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Abstract—**Moment invariants have been widely applied to image pattern recognition in a variety of applications due to its invariant features on image translation, scaling and rotation. The moments are strictly invariant for the continuous function. However, in practical applications images are discrete. Consequently, the moment invariants may change over image geometric transformation. To address this research problem, an analysis with respect to the variation of moment invariants on image geometric transformation is presented, so as to analyze the effect of image's scaling and rotation. Finally, the guidance is also provided for minimizing the fluctuation of moment invariants.**

Keywords-Pattern recognition, Hu's moment invariant, Image transformation, Sapatial resolution

I. INTRODUCTION

Moments and the related invariants have been extensively analyzed to characterize the patterns in images in a variety of applications. The well-known moments include geometric moments[1], zernike moments[2], rotational moments[3], and complex moments [4].

Moment invariants are firstly introduced by Hu. In[1], Hu derived six absolute orthogonal invariants and one skew orthogonal invariant based upon algebraic invariants, which are not only independent of position, size and orientation but also independent of parallel projection. The moment invariants have been proved to be the adequate measures for tracing image patterns regarding the images translation, scaling and rotation under the assumption of images with continuous functions and noise-free. Moment invariants have been extensively applied to image pattern recognition[4-7], image registration[8] and image reconstruction. However, the digital images in practical application are not continuous and noise-free, because images are quantized by finite-precision pixels in a discrete coordinate. In addition, the noise may be introduced by various factors such as camera. In this respect, errors are inevitably introduced during the computation of moment invariants. In other words, the moment invariants may vary with image geometric transformation. But how much are the variation? To our knowledge, this issue has never been quantitatively studied for image geometric transformation.

Salama[9] analyzed the effect of spatial quantization on moment invariants. He found that the error decreases when the image size increases and the sampling intervals decrease, but does not decrease monotonically in general. Teh[10] analyzed three fundamental issues related to moment invariants, concluding that: 1) Sensitivity to image noise; 2) Aspects of information redundancy; and 3) Capability for image representation. Teh declared that the higher order moments are the more vulnerable to noise. Computational errors on moment invariants can be caused by not only the quantization and pollution of noise, but also transformations such as scaling and rotation. When the size of images are increased or decreased, the pixels of images will be interpolated or deleted. Moreover, the rotation of images also causes the change of image function, because it involves rounding pixel values and coordinates[11]. Therefore, moment invariants may change as images scale or rotate.

This paper quantitatively analyzes fluctuation of moment invariants on image scaling and rotation. Empirical studies have been conducted with various images. The major contributions of this paper include the findings the relationship among the image scaling, rotation and resolution, and the guidance of minimizing the errors of moment invariants for image scaling and rotation.

The rest of paper is organized as follows: Section II describes seven Hu's moment invariants. Section III discusses the variation of moment invariants when images scale and rotate. Section IV analyzes the relationship between moment invariants and computation. Finally, section V concludes the paper.

II. MOMENT INVARIANTS

Two-dimensional (p+q)th order moment are defined as follows:

$$m_{pq} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^p y^q f(x,y)dxdy \qquad p,q = 0,1,2,\cdots \qquad (1)$$

If the image function f(x,y) is a piecewise continuous bounded function, the moments of all orders exist and the moment sequence $\{m_{pq}\}$ is uniquely determined by f(x,y);

and correspondingly, f(x,y) is also uniquely determined by the moment sequence $\{m_{pq}\}$.

One should note that the moments in (1) may be not invariant when f(x,y) changes by translating, rotating or scaling. The invariant features can be achieved using central moments, which are defined as follows:

$$\mu_{pq} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(x-\bar{x})^p(y-\bar{y})^q f(x,y)dxdy \qquad (2)$$
$$p,q = 0,1,2,\cdots$$

where

$$\bar{x} = \frac{m_{10}}{m_{00}} \qquad \bar{y} = \frac{m_{01}}{m_{00}}$$

The pixel point ($\bar{x}$, $\bar{y}$) are the centroid of the image f(x,y).

The centroid moments μ_{pq} computed using the centroid of the image f(x,y) is equivalent to the m_{pq} whose center has been shifted to centroid of the image. Therefore, the central moments are invariant to image translations.

