MSDM5058 final project: Portfolio Management Using Prediction Rules and Communication in Social Networks

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Abstract:
This project aims to research on Portfolio Management by using prediction rules and basing on different situation, like risk free interest, charge fee etc. We choose ***A UN Equity*** as our target stock which have more than 4500 data records. Firstly we generate rules and Bayes detector from our training data. Then we combine rules and Bayes detector together to simulate our portfolio in testing data. Considering different situation like transaction cost, risk-free interest, we will adopt different definitions of portfolio value. We compare among possible parameters of each portfolio value definition and examine which one is relatively better. We also explore the possible values of greed which inspires our future research direction.

Keywords: Portfolio Management, Association rules

1 Data processing

First of all, we select ***A UN Equity*** as our target stock, which include more than 4500 days data. After that, we calculate its daily return and this equation as below:

$$x(t) = \frac{s(t) - s(t-1)}{s(t-1)}$$

In order to use the tag to represent the up and down, we set $\varepsilon = 0.002$ for the criteria to generate digitize d(t) base on this transform:

$$d(t) = \begin{cases} D\ [x(t) <- \varepsilon] \\ H\ \ [x(t) <+ \varepsilon] \\ U\ (otherwise) \end{cases}$$

Then we need to split the data into 3:1 as a learning set (t ≤ 0) and a testing set (t > 0). Here, training data include 3912 data points, and testing data have 1304 data points.

They include alphabet, price and daily return which means we have three data respectively.

2 Cumulative Distribution Function

Given the return x(t) on one day, we would like to predict its value x(t+1) one day later. So we calculate conditional CDF FU(x) = CDF[x(t) | d(t+1) = U] and FD(x) = CDF[x(t) | d(t+1) = D] , then we get its daily return in this two situation. Therefore we can plot the following picture representing the conditional CDF of D alphabet and U alphabet posterior in t+1.

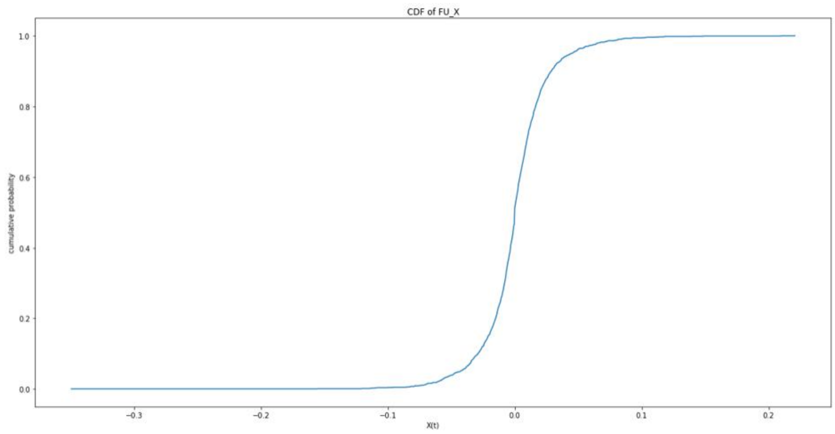

Fig. 1. Conditional cumulative distribution function for $d(t+1) = U$

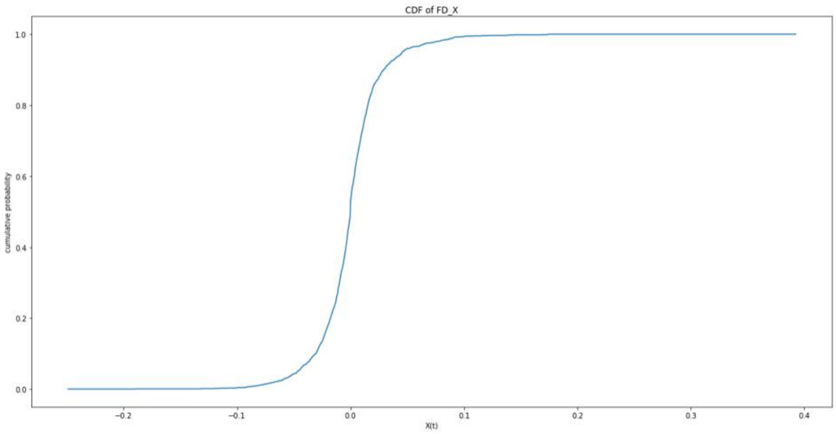

Fig. 2. Conditional cumulative distribution function for $d(t+1) = D$

3 Probability Density Function

3.1 Fermi-Dirac distribution

In this section, when we use Fermi-Dirac distribution, we need to calculate the value of b firstly.

$$F(x) = \frac{1}{1 + \exp[-b(x - x^*)]}$$

$$F^{'}(x) = f(x) = \frac{be^{-bx}}{\left(1 + e^{-bx}\right)^2}$$

Noticed that we calculate the f(0) by $\frac{\Delta \mathrm{y}}{\Delta t}$ near x=0, then we can get the value of b belong to FD_b and FU_b.

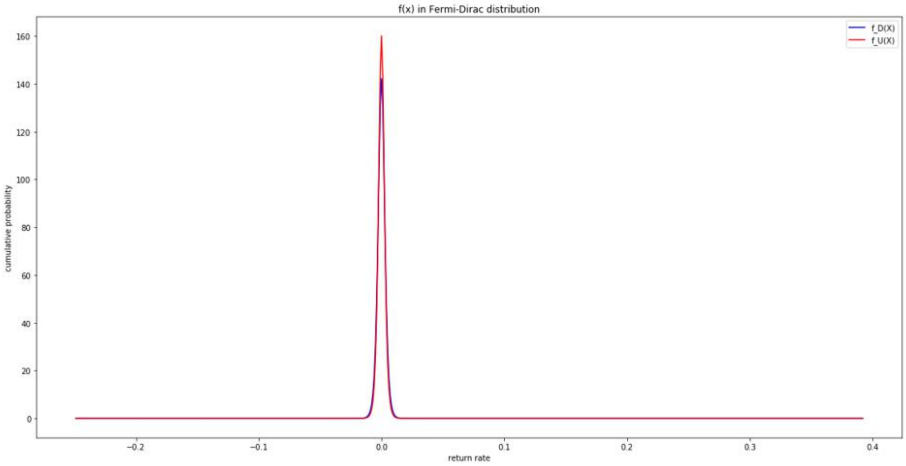

Fig. 3. PDF of $f_U(x)$ and $f_D(x)$ under Fermi-Dirac distribution

3.2 Gaussian distribution

On the other hand, when we choose the Gaussian distribution, we calculate the mean and standard error in this two different situation:

Function	Mean	Standard Deviation
$P[d(t+1) = U]$	6.043563e-07	0.0316
$P[d(t+1) = D]$	0.0006006	0.0292

Then base on the PDF of Gaussian distribution:

$$g(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[\frac{(x-\mu)^2}{2\sigma^2}\right]$$

we can get the distributions of FU(x) and FD(x). We plot them in the same graph and show as below:

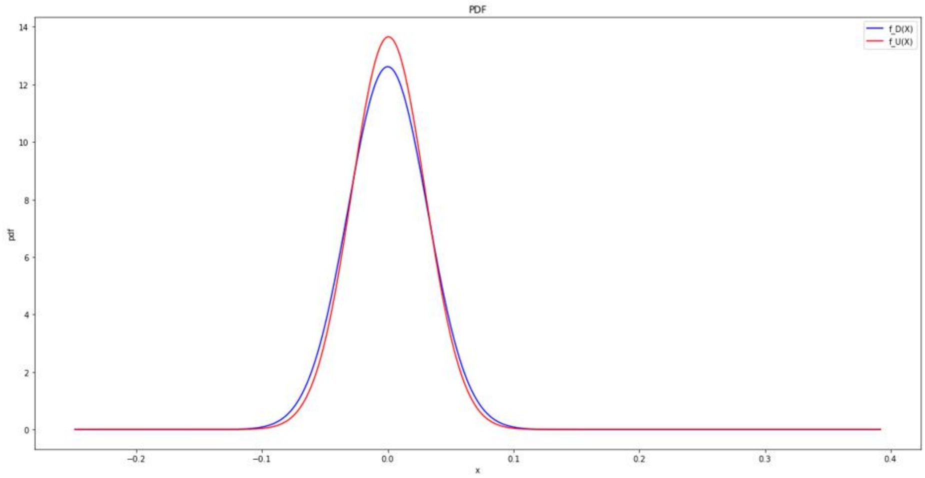

Fig. 4. PDF of $f_U(x)$ and $f_D(x)$ under Gaussian distribution

4 Bayes detector

4.1 Compute the probabilities P[d(t+1) = U] and P[d(t+1) = D]

In this task, we start with t=1, then we calculate how many D and how many U separately, and we divide it with the total number then we can get the probabilities. In this task the probabilities of U is 44.2% and for the D is 43.13%.

probabilities	value
P(U)	0.442
P(D)	0.431
P(H)	0.127

4.2 Construct the detector with the Fermi-Dirac

$$\Lambda(z) \equiv \frac{f_Z(z \mid H_1)}{f_Z(z \mid H_0)} \underset{H_1}{\overset{H_0}{\lessgtr}} \frac{c_{10} - c_{00}}{c_{01} - c_{11}} \frac{p}{q} \equiv \eta$$

Base on this equation, f_z is the Fermi-Dirac distribution and p and q is the FD_b and FU_b, so we calculate that x1= -0.00286025 and x2=0.00286025.
In general, x ∈ (x1, x2) accepts one hypothesis, when x < x1 or x > x2 accepts the other one. In term of trading, x ∈ (x1, x2) represents a buy decision. Similarly, x < x1 or x > x2 represents a sell decision.

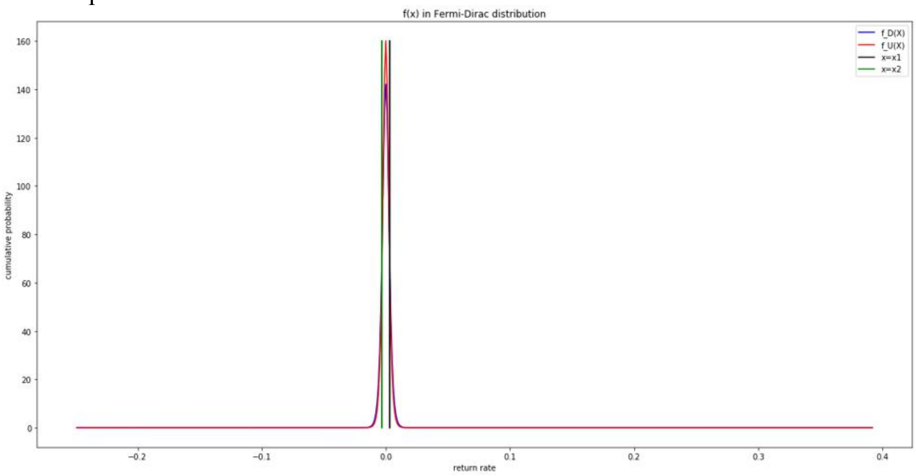

Fig. 5. Fermi-Dirac distribution with x1 and x2 from Bayes detector

4.3 Construct the detector with the Gaussian

Bayes detector can also applies to Gaussian Distribution. Similar to 4.2, we get x1=-0.0299996 and x2=0.00530494

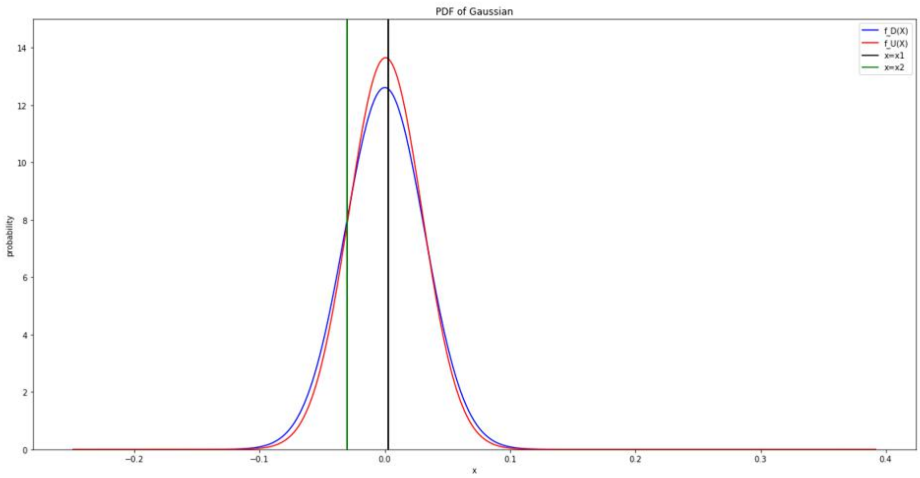

Fig. 6. Gaussian distribution with x1 and x2 from Bayes detector

5 Association Rules

5.1 Mine the two best 1-day Rule

For R_U^1 and R_D^1, we calculate confidence number and support number. For confidence number, it represents how many time it gets U or D in the next day and support number is the total number of this data. Therefore confidence rate is confidence number divides to the support number and the support rate is support number divides to the sum of support number.

Base on this algorithm, firstly we calculate R_D^1 and the result show as below:

Table 1. highest confidence rate in R_D^1

	confidence_number	support_number	confidence_rate	support_rate
D	755	1687	0.447540	0.431347
H	213	495	0.430303	0.126566
U	719	1729	0.415847	0.442086

Here we know that D is the best signal for predicting a D in t+1.

