淡江大學

航空太空工程學系研究所

高等工程數學

作業1

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Question 1

#1(a)

```
clear;clc;close all
fun1 = @(n) (-3/5)^(n-1);
x1 = limsum(fun1, 1);
```

```
Iteration times: 29
Value: 0.625000
Error: 0.000001
```

#1(b)

```
clear;clc;close all
fun2 = @(n) (4/9)^(n-1);
x2 = limsum(fun2, 1);
```

```
Iteration times: 19
Value: 1.800000
Error: 0.000000
```

#1(c)

```
clear;clc;close all
fun3 = @(n) sin(n*pi()/2)/n;
x3 = limsum(fun3, 1);
```

```
Iteration times: 2
Value: 1.000000
Error: 0.000000
```

#1(d)

```
clear;clc;close all
sum = 1;
n = 1;
x = [];
while 1
    error = 1;
    for i = 1:2:2*n
        error = error*(i/(i+1));
    end
    error = (-1)^n*(1+4*n)*error^3;
    sum = sum + error;
    if abs(error) < 1e-3
        break
    end
    n = n+1;
end
fprintf("Iteration times: %d \nValue: %f \nError: %f\n\n", n, sum, error);
```

```
Iteration times: 516025
Value: 0.636120
Error: -0.001000
```

Question 2

#2(a)

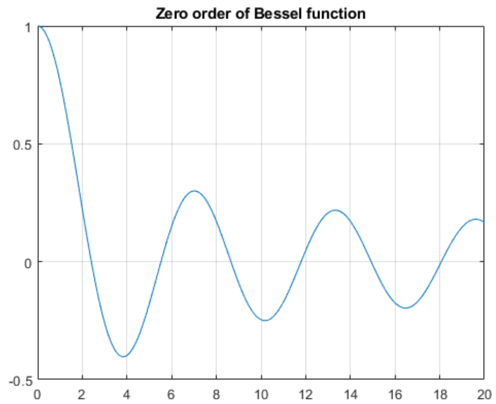
```
clear;clc;close all
z = 0:0.0001:20;
J0 = besselj(0,z);
plot(z,J0)
grid()
title("Zero order of Bessel function")

roots_ans = [];
count = 0;
for i = 1:length(J0)-1
    if J0(i)*J0(i+1)<0
        count = count+1;
        roots_ans(count) = i;
    end
end
for i = 1:3
```

```
    fprintf("lambda%d = %.4f\n",i,z(roots_ans(i)))
end
```

....

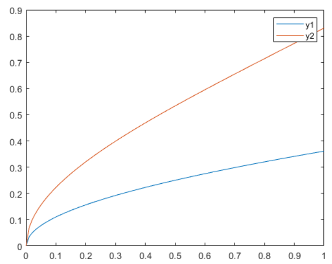
```
lambda1 = 2.4048
lambda2 = 5.5200
lambda3 = 8.6537
```

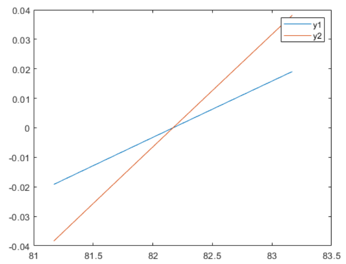


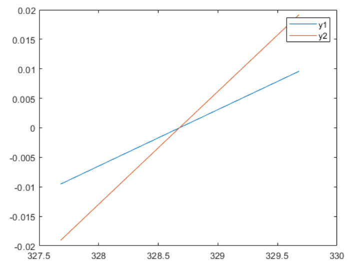
#2(b)

```
clear;clc;close all
lambda = 0:0.01:400;
y1 = tan(sqrt(lambda)*log(2)/2);
y2 = tan(sqrt(lambda)*log(2));
error = abs(y1-y2);
count = 1;
for i = 1:length(error)
   if error(i) <= 5e-5
       fprintf("lambda_%d = %f\n", count, lambda(i))
       count = count+1;
       figure()
       if i-100 <= 0
           plot(lambda(i:i+100),y1(i:i+100),lambda(i:i+100),y2(i:i+100))
       else
           plot(lambda(i-100:i+100),y1(i-100:i+100),lambda(i-100:i+100),y2(i-100:i+100))
       end
       legend('y1','y2')
   end
end
```

```
lambda_1 = 0.000000
lambda_2 = 82.170000
lambda_3 = 328.680000
```





