#2 (a)

$$f(\rho) = \frac{1}{2}\tan^{-1}\left(\frac{2\rho\sin\theta}{1-\rho^2}\right), \quad \text{Let } x(\rho) = \frac{2\rho\sin\theta}{1-\rho^2}$$

$$\Rightarrow f(x) = \frac{1}{2}\tan^{-1}(x)$$

From Maclaurin series,

$$f(\rho) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}\rho^n$$

for $n=0$,

$$f(0) = \frac{1}{2}\tan^{-1}(0) = 0$$

for $n=1$,

$$f'(\rho) = \frac{df}{d\rho} = \frac{df}{dx}\frac{dx}{d\rho} = \frac{1}{\cancel{4}(x^2+1)} \cdot \frac{\cancel{4}\sin\theta(1+\rho^2)}{(1-\rho^2)^2} = \frac{\sin\theta(\rho^2+1)}{1-2\rho^2+\rho^4+4\rho^2\sin^2\theta}$$

- $$\frac{df}{dx} = \frac{1}{2(x^2+1)}$$
- $$\frac{dx}{d\rho} = \frac{2\sin\theta}{1-\rho^2} + \frac{2\rho\sin\theta}{(1-\rho^2)^2}(-1)(-2\rho)$$

$$= \frac{2\sin\theta(1-\rho^2)+4\rho^2\sin\theta}{(1-\rho^2)^2} = \frac{2\sin\theta(1+\rho^2)}{(1-\rho^2)^2}$$

$$f'(0) = \frac{1}{2(x(0)^2+1)}(2\sin\theta) = \underline{\sin\theta}$$

for $n=2$,

$$f''(\rho) = \frac{2\rho\sin\theta}{1-2\rho^2+\rho^4+4\rho^2\sin^2\theta} - \frac{\sin\theta(\rho^2+1)}{(1-2\rho^2+\rho^4+4\rho^2\sin^2\theta)^2}(-4\rho+4\rho^3+8\rho\sin^2\theta)$$

$$= \frac{2\rho\sin\theta\left[-\rho^4-2\rho^2+2\cos(2\theta)+1\right]}{1-2\rho^2+\rho^4+4\rho^2\sin^2\theta}$$

$$\Rightarrow f''(0) = 0 - 0 = \underline{0}$$