Similarly, we can calculate R_U^1 and the result show as below:

Table 2. highest confidence rate in R_U^1

	confidence_number	support_number	confidence_rate	support_rate
U	772	1729	0.446501	0.442086
H	218	495	0.440404	0.126566
D	739	1687	0.438056	0.431347

Here we know that U is the best signal for predicting a U in t+1.

5.2 Mine the two best 5-day

One day rule is simple, while it can associates with more days, for example 5days which can included more information.

So we will use the training data to check for the occurrence of the 5 day rules. There are $3^5 = 243$ possible rules in total as there are 3 possible outcomes (U, H, D) for the five days rule.

Firstly, we consider the R_D^5, after calculation, we can see that ***DHUDH*** is the best rule of R_D^5, which have the highest confidence rate.

Table 3. highest confidence rate in R_D^5 of 5 days

	confidence_number	support_number	confidence_rate	support_rate
DHUDH	4	4	1.0	0.001024

Similarly, when we calculate R_U^5, we get ***HUHUH*** is the best rule in R_U^5. Its detail information show as below:

Table 4. highest confidence rate in R_U^5 of 5 days

	confidence_number	support_number	confidence_rate	support_rate
HUHUH	4	4	1.0	0.001024

6 Portfolio Management

6.1 Portfolio's value in Fermi-Dirac Bayes detector

It is important to simulate our trading rule to examine the worth of portfolio. In our simulation, base on the rule we get before, we iterate each day in the testing set to obtain the change of portfolio's worth if we follows the specific rule to predict the stock go up or down. More specifically, if the past days patterns equal with the rule meanwhile the result of Bayes detector have the same operation, we will do a sell or buy action respectively.

Here, we assume the gamma is 0.5 as at the beginning which will be change in the coming sections.

The portfolio's value $V_f(t)$ when we trade base on the Fermi-Dirac Bayes detector with 1 day rule show as follow :

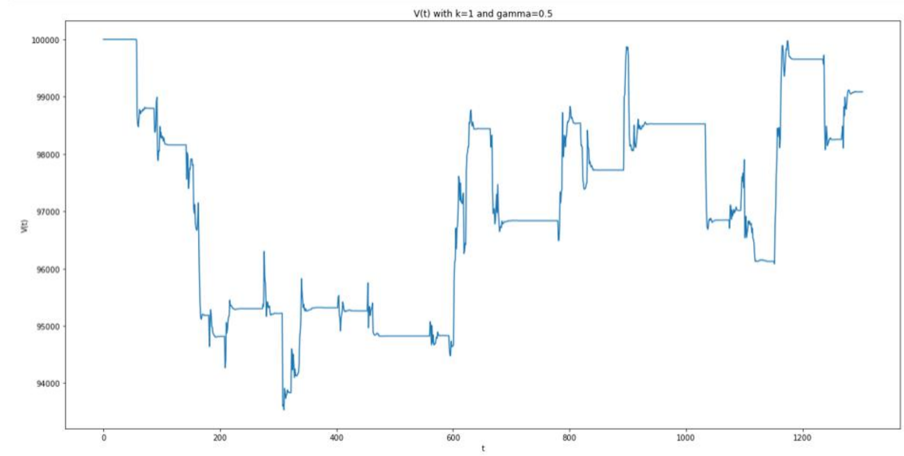

Fig. 7. $V_f(t)$ with 1-day rules & Fermi-Dirac Bayes detector

When we use Fermi-Dirac detector, the final portfolio value is 99083.95924908799.

6.2 Portfolio's value in Gaussian Bayes detector

Similar to the method in 6.1, here we use Gaussian Bayes detector instead, and the result show as follow:

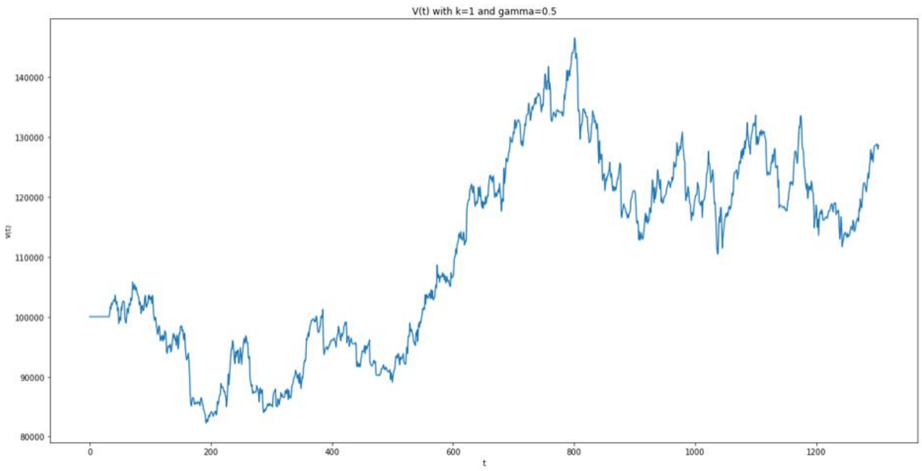

Fig. 8. $V_f(t)$ with 1-day rules & Gaussian Bayes detector

When we use Gaussian Bayes detector, the final portfolio value is 128574.93381814535

6.3 Comparison

According to the final portfolio values, it shows that the Gaussian Bayes detector performs extremely better than Fermi-Dirac one(128574 vs 99083). Besides the final portfolio value is lower than the beginning value when we use the Fermi-Dirac Bayes detector.

6.4 Repeat 6.1 and 6.2 with k=5

After simulate for one day rules, we estimate the portfolio's worth with 5 days rules. Same as before, we use Fermi-Dirac Bayes detector as well as the Gaussian Bayes detector to simulate it.

Firstly, we simulate the 5 days rules with Fermi-Dirac Bayes method and the result show as below:

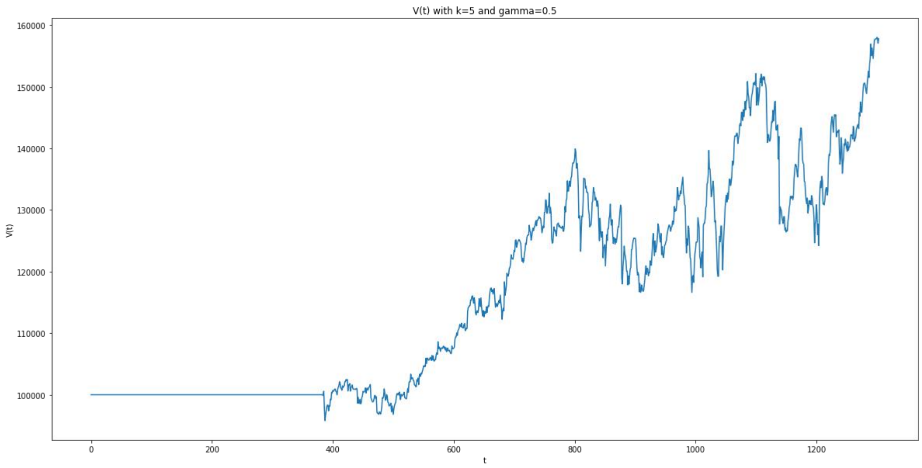

Fig. 9. $V_f(t)$ with 5-day rules & Fermi-Dirac Bayes detector

The final portfolio value is 157788.56594312267

Similarly, we can also simulate the 5 days rules with Gaussian Bayes method and the result show as below:

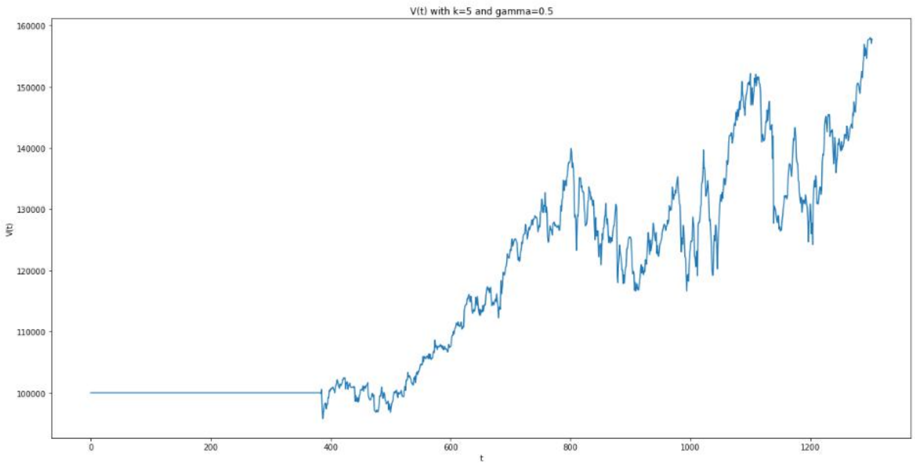

Fig. 10. Portfolio's value in Gaussian Bayes detector with k=5
The final portfolio value is 157788.56594312267

It is very interesting that the final portfolio values is equal for both detector. For the potential reasons, first of all, the occurrence of the five day rule is less than the one day rule. In our experiment, the 5 days rule only triggered in the 3 times. Because both 5 day rules and Bayes detector have to be satisfied then it can trigger the buy or sell decision. Secondly, the testing set is relatively small. There are 243 possible 5 days rules while the testing set only contains 1403 data.

According to the figure we plotted above, we known that the stock price increases for most of the time during the testing period(the upward trend).

As a result, basing on the 5 days rules, all of them go through the Fermi-Dirac Bayes detector or Gaussian Bayes detector which results in the same portfolio's value. Therefore, we can draw the conclusion that the Gaussian Bayes detector performs better or same as the Fermi-Dirac Bayes detector in the above testing case(one days rules is better, 5days rules is the same). Therefore, we choose the Gaussian Bayes detector as the superior method for the remaining simulation in the coming sections.

7 Transaction Cost

7.1 Portfolio's value with k=1 and cost (ξ) = 0.2%

In real life, most of the agents or trading platforms charge for a certain amount of money for the transaction cost. We hope to simulate realistic results more accurately. So in our model, we add this item by setting a fee required a certain percentage ξ of transaction amount.

So we update our model as the following equation:

$$\begin{cases} M(t) & \leftarrow & M(t) & - & m \\ N(t) & \leftarrow & N(t) & + & (1-\xi)m/s(t) \end{cases} \quad \text{for} \quad m = \gamma M(t)$$

$$\begin{cases} M(t) & \leftarrow & M(t) & + & (1-\xi)ns(t) \\ N(t) & \leftarrow & N(t) & - & n \end{cases} \quad \text{for} \quad n = \gamma N(t)$$

The portfolio's value $V^1(t)$ changes when we use the Gaussian Bayes detector and the 1 day rule show as below:

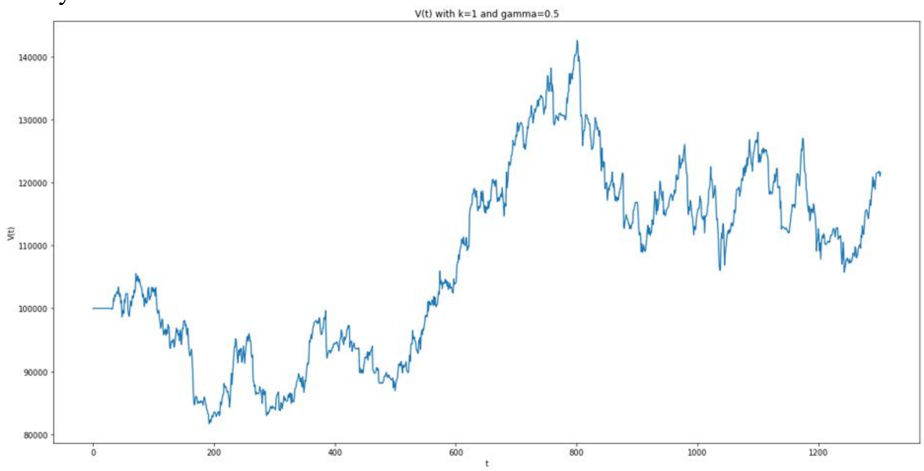

Fig. 11. Portfolio's value $V^1(t)$ with K=1 and ξ=0.2%

The final portfolio value is 121591.8629287736 and the trading activities happens 13.96% of the time.

7.2 Poportfolio's value with k=5

Similarly, we can use 5 days rules to simulate our result:

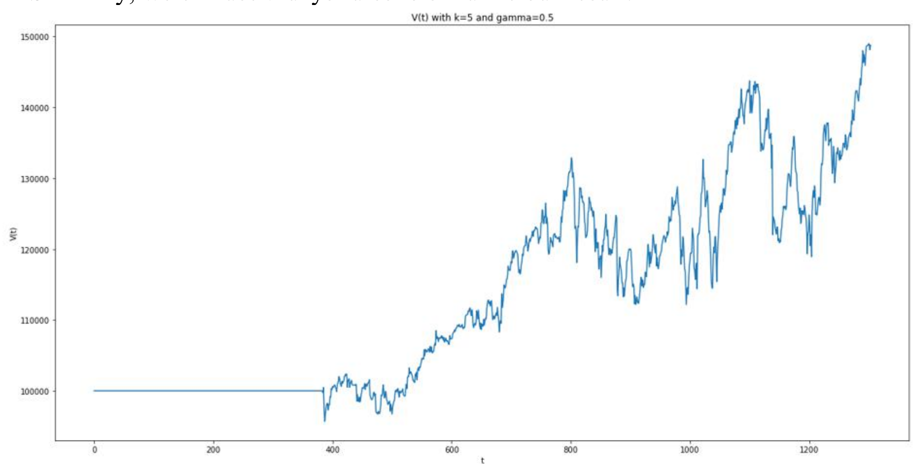

Fig. 12. Portfolio's value $V^5(t)$ with K=1 and ξ=0.2%

The final portfolio value is 148760.8040337521. and the trading activities happens 0.307% of the time.

