Optimal Control

HW9

班級：航太四 A
姓名：吳柏勳
學號：407430635
座號：3

1.

$$\min \quad t_f = \int_1^{t_f} 1 \, dt$$

s.t.

$$\begin{cases} \ddot{x} = -\dot{x} + u & \dot{x}(0) = 1 \\ \dot{x} = \dot{x} & x(0) = 1 \end{cases}$$

$$|u| \le 1$$

$$\begin{cases} \dot{x}(t_f) = 0 \\ x(t_f) = 0 \end{cases}$$

$$H = 1 + \lambda_1(-\dot{x} + u) + \lambda_2(\dot{x})$$

① $\begin{cases} \ddot{x} = -x + u \\ \dot{x} = \dot{x} \end{cases}$

② $\begin{cases} \dot{\lambda}_1 = -\lambda_1 + \lambda_2 \\ \dot{\lambda}_2 = 0 \end{cases}$

③ $\begin{bmatrix} \lambda_1(t_f) \\ \lambda_2(t_f) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

④ $\begin{cases} \dot{x}(t_f) = 0 \\ x(t_f) = 0 \end{cases}$

⑤ $\min_u \quad 1 + \lambda_1(-\dot{x} + u) + \lambda_2(\dot{x})$

$$\Rightarrow \min_{u \in \{-1, 1\}} \lambda_1 u$$

$$\Rightarrow \begin{cases} \text{if } \lambda_1 < 0, \quad u = 1 \\ \text{if } \lambda_1 \ge 0, \quad u = -1 \end{cases}$$

#

```
clear;clc;close all
[t, x] = ode45(@ODE, [4.8 0], [0 0 0 0]');

figure()
plot(t, x(:,1), t, x(:,2))
legend("$\dot{x}$", "$x$", 'Interpreter', 'latex')
grid on

function dxdt = ODE(~, x)
    % state define: [x_dot, x, lambda1, lambda2]'

    if x(3)<=0
        u = 1;
    else
        u = -1;
    end

    dxdt = [-x(2)+u
             x(1)
            -x(3)+x(4)
             0];
end
```