3.2 Graphing using the first and second derivatives

If $f''(x) > 0$ on I , then $f'(x)$ increases on I and the graph of $f(x)$ is concave up on I.

If $f''(x) < 0$ on I , then $f'(x)$ decreases on I and the graph of $f(x)$ is concave down on I.

Concaving and inflection points

On an interval,

$f'' > 0$ means that f is concave up

$f'' < 0$ means that f is concave down

An inflection point is where the concaving changes (f'' must be zero or undefined)

example 1

$f(x) = x^3 - 9x^2 + 24x$

$f'(x) = 3x^2 - 18x + 24 = 3(x-2)(x-4) = 0$

⇨ 1^{st} order C.N. $\begin{cases} x = 2 \\ x = 4 \end{cases}$

$f''(x) = 6x - 18 = 0$

⇨ 2^{nd} order C.N. $x = 3$

intervals	sign of f'	sign of f''	$\uparrow or \downarrow$	concavity	shape
$x < 2$	$+$	$-$	$\uparrow$	down	
$2 < x < 3$	$-$	$-$	$\downarrow$	down	
$3 < x < 4$	$-$	$+$	$\downarrow$	up	
$x > 4$	$+$	$+$	$\uparrow$	up	

1. $f(x)$ has a relative maximum at $x = 2$ and $f(2) = 20$

   $f(x)$ has a relative minimum at $x = 4$ and $f(4) = 16$
2. Inflection point = (3, 18)
3. graph : P. 185

example 2

$f(x) = 18x^{\frac{1}{3}}$

$f'(x) = 6x^{-\frac{2}{3}}$ is undefined at $x = 0$

⇨ 1^{st} order C.N. $x = 0$

$f''(x) = -4x^{-\frac{5}{3}}$ is undefined at $x = 0$

⇨ 2^{nd} order C.N. $x = 0$

intervals	sign of f'	sign of f''	$\uparrow or \downarrow$	concavity	shape
$x < 0$	$+$	$+$	$\uparrow$	up	
$x > 0$	$+$	$-$	$\uparrow$	down	

1. No relative extremum
2. Inflection point : (0,0)
3. graph : P. 206

Second – derivative test

If $x = c$ is a critical number of f at which f'' is defined, then

1. $f''(c) > 0$ means that f has a relative minimum at $x = c$
2. $f''(c) < 0$ means that f has a relative maximum at $x = c$
3. $f''(c) = 0$ means that the test is inconclusive.

example 4

$f(x) = x^3 - 9x^2 + 24x$

$f'(x) = 3x^2 - 18x + 24 = 3(x-2)(x-4) = 0$

⇨ C.N. $\begin{cases} x = 2 \\ x = 4 \end{cases}$

$f''(x) = 6x - 18$

$f''(2) = -6 < 0 \Rightarrow f(x)$ has a relative minimum at $x = 2$

$f''(4) = 6 > 0 \Rightarrow f(x)$ has a relative maximum at $x = 4$

Note :

If $(c, f(c))$ is an inflection point, then either $f''(c) = 0$ or $f''(c)$ is undefined.

If $f''(c) = 0$ or $f''(c)$ is undefined, $\nRightarrow$ $(c, f(c))$ is an inflection point, example : $f(x) = x^4$