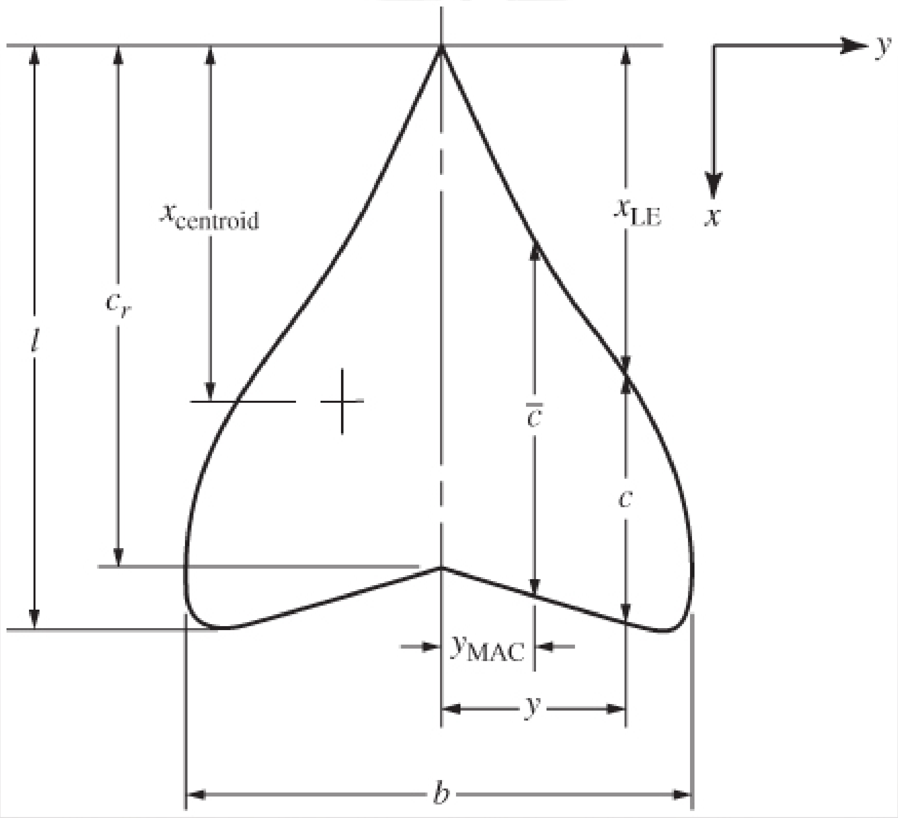
According to thin-airfoil theory, the change in moment due to deflection of a plain flap is given by

$$c_{m_\delta} = -2\sqrt{\frac{c_f}{c}\left(1-\frac{c_f}{c}\right)^3}\ /\ rad$$

Although the magnitude of the moment is affected bt flap deflection, the slope of the moment with angle of attack, c_{m_α} , is not appreciably affected. Hence, the location of the section's aerodynamic center, x_{ac} , is not appreciably affected by flap deflection.

Wing Planform Characteristics

- Lift for a Finite Wing



Finite-wing geometry

Variable	Definition
A	Aspect ratio $= b^2/S$
b	Wing span
$b/(2l)$	Wing-slenderness parameter
c	Chord (parallel to axis of symmetry) at any given span station y
$\bar{c}$	Mean aerodynamic chord (MAC) (or mean geometric chord) $\bar{c} = \frac{2}{S}\int_0^{b/2} c^2\, dy$
c_r	Root chord
l	Over-all length from wing apex to most aft point on trailing edge
p	Planform-shape parameter $= S/(bl)$
S	Wing area $= 2\int_0^{b/2} c\, dy$ Note: projected or planform area
x_{LE}	Chordwise location of leading edge at span station y
$x_{centroid}$	Chordwise location of centroid of area (chordwise distance from apex to $\bar{c}/2$) $x_{centroid} = \frac{2}{S}\int_0^{b/2} c\left(x_{LE} + \frac{c}{2}\right)dy$
y	General span station measured perpendicular to plane of symmetry
y_{MAC}	spanwise location of MAC (equivalent to spanwise location of centroid of area) $y_{MAC} = \frac{2}{S}\int_0^{b/2} cy\, dy$