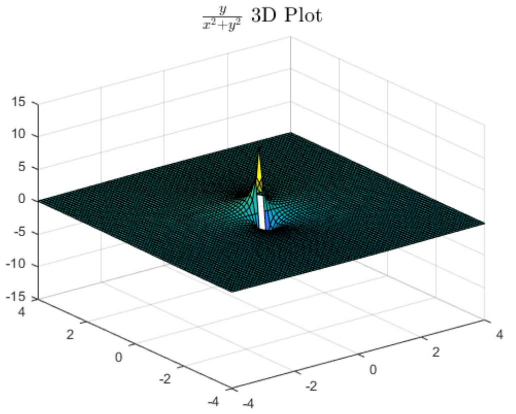


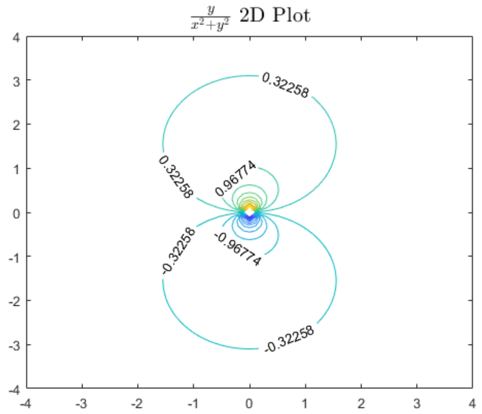
Question 3

#3(a)

```
clear;clc;close all
[x, y] = meshgrid(-4:0.1:4);
T = y./(x.^2+y.^2);
figure()
surf(x,y,T)
title("$\frac{y}{x^2+y^2}$ 3D Plot",'FontSize',15,'interpreter','latex')

figure()
contour(x,y,T, 30,'ShowText','on')
title("$\frac{y}{x^2+y^2}$ 2D Plot",'FontSize',15,'interpreter','latex')
```



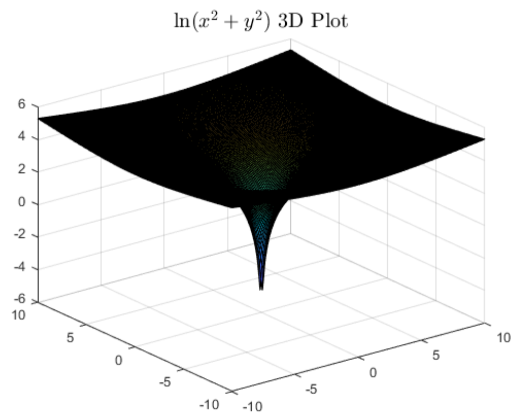


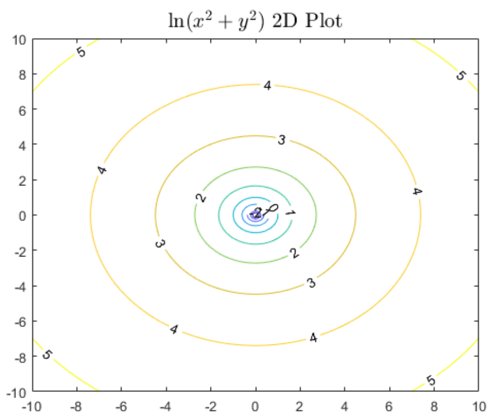
#3(b)

```
clear;clc;close all
[x, y] = meshgrid(-10:0.1:10);
T = log(x.^2+y.^2);
figure()
surf(x,y,T)
title("$\ln(x^2+y^2)$ 3D Plot",'FontSize',15,'interpreter','latex')

figure()
contour(x,y,T,'ShowText','on')
title("$\ln(x^2+y^2)$ 2D Plot",'FontSize',15,'interpreter','latex')
```

....



