PHYSICS 5300 Final Project

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Physical Problem: Double Spherical Pendulum

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\begin{aligned}
x_1 &= R_1 \sin\theta_1 \cos\phi_1 \\
y_1 &= R_1 \sin\theta_1 \sin\phi_1 \\
z_1 &= R_1 \cos\theta_1 \\
x_2 &= R_1 \sin\theta_1 \cos\phi_1 + R_2 \sin\theta_2 \cos\phi_2 \\
y_2 &= R_1 \sin\theta_1 \sin\phi_1 + R_2 \sin\theta_2 \sin\phi_2 \\
z_2 &= R_1 \cos\theta_1 + R_2 \cos\theta_2
\end{aligned}
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\begin{aligned}
\dot{x}_1 &= R_1(\dot{\theta}_1 \cos\theta_1 \cos\phi_1 - \dot{\phi}_1 \sin\theta_1 \sin\phi_1) \\
\dot{y}_1 &= R_1(\dot{\theta}_1 \cos\theta_1 \sin\phi_1 + \dot{\phi}_1 \sin\theta_1 \cos\phi_1) \\
\dot{z}_1 &= -R_1\dot{\theta}_1 \sin\theta_1 \\
\dot{x}_2 &= R_1(\dot{\theta}_1 \cos\theta_1 \cos\phi_1 - \dot{\phi}_1 \sin\theta_1 \sin\phi_1) + R_2(\dot{\theta}_2 \cos\theta_2 \cos\phi_2 - \dot{\phi}_2 \sin\theta_2 \sin\phi_2) \\
\dot{y}_2 &= R_1(\dot{\theta}_1 \cos\theta_1 \sin\phi_1 + \dot{\phi}_1 \sin\theta_1 \cos\phi_1) + R_2(\dot{\theta}_2 \cos\theta_2 \sin\phi_2 + \dot{\phi}_2 \sin\theta_2 \cos\phi_2) \\
\dot{z}_2 &= -R_1\dot{\theta}_1 \sin\theta_1 - R_2\dot{\theta}_2 \sin\theta_2
\end{aligned}
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\begin{aligned}
\dot{x}_1^2 &= R_1^2(\dot{\theta}_1^2 \cos^2\theta_1 \cos^2\phi_1 - 2\dot{\theta}_1\dot{\phi}_1 \cos\theta_1 \cos\phi_1 \sin\theta_1 \sin\phi_1 + \dot{\phi}_1^2 \sin^2\theta_1 \sin^2\phi_1) \\
\dot{y}_1^2 &= R_1^2(\dot{\theta}_1^2 \cos^2\theta_1 \sin^2\phi_1 + 2\dot{\theta}_1\dot{\phi}_1 \cos\theta_1 \sin\phi_1 \sin\theta_1 \cos\phi_1 + \dot{\phi}_1^2 \sin^2\theta_1 \cos^2\phi_1) \\
\dot{z}_1^2 &= R_1^2\dot{\theta}_1^2 \sin^2\theta_1 \\
\dot{x}_2^2 &= R_1^2(\dot{\theta}_1^2 \cos^2\theta_1 \cos^2\phi_1 - 2\dot{\theta}_1\dot{\phi}_1 \cos\theta_1 \cos\phi_1 \sin\theta_1 \sin\phi_1 + \dot{\phi}_1^2 \sin^2\theta_1 \sin^2\phi_1) \\
&\quad + 2R_1R_2(\dot{\theta}_1\dot{\theta}_2 \cos\theta_1 \cos\theta_2 \cos\phi_1 \cos\phi_2 - \dot{\theta}_1\dot{\phi}_2 \cos\theta_1 \cos\phi_1 \sin\theta_2 \sin\phi_2 \\
&\quad - \dot{\phi}_1\dot{\theta}_2 \sin\theta_1 \sin\phi_1 \cos\theta_2 \cos\phi_2 + \dot{\phi}_1\dot{\phi}_2 \sin\theta_1 \sin\phi_1 \sin\theta_2 \sin\phi_2) \\
&\quad + R_2^2(\dot{\theta}_2^2 \cos^2\theta_2 \cos^2\phi_2 - 2\dot{\theta}_2\dot{\phi}_2 \cos\theta_2 \cos\phi_2 \sin\theta_2 \sin\phi_2 + \dot{\phi}_2^2 \sin^2\theta_2 \sin^2\phi_2) \\
\dot{y}_2^2 &= R_1^2(\dot{\theta}_1^2 \cos^2\theta_1 \sin^2\phi_1 + 2\dot{\theta}_1\dot{\phi}_1 \cos\theta_1 \sin\phi_1 \sin\theta_1 \cos\phi_1 + \dot{\phi}_1^2 \sin^2\theta_1 \cos^2\phi_1) \\