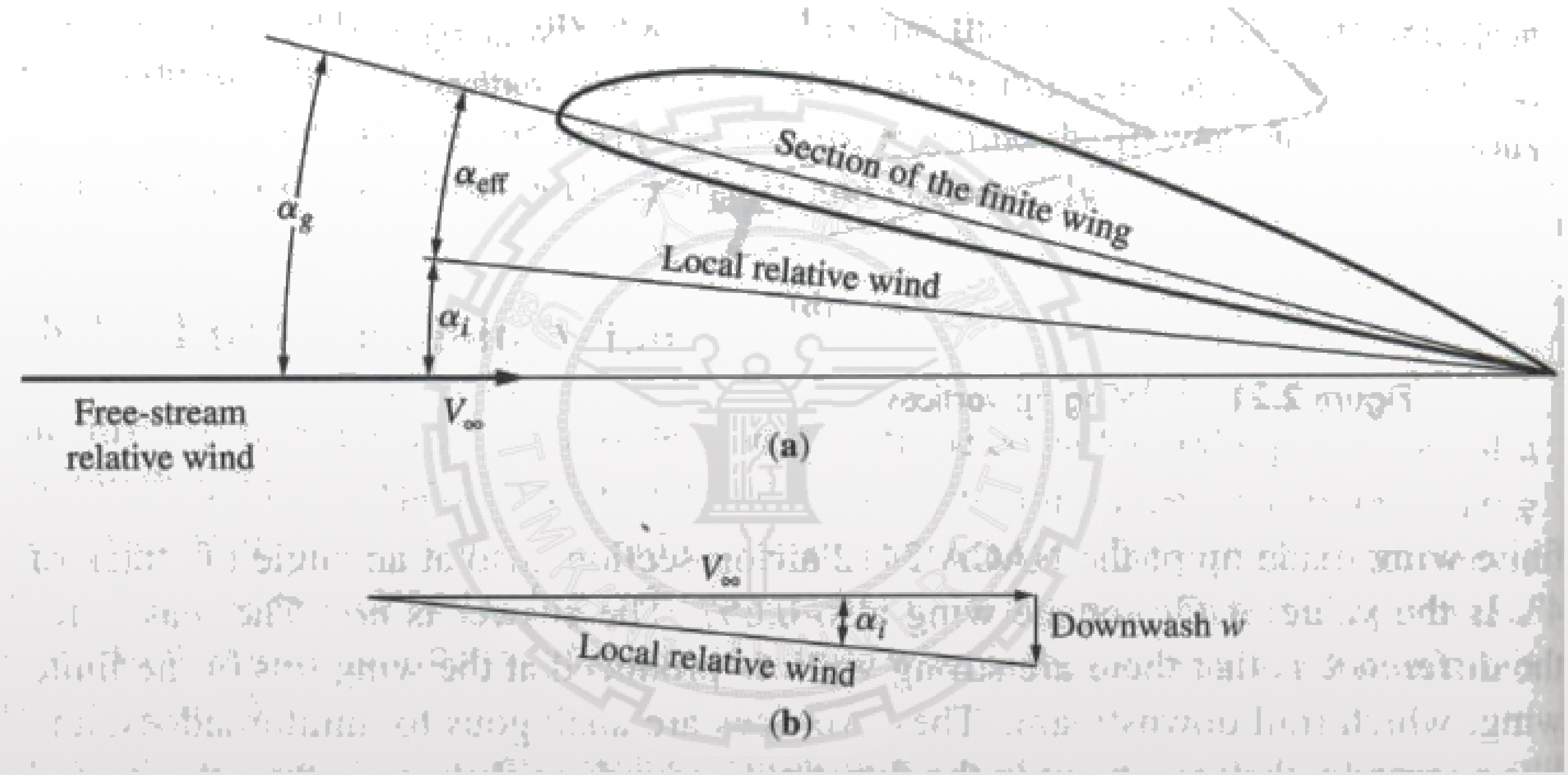