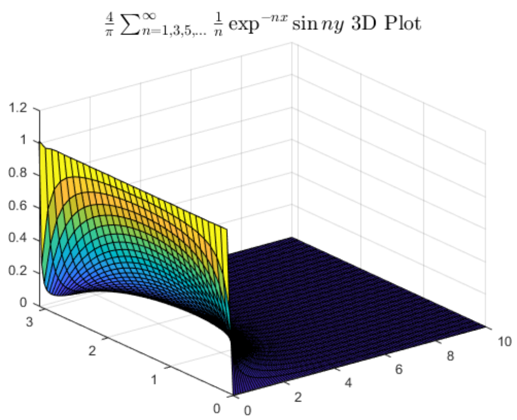


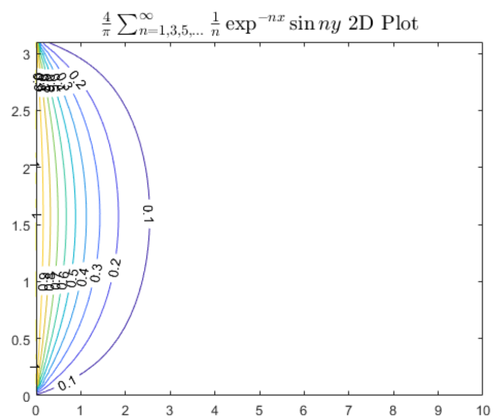
#3(c)

```
clear;clc;close all
[x, y] = meshgrid(0:0.1:10,0:0.1:pi());
len = size(x);
for i = 1:len(1)
    for j = 1:len(2)
        f = @(n) exp(-(2.*n-1).*x(i,j)).*sin((2.*n-1).*y(i,j))./(2.*n-1);
        T(i,j) = 4*limsum(f,0)/pi();
    end
end
figure()
surf(x,y,T)
title("$\frac{4}{\pi}\sum_{n=1,3,5,...}^{\infty}\frac{1}{n}\exp^{-nx}\sin{ny}$ 3D Plot",...
'FontSize',15,'interpreter','latex')

figure()
contour(x,y,T,'ShowText','on')
title("$\frac{4}{\pi}\sum_{n=1,3,5,...}^{\infty}\frac{1}{n}\exp^{-nx}\sin{ny}$ 2D Plot",...
'FontSize',15,'interpreter','latex')
```

....





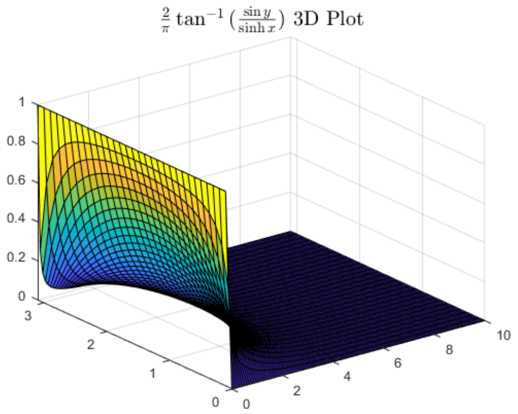
#3(d)

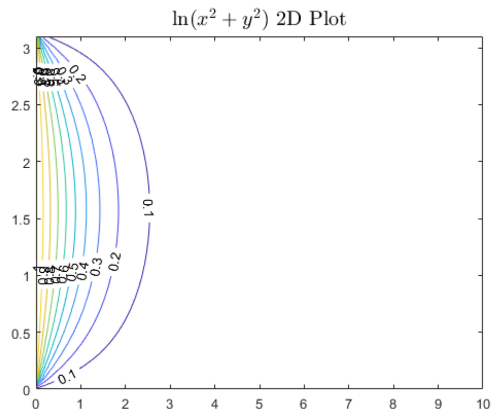
```
clear;clc;close all
[x, y] = meshgrid(0:0.1:10,0:0.1:pi());
T = 2.*atan2(sin(y),sinh(x))./pi();
figure()
surf(x,y,T)
title("$\frac{2}{\pi}\tan^{-1}{(\frac{\sin{y}}{\sinh{x}})}$ 3D Plot",...
```

```
'FontSize',15,'interpreter','latex')

figure()
contour(x,y,T,'ShowText','on')
title("$\ln(x^2+y^2)$ 2D Plot",'FontSize',15,'interpreter','latex')
```