Scale invariance can be obtained by normalization. The normalized central moments are defined as follows:

$$\eta_{pq} = \frac{\mu_{pq}}{\mu_{00}^{\gamma}},\ \gamma = (p+q+2)/2,\ \text{p+q}=2,3,\cdots \ (3)$$

Based on normalized central moments, Hu[1] introduced seven moment invariants:

$$\phi_1 = \eta_{20} + \eta_{02}$$
$$\phi_2 = (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2$$
$$\phi_3 = (\eta_{30} - 3\eta_{12})^2 + (3\eta_{21} - \mu_{03})^2$$
$$\phi_4 = (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \mu_{03})^2$$
$$\phi_5 = (\eta_{30} - 3\eta_{12})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2]$$
$$+ (3\eta_{21} - \eta_{03})(\eta_{21} + \eta_{03})[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2]$$
$$\phi_6 = (\eta_{20} - \eta_{02})[(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2]$$
$$+ 4\eta_{11}(\eta_{30} + \eta_{12})(\eta_{21} + \eta_{03})$$
$$\phi_7 = (3\eta_{21} - \eta_{03})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2]$$
$$- (\eta_{30} - 3\eta_{12})(\eta_{21} + \eta_{03})[(3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2]$$

The seven moment invariants are useful properties of being unchanged under image scaling, translation and rotation.

III. MOMENT INVARIANTS AND IMAGE TRANSFORMATION

Image geometric transformation is a popular technique in image processing, which usually involves image translation, scaling and rotation. The translation operator maps the position of each pixel in an input image into a new position in an output image; the scale operator is used to shrink or zoom the size of an image, which is achieved by subsampling or interpolating to the input image; the rotation operator maps the position of each pixel onto a new position by rotating it through a specified angle, which may produce non-integer coordinates. In order to generate the intensity of the pixels at each integer position, different interpolation methods can be employed such as nearest-neighbor interpolation, bilinear interpolation and bicubic interpolation.

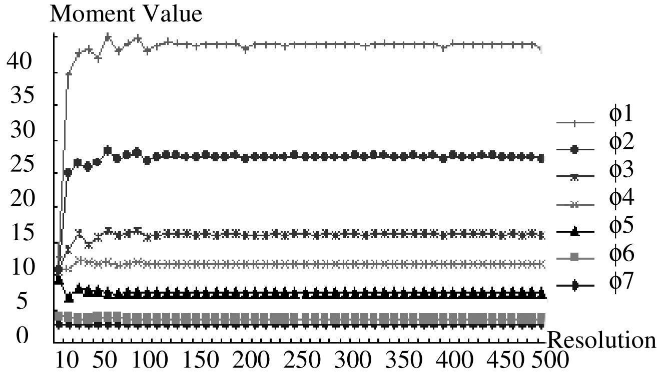
From the description in last paragraph, we can see that translation only shifts the position of pixels, but scaling and rotation not only shift the position of pixels but also change the image function itself. Therefore, only the scaling and rotation cause the errors of moment invariants. We employ two images with 250 x 250 resolutions to study what extent the image scaling and rotation can impact the moment invariants, One is a complex image without principal orientation (image A); the other is a simple image with principal orientation (image B). Figure 1 shows the two images.

Image A Image B

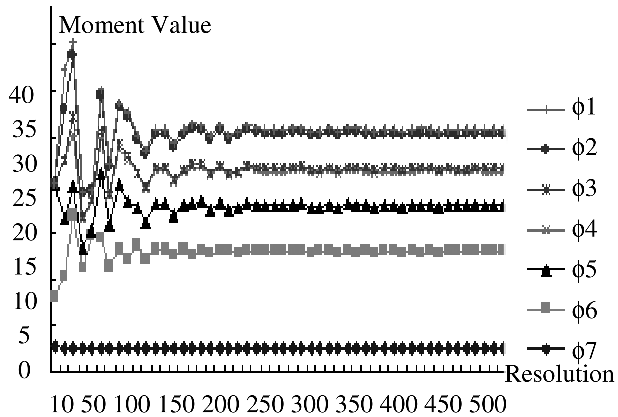
Figure 1. Images for Experiments

A. *Moment Invariants and Image Scaling*

As described above, image scaling may cause change of image function, so the moment invariants may also change correspondingly. Therefore, it is very necessary to study the relationship between moment invariants and image scaling. We conduct the research by computing seven moment invariants on image A and image B with resolution from 10x10 to 500x500 in 10 increments. The Figure 2(a), (b) separately shows the results of moment invariant for image A and B. From the diagrams we see that the resolution of 130x130 is the threshold for image A. The values have apparent fluctuations when the resolution less than the threshold but stability when the resolution more than the threshold. Correspondingly, the threshold of image B is 150x150.



