Robust Bayesian Allocation

Ricky

TradingValley

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Overview

Motivation

- Typically, asset allocation is a two-step approach: estimate market distribution (μ, Σ), and then perform optimization for asset allocation (w).
- However, the second step is very sensitive due to input parameters (μ, Σ).
- In order to deal with this issue, robust method provides different approach for estimation risk by giving a uncertain range for parameter.
- Bayesian approach is quite nature to identify a suitable uncertainty range for the market parameters μ, Σ (location-dispersion ellipsoid of the posterior distribution)

Allocation

- Recall, classical approach

$$w = \text{argmax } w'\mu$$

$$s.t.\ w \in C, max\{w'\Sigma w\} \leq v^i$$

- Robust:

$$w = argmax\{min\{w'\mu\}\}$$

$$s.t.\ w \in C, max\{w'\Sigma w\} \leq v^i, \Sigma \in \Theta_\Sigma$$

- Question: how to measure uncertainty range for μ, Σ?

- $$\hat{\Theta}_\mu = \{\mu : (\mu - \hat{\mu}_{ce})' S_\mu^{-1} (\mu - \hat{\mu}_{ce}) \leq q_\mu^2\}$$

$$\hat{\Theta}_\Sigma = \{\Sigma : vech[\Sigma - \hat{\Sigma}_{ce}]' S_\Sigma^{-1} vech[\Sigma - \hat{\Sigma}_{ce}] \leq q_\Sigma^2\}$$

$$\text{where } q^2 \sim x^2$$

- In order to determine $\hat{\Theta}_\mu$, $\hat{\Theta}_\Sigma$, we have to compute posterior of μ, Σ first
- Assume returns follow normal distribution:

$$R_t | \mu, \Sigma \sim N(\mu, \Sigma)$$

Prior follows:

$$\mu | \Sigma \sim N(\mu_0, \frac{\Sigma}{T_0})$$

,

$$\Sigma^{-1} \sim W(v_0, \frac{\Sigma_0^{-1}}{v_0})$$

- Respective posterior parameters:

$$T_1 = T_0 + T$$

$$\mu_1 = \frac{1}{T_1}[T_0\mu_0 + T\hat{\mu}]$$

$$v_1 = v_0 + T$$

$$\Sigma_1 = \frac{1}{v_1}[v_0\Sigma_0 + T\hat{\Sigma} + \frac{(\mu_0 - \hat{\mu})(\mu_0 - \hat{\mu})'}{\frac{1}{T}\frac{1}{T_0}}]$$

- Therefore:

$$\hat{\mu}_{ce} = \mu_1$$

$$S_\Sigma = \frac{1}{T_1}\frac{v_1}{v_1 - 2}\Sigma_1$$

$$\hat{\Sigma}_{ce} = \frac{v_1}{v_1 + N + 1}\Sigma_1$$

$$S_\Sigma = \frac{2v_1^2}{(v_1 + N + 1)^3}(D_N'(\Sigma_1^{-1} \otimes \Sigma_1^{-1})D_N)^{-1}$$

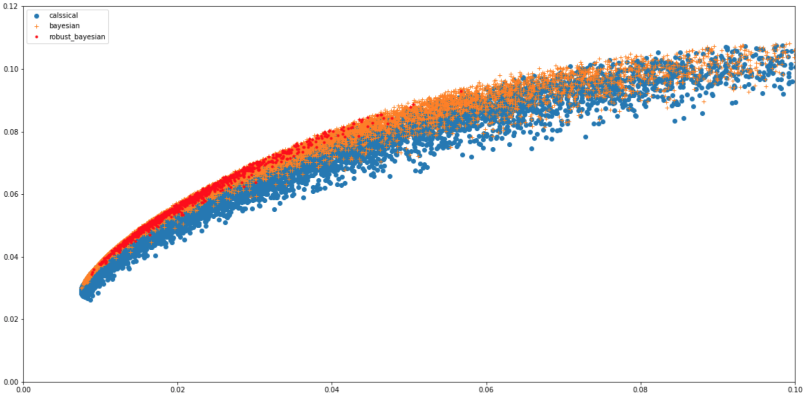
- Then, robust bayesian allocation becomes:

$$W^i_{rB} = \text{argmax}\{w'\mu_1 - r_\mu\sqrt{w'\Sigma_1 w}\}$$

$$s.t.\ w \in C, w'\Sigma w \leq r^i_\Sigma$$

$$where\ r_\mu = \sqrt{\frac{q^2_\mu}{T_1}\frac{v_1}{v_1 - 2}},\ r^i_\Sigma = \frac{v^i}{\frac{v_1}{v_1+N+1} + \sqrt{\frac{2v_1^2 q^2_\Sigma}{(v1+N+1)^3}}}$$

Result



calssical
bayesian
robust_bayesian
0.12
0.10
0.08
0.06
0.04
0.02
0.00
0.00
0.02
0.04
0.06
0.08
0.10