7.3 Comparison

Considering their final portfolio, 5 days rules is better than 1 days rule (148760 vs 121591). When we look back to Q6, the 5 days rules also earn more than 1 days rule. However, the gap is narrow in Q7, one potential reason is the transaction fee which reduce their earn totally.

7.4 Extra: Simulation with transaction cost (ξ) = 0.1% or 0.5%

We want to simulate that in different transaction cost, how will our earn change. So we simulate again with ξ=0.1$ firstly and the result show as below:

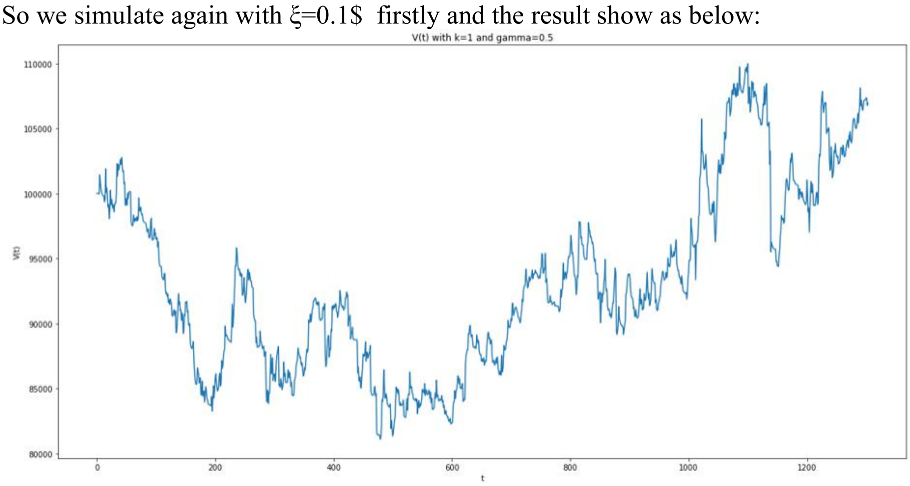

Fig. 13. Portfolio's value $V^1(t)$ with K=1 and ξ=0.1% and day=1

In this case, the final portfolio value is 125035.16829100116.

Similarly, we can plot the figure when we use 5 days rules.

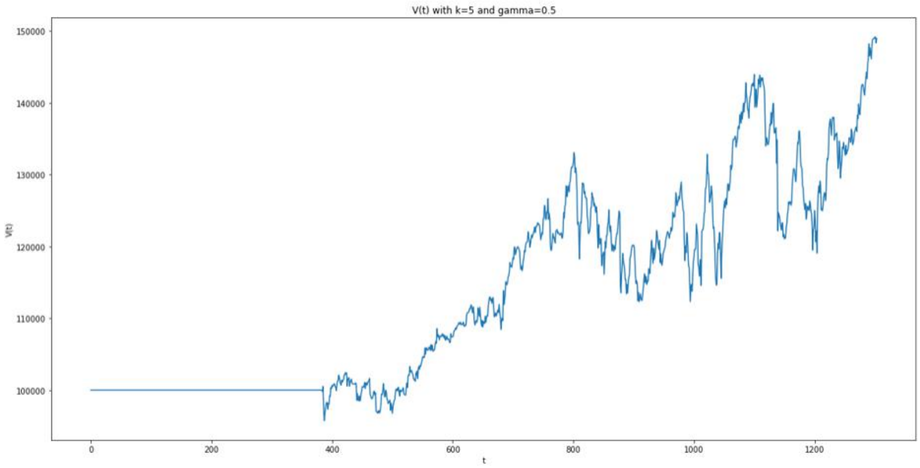

Fig. 14. Portfolio's value $V^5(t)$ with K=1 and ξ=0.1% and day=5

In this case, the final portfolio value is 157643.27737717956.

When ξ=0.5%, we can also simulate like the above way:

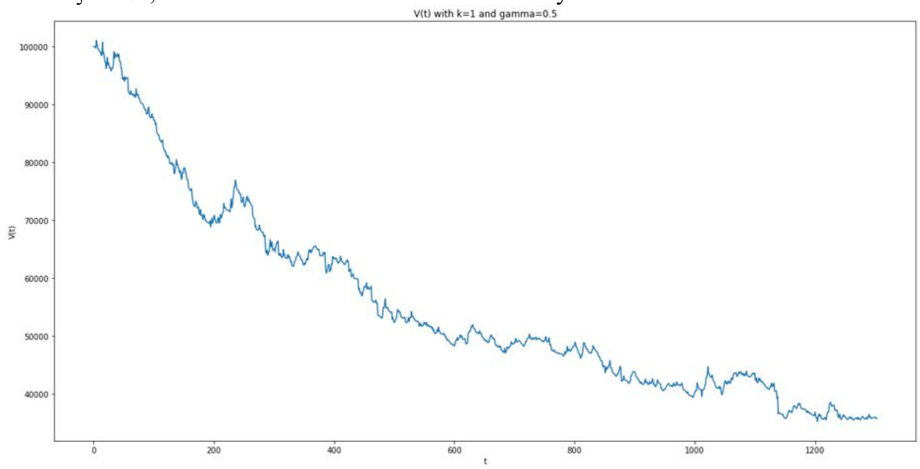

Fig. 15. Portfolio's value $V^1(t)$ with K=1 and ξ=0.5% and day=1

In this case, the final portfolio value is 111815.04476229992.

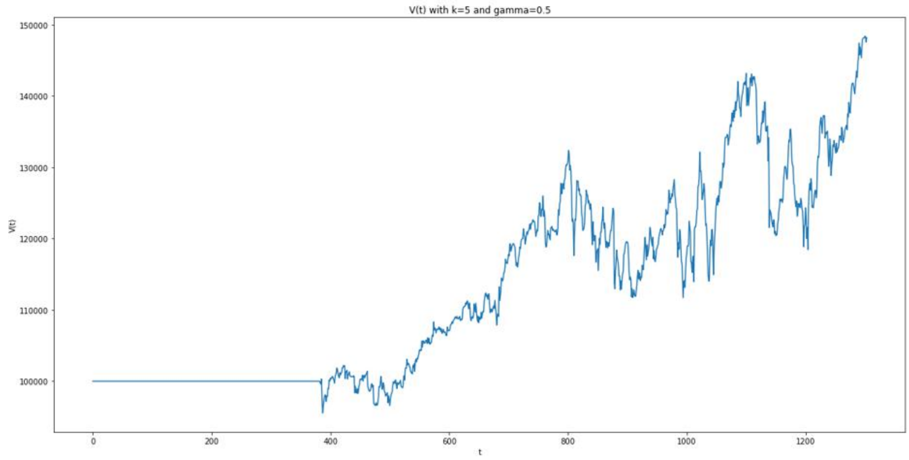

Fig. 16. Portfolio's value $V^5(t)$ with K=1 and ξ=0.5% and day=5

In this case, the final portfolio value is 157062.12311340705.

According to the final portfolio, it is reasonable that in the same rules(one day or five days), the ξ higher and the portfolio will be lower because the charge fee is higher. On the other hand, the final portfolio of 5 days rule is higher than the portfolio of 1 day rule no matter the ξ is 0.1% or 0.5%.

8 Risk-Free Interest

In real world, not only it will have the charge of transaction but also have the interest rate of our asset. We want to simulate it more realistic so we update M(t) of risk-free interest, we can get new portfolio, the update equation show as below:

$$M(t) \leftarrow M(t)\,(1+r)$$

8.1 Plot the portfolio's value

When we update the equation of M(t), we simulate it in

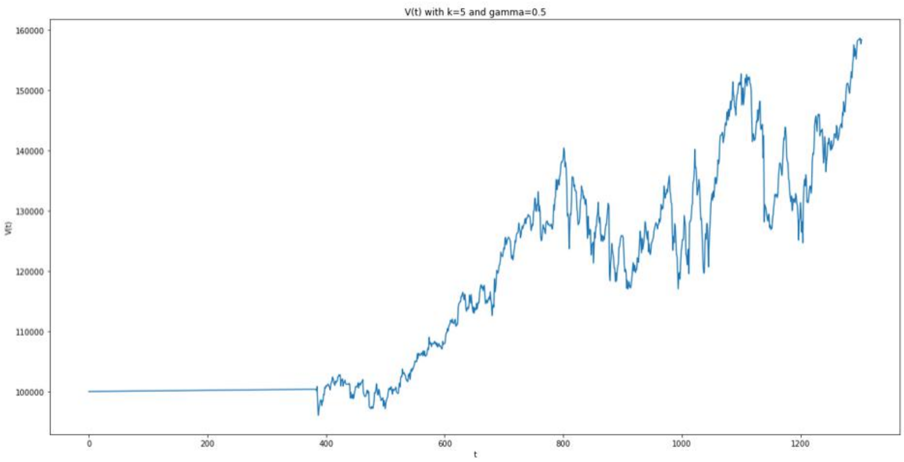

Fig. 17. $V_{\mathrm{r}}(t)$ based on 5-day rules, 0.5% transaction cost and 0.001% risk free interest

The final portfolio value is 158350.75564380878

8.2 Plot the portfolio's value with adjust M(t)

If we don't trade at all, our portfolio value will be

$$M_{\mathrm{r}}(t) = M(0)(1+r)^t$$

Under this equation, we define

$$\rho_{\mathrm{r}}(t) = V_r(t) / M_r(t)$$

Then we can plot the curve of $\rho_{\mathrm{r}}(t)$

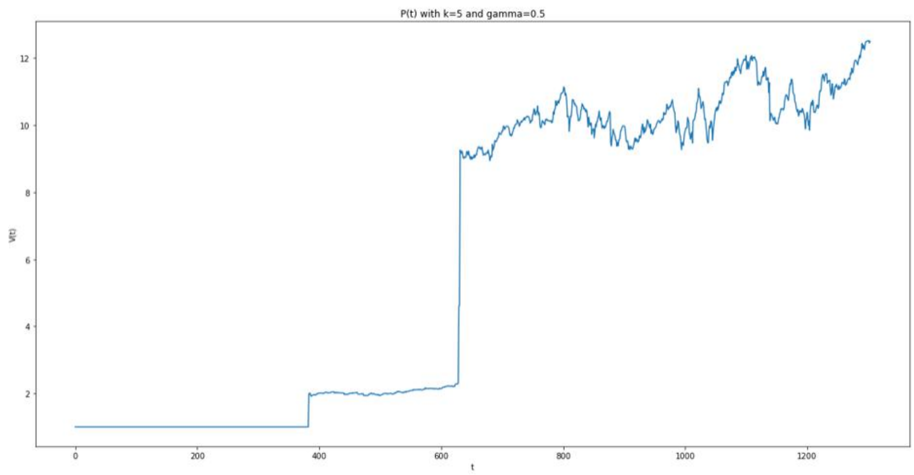

Fig. 18. $\rho_{\mathrm{r}}(t)$ based on 5-day rules, 0.5% transaction cost and 0.001% risk free interest

8.3 Extra

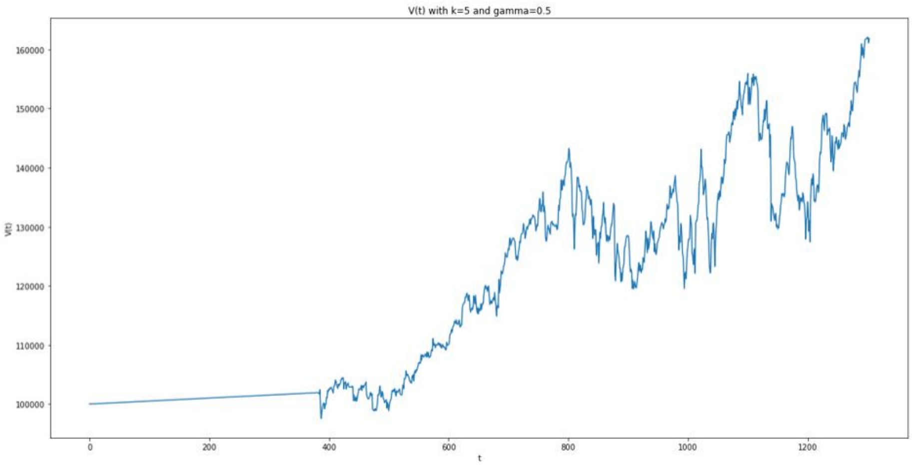

Fig. 19. $V_{\mathrm{r}}(t)$ based on 5-day rules with 0.5% transaction cost and 0.005% risk free interest

