Advanced Dynamics

HW9

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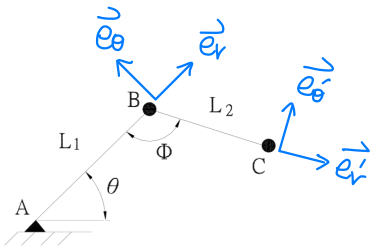
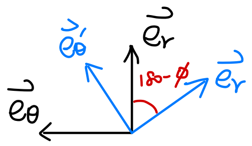
#1





The velocity of partical.

$$\vec{r}_B = L_1 \vec{e}_r$$

$$\vec{v}_B = \frac{d}{dt}\vec{r}_B = \frac{d}{dt}(L_1\vec{e}_r) = \cancelto{0}{\dot{L}_1}\vec{e}_r + L_1\dot{\vec{e}}_r$$

$$= L_1\dot{\theta}\vec{e}_\theta$$

$$\vec{e}_r' = \cos(180^\circ-\phi)\vec{e}_r - \sin(180^\circ-\phi)\vec{e}_\theta$$

$$= -\cos\phi\,\vec{e}_r - \sin\phi\,\vec{e}_\theta$$

$$\vec{e}_\theta' = \sin(180^\circ-\phi)\,\vec{e}_r + \cos(180^\circ-\phi)\,\vec{e}_\theta$$

$$= \sin\phi\,\vec{e}_r + \cos\phi\,\vec{e}_\theta$$

$$\vec{r}_C = L_1\vec{e}_r + L_2\vec{e}_r'$$

$$= L_1\vec{e}_r + L_2(-\cos\phi\,\vec{e}_r - \sin\phi\,\vec{e}_\theta)$$

$$= (L_1 - L_2\cos\phi)\vec{e}_r - L_2\sin\phi\,\vec{e}_\theta$$

$$\vec{v}_C = \cancelto{0}{\dot{L}_1}\vec{e}_r + L_1\dot{\vec{e}}_r + \cancelto{0}{\dot{L}_2}\vec{e}_r' + L_2\dot{\vec{e}}_r'$$

$$= L_1\dot{\theta}\vec{e}_\theta + L_2\dot{\phi}\vec{e}_\theta'$$

$$= L_1\dot{\theta}\vec{e}_\theta + L_2\dot{\phi}(\sin\phi\,\vec{e}_r + \cos\phi\,\vec{e}_\theta)$$

$$= L_1\dot{\theta}\vec{e}_\theta + L_2\dot{\phi}\sin\phi\,\vec{e}_r + L_2\dot{\phi}\cos\phi\,\vec{e}_\theta$$

$$= L_2\dot{\phi}\sin\phi\,\vec{e}_r + (L_1\dot{\theta} + L_2\dot{\phi}\cos\phi)\vec{e}_\theta$$

$$\|\vec{v}_B\|^2 = (L_1\dot{\theta})^2$$

$$\|v_C\|^2 = (L_2\dot{\phi}\sin\phi)^2 + (L_1\dot{\theta} + L_2\dot{\phi}\cos\phi)^2 = (L_2\dot{\phi})^2\sin^2\phi + (L_1\dot{\theta})^2 + 2L_1L_2\dot{\theta}\dot{\phi}\cos\phi + (L_2\dot{\phi})^2\cos^2\phi$$

$$= (L_1\dot{\theta})^2 + 2L_1L_2\dot{\theta}\dot{\phi}\cos\phi + (L_2\dot{\phi})^2\cancelto{1}{(\sin^2\phi + \cos^2\phi)}$$

$$= (L_1\dot{\theta})^2 + 2L_1L_2\dot{\theta}\dot{\phi}\cos\phi + (L_2\dot{\phi})^2$$

Find the Kinematic energy of system.

$$T = \sum_{i=1}^{2}\frac{1}{2}m v_i^2 = \frac{1}{2}m v_B^2 + \frac{1}{2}m v_C^2 = \frac{1}{2}m(v_B^2 + v_C^2)$$

$$= \frac{1}{2}m\left[(L_1\dot{\theta})^2 + (L_1\dot{\theta})^2 + 2L_1L_2\dot{\theta}\dot{\phi}\cos\phi + (L_2\dot{\phi})^2\right]$$

$$= \frac{1}{2}m\left[2(L_1\dot{\theta})^2 + 2L_1L_2\dot{\theta}\dot{\phi}\cos\phi + (L_2\dot{\phi})^2\right]$$

Find the Potential energy of the system,

$$V = \sum_{i=1}^{2} mgh_i = mgh_B + mgh_C$$

$$= mg\,L_1 \sin\theta + mg\left[L_1 \sin\theta - L_2 \sin(\theta+\phi)\right]$$

$$= mg\left[2L_1 \sin\theta - L_2 \sin(\theta+\phi)\right]$$

Aside:

$$h_B = \vec{r}_B \cdot \vec{j} = (L_1 \vec{e}_r)\cdot(\sin\theta\,\vec{e}_r + \cos\theta\,\vec{e}_\theta)$$

$$= L_1 \sin\theta$$

$$h_C = \vec{r}_C \cdot \vec{j}$$

$$= \left[(L_1 - L_2\cos\phi)\vec{e}_r - L_2 \sin\phi\,\vec{e}_\theta\right]\cdot\left[\sin\theta\,\vec{e}_r + \cos\theta\,\vec{e}_\theta\right]$$

$$= L_1\sin\theta - L_2\sin\theta\cos\phi - L_2\cos\theta\sin\phi$$

$$= L_1\sin\theta - L_2(\sin\theta\cos\phi + \cos\theta\sin\phi)$$

$$= L_1\sin\theta - L_2\sin(\theta+\phi)$$

Find the Lagrangian function

$$L = T - V$$

$$= \frac{1}{2}m\left[2(L_1\dot\theta)^2 + 2L_1L_2\dot\theta\dot\phi\cos\phi + (L_2\dot\phi)^2\right]$$

$$- mg\left[2L_1\sin\theta - L_2\sin(\theta+\phi)\right]$$

Substitude into Lagrangian Equation.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot q_i}\right) - \frac{\partial L}{\partial q_i} = 0$$

