Lecture 1

System of linear equations

Vector equation + two operations

Matrix equation

a2 a3

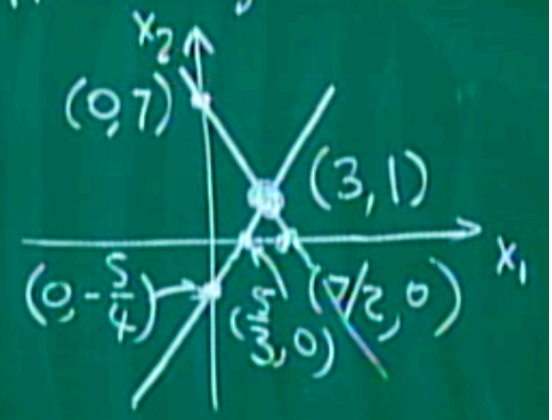
2 equations in 2 unknowns

$$2x_1 + x_2 = 7$$

$$-3x_1 + 4x_2 = -5$$

a1

Picture by rows



b1

Solving the linear equations

= finding the intersection of two lines

$$\begin{bmatrix} 2x_1 + x_2 \\ -3x_1 + 4x_2 \end{bmatrix} = \begin{bmatrix} 7 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} 2x_1 \\ -3x_1 \end{bmatrix} + \begin{bmatrix} x_2 \\ 4x_2 \end{bmatrix} = x_1 \begin{bmatrix} 2 \\ -3 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

Vector Equation

$$x_1 \begin{bmatrix} 2 \\ -3 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 7 \\ -5 \end{bmatrix}$$

b2

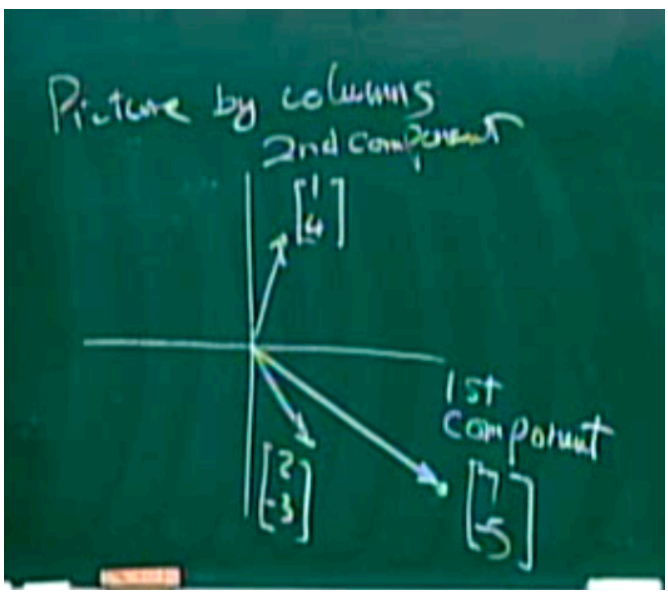
Two Basic Vector operations

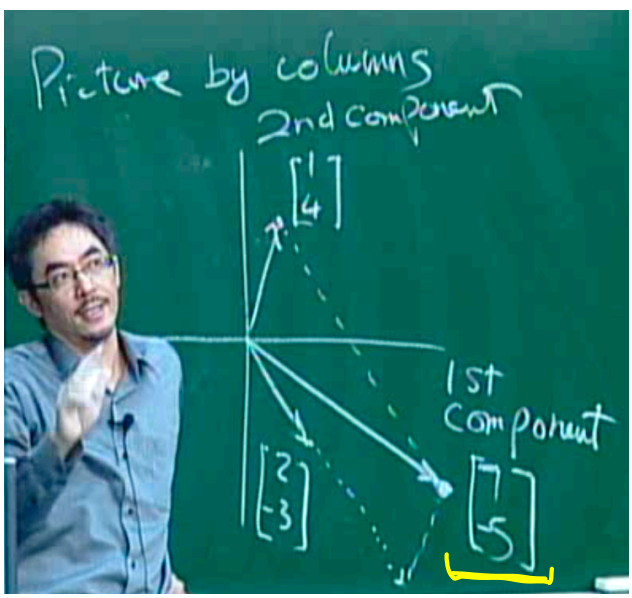
1) Vector Addition

$$\begin{bmatrix}1\\5\end{bmatrix}+\begin{bmatrix}2\\-3\end{bmatrix}=\begin{bmatrix}3\\2\end{bmatrix}$$