The final portfolio value is 161818.51508951496

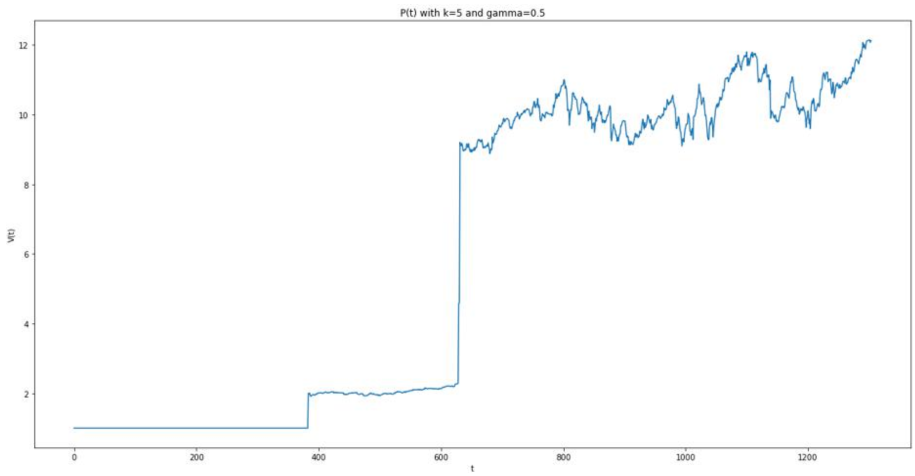

Fig. 20. $\rho_{\mathrm{r}}(t)$ based on 5-day rules with 0.5% transaction cost and 0.005% risk free interest

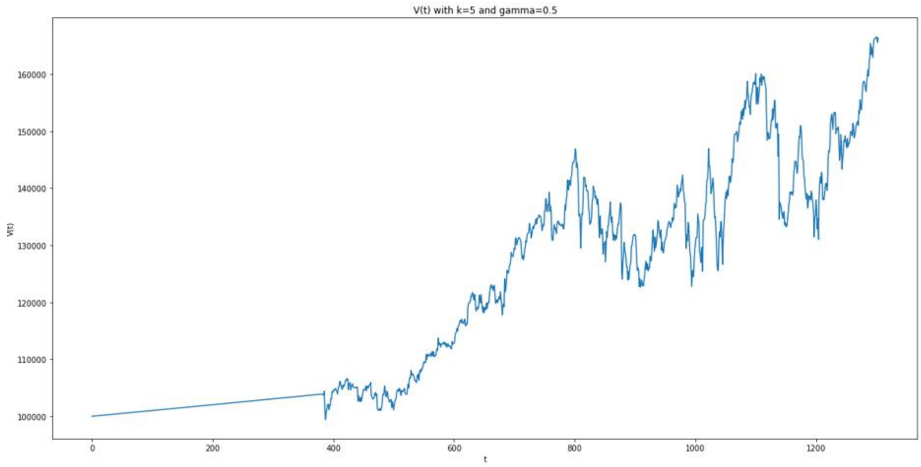

Fig. 21. $V_{\mathrm{r}}(t)$ based on 5-day rules with 0.5% transaction cost and 0.01% risk free interest

The final portfolio value is 166284.36489640595

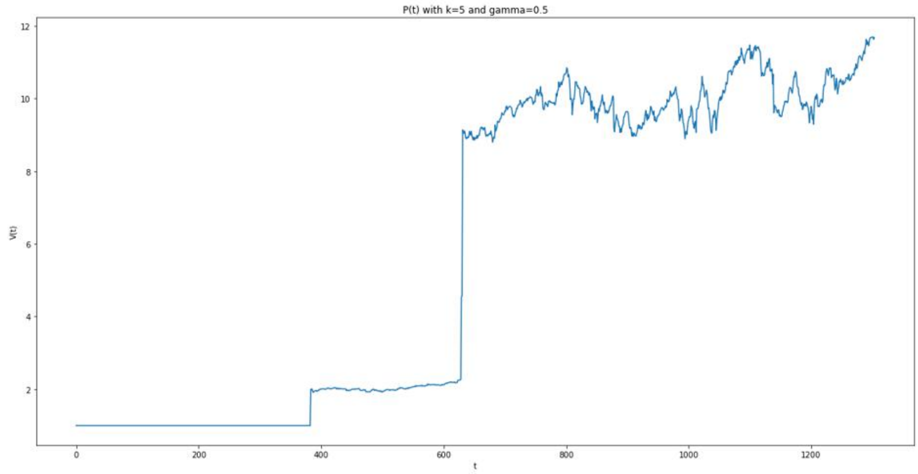

Fig. 22. $\rho_{\mathrm{r}}(t)$ based on 5-day rules with 0.5% transaction cost and 0.01% risk free interest

For $V_{\mathrm{r}}(t)$, it is reasonable that the portfolio with 0.01% risk free interest higher than that of 0.005%. On the other hand, $\rho_{\mathrm{r}}(t)$ with 0.005% risk free interest higher than that of 0.01%. So we know that $\rho_{\mathrm{r}}(t)$ and risk free interest are inversely proportional.

9 Greed

In stock market, someone is conservative and other may aggressive which reflect their greed.

9.1 Trade and plot the portfolio’s value $V_r(t)$

Base on the changing gamma of [0.1, 0,3, 0.5, 0.7, 0.9], we can have different simulation and show as follow:

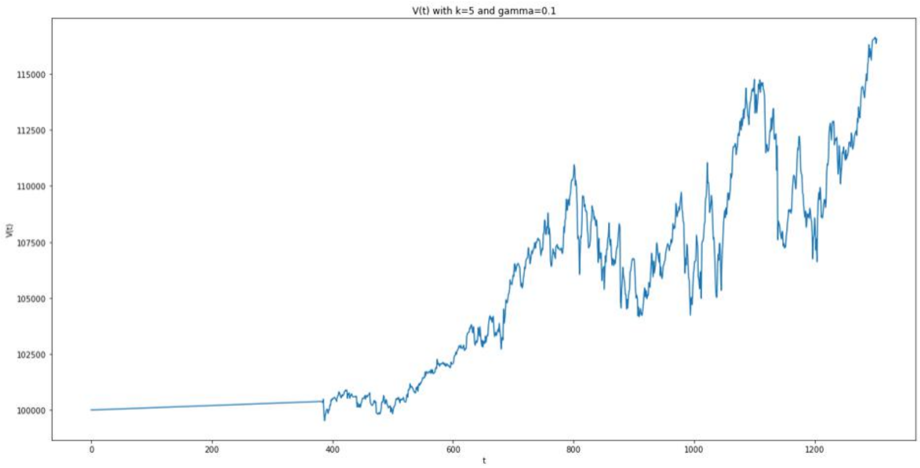

Fig. 23. portfolio’s value $V_r(t)$ when gamma=0.1

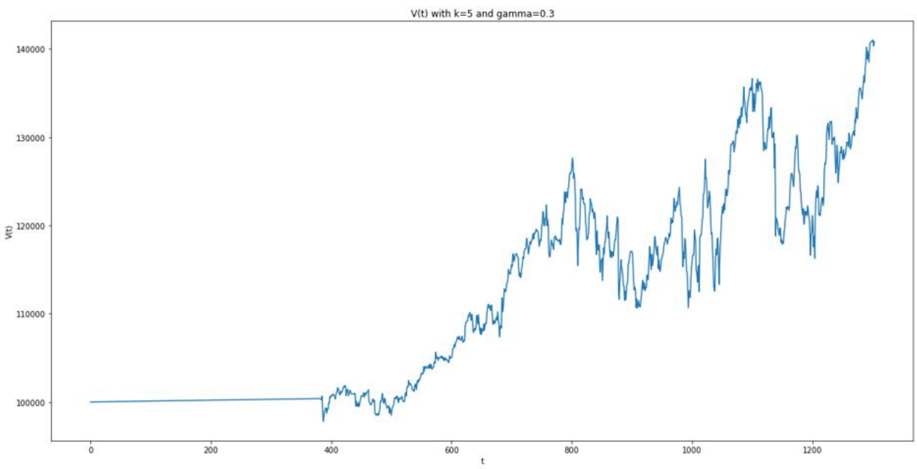

Fig. 24. portfolio’s value $V_r(t)$ when gamma=0.3

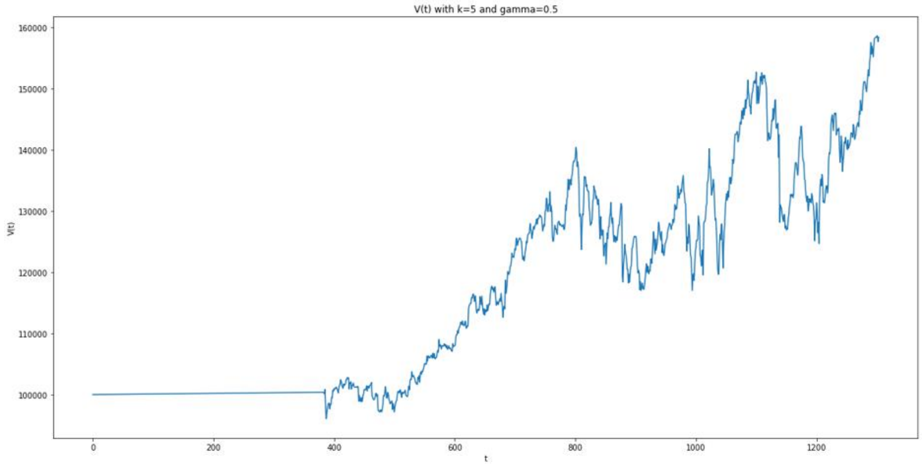

Fig. 25. portfolio's value $V_{\mathrm{r}}(t)$ when gamma=0.5

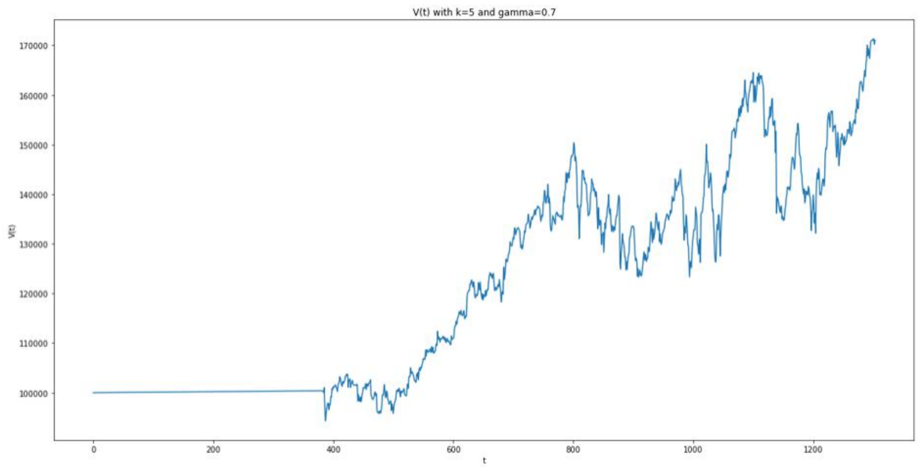

Fig. 26. portfolio's value $V_{\mathrm{r}}(t)$ when gamma=0.7

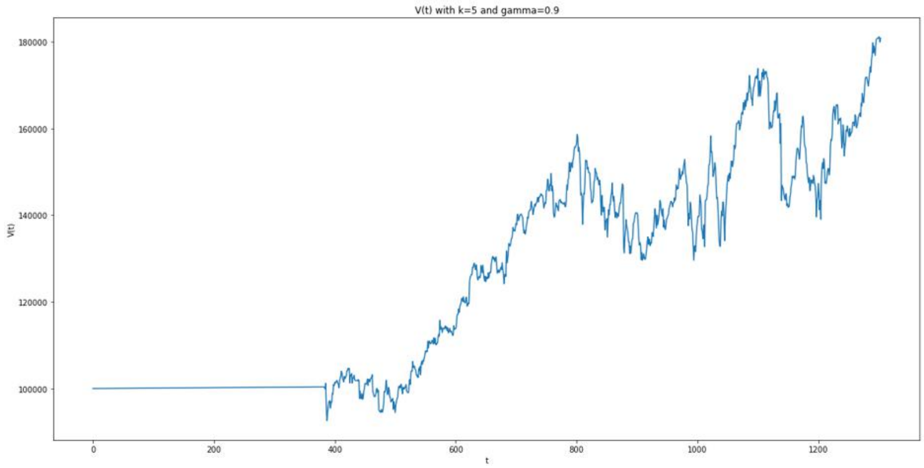

Fig. 27. portfolio's value $V_r(t)$ when gamma=0.9

Comparing with the 5 figures above, we know that the high the gamma value, the higher this portfolio.

9.2 Plot the last portfolio's value $V_r(t)$

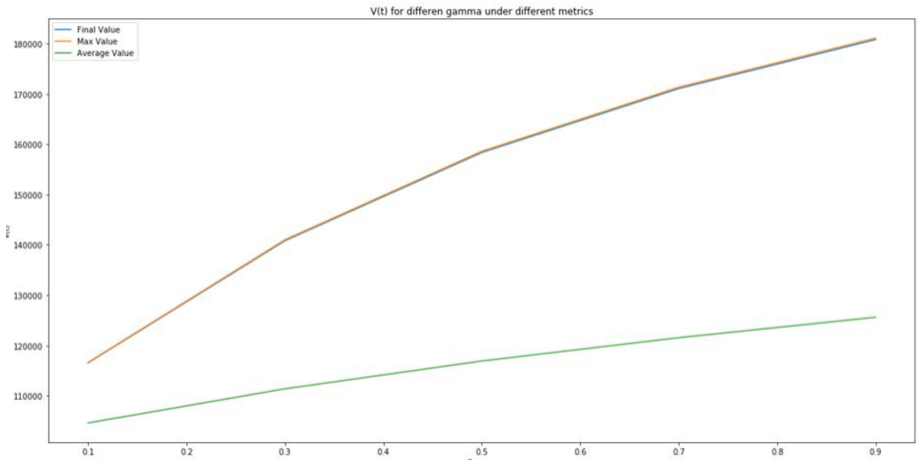

Fig. 28. portfolio's value $V_r(t)$ with max value, final value and average value

We can see that final value and max value remain the same, that is because those portfolio get highest in the final step. Besides, we can see that final value and max value are higher than average value.

