1.

$$\min \quad t_f = \int_{1}^{t_f} 1 \, dt$$

s.t.

$$\begin{cases} \ddot{x} = -\dot{x} + u & \dot{x}(0) = 1 \\ \dot{x} = \dot{x} & x(0) = 1 \end{cases}$$

$$|u| \leq 1$$

$$\begin{cases} \dot{x}(t_f) = 0 \\ x(t_f) = 0 \end{cases}$$

$$H = 1 + \lambda_1(-\dot{x} + u) + \lambda_2(\dot{x})$$

① $\begin{cases} \ddot{x} = -x + u \\ \dot{x} = \dot{x} \end{cases}$

② $\begin{cases} \dot{\lambda}_1 = -\lambda_1 + \lambda_2 \\ \dot{\lambda}_2 = 0 \end{cases}$

③ $\begin{bmatrix} \lambda_1(t_f) \\ \lambda_2(t_f) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

④ $\begin{cases} \dot{x}(t_f) = 0 \\ x(t_f) = 0 \end{cases}$

⑤ $\min\limits_{u} \quad 1 + \lambda_1(-\dot{x} + u) + \lambda_2(\dot{x})$

$$\Rightarrow \min_{u \in \{-1, 1\}} \lambda_1 u$$

$$\Rightarrow \begin{cases} \text{if } \lambda_1 < 0, & u = 1 \\ \text{if } \lambda_1 \geq 0, & u = -1 \end{cases}$$

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