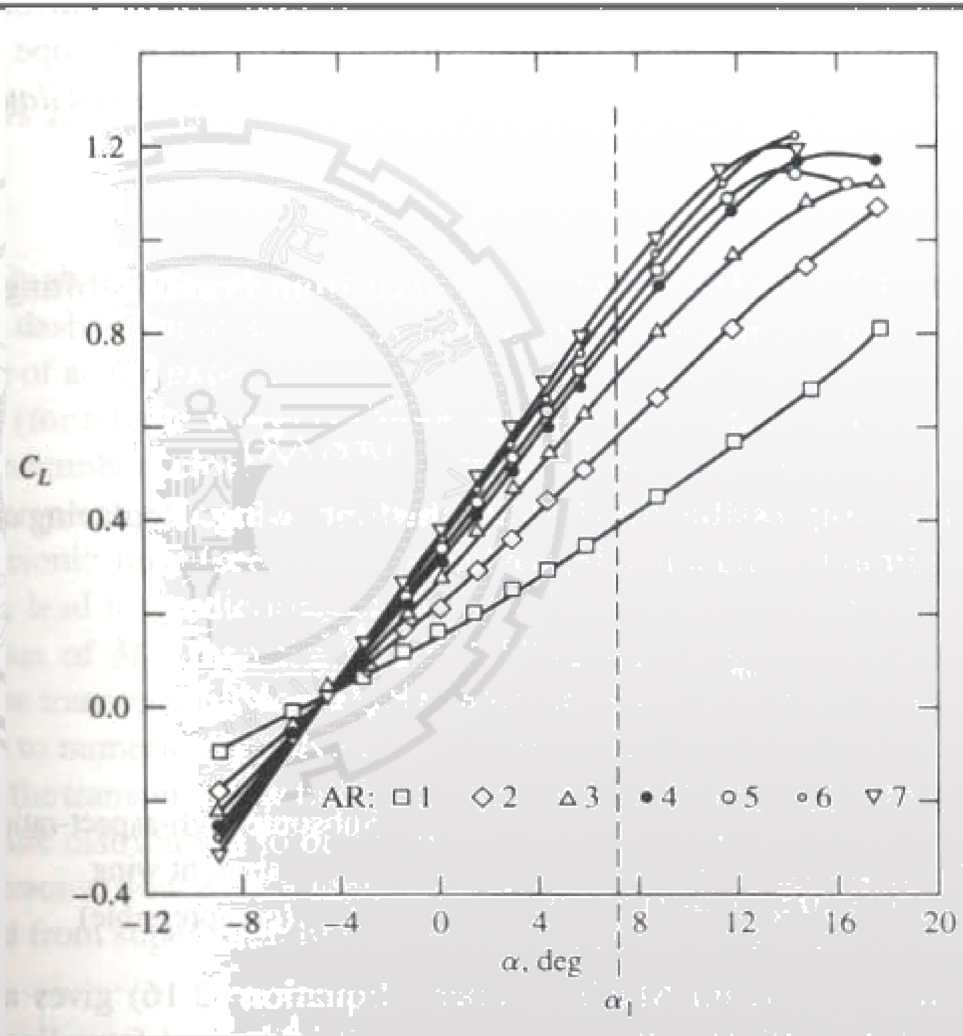