9.3 Extra: Simulation with greed (γ) for {0.1, 0.15, 0.2, 0.25…..0.95}

We generate gamma from 0 to 1 interval is 0.05. Then we can have 15 different gamma. According to each gamma, we can generate its final value, max value and average value. All this 15 figures show as below:

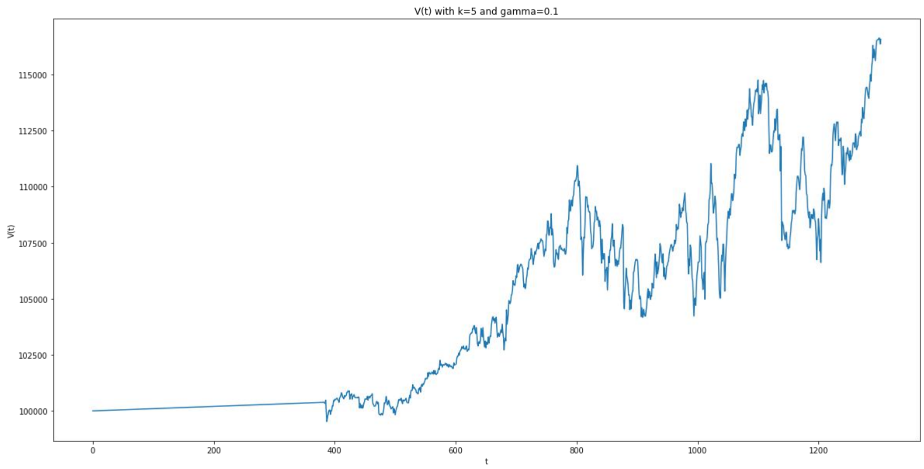

Fig. 29. based on 5-day rules with 0.5% ξ, 0.01% r & γ = 0.1

The final portfolio value is 116564.0233321185 $V_r(t)$ 2

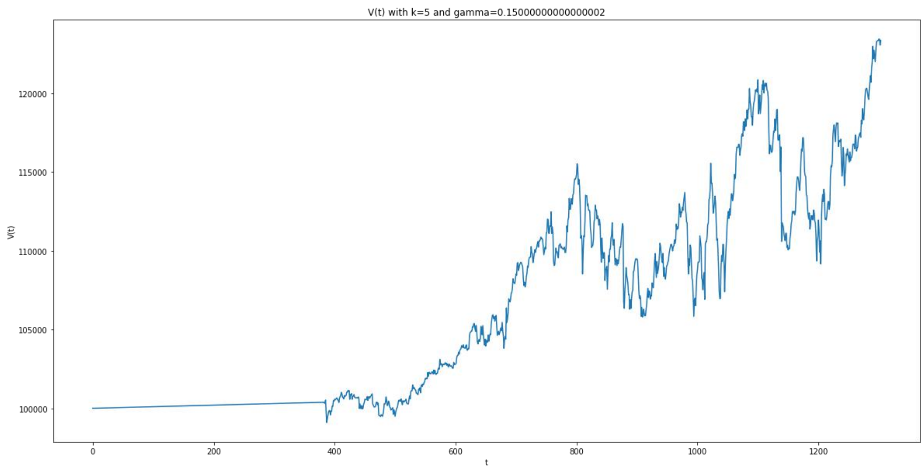

Fig. 30. $V_r(t)$ based on 5-day rules with 0.5% ξ, 0.01% r & γ = 0.15

The final portfolio value is 123365.1396507225

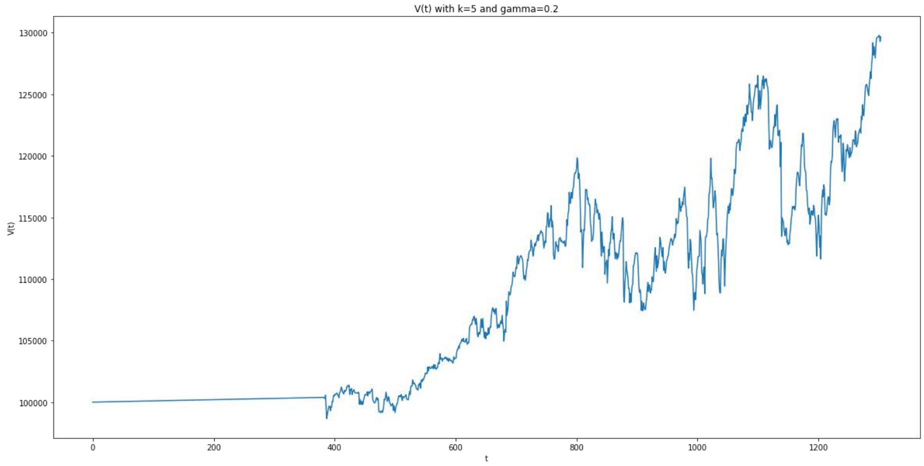

Fig. 31. $V_{\mathrm{r}}(t)$ based on 5-day rules with 0.5% ξ, 0.01% r & γ = 0.2

The final portfolio value is 129656.0872228656

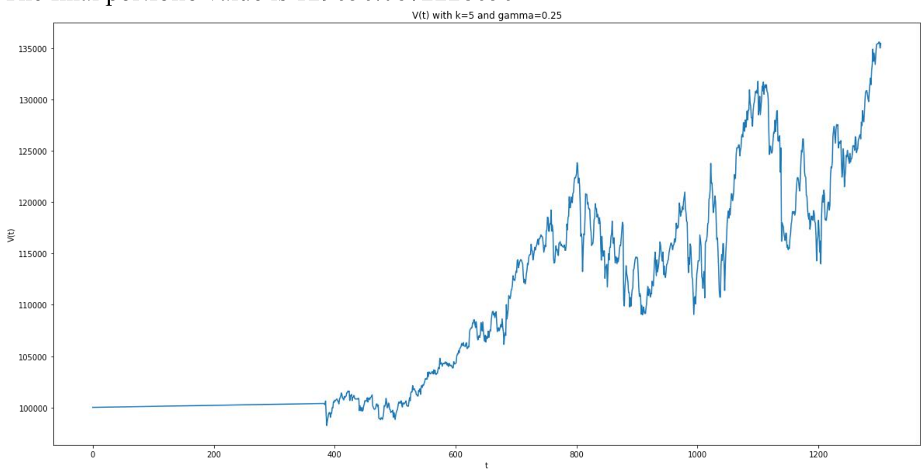

Fig. 32. $V_{\mathrm{r}}(t)$ based on 5-day rules with 0.5% ξ, 0.01% r & γ = 0.25

The final portfolio value is 135466.55402468628

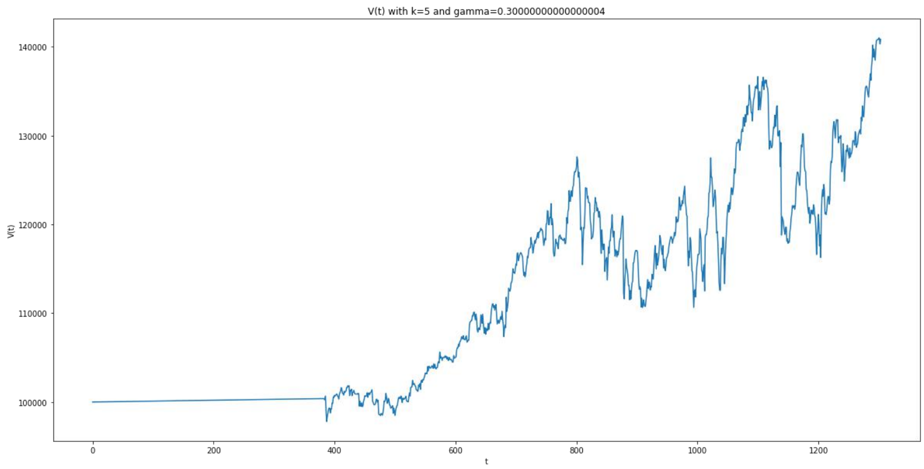

Fig. 33. $V_{\mathrm{r}}(t)$ based on 5-day rules with 0.5% ξ, 0.01% r & γ = 0.3

The final portfolio value is 140826.22803232347

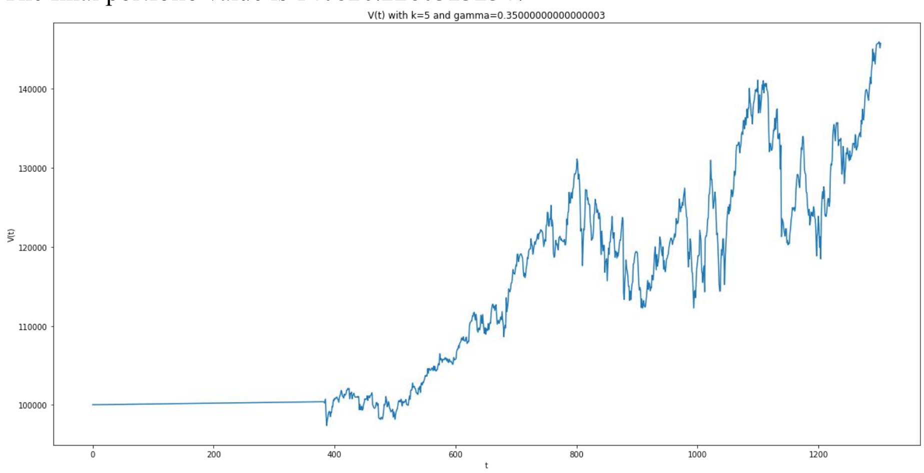

Fig. 34. $V_{\mathrm{r}}(t)$ based on 5-day rules with 0.5% ξ, 0.01% r & γ = 0.35

