

1.

$$\min -x(t_f)$$

s.t.

$$\dot{x} = x + \sin u, \quad x(0) = 0 \quad \#$$

$$H = \lambda(x + \sin u), \quad \phi(x) = -x, \quad \psi(x) = 0$$

$$\begin{cases} \dot{x} = x + \sin u & - \textcircled{1} \\ \dot{\lambda} = -\lambda & - \textcircled{2} \\ H_u = \lambda \cos u = 0 & - \textcircled{3} \\ \lambda(t_f) + 1 = 0 & - \textcircled{4} \end{cases}$$

From $\textcircled{3}$

$$\begin{aligned} \lambda \cos u = 0 &\Rightarrow \cos u(t) = 0 \\ &\Rightarrow \sin u(t) = 1 \quad \# \end{aligned}$$

From $\textcircled{1}$

$$\begin{aligned} \dot{x} &= x + \sin u = x + 1 \\ \frac{dx}{dt} &= x + 1 \Rightarrow \int_0^{x(t)} \frac{dx}{x+1} = \int_0^t dz \\ &\Rightarrow \ln(x+1) \Big|_0^{x(t)} = z \Big|_0^t \\ &\Rightarrow \ln(x(t)+1) - \ln(1) = t - 0 \\ &\Rightarrow x(t)+1 = e^t \Rightarrow x(t) = e^t - 1 \quad \# \end{aligned}$$