(a) Moment Invariants for Image A



(b) Moment Invariants for Image B

Figure 2. Moment invariants on image scaling

Table Ⅰ. Fluctuation Ratio on Image A

Moment Invariants	Resolution <Threshold	Resolution ≥Threshold
ϕ1	0.58%	0.04%
ϕ2	47.02%	1.06%
ϕ3	60.25%	1.41%
ϕ4	88.26%	1.40%
ϕ5	124.38%	5.68%
ϕ6	84.43%	1.84%
ϕ7	102.94%	2.59%

The equation (4) is used to measure the fluctuation a series of data x.

$$R = \frac{\max(x) - \min(x)}{|\,\mathrm{mean}(x)\,|} \times 100\% \qquad (4)$$

Where, max(x), min(x) and mean(x) separately note the maximum, minimum and mean in data x. The data in Figure 2(a) are divided into two groups by the threshold 130x130 to separately compute the fluctuation by equation (4). The results are displayed in table Ⅰ. The maximum of fluctuation comes up to 124.38% in ϕ5 when resolution of image less than threshold. But it decreases to 5.68% in ϕ5 when resolution of image greater then threshold.

B. Moment Invariants and Image Rotation

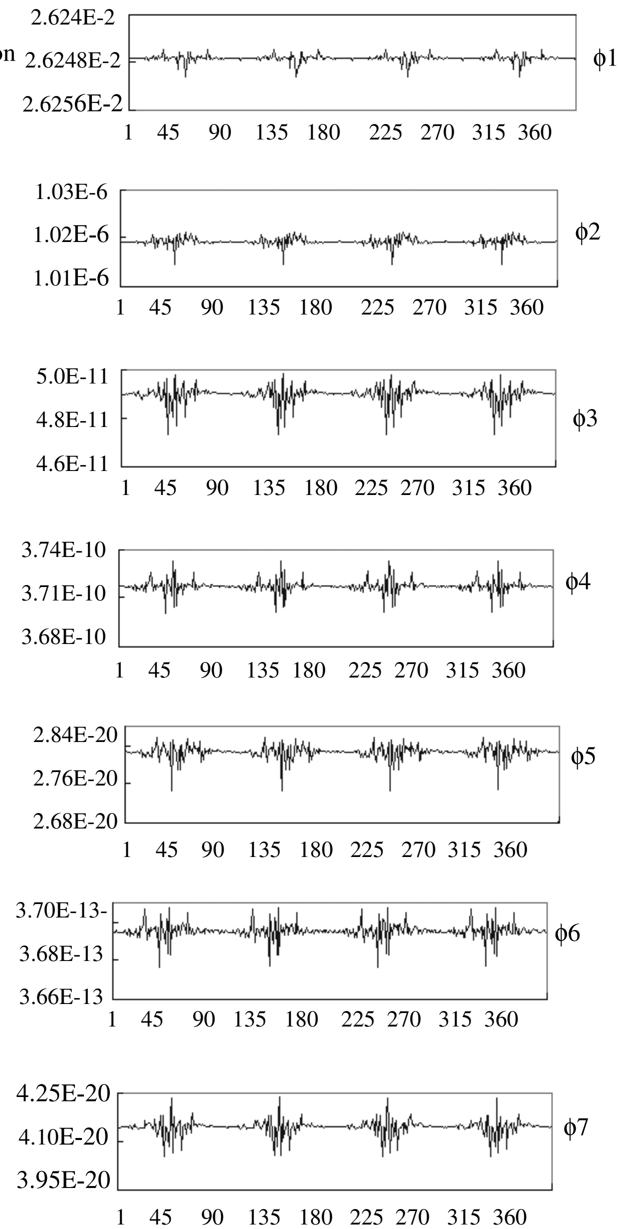


Figure 3. Moment Invariants on Image B Rotated from 1° to 360° in 1° Increments

Since the rotation of an image can change the function of the image more or less, the moment invariants keep changing as the image is rotated. In order to investigate the fluctuation of moment invariants result from the image rotation, we conduct the two kinds of experiments, i.e., one kind is the different images (A, B) with the same resolution; another is the different resolutions of the same image. Each input image is rotated from 1^o to 360^o in 1^o increments.

Figure 3 display the results of moment invariants for image B rotated from 1^o to 360^o, it shows the fluctuation of moment invariants becomes strong when the rotated angle near to 45,135, 225 and 315 degrees, while becomes weak as the rotated angle near to 90,180, 270 and 360 degree.

In Section III A, we know the threshold of resolution on Image B is 150x150. Does the same image have same threshold between scaling and rotation? In order to answer this question, we produce a series of images with resolutions from 60x60 to 330x330 in 30 increments using the same original image, and compute the moment invariants for each image rotated from 1^o to 360^o. Then, the fluctuation is computed by equation (4) for each 360 images. Figure 4 shows the trend of fluctuation of moment invariants. From the diagram, we can see that the threshold is different between image scaling and image rotation. The threshold of image rotation on Image B is not 150x150 but 240x240.