The final portfolio value is 145764.7972219157

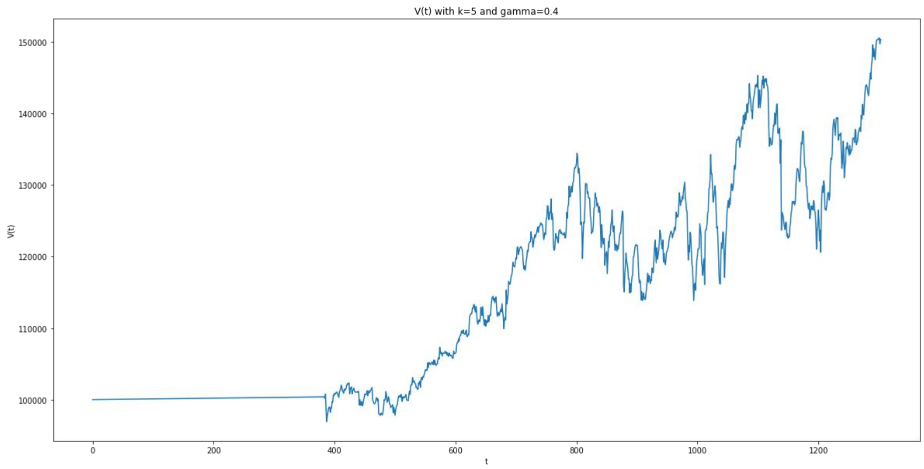

Fig. 35. $V_r(t)$ based on 5-day rules with 0.5% ξ, 0.01% r & γ = 0.4

The final portfolio value is 150311.94956960148

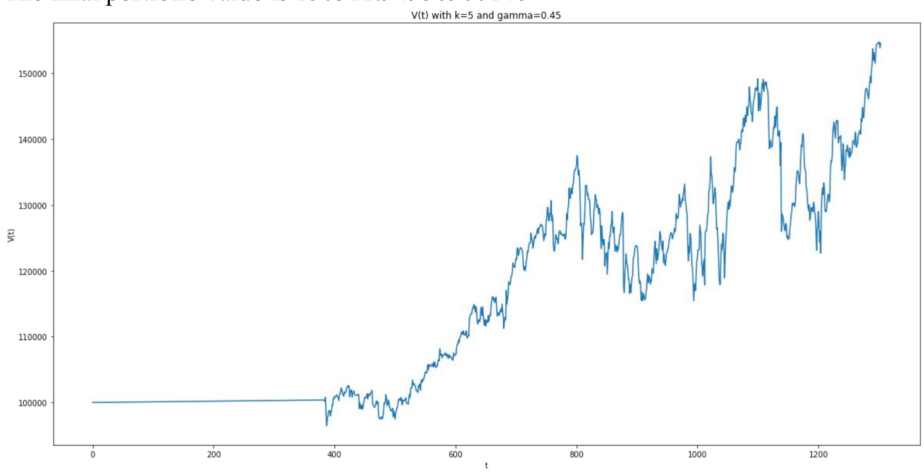

Fig. 36. $V_r(t)$ based on 5-day rules with 0.5% ξ, 0.01% r & γ = 0.45

The final portfolio value is 154497.3730515196

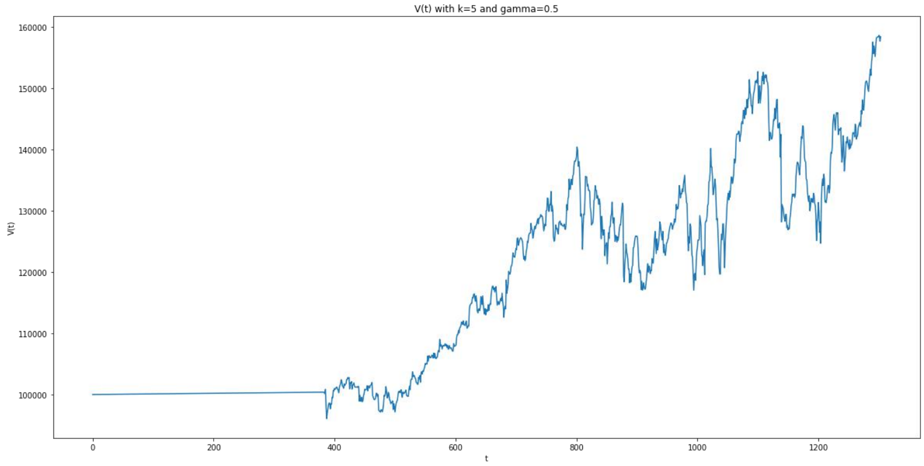

Fig. 37. $V_{\mathrm{r}}(t)$ based on 5-day rules with 0.5% ξ, 0.01% r & γ = 0.5

The final portfolio value is 158350.75564380878

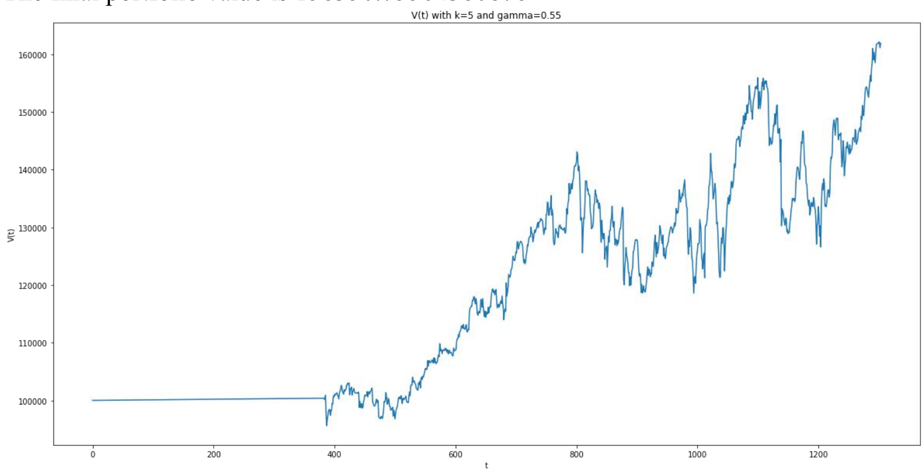

Fig. 38. $V_{\mathrm{r}}(t)$ based on 5-day rules with 0.5% ξ, 0.01% r & γ = 0.55

The final portfolio value is 161901.78532260744

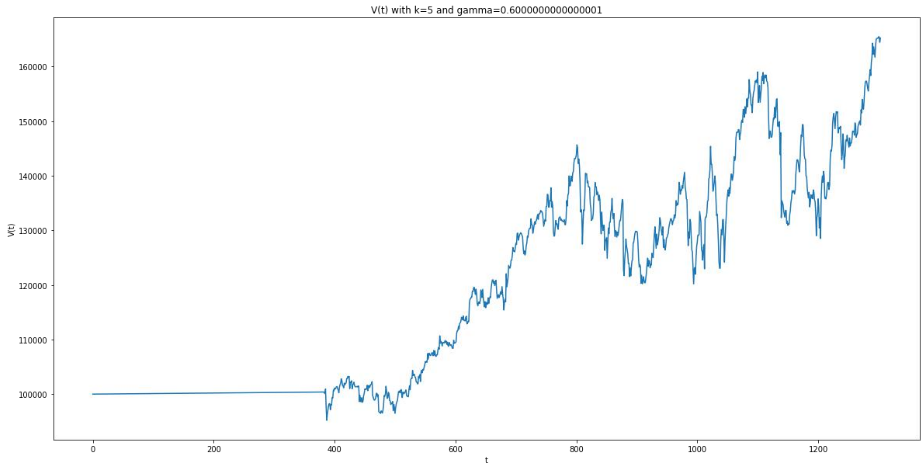

Fig. 39. $V_{\mathrm{r}}(t)$ based on 5-day rules with 0.5% ξ, 0.01% r & γ = 0.6

The final portfolio value is 165180.1500640543

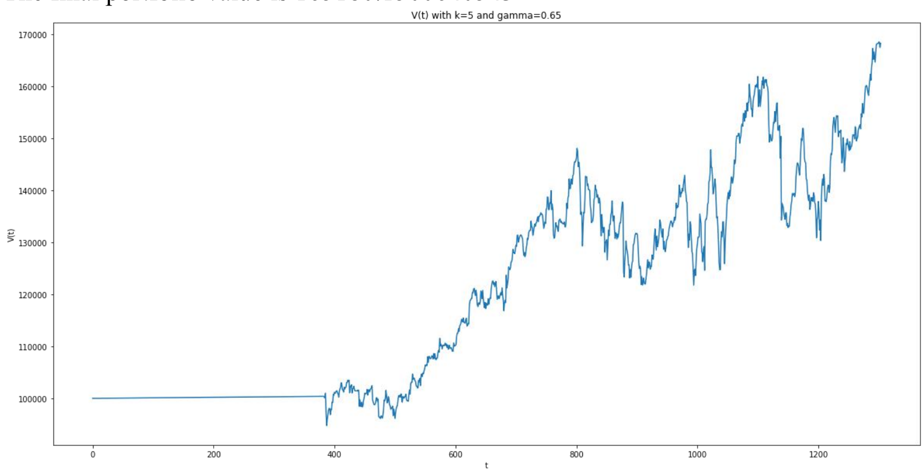

Fig. 40. $V_{\mathrm{r}}(t)$ based on 5-day rules with 0.5% ξ, 0.01% r & γ = 0.65

The final portfolio value is 168215.53784428796

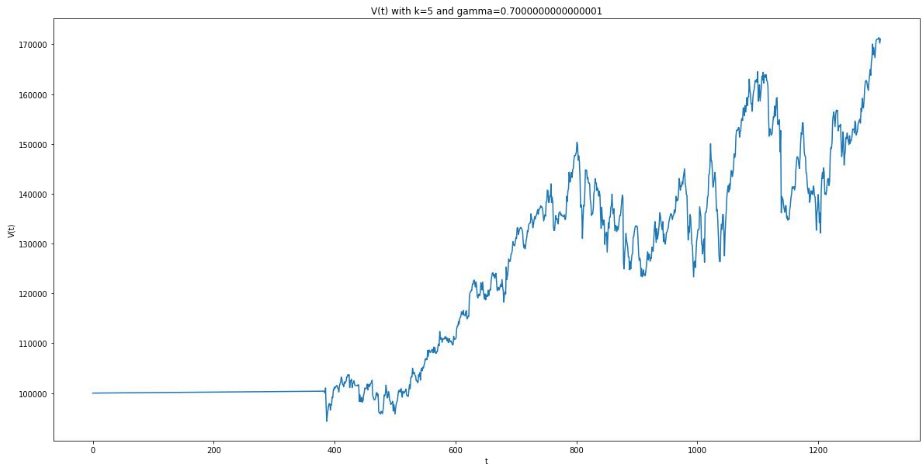

Fig. 41. $V_r(t)$ based on 5-day rules with 0.5% ξ, 0.01% r & γ = 0.7

The final portfolio value is 171037.6366394471

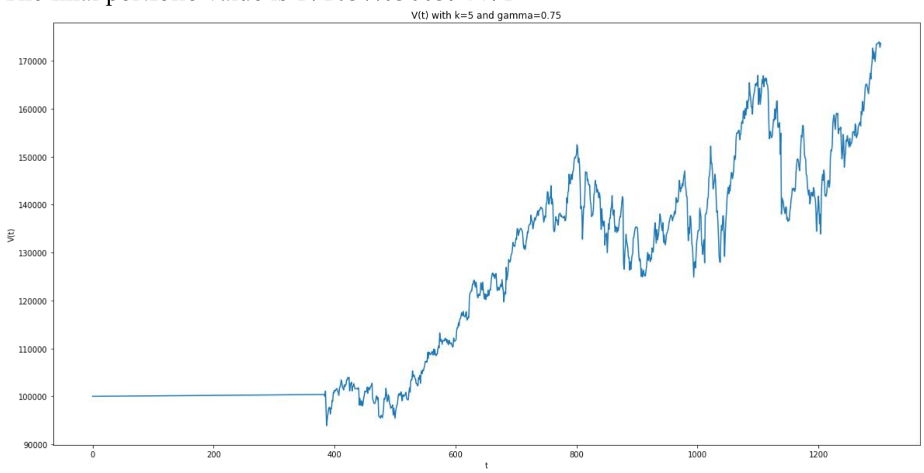

Fig. 42. $V_r(t)$ based on 5-day rules with 0.5% ξ, 0.01% r & γ = 0.75

The final portfolio value is 173676.1344256704

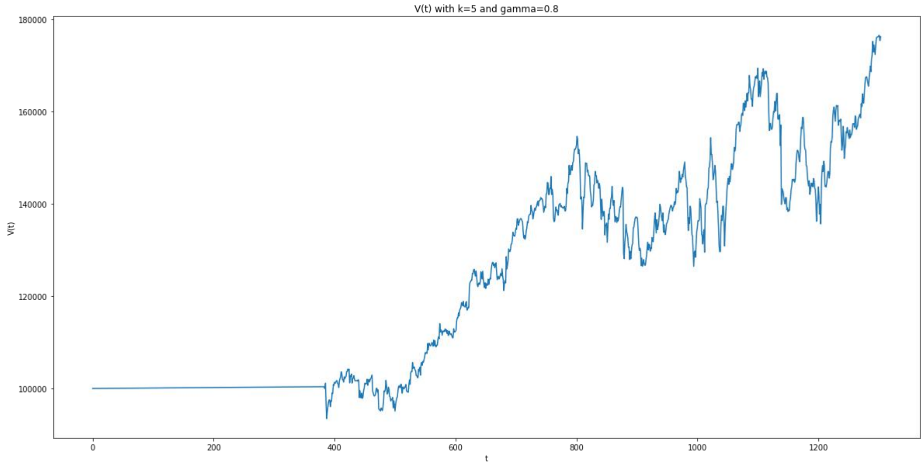

Fig. 43. $V_{\mathrm{r}}(t)$ based on 5-day rules with 0.5% ξ, 0.01% r & γ = 0.8

The final portfolio value is 176160.71917909637

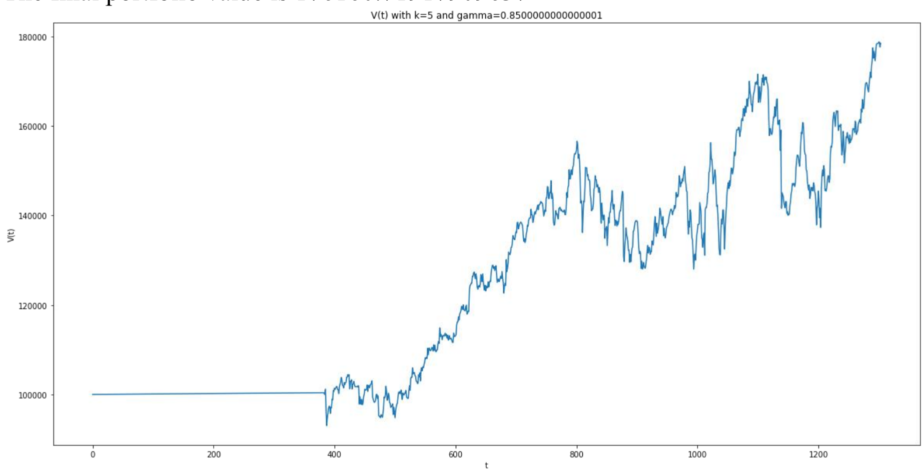

Fig. 44. $V_{\mathrm{r}}(t)$ based on 5-day rules with 0.5% ξ, 0.01% r & γ = 0.85

