#5.

$$y_{A_1}(x) = x(1-x) = -x^2+x$$

$$\tilde{\lambda}_1 = \frac{\int_0^1 (1+x^2)\,y_{A_1}'^{\,2}(x)\,dx}{\int_0^1 2x\,y_{A_1}^2(x)\,dx} = \frac{\frac{7}{15}}{\frac{1}{30}} = \frac{7\times \cancel{30}^{\,2}}{\cancel{15}} = \underline{14}\ \#$$

Aside:

$$(1+x^2)(4x^2-4x+1)$$

$$= 4x^2-4x+1+4x^4-4x^3+x^2$$

$$= 4x^4-4x^3+5x^2-4x+1$$

Num:

$$y_{A_1}' = -2x+1$$

$$\int_0^1 (1+x^2)(-2x+1)^2\,dx = \int_0^1 (1+x^2)(4x^2-4x+1)\,dx$$

$$= \int_0^1 4x^4-4x^3+5x^2-4x+1\,dx = \frac{4}{5}x^5 - x^4 + \frac{5}{3}x^3 - 2x^2 + x\Big|_0^1$$

$$= \frac{4}{5} - 1 + \frac{5}{3} - 2 + 1 = \frac{7}{15}$$

Dem:

$$\int_0^1 2x(-x^2+x)^2\,dx = \int_0^1 2x(x^4-2x^3+x^2)\,dx = \int_0^1 2x^5-4x^4+2x^3\,dx$$

$$= \frac{1}{3}x^6 - \frac{4}{5}x^5 + \frac{1}{2}x^4\Big|_0^1 = \frac{1}{3} - \frac{4}{5} + \frac{1}{2} = \frac{1}{30}$$