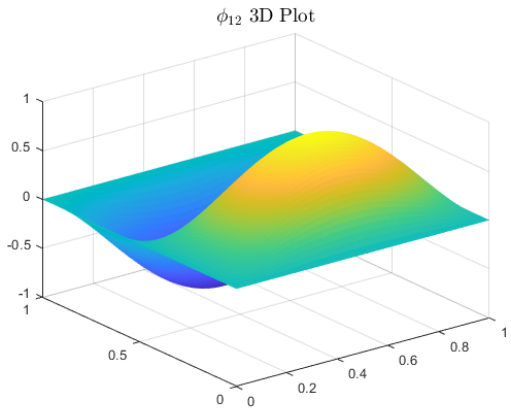
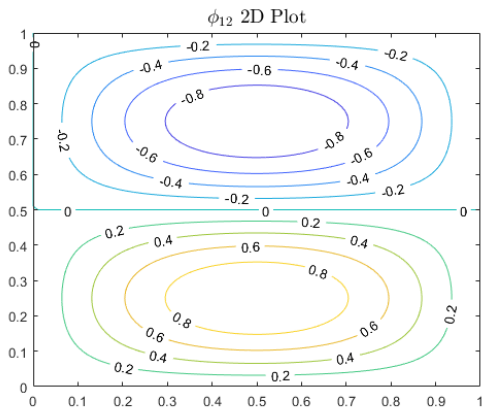
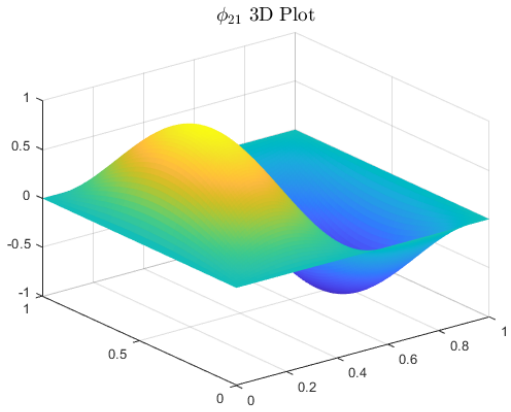
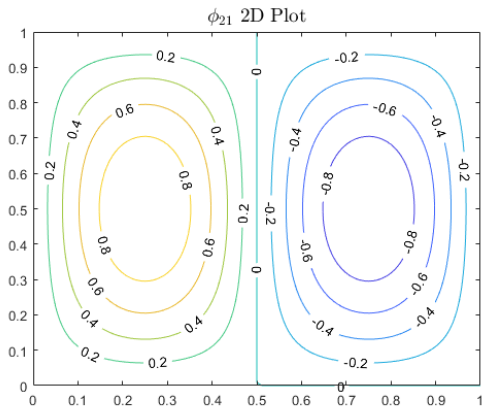
#2(a)

```
clear;clc;close all
[x, y] = meshgrid(0:0.01:1,0:0.01:1);
len = size(x);
phi_12 = sin(1*pi*x).*sin(2*pi*y);
phi_21 = sin(2*pi*x).*sin(1*pi*y);
phi_13 = sin(1*pi*x).*sin(3*pi*y);
phi_31 = sin(3*pi*x).*sin(1*pi*y);

figure()
surf(x,y,phi_12, 'edgecolor', 'none')
title("$\phi_{12}$ 3D Plot", "fontsize", 14, "interpreter", "latex")
figure()
surf(x,y,phi_21, 'edgecolor', 'none')
title("$\phi_{21}$ 3D Plot", "fontsize", 14, "interpreter", "latex")

figure()
contour(x,y,phi_12,'ShowText','on')
title("$\phi_{12}$ 2D Plot", "fontsize", 14, "interpreter", "latex")
figure()
contour(x,y,phi_21,'ShowText','on')
title("$\phi_{21}$ 2D Plot", "fontsize", 14, "interpreter", "latex")
```