The final portfolio value is 178521.07887586384

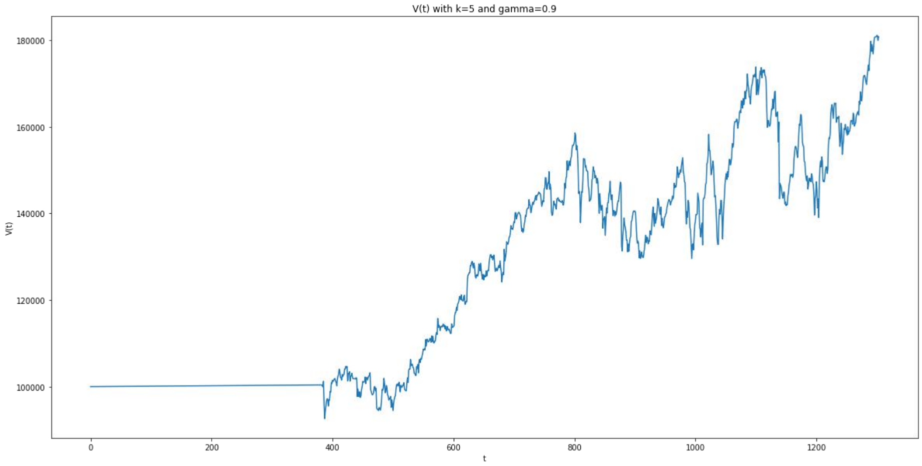

Fig. 45. $V_{\mathrm{r}}(t)$ based on 5-day rules with 0.5% ξ, 0.01% r & γ = 0.9

The final portfolio value is 180786.90149211124

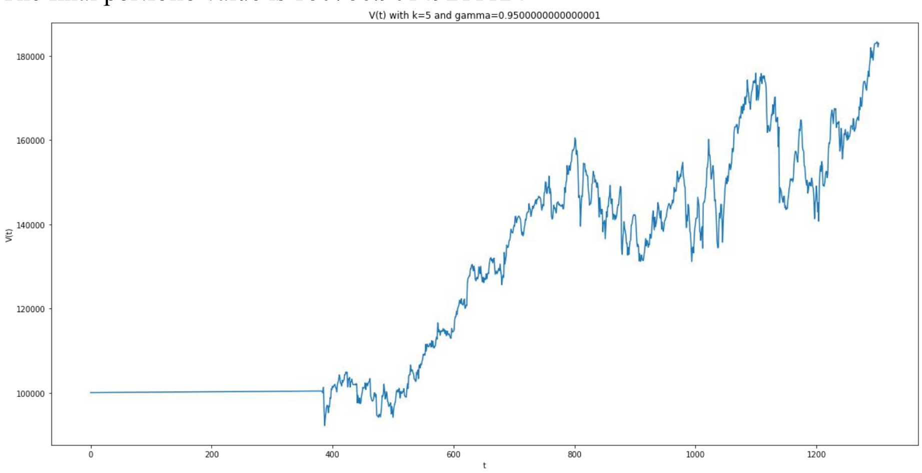

Fig. 46. $V_{\mathrm{r}}(t)$ based on 5-day rules with 0.5% ξ, 0.01% r & γ = 0.95

The final portfolio value is 182987.87500397736

After simulating with the γ = [0.1, 0.15, 0.2…. 0.95]. It shows that the high the gamma value, the larger portfolio it is.

10 Alternative Portfolio Measure

10.1 Compute portfolio's value $V_r(t)$ and $W_r(t)$ with different gamma

We compute $V_r(t)$ and $W_r(t)$ by changing different gamma, under different situation, the final result of $V_r(t)$ and $W_r(t)$ show as below:

Gamma	Final V(t)	Final W(t)
0.1	116564.023332	1366.35826201
0.3	140826.228032	1650.7587391
0.5	158350.755643	1856.18046705
0.7	171037.636639	2004.89551799
0.9	180786.901492	2119.17596404

10.2 Plot v(t) = ln[Vγ(t)/V(0)] and w(t) = ln[Wγ(t)/W(0)]

We have new definition of v(t) and w(t) by the following equation

$$v(t) = \ln[V_\gamma(t)/V(0)]$$
$$w(t) = \ln[W_\gamma(t)/W(0)]$$

For v(t), it means whether the portfolio value at time t worth more than the portfolio value at time 0.

On the other hand, w(t) represent whether the number of stock worth of portfolio value is higher at time t than it is at time 0.

So we have to generate v(t) and w(t) and shows as below.