Illustration of induced and effective angles of attack, and downwash

Effect of aspect ratio on the lift curve.



High Aspect Ratio Straight Wing

The high aspect ratio straight wing is the choice for relatively low speed subsonic airplanes. By the *Prandtl's lifting line theory*, the estimate of the lift slope for a finite wing in terms of the lift slope of the airfoil section as

$$a = \frac{a_0}{1 + a_0/(\pi e AR)} \qquad \text{(incompressible)}$$

where a and a_0 are the lift slope *per radian* and e is a factor that depends on the geometric shape of the wing, including the aspect ratio and taper ratio. Values of e are typically on the order of 0.95.

Compressible subsonic high aspect ratio straight wing

$$a_{comp} = \frac{a_0}{\sqrt{1 - M_\infty^2} + a_0/(\pi e AR)}$$

Supersonic high aspect ratio straight wing

$$a_{comp} = \frac{4}{\sqrt{M_\infty^2 - 1}}$$

Example – Consider a straight wing of aspect ratio 6 with an NACA 2412 airfoil. Assuming low speed flow, calculate the lift coefficient at an angle of attack of 6°. For this wing, the span effectiveness factor $e = 0.95$.

From the NACA 2412 Data, $a_0 = 0.105$ per degree and $\alpha_{L0} = -2.2^o$. The lift slope is given by

$$a = \frac{a_0}{1 + a_0/(\pi e AR)}$$

where a and a_0 are the lift slope *per radian.*

$a_0 = 0.105$ per degree $= (0.105)(57.3) = 6.02$ per radian.

Hence

$$a = \frac{6.02}{1 + 6.02/(\pi(.95)(6))} = 4.51 \quad \text{per radian}$$

or

$$a = \frac{4.51}{57.3} = 0.079 \quad \text{per degree}$$

$$C_L = a(\alpha - \alpha_{L0}) = 0.079(6 - (-2.2)) = 0.648$$

Note: Comparing the above result with that for the airfoil : $C_L = 0.85$. As expected, the finite wing reduces the lift coefficient.

Low Aspect Ratio Straight Wing:

Instead of a single spanwise lifting line, the low aspect ratio wing (AR < 4) must be modeled by a large number of spanwise vortices, each located at a different chordwise station. This is the essence of *lifting surface theory*.

Based on a lifting surface solution for elliptic wings, Helmbold's equation is

$$a = \frac{a_0}{\sqrt{1+[a_0/(\pi AR)]^2} + a_0/(\pi AR)}$$

(incompressible)

Compressible subsonic low aspect ratio straight wing

$$a_{comp} = \frac{a_0}{\sqrt{1 - M_\infty^2 + [a_0/(\pi AR)]^2} + a_0/(\pi AR)}$$

Supersonic low aspect ratio straight wing

$$a_{comp} = \frac{4}{\sqrt{M_\infty^2 - 1}}\left(1 - \frac{1}{2AR\sqrt{M_\infty^2 - 1}}\right)$$

Example – Consider a straight wing of aspect ratio 2 with an NACA 2412 airfoil. Assuming low speed flow, calculate the lift coefficient at an angle of attack of 6°. For this wing, the span effectiveness factor $e = 0.95$.

From the NACA 2412 Data, $a_0 = 0.105$ per degree and $\alpha_{L0} = -2.2^{o}$. We have

$$\frac{a_0}{\pi AR} = \frac{6.02}{\pi(2)} = 0.955$$

Hence,

$$a = \frac{a_0}{\sqrt{1+[a_0/(\pi AR)]^2} + a_0/(\pi AR)} = \frac{6.02}{\sqrt{1+(0.955)^2} + 0.955}$$

$$= 2.575 \quad \text{per radian}$$

or

$$a = \frac{2.575}{57.3} = 0.0449 \quad \text{per degree}$$

$$C_L = a(\alpha - \alpha_{L0}) = 0.0449(6-(-2.2)) = 0.368$$

Note: Comparing the above result with that for the aspect ratio of 6, the wing with aspect ratio of 2, reduces the lift coefficient by 43%.

Swept Wing:

Simply stated, a swept wing has a lower lift coefficient than a straight wing, everything else being equal. For the tapered wing, the sweep angle Λ is referred to the half-chord line. By using the half-chord line as reference, the lift slope for a swept wing becomes independent of taper ratio.