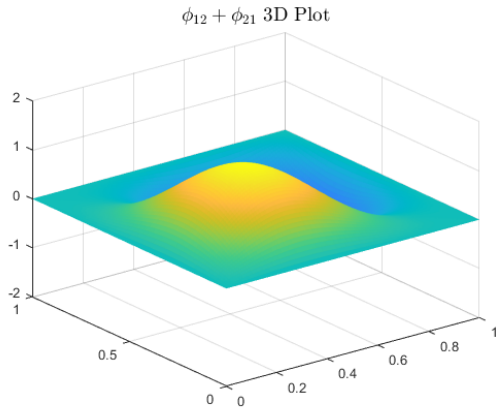


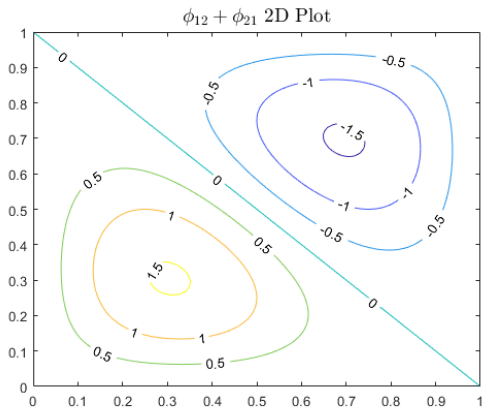


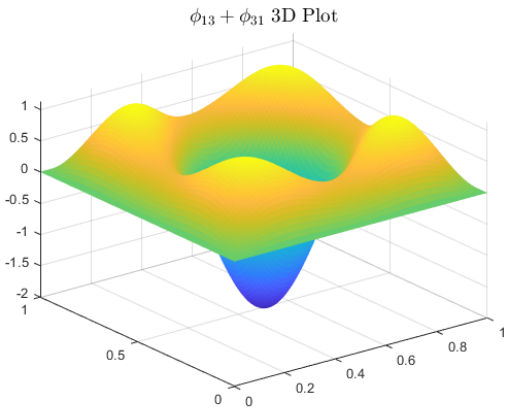
#2(b)(c)(d)(e)

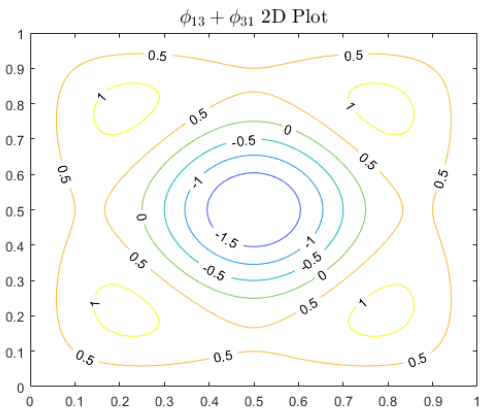
```
clear;clc;close all
[x, y] = meshgrid(0:0.01:1,0:0.01:1);
len = size(x);
phi_12 = sin(1*pi*x).*sin(2*pi*y);
phi_21 = sin(2*pi*x).*sin(1*pi*y);
phi_13 = sin(1*pi*x).*sin(3*pi*y);
phi_31 = sin(3*pi*x).*sin(1*pi*y);

figure()
surf(x,y,phi_12+phi_21, 'edgecolor', 'none')
title("$\phi_{12}+\phi_{21}$ 3D Plot", "fontsize", 14, "interpreter", "latex")
figure()
surf(x,y,phi_13+phi_31, 'edgecolor', 'none')
title("$\phi_{13}+\phi_{31}$ 3D Plot", "fontsize", 14, "interpreter", "latex")
figure()
surf(x,y,phi_13-phi_31, 'edgecolor', 'none')
```

