#1.

$$\min \; -x(t_f)$$

s.t.

$$\dot{x} = x + \sin u, \quad x(0) = 0 \quad \#$$

$$H = \lambda(x + \sin u), \quad \phi(x) = -x, \quad \psi(x) = 0$$

$$\begin{cases} \dot{x} = x + \sin u & — ① \\ \dot{\lambda} = -\lambda & — ② \\ H_u = \lambda \cos u = 0 & — ③ \\ \lambda(t_f) + 1 = 0 & — ④ \end{cases}$$

From ③

$$\lambda \cos u = 0 \Rightarrow \cos u(t) = 0$$

$$\Rightarrow \sin u(t) = 1 \quad \#$$

From ①

$$\dot{x} = x + \sin u = x + 1$$

$$\frac{dx}{dt} = x + 1 \Rightarrow \int_0^{x(t)} \frac{1}{x+1}\, dx = \int_0^t d\tau$$

$$\Rightarrow \ln(x+1)\Big|_0^{x(t)} = \tau\Big|_0^t$$

$$\Rightarrow \ln(x(t)+1) - \cancel{\ln(1)}^{0} = t - 0$$

$$\Rightarrow x(t) + 1 = e^t \Rightarrow x(t) = e^t - 1 \quad \#$$