

Proof

- Verification of propositions by a chain of logical deductions from a set of Axioms

$$\boxed{\forall n \in \mathbb{N}, n^2 + n + 41 = 2 \text{ prime numbers}}$$

\forall for all natural number

\mathbb{N} : a set of natural numbers
(0, 1, 2, 3, ...)

predicate

predicate: a kind of proposition whose truth value depends on one or more variables

note: $n^2 + n + 41 = \text{prime} \Rightarrow \text{false statement}$

when $n: 40, 40^2 + 40 + 41$

$$= 40(40 + 1) + 40 + 1$$

$$= 41(40 + 1) = \boxed{41 \cdot 41} \neq \text{prime}.$$

$$\exists a, b, c, d \in \mathbb{N}^+ \quad a^4 + b^4 + c^4 = d^4$$

Note \forall & \exists (for all, there exists): quantifier

$P \rightarrow Q$, implication



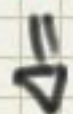
P implies Q or if P , then Q .

Implication Truth table.

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

e.g. if pigs fly, then you'll be awesome.

$F = \boxed{P} \Rightarrow Q$



From the truth table above when $P = F$,
then $P \rightarrow Q$ always T

$$\forall n \in \mathbb{Z} \quad (n \geq 2) \iff (n^2 \geq 4)$$

$P \iff Q$, P if and only if Q . (iff).

P	Q	$P \rightarrow Q$	$Q \rightarrow P$	$P \iff Q$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

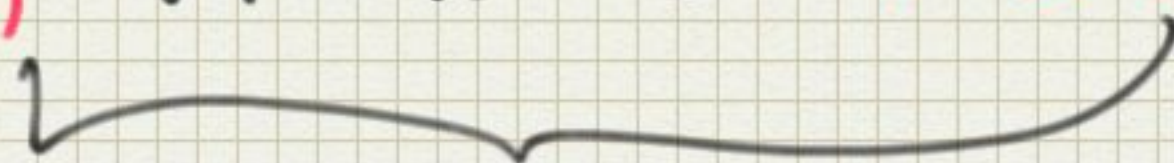
Note: for $P \iff Q$ is true, provided
 P & Q are both true or both false.

e.g. $(n \geq 2) \iff (n^2 \geq 4)$

T, when $n = 1$ F \iff F

T, when $n = 3$ T \iff T

F, when $n = -3$ F \iff T



as a whole: F

Axioms

a proposition, assumed to be true because it's somehow reasonable

e.g. if $a = b$, and $b = c$, then $a = c$

axiom $\left\{ \begin{array}{l} \text{Consistent: } \nexists \text{ proposition can be proved both true and false} \\ \text{Complete: can be proved or disproved} \end{array} \right.$

Logical deduction (inference rules)

- combine axioms and true propositions in order to form more true propositions.

One fundamental inference rule: modus ponens.

if P is true and $P \rightarrow Q$ is true, then Q is also true

$$\begin{array}{r} P \\ P \Rightarrow Q \\ \hline Q \end{array} \quad \left. \begin{array}{l} T \\ T \end{array} \right\} T$$

$$(P \wedge (P \Rightarrow Q)) \Rightarrow Q$$

if P and $P \rightarrow Q$ is true, then Q is true

$$(P \Rightarrow Q) \wedge (Q \Rightarrow R) \Rightarrow (P \Rightarrow R)$$

$$\begin{array}{c} P \Rightarrow Q \\ Q \Rightarrow R \\ \hline P \Rightarrow R \end{array}$$

$$(P \Rightarrow Q) \wedge \neg Q \Rightarrow \neg P$$

$$\begin{array}{c} P \Rightarrow Q \\ \neg Q \\ \hline \neg P \end{array}$$

A tautology:

$$(X \Rightarrow Y) \Leftrightarrow (\neg Y \Rightarrow \neg X)$$

Contrapositive of $X \Rightarrow Y$

'An implication is true if and only if its contrapositive is true.'

e.g. "if you are wise, then you attend recitation"

\Rightarrow

"if you don't attend recitation, then you're not wise"

Proof $(X \Rightarrow Y) \Leftrightarrow (\neg Y \Rightarrow \neg X)$

X	Y	$X \Rightarrow Y$	$\neg Y \Rightarrow \neg X$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

Showing: the left side is logically equivalent to the right side for every setting of the variables X & Y .

thus,

$(X \Rightarrow Y) \Leftrightarrow (\neg Y \Rightarrow \neg X)$ is true in every case (tautology)

Inference rules

$$\frac{P \rightarrow Q}{\neg Q \rightarrow \neg P}$$

$$\frac{\neg Q \rightarrow \neg P}{P \rightarrow Q}$$

A Proof by Contradiction. (Indirect proof).

Idea: to assume that the desired conclusion is false and then show that that assumption leads to an absurdity or contradiction.

, meaning: the assumption: wrong, so the desired conclusion is actually true

$$\frac{\neg P \Rightarrow \text{False.}}{P}$$

if $\neg P$ implies some falsehood, P must actually be true.

P	$(\neg P \Rightarrow \text{False}) \Rightarrow P$
T	T
F	T

$\sqrt{2}$ is an irrational number.

assume $\sqrt{2}$ is rational.

$$\sqrt{2} = a/b, \quad a \& b \text{ integers} \\ \& b \neq 0.$$

$$2 = \frac{a^2}{b^2}$$

\Rightarrow the fraction is in lowest form

$$2b^2 = a^2.$$

: a is even, 2 is a multiple of 2.

\Downarrow
 a^2 is a multiple of 4.

because of the equality.

$2b^2 = a^2$, $2b^2$ must be a multiple of 4.

\Downarrow

b^2 is even

* a & b are both even

, so the fraction a/b is not in lowest terms

so the assumption $\sqrt{2}$ is rational: F
(False)