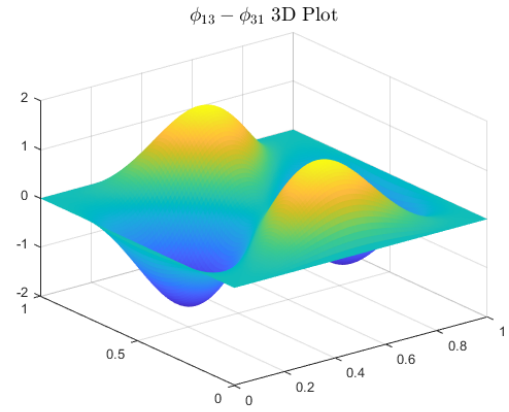
```
title("$\phi_{13}-\phi_{31}$ 3D Plot", "fontsize", 14, "interpreter", "latex")
figure()
surf(x,y,phi_13+phi_31/3, 'edgecolor', 'none')
title("$\phi_{13}+\frac{1}{3}\phi_{31}$ 3D Plot", "fontsize", 14, "interpreter", "latex")

figure()
contour(x,y,phi_12+phi_21,'ShowText','on')
title("$\phi_{12}+\phi_{21}$ 2D Plot", "fontsize", 14, "interpreter", "latex")
figure()
contour(x,y,phi_13+phi_31,'ShowText','on')
title("$\phi_{13}+\phi_{31}$ 2D Plot", "fontsize", 14, "interpreter", "latex")
figure()
contour(x,y,phi_13-phi_31,'ShowText','on')
title("$\phi_{13}-\phi_{31}$ 2D Plot", "fontsize", 14, "interpreter", "latex")
figure()
contour(x,y,phi_13+phi_31/3,'ShowText','on')
title("$\phi_{13}+\frac{1}{3}\phi_{31}$ 2D Plot", "fontsize", 14, "interpreter", "latex")
```

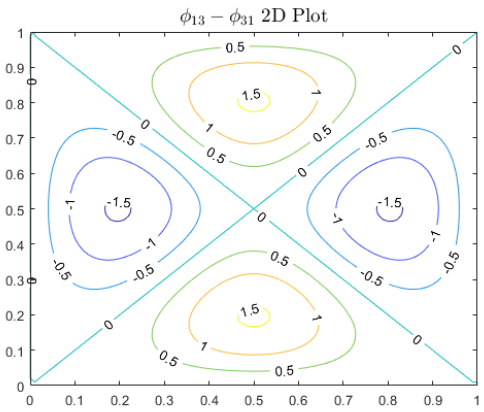


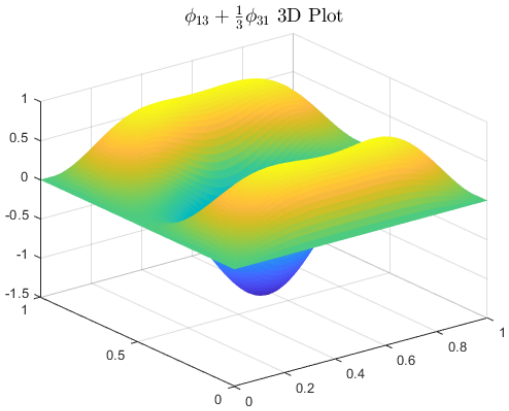


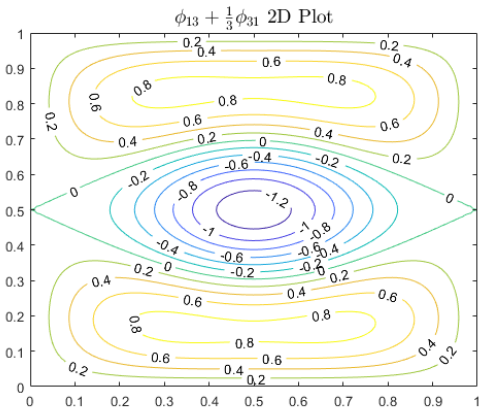












```
clear;clc;close all
syms x c1 c2
y = c1*x*(pi/2-x)+c2*x^2*(pi/2-x);
```

```
I = int(2*x*y-y^2+diff(y,x)^2, x, 0, pi/2);

I_c1 = diff(I,c1);
I_c2 = diff(I,c2);

result = vpasolve([I_c1==0; I_c2==0], [c1 c2]);
c1_ = result.c1
c2_ = result.c2
I_min = vpa(subs(I, [c1, c2], [c1_, c2_]))
```

```
c1_ =

-0.38226308020454397830452040163668


c2_ =

-0.17706905679930450830512698974608


I_min =

-0.27860356386806082489265488067732
```