....





Question 4

#4(a)

#4

(a)

Let $x(t) = e^{\lambda t}$

$$\lambda^2 e^{\lambda t} + \omega_0^2 e^{\lambda t} = 0$$

$$\Rightarrow \lambda^2 + \omega_0^2 = 0$$

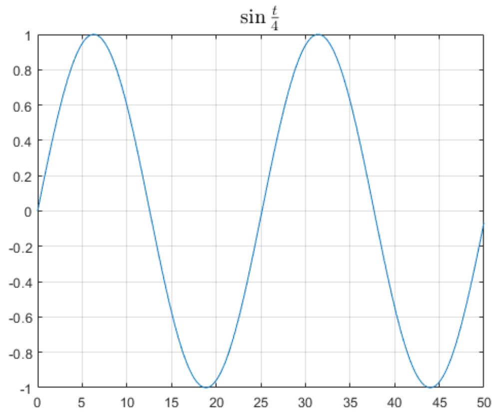
$$\Rightarrow \lambda = \pm \omega_0 i$$

$$x_p(t) = C e^{\pm \omega_0 t i} = \underline{C_1 \cos \omega_0 t + C_2 \sin \omega_0 t} \quad \#$$

#4(b)(1)

```
clear;clc;close all
t = 0:0.01:50;
x = sin(t./4);
plot(t,x)
grid()
title("$\sin{\frac{t}{4}}$",'FontSize',15,'interpreter','latex')
```

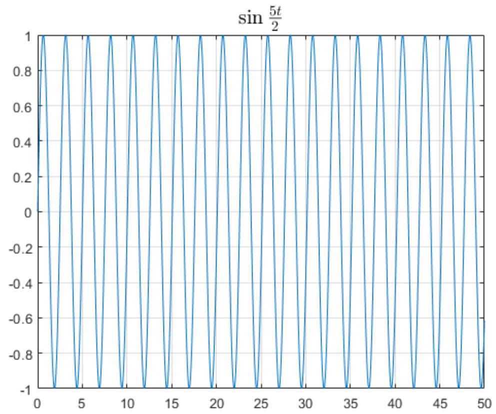
....



#4(b)(2)

```
clear;clc;close all
t = 0:0.01:50;
x = sin(5.*t./2);
plot(t,x)
grid()
title("$\sin{\frac{5t}{2}}$",'FontSize',15,'interpreter','latex')
```

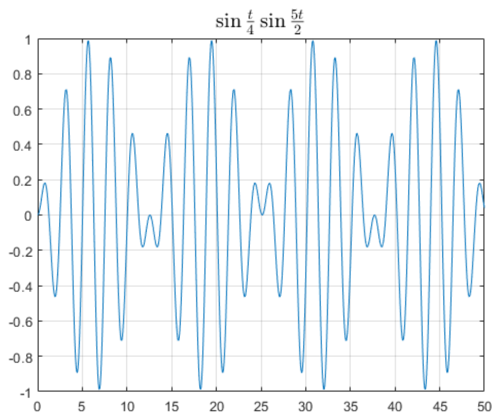
....



#4(b)(3)

```
clear;clc;close all
t = 0:0.01:50;
x = sin(t./4).*sin(5.*t./2);
plot(t,x)
grid()
title("$\sin{\frac{t}{4}}\sin{\frac{5t}{2}}$",'FontSize',15,'interpreter','latex')
```

....

