

Lecture 1

System of linear equations

Vector equation + two operations

Matrix equation

a2

a3

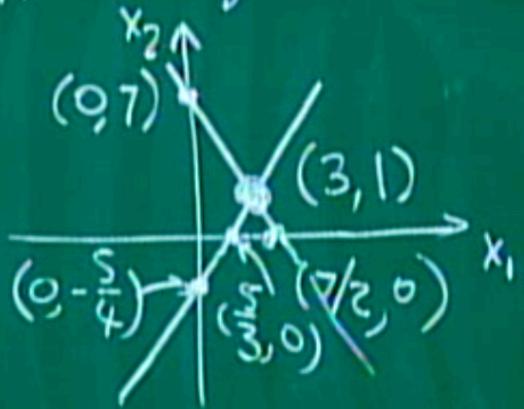
$\overbrace{2 \text{ equations in } 2 \text{ unknowns}}$

$$2x_1 + x_2 = 7$$

$$-3x_1 + 4x_2 = -5$$

a1

Picture by rows



b1

Solving the linear equations
= finding the intersection of two lines

$$\begin{bmatrix} 2x_1 + x_2 \\ -3x_1 + 4x_2 \end{bmatrix} = \begin{bmatrix} 7 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} 2x_1 \\ -3x_1 \end{bmatrix} + \begin{bmatrix} x_2 \\ 4x_2 \end{bmatrix} = x_1 \begin{bmatrix} 2 \\ -3 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

Vector Equation

$$x_1 \begin{bmatrix} 2 \\ -3 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 7 \\ -5 \end{bmatrix}$$

b2

Two Basic Vector Operations

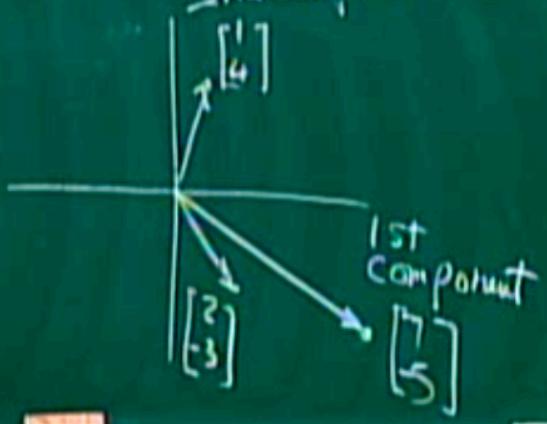
1) Vector Addition

$$\begin{bmatrix} 1 \\ 5 \end{bmatrix} + \begin{bmatrix} 2 \\ -3 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

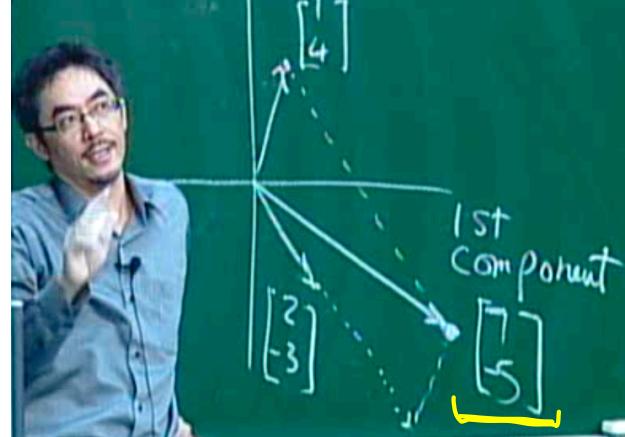
2) Scalar multiplication

$$5 \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 10 \\ 15 \end{bmatrix}, 0 \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Picture by columns
2nd component



Picture by columns
2nd component



Linear combination of columns

$$2 \begin{bmatrix} 2 \\ -3 \end{bmatrix} + 3 \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$$

Coefficients

$$3 \begin{bmatrix} 2 \\ -3 \end{bmatrix} + 1 \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 7 \\ -5 \end{bmatrix}$$

Solving the vector equation

= finding the coefficients of
Combination of two columns

且是唯一的

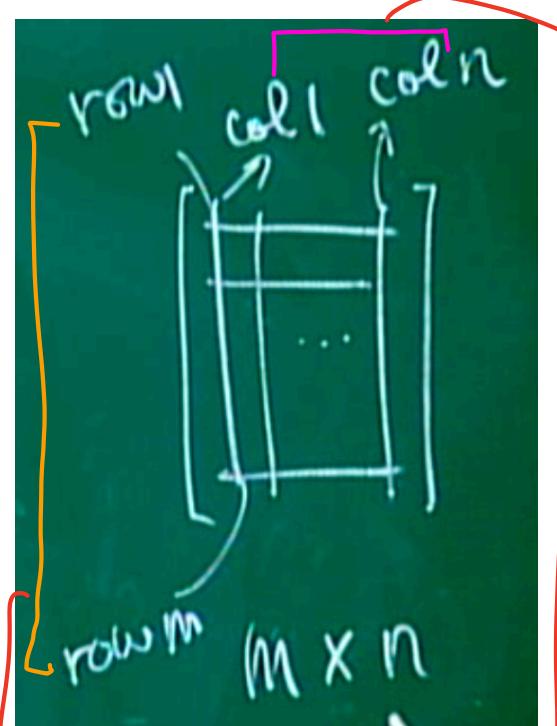
Matrix Equation

$$\left[\begin{array}{cc|c} 2 & 1 & 7 \\ -3 & 4 & -5 \end{array} \right]$$

$$A \quad x = b$$

^{2x2} Coefficient matrix Unknown vector

from a1



ref : a2

for b1

Matrix-Vector Multiplication

i) By rows (linear eq's)

$$\left[\begin{array}{cc|c} 2 & 1 & 2 \cdot x_1 + 1 \cdot x_2 \\ -3 & 4 & -3 \cdot x_1 + 4 \cdot x_2 \end{array} \right]$$

Ax is computed by the dot product of each row + x column

ref: a3

2) By columns (vector equation)

$$\begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_1 \begin{bmatrix} 2 \\ -3 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

Ax is a linear combination
of columns of A with coefficients x_i 's

for b_2

Ex:

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = 3 \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

Identity matrix $Ix = x$

$$C \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} C = \begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix}$$