2) Scalar multiplication

$$5\begin{bmatrix}2\\3\end{bmatrix}=\begin{bmatrix}10\\15\end{bmatrix},\quad 0\cdot\begin{bmatrix}2\\3\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$



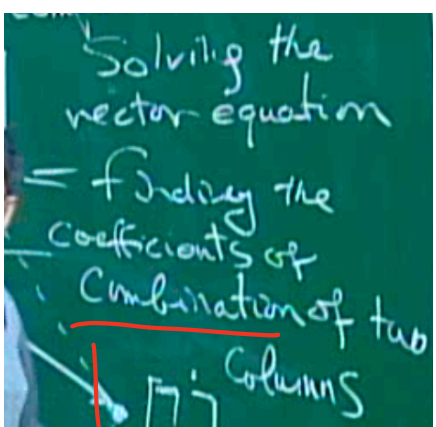


Linear combination of columns

$$2\begin{bmatrix}2\\-3\end{bmatrix}+3\begin{bmatrix}1\\4\end{bmatrix}=\begin{bmatrix}7\\6\end{bmatrix}$$

Coefficients

$$3\begin{bmatrix}2\\-3\end{bmatrix}+1\cdot\begin{bmatrix}1\\4\end{bmatrix}=\begin{bmatrix}7\\-5\end{bmatrix}$$



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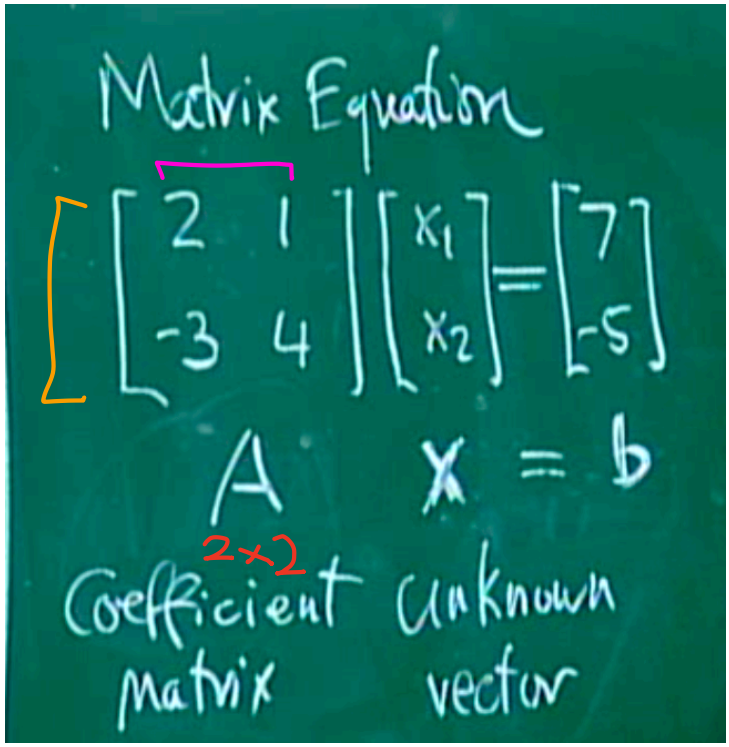
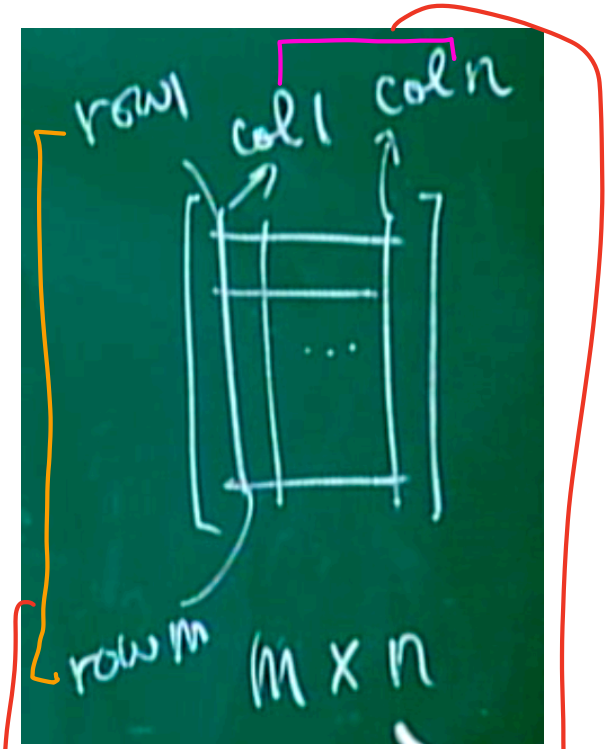


