Induction 1

Principle of induction.

Let $P(n)$ be a predicate. if

- $P(0)$ is true, and
- for all $n \in \mathbb{N}$, $P(n)$ implies $P(n+1)$

, then $P(n)$ is true for all $n \in \mathbb{N}$

Theorem For all $n \in \mathbb{N}$

$$1+2+3+\ldots+n = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i \quad \text{or} \quad \sum_{1 \leq i \leq n} i \quad \text{or} \quad \sum_{i \in \{1 \ldots n\}} i$$

Proof: $P(n)$ be the predicate

$$1+2+3+\dots+n = \frac{n(n+1)}{2}$$

- Base case: $P(0)$ is true $0 = 0$
  , both sides of the equation are zero.

- inductive step: assume that $p(n)$ is true,
  n is any natural number.
  $p(n+1)$ is also true,

$$1+2+3+\dots+n+n+1 = \frac{n(n+1)}{2} + n+1$$

$$= \frac{n(n+1)+2(n+1)}{2}$$

$$= \frac{n+1(n+2)}{2}.$$

$$1+2+3+\dots+n = \frac{n(n+1)}{2}$$

$$n+1 \rightarrow \frac{(n+1)(n+2)}{2} = \frac{n+1(n+2)}{2}$$

therefore, $p(n)$ is true for all natural n by induction

$\forall n \in \mathbb{N} \quad 3 \mid (n^3 - n)$

predicate "$3 \mid (n^3 - n)$.

$P(0) =$ true

$3 \mid 0^3 - 0$

$3 \mid 1 \quad =$ True.

Note: 0 is a multiple of 3 specifically $3 \cdot 0 = 0$

Faulty Induction Proof

e.g to proof with induction, not $\forall n \in \mathbb{N}$, $n \geq 0$ but $n \geq 1$, the induction

, proof $P(1) =$ True

, $p(n)$ implies $p(n+1)$ for $n \geq 1$.

False Theorem 14. all horses are the same colour.

Proof. $P(n)$: the proposition that in every set of n horses, all are the same colour

- Base case $P(1)$ is true, because all horses in a set of 1 must be the same colour

- inductive step: , consider a set of $n+1$ horses.

$$\underbrace{h_1, h_2, \ldots h_n}_{\text{same colour by assumption}}, h_{n+1}.$$

$P(n)$: every set of n horses, all are the same colour.

$$h_1, \underbrace{h_2, \ldots, h_n, h_{n+1}}_{\text{the last } n \text{ horses are the same color}}.$$

Therefore, $h_1, h_2, \ldots h_{n+1}$ must be the same color

$P(n+1)$ is true, thus $P(n)$ implies $P(n+1)$.

Note: $\boxed{h_1}, h_2, \ldots h_n$ $\emptyset$ overlap $\boxed{h_2}, \ldots h_n, h_{n+1}$.

$n=1$, the first set : $\boxed{h_1}$] showing do not overlap.

the second set : $\boxed{h_2}$

$P(1) \not\Rightarrow P(2)$ so the statement is false.!

for all $n \geq 0$ there exists a tiling of a $2^n \times 2^n$ courtyard w/ Bill in a central square.

fuzzy logic: truth $\not\in$ 0/1 but represented by a real value between 0 & 1.