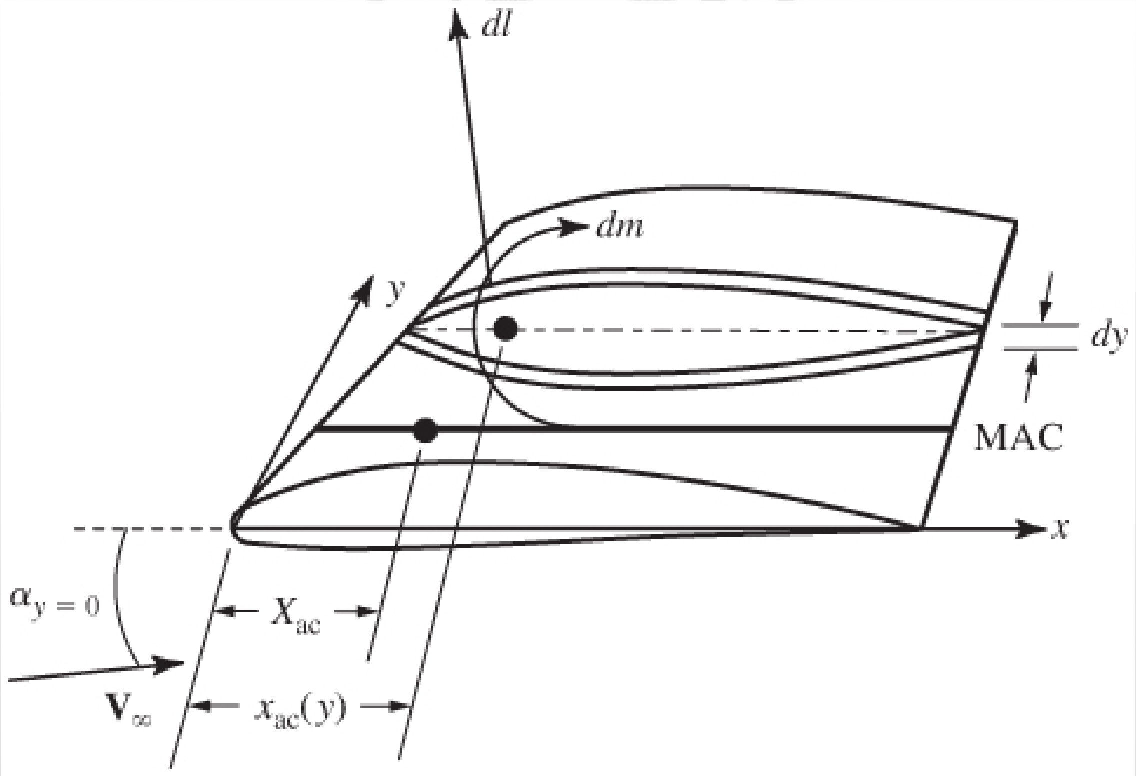
Incompressible:

$$a = \frac{a_0 \cos \Lambda}{\sqrt{1 + [(a_0 \cos \Lambda)/(\pi AR)]^2} + (a_0 \cos \Lambda)/(\pi AR)}$$

Compressible subsonic swept wing

$$a_{comp} = \frac{a_0 \cos\Lambda}{\sqrt{1 - M_\infty^2 \cos^2\Lambda + [(a_0 \cos\Lambda)/(\pi AR)]^2} + (a_0 \cos\Lambda)/(\pi AR)}$$

- Wing Pitching Moment and aerodynamic center



Right half of straight wing with aerodynamic center

The moment about a line through the wing aerodynamic center generated by the infinitesimal strip at span location y is