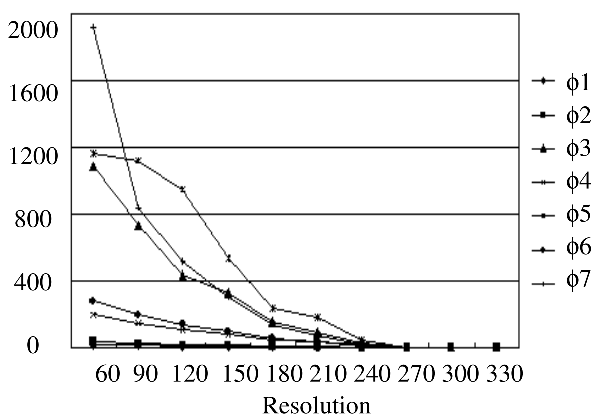


Figure 4. Fluctuation of Moment Invariants on Image B with Different Resolution

Table Ⅱ shows the values of fluctuation for seven moment invariants on different resolution from 60x60 to 330x330. We can see that the fluctuation decreases as the image spatial resolution increases. The fluctuation almost comes up to 1921.1% when the resolution is only 60x60, but rapidly decreases to 1.1% when the resolution is 270x270. The fluctuation obviously decreases as the resolution increases until to the threshold. However, the fluctuation does not monotonically decrease any more when the resolution greater than 270x270.

IV. MOMENT INVARIANTS AND COMPUTATION

As discussed in Section III, we can reach the conclusion: The higher the resolution is, the lower the fluctuation is. However, the computation of moment invariants will increase when the resolution increases. As a consequence, the research of relationship between the resolution of images and computation is necessary.

We calculate the computation of moment invariants for the resolutions from 10x10 to 1500x1500 in 30 increments on a PC (CPU P4 2.0G, RAM 1G). Figure 5 shows the results. From the diagram, we can see that the relationship between the resolution and the computation is non-linear. Therefore, we must select an acceptable resolution to balance computation and resolution on the real application.

Table Ⅱ. Fluctuation of Moment Invariants on Different Resolution of Image B

Resolution	ϕ1	ϕ2	ϕ3	ϕ4	ϕ5	ϕ6	ϕ7
60x60	18.7	39.9	1084.7	193.8	1157.5	280.6	1921.1
90x90	13.3	26.5	730.9	145.3	1118,1	194.7	842.0
120x120	10.7	19.1	436.0	109.9	947.6	140.7	517.4
150x150	7.4	13.6	328.0	86.3	532.0	98.9	302.1
180x180	4.5	8.2	159.2	51.5	237.3	57.7	140.0
210x210	3.2	5.6	88.1	36.2	179.3	38.9	75.5
240x240	1.1	1.9	21.4	12.3	46.2	12.8	19.7
270x270	0.2	0.3	1.8	0.4	2.9	0.5	1.1
300x300	0.2	0.5	1.4	0.3	2.0	0.5	1.3
330x330	0.1	0.3	1.9	0.2	1.2	0.2	1.7

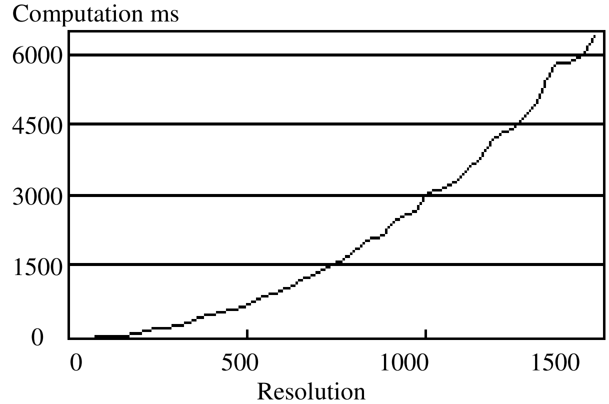


Figure 5. Computation of Moment Invariants for Different Resolution

V. CONCLUSION

This paper has presented an analysis of fluctuation of Hu's moment invariants on image scaling and rotation. Our findings may be summarized as follows: (1) the moment invariants change as images scale or rotate, because images are not continuous function or polluted by noise; (2) the fluctuation decreases when the spatial resolution of images increases. However, the change will not remarkably decrease as resolution increases if the resolution greater than the threshold; (3) the computation increases quickly as resolution increases.

From the experimental studies, we find that the choice of image spatial resolution is very important to keep invariant features. To decrease the fluctuation of moment invariants, the image spatial resolution must be higher than the threshold of scaling and rotation. However, the resolution cannot be too high, because the computation will remarkably increase as the resolution increases. Therefore, the choice of resolution must balance computation and resolution on the real application

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