3.6 Implicit differentiation and related rates

A function is said to be defined explicitly, meaning that y is defined by a rule or formula $f(x)$ in x alone is to be defined implicitly, meaning that y is defined by an equation in x and y.

example : $y = x^5 + 7x$

$x^2 + y^2 = 25$

Find $\frac{dy}{dx}$ by implicit differentiation

1. Differentiate both sides of the equation with respect to x, when differentiating a y, include $\frac{dy}{dx}$.
2. Collect all terms involving $\frac{dy}{dx}$ on one side, and all others on the other side.
3. Factor out the $\frac{dy}{dx}$ and solve for it by dividing.

example 1,2

$x^2 + y^2 = 25$

$\frac{d}{dx}x^2 + \frac{d}{dx}y^2 = \frac{d}{dx}25$

$2x + 2y\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{x}{y}$

The slope of the circle $x^2 + y^2 = 25$

at (3, 4) is $\frac{dy}{dx} = -\frac{x}{y} = -\frac{3}{4}$

at (3, -4) is $\frac{dy}{dx} = -\frac{x}{y} = \frac{3}{4}$

example 4

$y^4 + x^4 - 2x^2y^2 = 9$

$4y^3\frac{dy}{dx} + 4x^3 - 4xy^2 - 4x^2y\frac{dy}{dx} = 0$

$$(4y^3 - 4x^2y)\frac{dy}{dx} = -4x^3 + 4xy^2$$

$$\Longrightarrow \frac{dy}{dx} = \frac{-x^3 + xy^2}{y^3 - 4x^2y}$$

At (2, 1), $\frac{dy}{dx} = \frac{-6}{-3} = 2$

example 5

demand function $x = \sqrt{(1900 - p^3)}$

$$x^2 = 1900 - p^3$$

$$\frac{d}{dx}(x^2) = \frac{d}{dx}(1900 - p^3)$$

$$2x = -3p^2\frac{dp}{dx}$$

$$\Rightarrow \frac{dp}{dx} = \frac{-2x}{3p^2}$$

At $p = 10,\ x = \sqrt{(1900 - 10^3)} = 30$

$$\Rightarrow \frac{dp}{dx} = -\frac{60}{300} = -0.2$$

It says that the rate of change of price with respect to quantity is -0.2