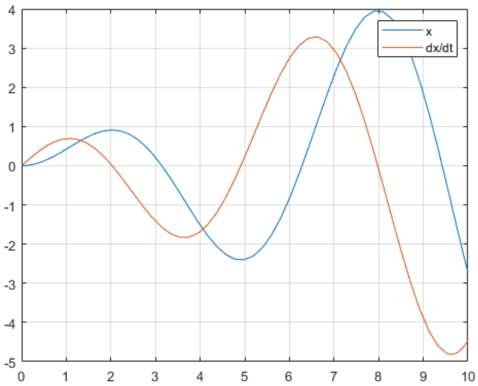


#4(c)

```
clear;clc;close all
global w0 omega
w0 = 1;
omega = 1;
```

```
[t, x] = ode45(@vibration_eqn, [0 10], [0; 0]);
plot(t,x)
legend('x','dx/dt')
grid()
title("")
```

....



Question 5

#5(a)

5

(a)

$$T_{xx} + T_{yy} = 0$$

Let $T(x,y) = X(x)\,Y(y)$

$$\frac{\partial^2 X(x)Y(y)}{\partial x^2} + \frac{\partial^2 X(x)Y(y)}{\partial y^2} = 0$$

$$\Rightarrow Y(y)\frac{\partial^2 X(x)}{\partial x^2} + X(x)\frac{\partial^2 Y(y)}{\partial y^2} = 0$$

$$\div (X(x)Y(y)) \Rightarrow \frac{\frac{d^2X(x)}{dx^2}}{X(x)} + \frac{\frac{d^2Y(y)}{dy^2}}{Y(y)} = 0$$

$$\Rightarrow \frac{\frac{d^2X(x)}{dx^2}}{X(x)} = -\frac{\frac{d^2Y(y)}{dy}}{Y(y)} = \lambda^2$$

$$\Rightarrow \begin{cases} \dfrac{d^2X(x)}{dx^2} - \lambda^2 X(x) = 0 \\ \dfrac{d^2Y(y)}{dy^2} + \lambda^2 Y(y) = 0 \end{cases}$$

$$\begin{cases} X(x) = e^{-nx} \\ Y(y) = C_1\cos\lambda y + C_2\sin\lambda y \end{cases}$$

$$T(x,0) = 0 = e^{-nx}(C_1\cdot 1 + C_2\cdot 0) \Rightarrow C_1 = 0$$

$$T(x,\pi) = 0 = e^{-nx}(C_2\sin\lambda\pi)$$

$$\Rightarrow \because C_2 \neq 0,\ \therefore \sin\lambda\pi = 0 \Rightarrow \lambda = 1,2,3,\cdots$$

$$\Rightarrow Y(y) = \sin ny,\ n = 1,2,3,\cdots \quad \#$$

$$T(x,y) = \sum_{n=1}^{\infty} A_n e^{-nx}\sin ny$$

$$T_{xx} + T_{yy} = 0$$

$$\Rightarrow \frac{\partial^2 f}{\partial \eta^2}\left[(-\sin y \coth x \operatorname{csch} x)^2 + (\cos y \operatorname{csch} x)^2\right]$$

$$+ \frac{\partial f}{\partial \eta}\left[+\sin y(+\operatorname{csch}^3 x + \coth^2 x \operatorname{csch} x) + (-\sin y \operatorname{csch} x)\right] = 0$$

$$\Rightarrow \frac{\partial^2 f}{\partial \eta^2}\underline{\left[\operatorname{csch}^2 x(\sin^2 y \coth^2 x + \cos^2 y)\right]} + \frac{\partial f}{\partial \eta}\underline{\left[\sin y \operatorname{csch} x(\operatorname{csch}^2 x + \coth^2 x - 1)\right]} = 0$$

$$= \operatorname{csch}^2 x\left(\sin^2 y \frac{\cosh^2 x}{\sinh^2 x} + \cos^2 y\right)$$

