National Tsing Hua University
Department of Electrical Engineering
EE3662 Digital Signal Processing Laboratory, Fall 2021

Lab 7 Image Filtering and Corner Detection

Assigned on Nov 15, 2021

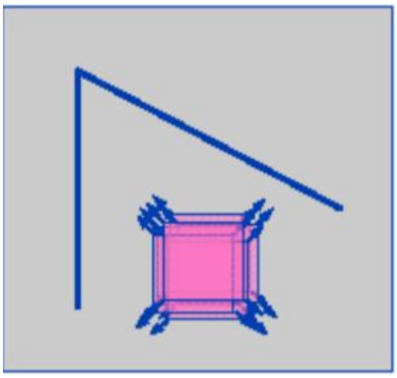
Due by Nov 22, 2021

Overview

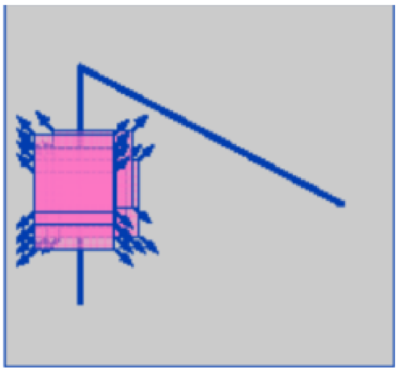
The goal of this lab is to use image filter to calculate image gradients. Then, we combine image gradients and apply Gaussian filter to compute second moment matrix at each pixel location. Next, we can compute the corner response function. Finally, we apply a threshold and non-maximum suppression to obtain distinctive corner locations.

(Left: original image. Right: image overlap with corners.)

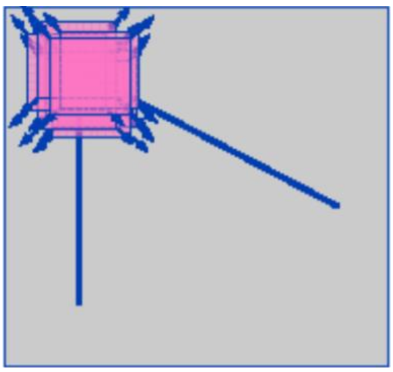
Ideas and Derivation



Flat region → no change in all directions



Edge → no change along the edge direction.



Corner → significant change in all directions

The change of intensity for the shift E(u,v) is given by

$$E(u,\mathrm{v}) = \sum_{x.y} w(x,y)|I(x+u,y+v) - I(x,y)|^2,$$

where w(x,y) is window function. By the first order approximation of Taylor Series for 2D functions, we have

$$I(x+u,y+v) \cong I(x,y) + uI_x(x,y) + vI_y(x,y),$$

and then we can get the equation as follow

$$E(u,\mathrm{v}) = \sum_{x,y} w(x,y)(u^2 I_x^2 + 2uvI_xI_y + v^2 I_y^2).$$

Rewrite it as matrix equation

$$E(u,\mathrm{v}) = \sum_{x,y} (u\ v) w(x,y) \begin{pmatrix} I_x^2 & I_xI_y \\ I_xI_y & I_y^2 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}$$

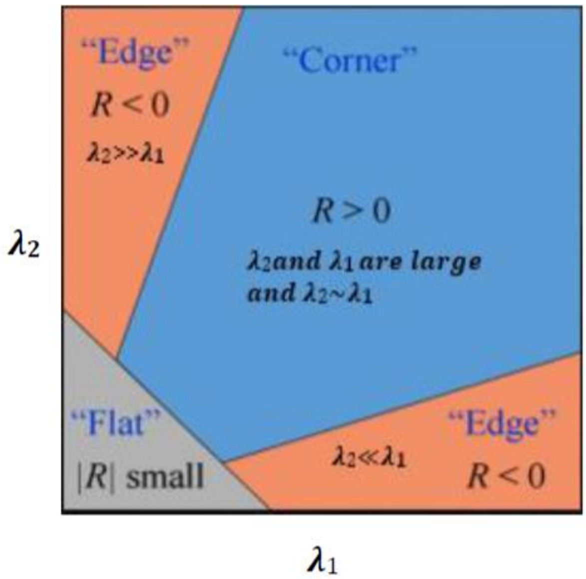
where $\mathrm{A} = \sum_{x,y} w(x,y) \begin{pmatrix} I_x^2 & I_xI_y \\ I_xI_y & I_y^2 \end{pmatrix}$, and $\begin{pmatrix} I_x^2 & I_xI_y \\ I_xI_y & I_y^2 \end{pmatrix}$ is called Harris Matrix.

Measurement of corner response is given by

$$\mathrm{R} = \det(\mathrm{A}) - \mathrm{k}\big(\mathrm{trace}(\mathrm{A})\big)^2 = \lambda_1\lambda_2 - k(\lambda_1 + \lambda_2)^2,$$

where λ_1, λ_1 are the eigenvalues of $\begin{pmatrix} I_x^2 & I_xI_y \\ I_xI_y & I_y^2 \end{pmatrix}$.

We use eigenvalues to classify image points as below



Procedure

I. Grayscale

Change the RGB to Grayscale by $R * 0.299 + G * 0.587 + B * 0.114$.

II. Get Image gradient

Use horizontal and vertical gradient filter to get I_x, I_y, I_{xy}.