For $q_i = \theta$,

$$\frac{\partial L}{\partial \dot\theta} = \frac{\partial}{\partial \dot\theta}\left\{\frac{1}{2}m\left[2(L_1\dot\theta)^2 + 2L_1L_2\dot\theta\dot\phi\cos\phi + (L_2\dot\phi)^2\right] - mg\left[2L_1\sin\theta - L_2\sin(\theta+\phi)\right]\right\}$$

$$= \frac{1}{2}m\left(4L_1^2\dot\theta + 2L_1L_2\dot\phi\cos\phi\right) = 2mL_1^2\dot\theta + mL_1L_2\dot\phi\cos\phi$$

$$\frac{\partial L}{\partial \theta} = \frac{\partial}{\partial \theta}\left\{\frac{1}{2}m\left[2(L_1\dot\theta)^2 + 2L_1L_2\dot\theta\dot\phi\cos\phi + (L_2\dot\phi)^2\right] - mg\left[2L_1\sin\theta - L_2\sin(\theta+\phi)\right]\right\}$$

$$= -mg\left[2L_1\cos\theta - L_2\cos(\theta+\phi)\right] = -2mgL_1\cos\theta + mgL_2\cos(\theta+\phi)$$

$$\Rightarrow \frac{d}{dt}\left[2mL_1^2\dot\theta + mL_1L_2\dot\phi\cos\phi\right] - \left[-mgL_1\cos\theta + mgL_2\cos(\theta+\phi)\right] = 0$$

$$2mL_1^2\ddot\theta + mL_1L_2\left(\ddot\phi\cos\phi - \dot\phi^2\sin\phi\right) + mgL_1\cos\theta - mgL_2\cos(\theta+\phi) = 0$$

$$\Rightarrow m\left[2L_1^2\ddot\theta + L_1L_2\ddot\phi\cos\phi - L_1L_2\dot\phi^2\sin\phi\right] + mg\left[L_1\cos\theta - L_2\cos(\theta+\phi)\right] = 0 \quad \#$$

For $q_i = \phi$,

$$\frac{\partial L}{\partial \dot\phi} = \frac{\partial}{\partial \dot\phi}\left\{\frac{1}{2}m\left[2(L_1\dot\theta)^2 + 2L_1L_2\dot\theta\dot\phi\cos\phi + (L_2\dot\phi)^2\right] - mg\left[2L_1\sin\theta - L_2\sin(\theta+\phi)\right]\right\}$$

$$= \frac{1}{2}m\left(2L_1L_2\dot\theta\cos\phi + 2L_2^2\dot\phi\right) = mL_1L_2\dot\theta\cos\phi + mL_2^2\dot\phi$$

$$\frac{\partial L}{\partial \phi} = \frac{\partial}{\partial \phi}\left\{\frac{1}{2}m\left[2(L_1\dot\theta)^2 + 2L_1L_2\dot\theta\dot\phi\cos\phi + (L_2\dot\phi)^2\right] - mg\left[2L_1\sin\theta - L_2\sin(\theta+\phi)\right]\right\}$$

$$= -mg\left[-L_2\cos(\theta+\phi)\right] = mgL_2\cos(\theta+\phi)$$

$$\Rightarrow \frac{d}{dt}\left[mL_1L_2\dot\theta\cos\phi + mL_2^2\dot\phi\right] - mgL_2\cos(\theta+\phi) = 0$$

$$mL_1L_2(\ddot\theta\cos\phi - \dot\theta\dot\phi\sin\phi) + mL_2^2\ddot\phi - mgL_2\cos(\theta+\phi) = 0$$

$$\Rightarrow m\left[L_1L_2\ddot\theta\cos\phi + L_2^2\ddot\phi - L_1L_2\dot\theta\dot\phi\sin\phi\right] - mgL_2\cos(\theta+\phi) = 0 \quad \#$$

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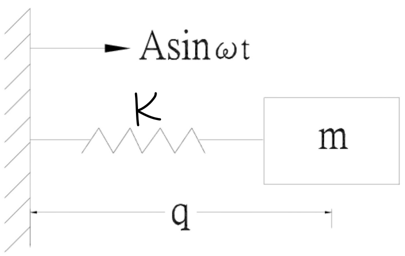

$$r_m = q + A\sin\omega t$$

$$V_m = \dot q + A\omega\cos\omega t$$

$$T = \frac{1}{2}m(\dot q + A\omega\cos\omega t)^2$$

$$V = \frac{1}{2}K(q + A\sin\omega t)^2$$

$$L = T - V$$

$$= \frac{1}{2}m(\dot q + A\omega\cos\omega t)^2 - \frac{1}{2}K(q + A\sin\omega t)^2$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot q}\right) + \frac{\partial L}{\partial q} = 0$$

$$\Rightarrow \frac{d}{dt}\left[m(\dot q + A\omega\cos\omega t)\right] - K(q + A\sin\omega t) = 0$$

$$\Rightarrow m(\ddot q - A\omega^2\sin\omega t) - K(q + A\sin\omega t) = 0$$

$$\Rightarrow m\ddot q - kq = mA\omega^2\sin\omega t + kA\sin\omega t \quad \#$$