$$= \operatorname{csch}^2 x\left[\eta^2(1+\sinh^2 x) + 1 - \sin^2 y\right]$$

$$= \operatorname{csch}^2 x\left[\eta^2 + 1 + \cancel{\eta^2 \sinh^2 x} - \cancel{\sin^2 y}\right]$$

$$= \operatorname{csh}^2 x(\eta^2 + 1)$$

$$= \eta(\operatorname{csch}^2 x + \operatorname{csch}^2 x)$$

$$= 2\eta \operatorname{csch}^2 x$$

$$\Rightarrow \underline{(1+\eta^2)\frac{d^2 f}{d\eta^2} + 2\eta \frac{df}{d\eta} = 0 \quad \#}$$

$$f(\eta) = C_1 \tan^{-1}\eta + C_2$$

$$T(\infty, y) = f(0) = 0 = C_2$$

$$T(0, y) = f(\infty) = 1 = C_1 \cdot \frac{\pi}{2} \Rightarrow C_1 = \frac{2}{\pi}$$

$$f(\eta) = \underline{\frac{2}{\pi}\tan^{-1}\eta \quad \#}$$

$$T(0,y) = f(y) = \sum_{n=1}^{\infty} A_n \sin ny$$

$$\Rightarrow A_n = \frac{\int_0^{\pi} f(y)\,dy}{\int_0^{\pi} \sin ny\,dy} = \frac{\int_0^{\pi} f(y)\sin ny\,dy}{\int_0^{\pi} \sin^2 ny\,dy}$$

$$= \frac{2}{\pi}\int_0^{\pi} \cancelto{1}{f(y)} \sin ny\,dy = \frac{2}{\pi}\int_0^{\pi} \sin ny\,dy$$

$$= \frac{2}{\pi}\left(\frac{-1}{n}\cos ny\right)\Bigg|_0^{\pi} = \frac{2}{\pi}\left[\frac{-1}{n}\cos n\pi - \frac{-1}{n}\cos 0\right]$$

$$= \frac{2}{n\pi}\left(1-\cos n\pi\right), \quad n=1,2,3,\cdots$$

$$= \frac{4}{n\pi}, \quad n=1,3,5,\cdots \quad \#$$

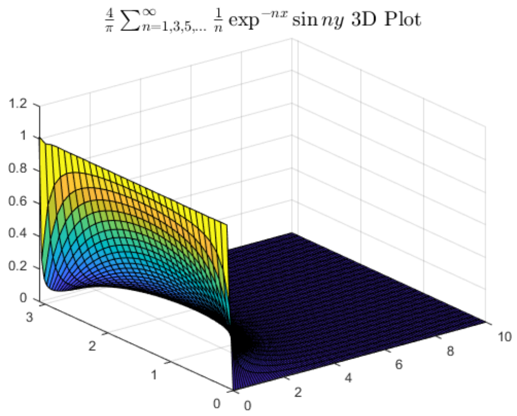
$$T(x,y) = \sum_{n=1,3,5,\cdots}^{\infty} \frac{4}{n\pi} e^{-nx} \sin ny$$

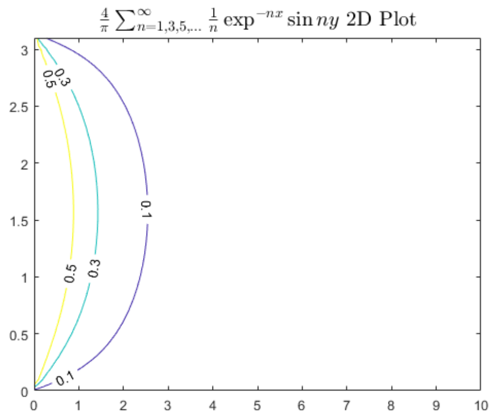
$$\boxed{\begin{aligned} &\int_0^{\pi} \sin^2 ny\,dy \\ &= \int_0^{\pi} \frac{1}{2}\,dy - \int_0^{\pi} \frac{\cos 2ny}{2}\,dy \\ &= \frac{\pi}{2} - \frac{1}{4n}\left[\sin(2n\pi) - \sin(0)\right] \\ &= \frac{\pi}{2} \end{aligned}}$$