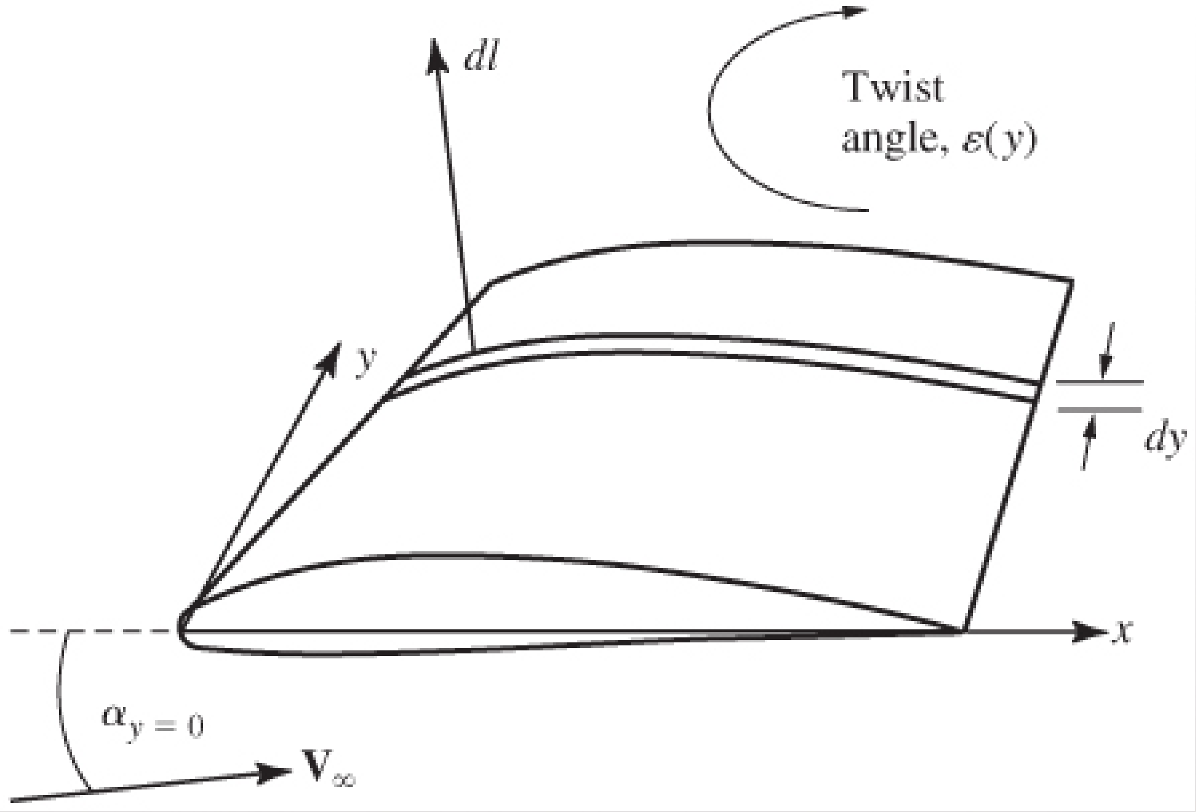
$$dm_{ac} = dm - dl(x_{ac}(y) - X_{ac})$$

where dm is the local section's pitching moment about its ac, or

$$dm = c_{m_{ac}}(y)q(y)dSc(y) = c_{m_{ac}}(y)q(y)c^2(y)dy$$

and the local section lift dl is given by the equation

$$dl = c_l(y)q(y)dS = c_{l_\alpha}(y)(\alpha(y) - \alpha_0(y))q(y)dS$$
$$= c_{l_\alpha}(y)(\alpha_{y=0} + \varepsilon(y) - \alpha_0(y))q(y)c(y)dy$$



Right half of straight, swept wing with strip shown

Integrating the local moment over the span of wing yields the expression for the wing pitching moment about its *ac*, or

$$M_{ac} = 2\left(\int_0^{b/2} c_{m_{ac}} q(y)c^2(y)dy - \int_0^{b/2} c_{l_\alpha}(y)(\alpha_{wing} + \varepsilon(y) - \alpha_0(y))(x_{ac}(y) - X_{ac})q(y)c(y)dy\right)$$

$$= C_{M_{ac}} q_\infty S\overline{c}_{mac}$$

or

$$C_{M_{ac}} = \frac{2}{S\overline{c}_{mac}}\left(\int_0^{b/2} c_{m_{ac}} c^2(y)dy - \int_0^{b/2} c_{l_\alpha}(y)(\alpha_{wing} + \varepsilon(y) - \alpha_0(y))(x_{ac}(y) - X_{ac})c(y)dy\right)$$

assuming the dynamic pressure independent of wing span.

