Good Proofs & Bad Proofs.

Principle of strong Induction.

Let $P(n)$ be a predicate. If

- $P(0)$ is true, and
- for all $n \in \mathbb{N}$, $P(0), P(1), \ldots, P(n)$ imply $P(n+1)$

then $P(n)$ is true for all $n \in \mathbb{N}$

Ordinary Induction:

→ $P(n)$ is true & try to prove that $P(n+1)$ is true

Strong Induction:

$P(0), P(1), \ldots P(n-1), P(n)$ are all true when you go to prove $P(n+1)$.

Strong Induction (stacking game)

stack heights									score
10									0
5	5								25
5	3	2							6
4	3	2	1						4
2	3	2	1	2					4
2	2	2	1	2	1				2
1	2	2	1	2	1	1			1
1	1	2	1	2	1	1	1		1
1	1	1	1	2	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1
									45 points

Theorem. Every way of unstacking n blocks gives a score of $\frac{n(n-1)}{2}$ points.

$P(1)$ is a base case.

$P(1), \ldots P(n-1)$ imply $P(n)$ for all $n \geq 2$

$P(n)$: proposition that every way of unstacking n blocks gives a score of $\frac{n(n-1)}{2}$

Base case: If $n=1$, then there is only one box, no move, total score: $\frac{0(0-1)}{2} = 0$.

$P(1)$ is then true.

Inductive case: $P(1), \ldots, P(n-1)$ imply $P(n)$ for all $n \geq 2$

assume: $P(1), \ldots P(n-1)$ are all true & that we have a stack of n blocks

The first move must split this stack w/ sizes k & $(n-k)$

$k = 0 - n$

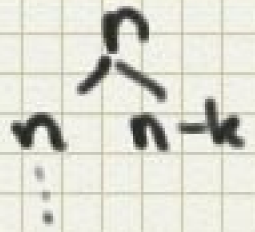
now the total score for the game:

the sum of points for this first move plus points obtained by unstacking the two resulting substacks:

totalscore = (score 1st move)

+ (score for unstacking k blocks)

+ (score for unstacking n-k blocks)



$$= k(n-k) + \frac{k(k-1)}{2} + \frac{(n-k)(n-k-1)}{2}$$

$$= nk - k^2 + \frac{k^2-k}{2} + \frac{n^2-nk-n-nk+k^2+k}{2}$$

$$= \frac{\cancel{2nk} - \cancel{2k^2} + \cancel{k^2} - \cancel{k} + n^2 - \cancel{nk} - n - \cancel{nk} + \cancel{k^2} + \cancel{k}}{2}$$

$$= \frac{n^2-n}{2} = \frac{n(n-1)}{2}.$$