

Induction 1

Principle of induction.

Let $P(n)$ be a predicate. if

- $P(0)$ is true, and

- for all $n \in \mathbb{N}$, $P(n)$ implies $P(n+1)$

, then $P(n)$ is true for all $n \in \mathbb{N}$

Theorem

For all $n \in \mathbb{N}$

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^n i \quad \text{or} \quad \sum_{1 \leq i \leq n} i \quad \text{or} \quad \sum_{i \in \{1, \dots, n\}} i$$

Proof: $P(n)$ be the predicate

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

• Base case: $P(0)$ is true $0 = 0$
, both sides of the equation are zero.

• inductive step: assume that $P(n)$ is true,
 n is any natural number.
 $P(n+1)$ is also true,

$$\begin{aligned} 1 + 2 + 3 + \dots + n + n + 1 &= \frac{n(n+1)}{2} + n + 1 \\ &= \frac{n(n+1) + 2(n+1)}{2} \\ &= \frac{(n+1)(n+2)}{2} \end{aligned}$$

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

$$n+1 \rightarrow \frac{(n+1)(n+2)}{2} = \frac{(n+1)(n+2)}{2}$$

therefore, $P(n)$ is true for all natural n by induction

$$\forall n \in \mathbb{N} \quad 3 \mid (n^3 - n)$$

predicate " $3 \mid (n^3 - n)$ ".

$$P(0) = \text{true}$$

$$3 \mid 0^3 - 0$$

$$3 \mid 0 = \text{True.}$$

Note: 0 is a multiple
of 3
specifically
 $3 \cdot 0 = 0$

Faulty Induction Proof

e.g. to proof with induction, not $\forall n \in \mathbb{N}, n \geq 0$

but $n \geq 1$, the induction

, proof $P(1) = \text{True}$

, $P(n)$ implies $P(n+1)$ for $n \geq 1$.

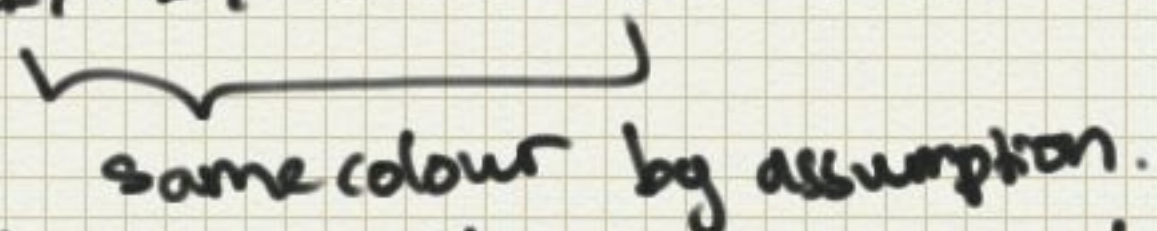
False Theorem 14. All horses are the same colour.

Proof. $P(n)$: the proposition that in every set of n horses, all are the same colour

Base case $P(1)$ is true, because all horses in a set of 1 must be the same colour

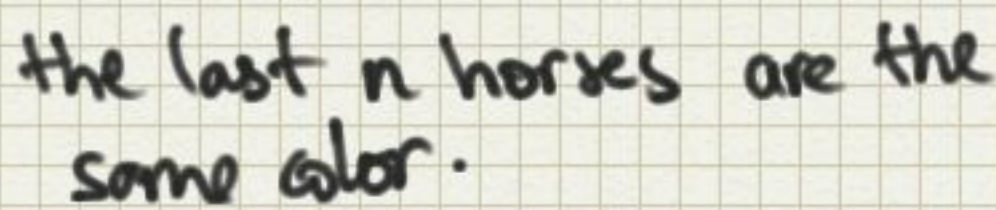
Inductive step: , consider a set of $n+1$ horses.

$h_1, h_2, \dots, h_n, h_{n+1}$.

 same colour by assumption.

$P(n)$: every set of n horses, all are the same colour.

$h_1, h_2, \dots, h_n, h_{n+1}$.

 the last n horses are the same color.

Therefore, h_1, h_2, \dots, h_{n+1} must be the same color

$P(n+1)$ is true, thus $P(n)$ implies $P(n+1)$.

Note: h_1, h_2, \dots, h_n & h_2, \dots, h_n, h_{n+1} .

$n=1$, the first set: h_1
the second set: h_2] showing do not overlap.

$P(1) \not\Rightarrow P(2)$ so the statement is false!

for all $n \geq 0$ there exists a tiling of a $2^n \times 2^n$
Court yard w/ Bill in a central square.

fuzzy logic: truth of 0/1 but represented by
 \Rightarrow real value between 0 & 1.