Recall the definition of the aerodynamic center, we have for the wing

$$\frac{\partial C_{M_{ac}}}{\partial \alpha_{wing}} = \int_0^{b/2} c_{l_\alpha}(y)(x_{ac}(y) - X_{ac})c(y)dy = 0$$

Since X_{ac} is invariant with respect to the span-wise integration, we may bring it outside the integral and solve for it yielding

$$X_{ac} = \frac{\int_0^{b/2} c_{l_\alpha}(y)x_{ac}(y)c(y)dy}{\int_0^{b/2} c_{l_\alpha}(y)c(y)dy}$$

If the section lift curve slop is approximately constant with span, the equation may ne simplified to yield

$$X_{ac} = \frac{\int_0^{b/2} x_{ac}(y)c(y)dy}{\int_0^{b/2} c(y)dy} = \frac{2}{S}\int_0^{b/2} x_{ac}(y)c(y)dy$$

Refer to the equation of wing's pitching moment coefficient

$$C_{M_{ac}} = \frac{2}{S\overline{c}_{mac}} \left(\int_0^{b/2} c_{m_{ac}} c^2(y)dy - \int_0^{b/2} c_{l_\alpha}(y)(\alpha_{wing} + \varepsilon(y) - \alpha_0(y))(x_{ac}(y) - X_{ac})c(y)dy \right)$$

By definition the wing's moment about its aerodynamic center is invariant with angle of attack, we may choose the wing's angle of attack in the above expression to be its zero-lift angle of attack, $\alpha_{0_{wing}}$, thus

$$C_{M_{ac}} = \frac{2}{S\overline{c}_{mac}} \left(\int_0^{b/2} c_{m_{ac}} c^2(y)dy - \int_0^{b/2} c_{l_\alpha}(y)(\alpha_{0_{wing}} + \varepsilon(y) - \alpha_0(y))(x_{ac}(y) - X_{ac})c(y)dy \right)$$

For the wing with on twist and constant section characteristics with span,

$$C_{M_{ac}} = \frac{2}{S\overline{c}_{mac}} \int_0^{b/2} c_{m_{ac}} c^2(y)dy = c_{m_{ac}} \quad \text{and} \quad \overline{c}_{mac} = \frac{2}{S} \int_0^{b/2} c^2(y)dy$$

For wings with linear taper the mean aerodynamic chord and its location are given by

$$\bar{c}_{mac} = \frac{2}{3}\frac{1+\lambda+\lambda^2}{1+\lambda}c_r, \quad \text{located at} \quad \frac{y_{mac}}{b} = \frac{1}{6}\frac{1+2\lambda}{1+\lambda}$$

- Wing Drag
  - There are two sources of aerodynamic force on a body moving through a fluid – the pressure and the shear stress distribution acting over the body surface. Therefore, there are only two general types of drag:
    - Pressure drag – due to a net imbalance of surface pressure acting in the drag direction.
    - Friction drag – due to the net effect of shear acting in the drag direction.

– Subsonic drag
  - Airfoil
    – The drag coefficient in the airfoil data is labeled the section drag coefficient; it is also frequently called the *profile drag coefficient*. Profile drag is combination of two types of drag:
      » Skin-friction drag – due to the frictional shear stress acting on the surface of the airfoil.

