Optimal Control

HW7

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#1.

$$\min \quad t_f$$

s.t.

$$\ddot{r} = 3w^2 r + 2Rw\dot{\theta} + \frac{T}{m}\sin\delta$$
$$\dot{r} = \dot{r}$$
$$\ddot{\theta} = -\frac{2w}{R}\dot{r} + \frac{T}{mR}\cos\delta$$
$$r(t_f) - 600 = 0$$

state : $\dot{r}, r, \dot{\theta}$

control : δ

parameter : $R = 300$ km

$$w = \sqrt{\frac{3.986\times10^5}{300^3}} = 0.1215 \text{ km/s}$$

$T = 100$ N

$m = 500$ kg

#2.

$$H = \lambda_1\left(3w^2 r + 2Rw\dot{\theta} + \frac{T}{m}\sin\delta\right) + \lambda_2(\dot{r}) + \lambda_3\left(-\frac{2w}{R}\dot{r} + \frac{T}{mR}\cos\delta\right)$$

① $$\begin{cases} \ddot{r} = 3w^2 r + 2Rw\dot{\theta} + \frac{T}{m}\sin\delta \\ \dot{r} = \dot{r} \\ \ddot{\theta} = -\frac{2w}{R}\dot{r} + \frac{T}{mR}\cos\delta \end{cases}$$

② $$\begin{cases} \dot{\lambda}_1 = -\frac{\partial H}{\partial r} = -\lambda_2 - \lambda_3\left(-\frac{2w}{R}\right) \\ \dot{\lambda}_2 = -\frac{\partial H}{\partial r} = \lambda_1(3w^2) \\ \dot{\lambda}_3 = -\frac{\partial H}{\partial \dot{\theta}} = -\lambda_1(2Rw) \end{cases}$$

③ $$H|_\delta = 0 = \lambda_1\left(\frac{T}{m}\cos\delta\right) + \lambda_3\left(-\frac{T}{mR}\sin\delta\right) \Rightarrow \quad \delta = \tan^{-1}\left(\frac{\lambda_1}{\lambda_3}R\right)$$

④ $$\begin{bmatrix}\lambda_1(t_f)\\ \lambda_2(t_f)\\ \lambda_3(t_f)\end{bmatrix} - \begin{bmatrix}0\\0\\0\end{bmatrix} = \nu\begin{bmatrix}0\\1\\0\end{bmatrix}$$

⑤ $$r(t_f) - 600 = 0$$

⑥ $$H|(t_f) = -1 = \lambda_1(t_f)\left[3w^2 r(t_f) + 2Rw\dot{\theta}(t_f) + \frac{T}{m}\sin\delta(t_f)\right] + \lambda_2(t_f)\,r(t_f) + \lambda_3(t_f)\left[-\frac{R}{2w}\ddot{\theta}(t_f) + \frac{T}{2mw}\cos\delta(t_f)\right]$$