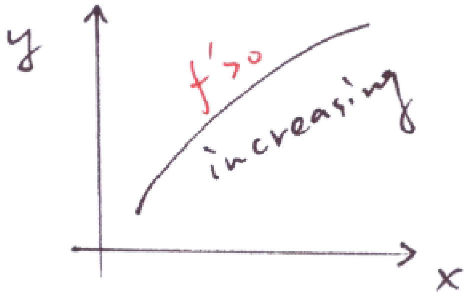
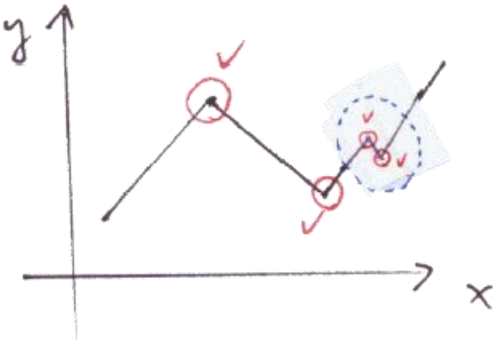
3.1 Graphing using the first derivative

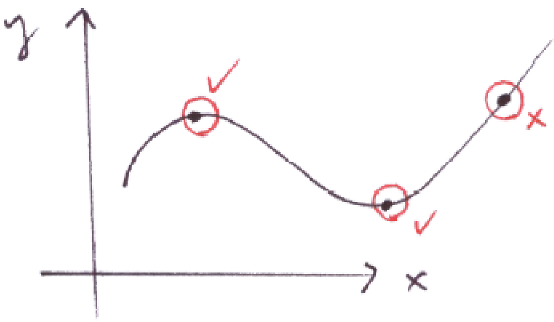
A function is said to be increasing if its graph is rising as x increase i and decreasing if its graph is falling as x increases.

If $f'(x) > 0$ for $a < x < b$, then $f(x)$ is increasing for $a < x < b$.
If $f'(x) < 0$ for $a < x < b$, then $f(x)$ is decreasing for $a < x < b$.



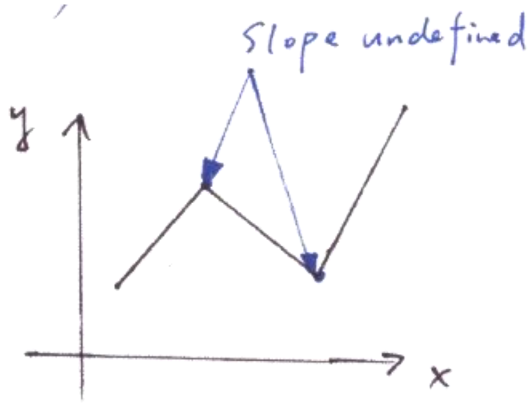
On a graph, a relative maximum point is a point that is at least as high as high as the neighboring points of the curve on either side ; and a relative minimum point is a point that is at least as low as the neighboring points on either side.

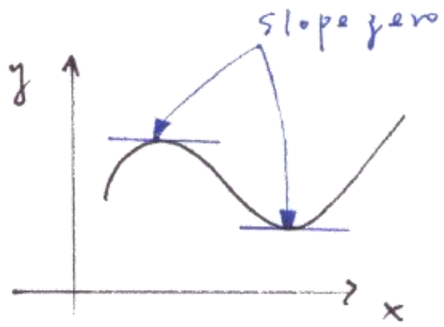




Critical number

A critical number of a function f is an x-value in the domain of f at which either $f'(x) = 0$ or $f'(x)$ is undefined. (Derivative is zero or undefined)





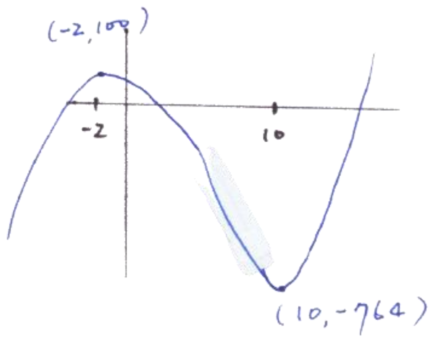
example 1

Determine where the function $f(x) = x^3 - 12x^2 - 60x + 36$ is increasing and where is decreasing, find its relative extrema, and draw the graph.

$$f'(x) = 3x^2 - 24x - 60 = 3(x+2)(x-10) = 0$$

⇨ Critical numbers C.N. $\begin{cases} x = -2 \\ x = 10 \end{cases}$

intervals	sign of f'	increasing or decreasing
$x < -2$	$+$	↑
$-2 < x < 10$	$-$	↓
$x > 10$	$+$	↑



1. $f(x)$ is ↑ for $x < -2$ and $x > 10$
   $f(x)$ is ↓ for $-2 < x < 10$
2. $f'(x)$ has a relative maximum at $x = -2$ and $f(-2) = 100$
   $f'(x)$ has a relative minimum at $x = 10$ and $f(10) = -764$

First-derivative Test

If a function f has a critical number c, then at x=c, the function has a

$$\begin{cases} relative\ maximum\ if\ f' > 0\ just\ before\ c\ and\ f' < 0\ just\ after\ c \\ relative\ minimum\ if\ f' < 0\ just\ before\ c\ and\ f' > 0\ just\ after\ c \end{cases}$$

example2

Determine where the function $f(x) = -x^4 + 4x^3 - 20$ is increasing and where is decreasing, find its relative extrema, and draw the graph

$$f'(x) = -4x^3 + 12x^2 = -4x^2(x-3) = 0$$

⇨ Critical numbers C.N. $\begin{cases} x = 0 \\ x = 3 \end{cases}$

intervals	sign of f'	increasing or decreasing
$x < 0$	$+$	↑
$0 < x < 3$	$+$	↑
$x > 3$	$-$	↓

1. $f(x)$ is increasing for $x < 3$ and decreasing for $x > 3$
2. $f(x)$ has a relative maximum at $x = 3$, and $f(3) = 7$ No relative minimum
3. graph : P. 193

example 3

$f(x) = \frac{1}{x^2-4x}$ is undefined at $x = 0$ and $x = 4$

$f'(x) = \frac{-2(x-2)}{(x^2-4x)^2}$ is zero at $x = 2$ and undefined at $x = 0$ and $x = 4$

intervals	sign of f'	increasing or decreasing
$x < 0$	$+$	↑
$0 < x < 2$	$+$	↑
$2 < x < 4$	$-$	↓
$x > 4$	$-$	↓

1. $f(x)$ is increasing for $x < 2$ and $0 < x < 2$
   $f(x)$ is decreasing for $2 < x < 4$ and $x > 4$
2. $f(x)$ has a relative maximum at $x = 2$ and $f(2) = -\frac{1}{4}$. No relative maximum