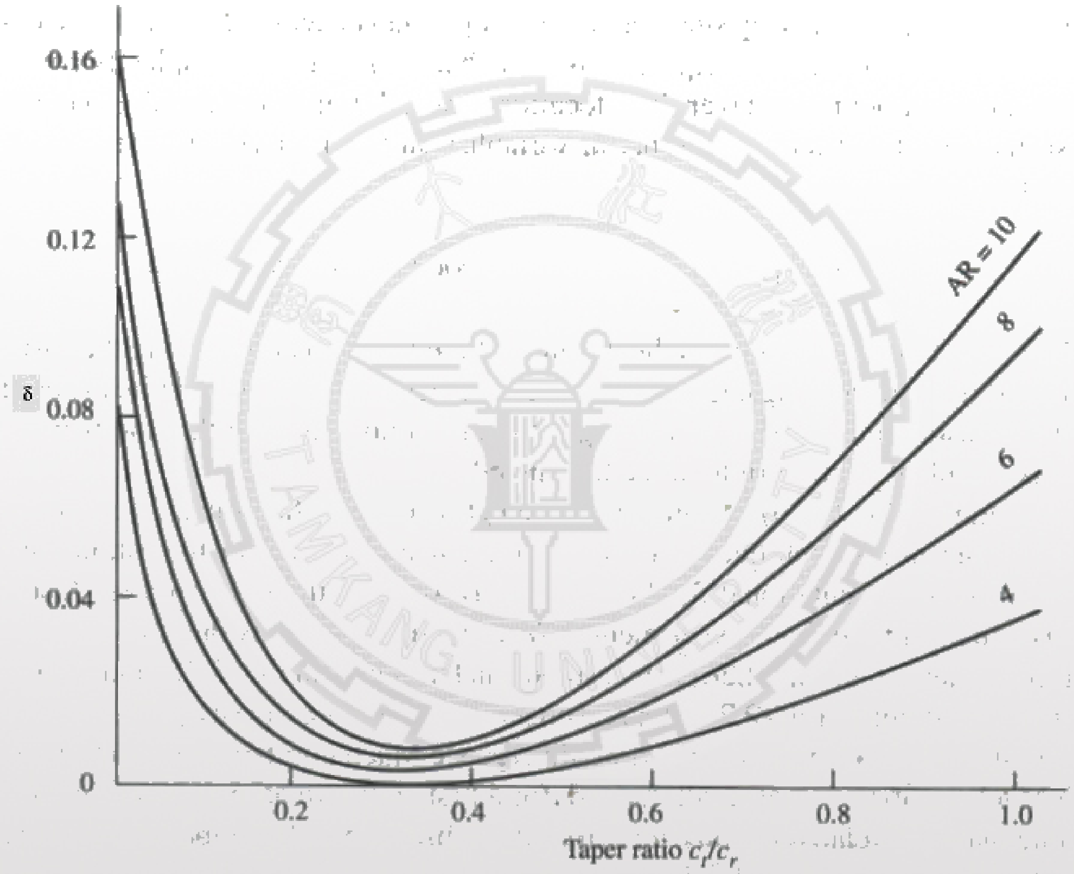
» Pressure drag –due to the flow separation is caused by the imbalance of the pressure distribution in drag direction when the boundary layer separates form the airfoil surface.

- Finite Wings
  - The same induced flow effects due to the wing-tip vortices result in an extra component of drag on a three-dimensional lifting body. This extra drag is called *induced drag*. Induced drag is purely a pressure drag. For a high aspect ratio straight wing, *Prandtl's lifting line theory* shows that the induced drag coefficient is given by

$$C_{D_i} = \frac{C_L^2}{\pi e AR}$$

where e is the *span efficiency factor*, given by

$$e = \frac{1}{1+\delta}$$



Induced drag factor as a function of taper ratio for wings of different aspect ratio

Example – Consider a straight wing of aspect ratio 6 with an NACA 2412 airfoil. Assuming low speed flow, calculate the drag coefficient at an angle of attack of 6°. For this wing, the span effectiveness factor $e = 0.95$.

From the result of the previous example at $\alpha = 6^o$, $C_L = 0.648$. Hence

$$C_{D_i} = \frac{C_L^2}{\pi e AR} = \frac{(0.648)^2}{\pi(0.95)(6)} = 0.0234$$

From the airfoil data, when the airfoil is at 6^o angle of Attack ($c_l = 0.85$), the value of c_d is 0.0076 (assuming a Reynolds number on the order of $9X10^6$). Hence, for the finite wing, the total drag coefficient is given by

$$C_D = c_d + C_{D_i} = 0.0076 + 0.0234 = 0.0312$$

The lift-to-drag ratio is

$$\frac{L}{D} = \frac{C_L}{C_D} = \frac{0.648}{0.031} = 20.9$$

Note: Comparing the above result with that for the airfoil at $\alpha = 6^o$, $L/D = 111.8$.