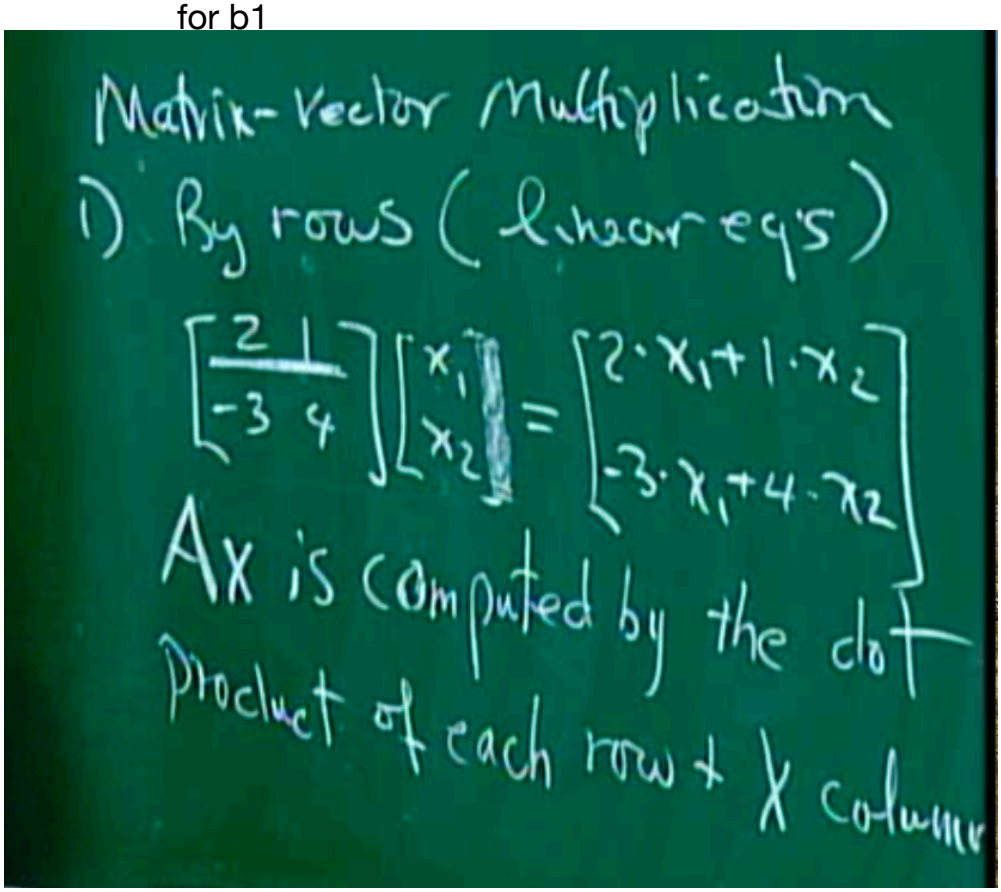
from a1



ref : a2

ref: a3

for b1



2) By columns (vector equation)

$$\begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_1\begin{bmatrix} 2 \\ -3 \end{bmatrix} + x_2\begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

Ax is a linear combination of columns of A with coefficients x_i's

for b2

Ex:

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = 3\cdot\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

Identity matrix $Ix = x$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix}$$