$$\boxed{\begin{aligned} f(n) &= 1-\cos n\pi, \quad n=1,2,3 \\ &= \begin{cases} 0, & n=2,4,6,\cdots \\ 2, & n=1,3,5,\cdots \end{cases} \end{aligned}}$$

```
clear;clc;close all
[x, y] = meshgrid(0:0.1:10,0:0.1:pi());
len = size(x);
for i = 1:len(1)
    for j = 1:len(2)
        f = @(n) exp(-(2.*n-1).*x(i,j)).*sin((2.*n-1).*y(i,j))./(2.*n-1);
        T(i,j) = 4/pi()*limsum(f,0);
    end
end
figure()
surf(x,y,T)
title("$\frac{4}{\pi}\sum_{n=1,3,5,...}^{\infty}\frac{1}{n}\exp^{-nx}\sin{ny}$ 3D Plot",...
'FontSize',15,'interpreter','latex')

figure()
contour(x,y,T,[.1 .3 .5],'ShowText','on')
title("$\frac{4}{\pi}\sum_{n=1,3,5,...}^{\infty}\frac{1}{n}\exp^{-nx}\sin{ny}$ 2D Plot",...
'FontSize',15,'interpreter','latex')
```

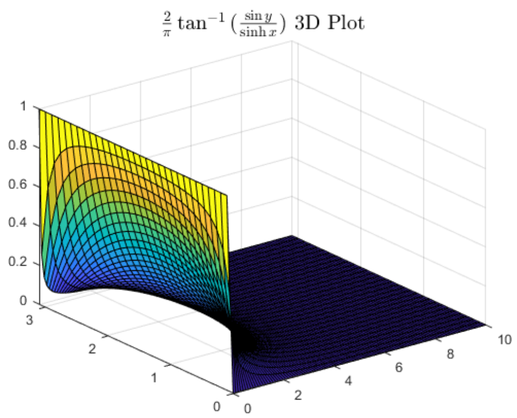


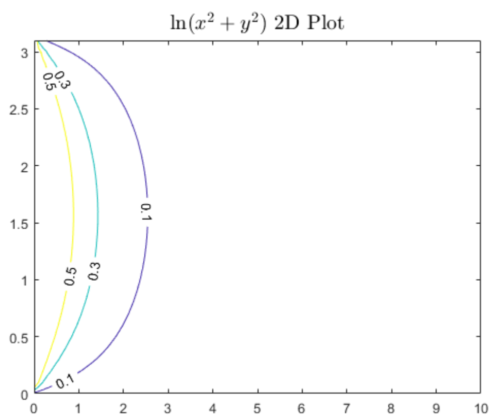


#5(b)

```
clear;clc;close all
[x, y] = meshgrid(0:0.1:10,0:0.1:pi());
T = 2./pi().*atan2(sin(y),sinh(x));
figure()
surf(x,y,T)
title("$\frac{2}{\pi}\tan^{-1}{(\frac{\sin{y}}{\sinh{x}})}$ 3D Plot",...
'FontSize',15,'interpreter','latex')

figure()
contour(x,y,T,[.1 .3 .5],'ShowText','on')
title("$\ln(x^2+y^2)$ 2D Plot",'FontSize',15,'interpreter','latex')
```





function of #1

```
function sum = limsum(f,output)
    sum = 0;
    n = 1;
    while 1
        error = f(n);
        sum = sum + error;
        if abs(error) < 1e-6
            break
        end
        n = n+1;
    end
    if output == 1
        fprintf("Iteration times: %d \nValue: %f \nError: %f\n\n", n, sum, error)
    end
end
```

function of #4

```
function dydt = vibration_eqn(t, y)
    global w0 omega
    dydt = [y(2); -w0^2*y(1)+cos(omega*t)];
end
```