III. Get Gaussian smoothed I_x, I_y, I_{xy}

Use Gaussian filter to get Gaussian smoothed I_x, I_y, I_{xy}.

IV. Calculate corner response R, and map R to 0 ~ 1000

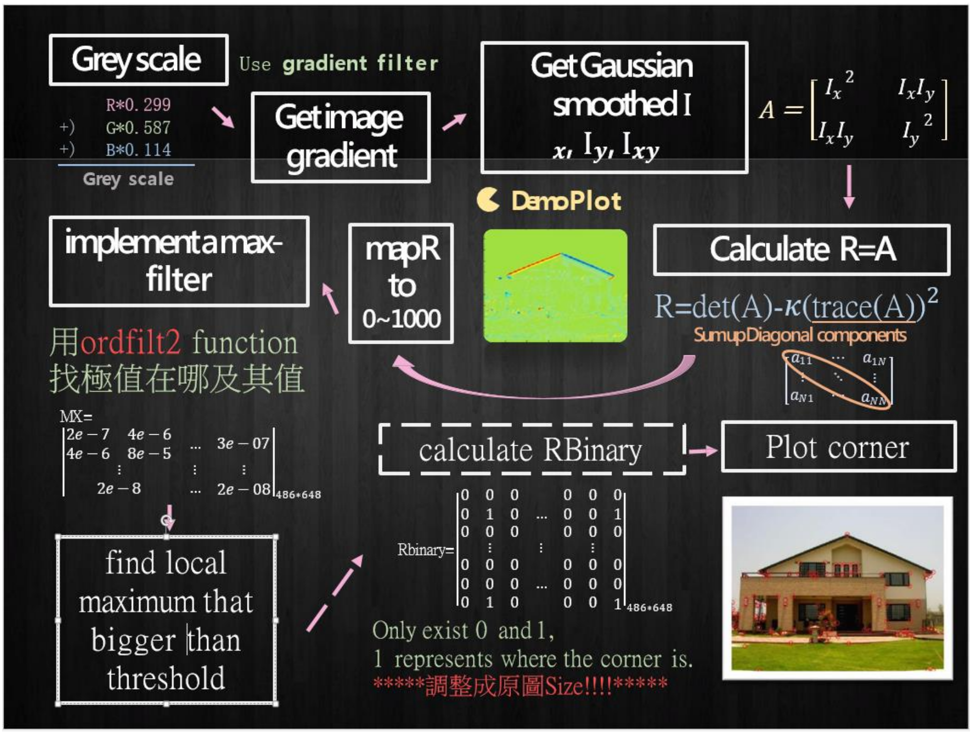
Calculate $R = det(A) - k\left(trace(A)\right)^2$, where k is 0.04 empirically and trace(A) is the sum of diagonal component of A.

V. Find local maximum

Use *ordfilt2* function to find the local maximum.

VI. Calculate RBinary and plot corners

RBinary = $\begin{vmatrix} 0 & 0 & 0 & & 0 & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 & 1 \\ 0 & 0 & 0 & & 0 & 0 & 0 \\ & \vdots & & \vdots & & \vdots & \\ 0 & 0 & 0 & & 0 & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & 0 & & 0 & 0 & 1 \end{vmatrix}$, and 1 represents where the corner is larger than the threshold, which stands for where the corner is.



(Overall procedure)

In-class Demo

1. Calculate and plot Gaussian smoothed I_{xy}. (35%)
2. Calculate RBinary and generate the image with corner detection. (15%)
3. Find all corners of the rectangle in the two provided images (img_1.png, img_2.png) to calculate how much the rectangle is shifted between img_1.png and img_2.png. (10%)

Report

1. (5%) Show at least two different corner image generation results and each needs to contain the input image, Gaussian smoothed I_{xy} and corner detection result.
2. (5%) Why using Gaussian window to get Gaussian smoothed I_x, I_y, I_{xy}?
3. (5%) Why do we need the threshold in procedure VI?
4. (10%) Analyze the result of using Rectangular window function and try to discuss the difference between Rectangular window function and Gaussian window function. Besides, please try your own window function and explain the reason why you choose it.
   Rectangular window function (use the same size as the provided Gaussian filter):

1	1	...	1
1	1	...	1
...	...	...	...
1	1	...	1

5. (10%) Analyze the result of using Sobel and Scharr gradient filter. Besides, please try to discuss the difference between Sobel and Scharr gradient filter. Below shows these two gradient filters.

Sobel

dx			dy		
1	0	-1	1	2	1
2	0	-2	0	0	0
1	0	-1	-1	-2	-1

Scharr

dx			dy		
3	0	-3	3	10	3
10	0	-10	0	0	0
3	0	-3	-3	-10	-3

6. (5%) Conclusion

Deliverable and file organization

Directory	Filename	Description
LAB7/code/	*.m	Matlab code
LAB7/data/	*.png / *.jpg	Your own source images
LAB7/results/	*.png / *.jpg	Your results
LAB7/report/	report.pdf	Your report

Please organize your files according to the above table and compress it as LAB7_10xxxxxxx.zip in ZIP format. (P.S. 10xxxxxxx is your student ID number)

Reference

[1] http://www.cse.psu.edu/~rtc12/CSE486/lecture06.pdf