&\quad + 2R_1R_2(\dot{\theta}_1\dot{\theta}_2 \cos\theta_1 \sin\phi_1 \cos\theta_2 \sin\phi_2 + \dot{\theta}_1\dot{\phi}_2 \cos\theta_1 \sin\phi_1 \sin\theta_2 \cos\phi_2 \\
&\quad + \dot{\phi}_1\dot{\theta}_2 \sin\theta_1 \cos\phi_1 \cos\theta_2 \sin\phi_2 + \dot{\phi}_1\dot{\phi}_2 \sin\theta_1 \cos\phi_1 \sin\theta_2 \cos\phi_2) \\
&\quad + R_2^2(\dot{\theta}_2^2 \cos^2\theta_2 \sin^2\phi_2 + 2\dot{\theta}_2\dot{\phi}_2 \cos\theta_2 \sin\phi_2 \sin\theta_2 \cos\phi_2 + \dot{\phi}_2^2 \sin^2\theta_2 \cos^2\phi_2) \\
\dot{z}_2^2 &= R_1^2\dot{\theta}_1^2 \sin^2\theta_1 + 2R_1R_2\dot{\theta}_1\dot{\theta}_2 \sin\theta_1 \sin\theta_2 + R_2^2\dot{\theta}_2^2 \sin^2\theta_2
\end{aligned}
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$$\begin{aligned}\mathcal{L} = \frac{1}{2}m_1\Big[&R_1^2(\dot\theta_1^2\cos^2\theta_1\cos^2\phi_1 - 2\dot\theta_1\dot\phi_1\cos\theta_1\cos\phi_1\sin\theta_1\sin\phi_1 + \dot\phi_1^2\sin^2\theta_1\sin^2\phi_1)\\ &+ R_1^2(\dot\theta_1^2\cos^2\theta_1\sin^2\phi_1 + 2\dot\theta_1\dot\phi_1\cos\theta_1\sin\phi_1\sin\theta_1\cos\phi_1 + \dot\phi_1^2\sin^2\theta_1\cos^2\phi_1) + R_1^2\dot\theta_1^2\sin^2\theta_1\Big]\\ &+ \frac{1}{2}m_2\Big[R_1^2(\dot\theta_1^2\cos^2\theta_1\cos^2\phi_1 - 2\dot\theta_1\dot\phi_1\cos\theta_1\cos\phi_1\sin\theta_1\sin\phi_1 + \dot\phi_1^2\sin^2\theta_1\sin^2\phi_1)\\ &+ 2R_1R_2(\dot\theta_1\dot\theta_2\cos\theta_1\cos\theta_2\cos\phi_1\cos\phi_2 - \dot\theta_1\dot\phi_2\cos\theta_1\cos\phi_1\sin\theta_2\sin\phi_2\\ &- \dot\phi_1\dot\theta_2\sin\theta_1\sin\phi_1\cos\theta_2\cos\phi_2 + \dot\phi_1\dot\phi_2\sin\theta_1\sin\phi_1\sin\theta_2\sin\phi_2)\\ &+ R_2^2(\dot\theta_2^2\cos^2\theta_2\cos^2\phi_2 - 2\dot\theta_2\dot\phi_2\cos\theta_2\cos\phi_2\sin\theta_2\sin\phi_2 + \dot\phi_2^2\sin^2\theta_2\sin^2\phi_2)\\ &+ R_1^2(\dot\theta_1^2\cos^2\theta_1\sin^2\phi_1 + 2\dot\theta_1\dot\phi_1\cos\theta_1\sin\phi_1\sin\theta_1\cos\phi_1 + \dot\phi_1^2\sin^2\theta_1\cos^2\phi_1)\\ &+ 2R_1R_2(\dot\theta_1\dot\theta_2\cos\theta_1\sin\phi_1\cos\theta_2\sin\phi_2 + \dot\theta_1\dot\phi_2\cos\theta_1\sin\phi_1\sin\theta_2\cos\phi_2\\ &+ \dot\phi_1\dot\theta_2\sin\theta_1\cos\phi_1\cos\theta_2\sin\phi_2 + \dot\phi_1\dot\phi_2\sin\theta_1\cos\phi_1\sin\theta_2\cos\phi_2)\\ &+ R_2^2(\dot\theta_2^2\cos^2\theta_2\sin^2\phi_2 + 2\dot\theta_2\dot\phi_2\cos\theta_2\sin\phi_2\sin\theta_2\cos\phi_2 + \dot\phi_2^2\sin^2\theta_2\cos^2\phi_2)\\ &+ R_1^2\dot\theta_1^2\sin^2\theta_1 + 2R_1R_2\dot\theta_1\dot\theta_2\sin\theta_1\sin\theta_2 + R_2^2\dot\theta_2^2\sin^2\theta_2\Big] + m_1gR_1\cos\theta_1 + m_2g(R_1\cos\theta_1 + R_2\cos\theta_2)\end{aligned}$$

