1

$$\ddot{x} + g(x) = 0 \qquad X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

$$g(x) = 5(1+9x^2)x$$

$$g(x) = 5x$$

$$g(x) = 5(1-9x^2)x$$

(a)

$$\ddot{x} + 5(1+9x^2)x = 0 \Rightarrow \ddot{x} + 5x + 45x^3 = 0$$

$$\dot{X} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} x_2 \\ (-5-45x_1^2)x_1 \end{bmatrix} = \underline{\begin{bmatrix} 0 & 1 \\ -5-45x_1^2 & 0 \end{bmatrix} X} \ \#$$

(b) $\ddot{x} + 5x = 0$

$$\dot{X} = \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} x_2 \\ -5x_1 \end{bmatrix} = \underline{\begin{bmatrix} 0 & 1 \\ -5 & 0 \end{bmatrix} X} \ \#$$

(c)

$$\dot{X} = \begin{bmatrix} x_2 \\ (-5+45x_1^2)x_1 \end{bmatrix} = \underline{\begin{bmatrix} 0 & 1 \\ -5+45x_1^2 & 0 \end{bmatrix} X} \ \#$$

2

(a)

$$f_1 = x_1^2 + x_2^2 + x_2\cos x_1$$

$$f_2 = (1+x_1)x_1 + (1+x_2)x_2 + x_1\sin x_2$$

$$\dot{X} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}_{\substack{x_1=0 \\ x_2=0}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$x_1 + x_1^2 + x_2 + x_2^2 + x_1 \sin x_2$

$$= \begin{bmatrix} 2x_1 - x_2\sin x_1 & 2x_2 + \cos x_1 \\ 1 + 2x_1 + \sin x_2 & 1 + 2x_2 + x_1\cos x_2 \end{bmatrix}_{\substack{x_1=0 \\ x_2=0}} X$$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} X$$

(b) Let $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$

$$f_1 = x_2$$

$$f_2 = -(3+x_2^2)x_2 + (1+x_1+x_1^2)u$$

$$\dot{X} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}_{\substack{x_1=0 \\ x_2=0 \\ u=0}} X + \begin{bmatrix} \frac{\partial f_1}{\partial u} \\ \frac{\partial f_2}{\partial u} \end{bmatrix}_{\substack{x_1=0 \\ x_2=0 \\ u=0}} u$$

$$= \begin{bmatrix} 0 & 1 \\ (1+2x_1)u & -3-3x_2^2 \end{bmatrix}_{\substack{x_1=0 \\ x_2=0 \\ u=0}} X + \begin{bmatrix} 0 \\ 1+x_1+x_1^2 \end{bmatrix}_{\substack{x_1=0 \\ x_2=0 \\ u=0}} u$$

$$= \begin{bmatrix} 0 & 1 \\ 0 & -3 \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

#

#3

(a)

$$\begin{cases} J\ddot{\theta} + B\dot{\theta} = K_T i_a \\ L_a \dot{i}_a + R_a i_a + K_\theta \dot{\theta} = e_a \end{cases}$$

. Let $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \theta \\ \dot{\theta} \\ i_a \end{bmatrix}$

$$\begin{cases} \dot{X} = \begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \\ \dot{i}_a \end{bmatrix} = \begin{bmatrix} x_2 \\ -\frac{B}{J}x_2 + \frac{K_T}{J}x_3 \\ -\frac{R_a}{L_a}x_3 - \frac{K_\theta}{L_a}x_2 + \frac{e_a}{L_a} \end{bmatrix} \\ \qquad = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{B}{J} & \frac{K_T}{J} \\ 0 & -\frac{K_\theta}{L_a} & -\frac{R_a}{L_a} \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \\ i_a \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L_a} \end{bmatrix} e_a \\ y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \\ i_a \end{bmatrix} \end{cases}$$

#

(b)

$$i_a = \frac{T}{K_T} = \frac{J}{K_T}\ddot{\theta} + \frac{B}{K_T}\dot{\theta}$$

$$\frac{di_a}{dt} = \frac{J}{K_T}\theta^{(3)} + \frac{B}{K_T}\ddot{\theta}$$

$$L_a \frac{di_a}{dt} + R_a i_a + K_\theta \dot{\theta} = e_a$$

$$\Rightarrow L_a \left(\frac{J}{K_T} \theta^{(3)} + \frac{B}{K_T} \ddot{\theta} \right) + R_a \left(\frac{J}{K_T} \ddot{\theta} + \frac{B}{K_T} \dot{\theta} \right) + K_\theta \dot{\theta} = e_a$$

$\cdot K_T$

$$\Rightarrow L_a \left(J \theta^{(3)} + B \ddot{\theta} \right) + R_a \left(J \ddot{\theta} + B \dot{\theta} \right) + K_\theta K_T \dot{\theta} = K_T e_a$$

$$\Rightarrow L_a J \theta^{(3)} + \left(L_a B + R_a J \right) \ddot{\theta} + \left(R_a B + K_\theta K_T \right) \dot{\theta} = K_T e_a$$

$\mathcal{L}\{\cdot\}$

$$\Rightarrow L_a J s^3 \Theta + \left(L_a B + R_a J \right) s^2 \Theta + \left(R_a B + K_\theta K_T \right) s \Theta = K_T E_a$$

$$\Rightarrow \frac{\Theta}{E_a} = \frac{K_T}{L_a J s^3 + (L_a B + R_a J) s^2 + (R_a B + K_\theta K_T) s} \quad \#$$