考慮定義 (3.1) : $a^{[1]} = x \in R^{n_1}$

(3.2) : $a^{[l]} = \sigma(W^{[l]}a^{[l-1]} + b^{[l]}) \in R^{n_l}$

$l = 2, 3, \dots L$

求 $\nabla a^{[L]}(x)$ if $n_L = 1$

i.e: 輸出層只有1個 neuron

$\longrightarrow a^{[L]}(x)$ 為 scalar

Goal : 找到純量輸出对輸入 x 之梯度

$\longrightarrow$ 反向傳播 (Backpropagation)

Forward pass : $z^{[l]} = W^{[l]}a^{[l-1]} + b^{[l]}$ $l = 2, 3 \dots L$

$a^{[l]} = \sigma(z^{[l]})$

Output layer error : $\delta^{[L]} = \sigma'(z^{[L]}) \circ (a^{[L]} - y)$

$\because n^L = 1$ $C(a^{[L]}) = a^{[L]}$, $\frac{\partial C}{\partial a^{[L]}} = 1$

$\therefore \delta^{[L]} = \sigma'(z^{[L]})$

hidden layer error : $\delta^{[l]} = \sigma'(z^{[l]}) \circ W^{[l+1]^T} \delta^{[l+1]}$

for $L-1 \longrightarrow L-2 \dots \longrightarrow 2$

$\delta^{[L-1]} = \sigma'(z^{[L-1]}) \circ W^{L^T} \delta^{[L]}$

$\vdots$

$\delta^{[2]} = \sigma'(z^{[2]}) \circ W^{3^T} \delta^{[3]}$

Partial Derivatives

$$\frac{\partial C}{\partial b^{[\ell]}} = \delta^{[\ell]} , \quad \frac{\partial C}{\partial W^{[\ell]}} = \delta^{[\ell]} (a^{[\ell-1]})^T , \quad 2 \leq \ell \leq L$$

$$\nabla a^{[L]}(x) = \frac{\partial a^{[L]}}{\partial x} = \frac{\partial a^{[L]}}{\partial a^{[1]}} = \underbrace{\frac{\partial a^{[L]}}{\partial z^{[2]}}}_{n^{[2]} \times 1} \underbrace{\frac{\partial z^{[2]}}{\partial a^{[1]}}}_{n^{[2]} \times n^{[1]}}$$

$$= (W^{[2]})^T \delta^{[2]}$$