$$\begin{aligned}\mathcal{L} = \frac{1}{2}m_1\Big[R_1^2(\dot\theta_1^2\cos^2\theta_1 + \dot\phi_1^2\sin^2\theta_1) + R_1^2\dot\theta_1^2\sin^2\theta_1\Big] + \frac{1}{2}m_2\Big[&R_1^2(\dot\theta_1^2\cos^2\theta_1 + \dot\phi_1^2\sin^2\theta)\\ &+ 2R_1R_2(\dot\theta_1\dot\theta_2\cos\theta_1\cos\theta_2\cos\phi_1\cos\phi_2 - \dot\theta_1\dot\phi_2\cos\theta_1\cos\phi_1\sin\theta_2\sin\phi_2\\ &- \dot\phi_1\dot\theta_2\sin\theta_1\sin\phi_1\cos\theta_2\cos\phi_2 + \dot\phi_1\dot\phi_2\sin\theta_1\sin\phi_1\sin\theta_2\sin\phi_2) + R_2^2(\dot\theta_2^2\cos^2\theta_2 + \dot\phi_2^2\sin^2\theta_2)\\ &+ 2R_1R_2(\dot\theta_1\dot\theta_2\cos\theta_1\sin\phi_1\cos\theta_2\sin\phi_2 + \dot\theta_1\dot\phi_2\cos\theta_1\sin\phi_1\sin\theta_2\cos\phi_2\\ &+ \dot\phi_1\dot\theta_2\sin\theta_1\cos\phi_1\cos\theta_2\sin\phi_2 + \dot\phi_1\dot\phi_2\sin\theta_1\cos\phi_1\sin\theta_2\cos\phi_2) + R_1^2\dot\theta_1^2\sin^2\theta_1\\ &+ 2R_1R_2\dot\theta_1\dot\theta_2\sin\theta_1\sin\theta_2 + R_2^2\dot\theta_2^2\sin^2\theta_2\Big] + (m_1+m_2)gR_1\cos\theta_1 + m_2gR_2\cos\theta_2\end{aligned}$$

$$\begin{aligned}\mathcal{L} = \frac{1}{2}m_1\Big[R_1^2(\dot\theta_1^2 + \dot\phi_1^2\sin^2\theta_1)\Big] + \frac{1}{2}m_2\Big[&R_1^2(\dot\theta_1^2 + \dot\phi_1^2\sin^2\theta) + 2R_1R_2\Big(\dot\theta_1\dot\theta_2\cos\theta_1\cos\theta_2(\cos\phi_1\cos\phi_2 + \sin\phi_1\sin\phi_2)\\ &+ \dot\theta_1\dot\phi_2\cos\theta_1\sin\theta_2(\sin\phi_1\cos\phi_2 - \cos\phi_1\sin\phi_2) + \dot\phi_1\dot\theta_2\sin\theta_1\cos\theta_2(\cos\phi_1\sin\phi_2 - \sin\phi_1\cos\phi_2)\\ &+ \dot\phi_1\dot\phi_2\sin\theta_1\sin\theta_2(\sin\phi_1\sin\phi_2 + \cos\phi_1\cos\phi_2)\Big) + R_2^2(\dot\theta_2^2 + \dot\phi_2^2\sin^2\theta_2) + 2R_1R_2\dot\theta_1\dot\theta_2\sin\theta_1\sin\theta_2\Big]\\ &+ (m_1+m_2)gR_1\cos\theta_1 + m_2gR_2\cos\theta_2\end{aligned}$$

$$\begin{aligned}\mathcal{L} = \frac{1}{2}(m_1+m_2)\Big[R_1^2(\dot\theta_1^2 + \dot\phi_1^2\sin^2\theta_1)\Big] + \frac{1}{2}m_2\Big\{&2R_1R_2\Big[\dot\theta_1\dot\theta_2\big(\cos\theta_1\cos\theta_2\cos(\phi_1-\phi_2) + \sin\theta_1\sin\theta_2\big)\\ &+ \dot\theta_1\dot\phi_2\cos\theta_1\sin\theta_2\sin(\phi_1-\phi_2) + \dot\phi_1\dot\theta_2\sin\theta_1\cos\theta_2\sin(\phi_2-\phi_1) + \dot\phi_1\dot\phi_2\sin\theta_1\sin\theta_2\cos(\phi_1-\phi_2)\Big]\\ &+ R_2^2(\dot\theta_2^2 + \dot\phi_2^2\sin^2\theta_2)\Big\} + (m_1+m_2)gR_1\cos\theta_1 + m_2gR_2\cos\theta_2\end{aligned}$$