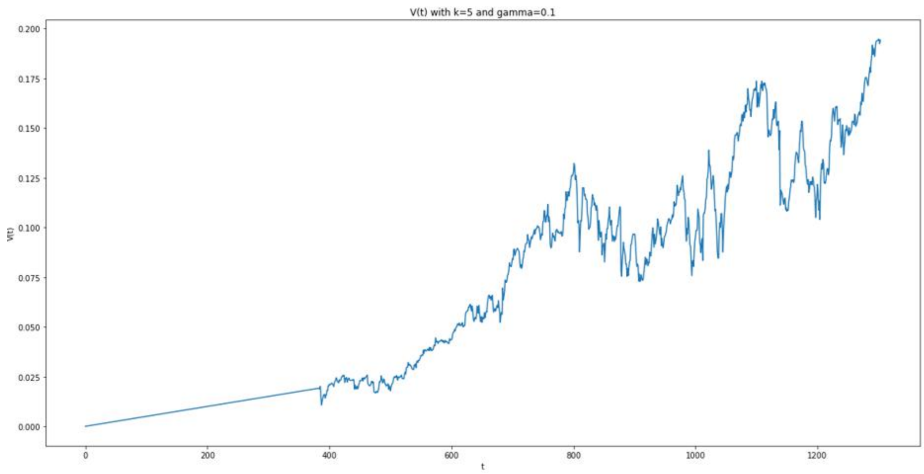

Fig. 47. v(t) = ln[Vγ(t)/V(0)] based on 5 days rules with γ = 0.1

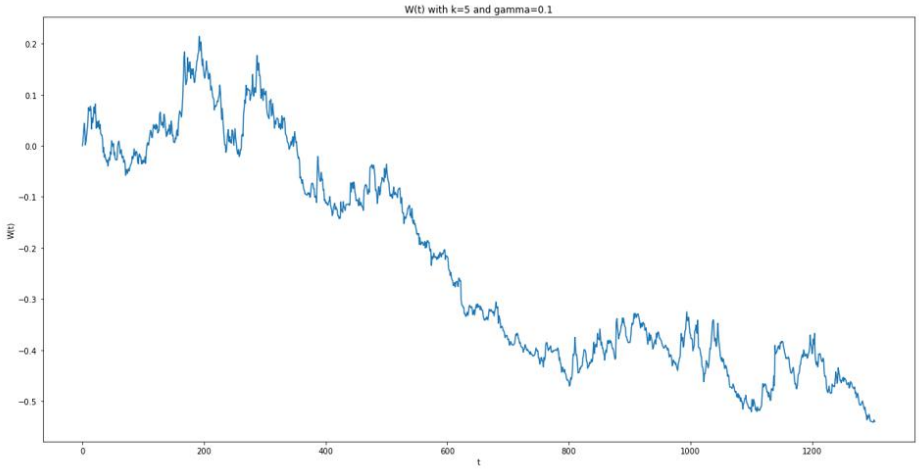

Fig. 48. w(t) = ln[Wγ(t)/W(0)]based on 5 days rules with γ = 0.1

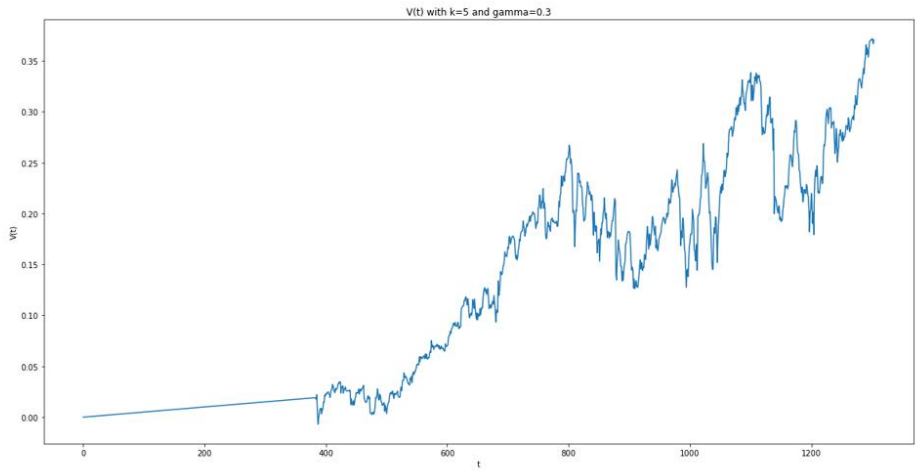

Fig. 49. v(t) = ln[Vγ(t)/V(0)] based on 5 days rules with γ = 0.3

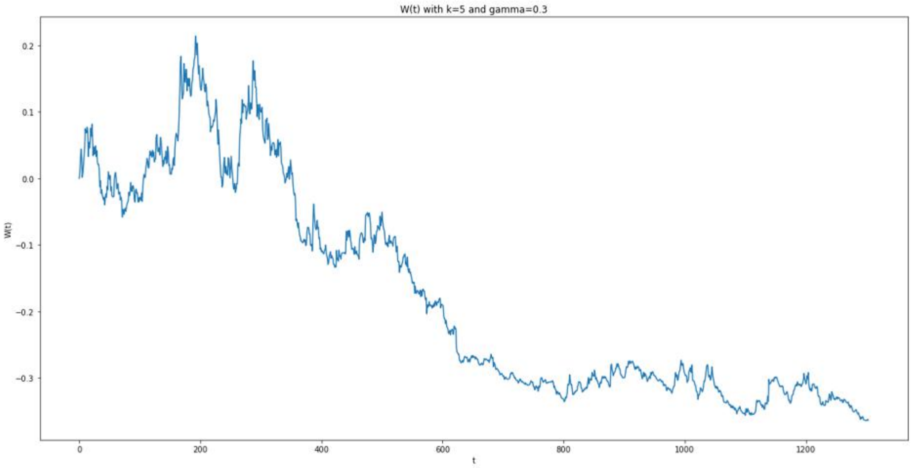

Fig. 50. w(t) = ln[Wγ(t)/W(0)]based on 5 days rules with γ = 0.3

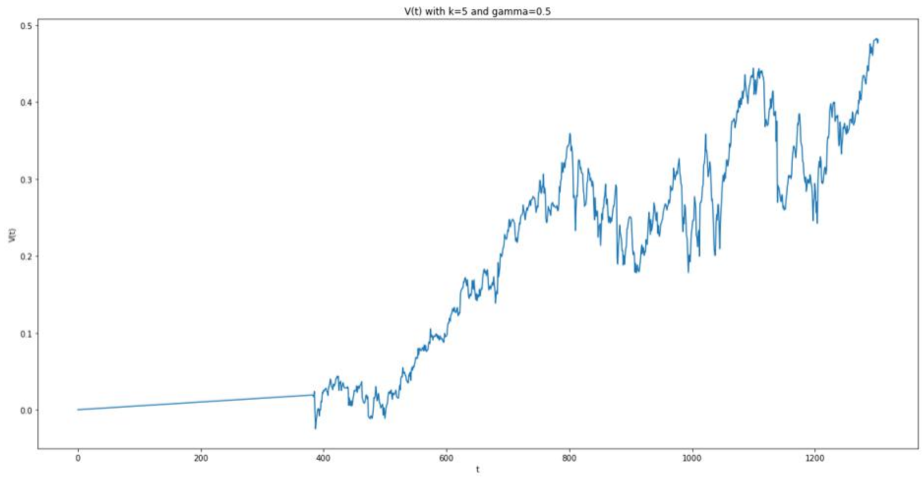

Fig. 51. v(t) = ln[Vγ(t)/V(0)] based on 5 days rules with γ = 0.5

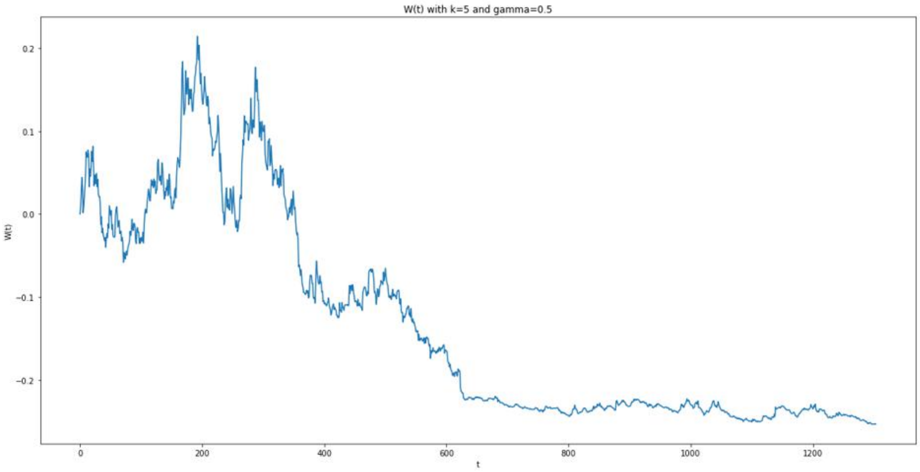

Fig. 52. $w(t) = \ln[W\gamma(t)/W(0)]$based on 5 days rules with $\gamma = 0.5$

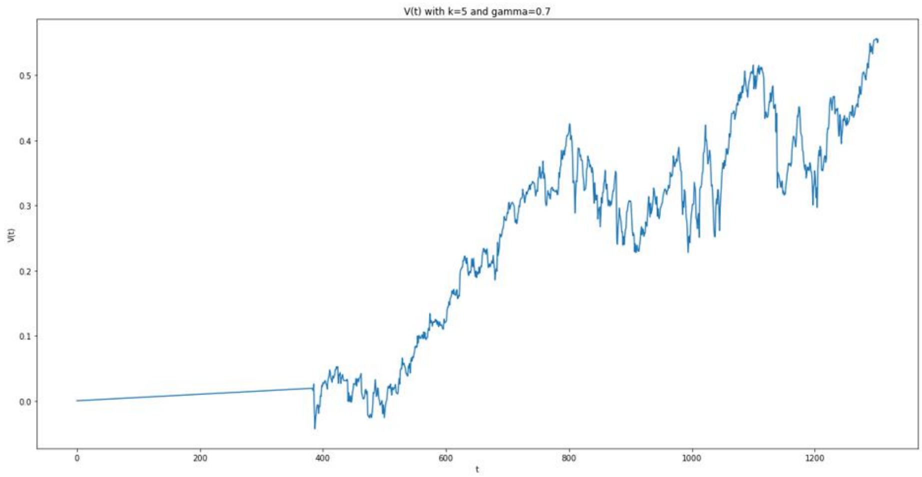

Fig. 53. $v(t) = \ln[V\gamma(t)/V(0)]$ based on 5 days rules with $\gamma = 0.7$

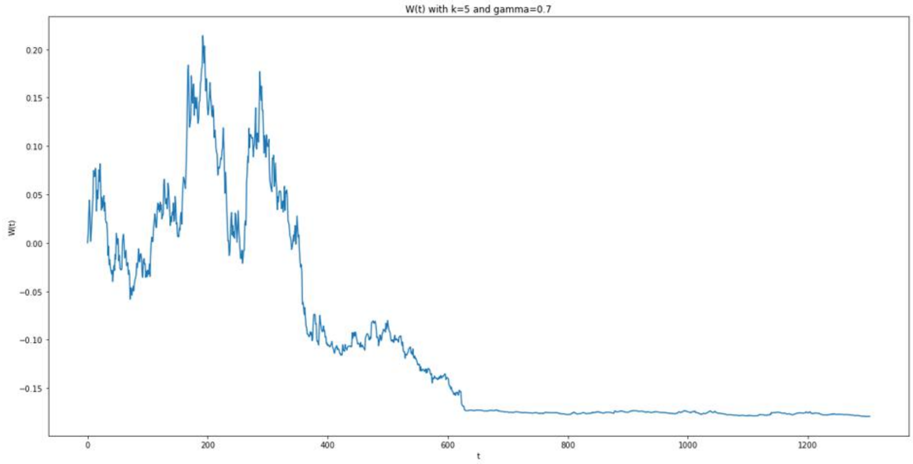

Fig. 54. w(t) = ln[Wγ(t)/W(0)]based on 5 days rules with γ = 0.7

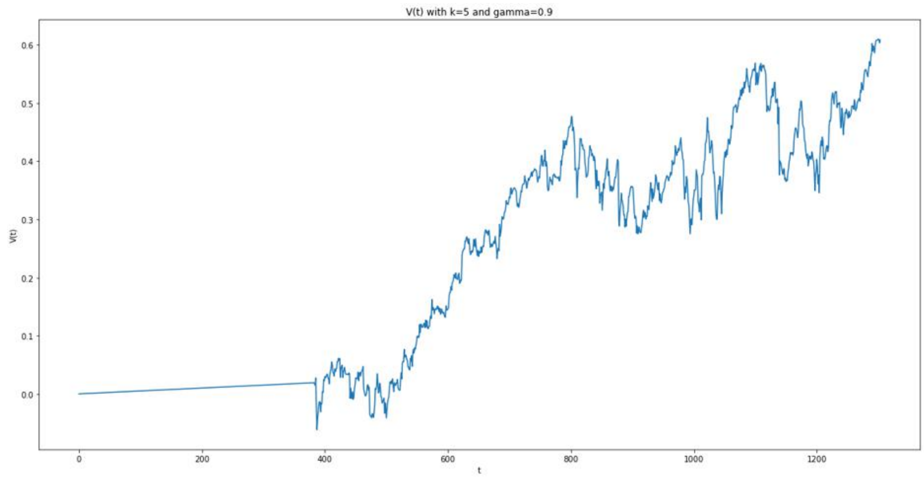

Fig. 55. v(t) = ln[Vγ(t)/V(0)] based on 5 days rules with γ = 0.9

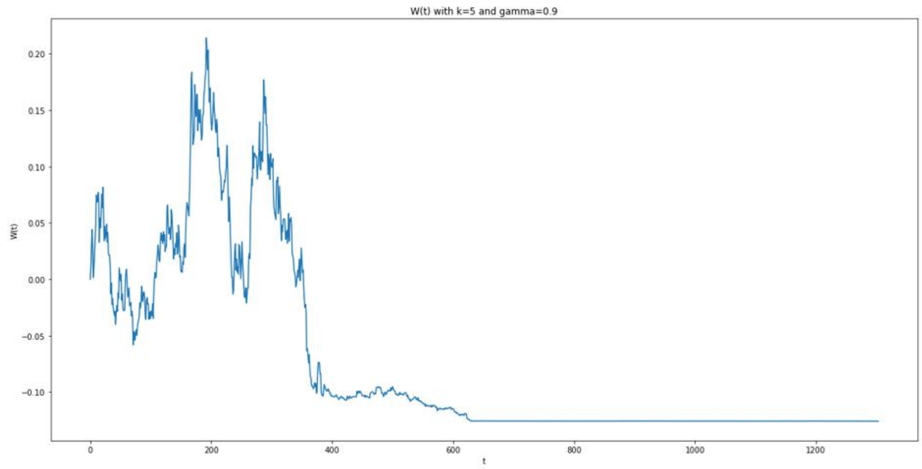

Fig. 56. w(t) = ln[Wγ(t)/W(0)]based on 5 days rules with γ = 0.7

Besides, we calculate v(t) and w(t) don't have the same sign in the same γ. It means that they don't synchronous.

10.3 Trade and plot the portfolio's value $V_r(t)$

We have new definition of v_i and w_i by the following equation

$$v_i = \ln[V_\gamma(t_i)/V_\gamma(t_i - 1)]$$
$$w_i = \ln[W_\gamma(t_i)/W_\gamma(t_i - 1)]$$

Here v_i means that whether the portfolio value at time t is larger than the portfolio value at time t-1. Similarly, w_i means that whether the stock worth at time t is larger than the stock worth at time t-1.

Base on this equation, we use different γ to simulate it.

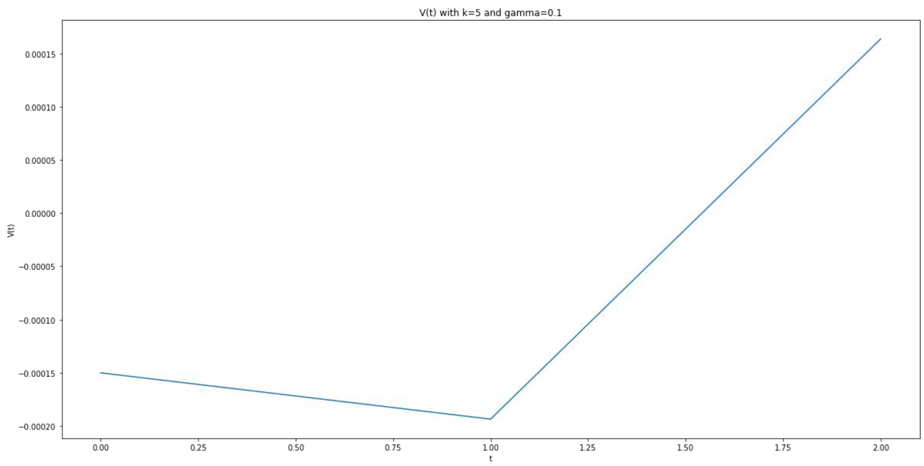

Fig. 57. vi = ln[Vγ(ti)/Vγ(ti–1)]based on 5 days rules with γ = 0.1

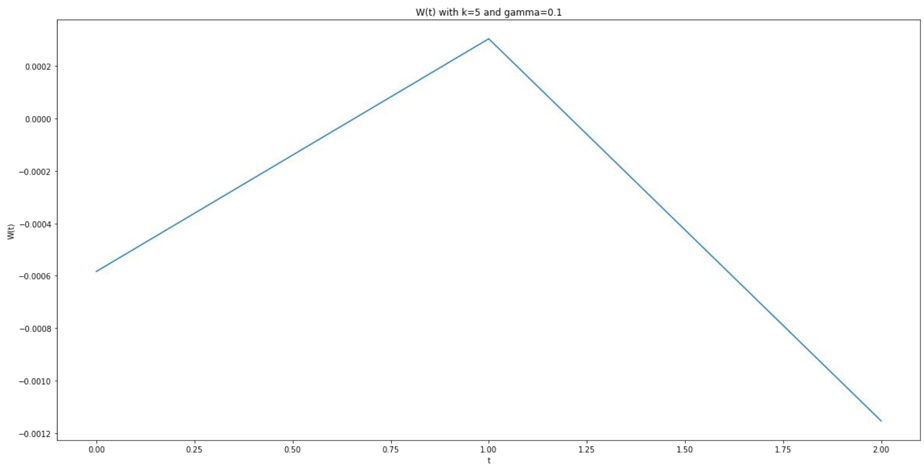

Fig. 58. wi = ln[Wγ(ti)/Wγ(ti–1)] based on 5 days rules with γ = 0.1

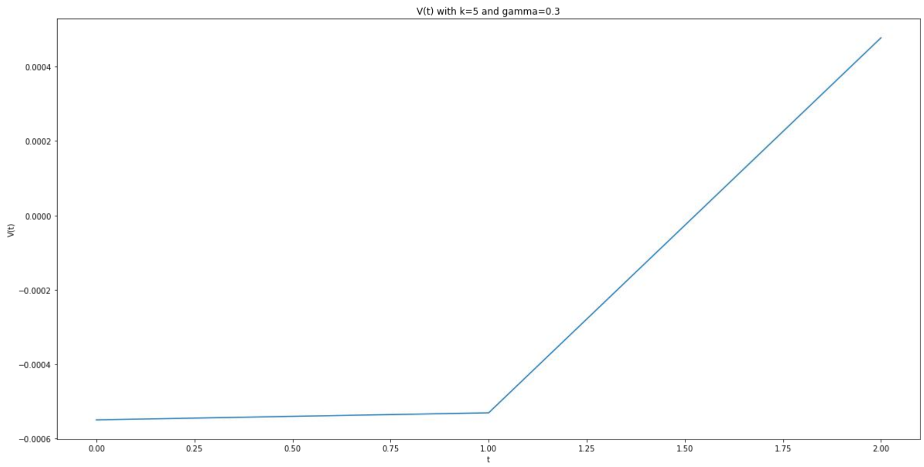

Fig. 59. vi = ln[Vγ(ti)/Vγ(ti–1)]based on 5 days rules with γ = 0.3

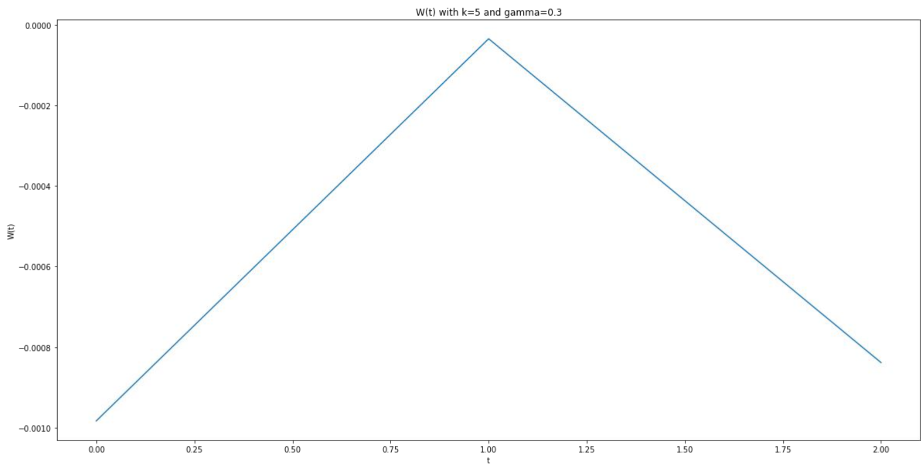

Fig. 60. wi = ln[Wγ(ti)/Wγ(ti–1)] based on 5 days rules with γ = 0.3

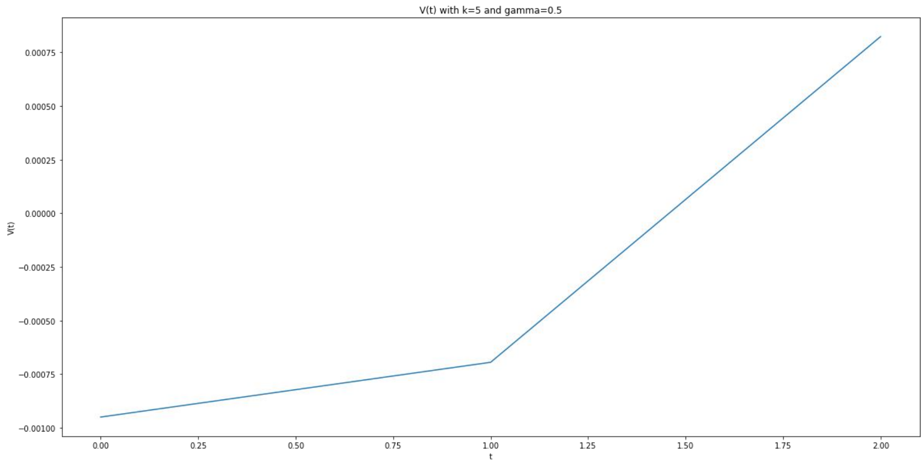

Fig. 61. vi = ln[Vγ(ti)/Vγ(ti–1)]based on 5 days rules with γ = 0.**5**

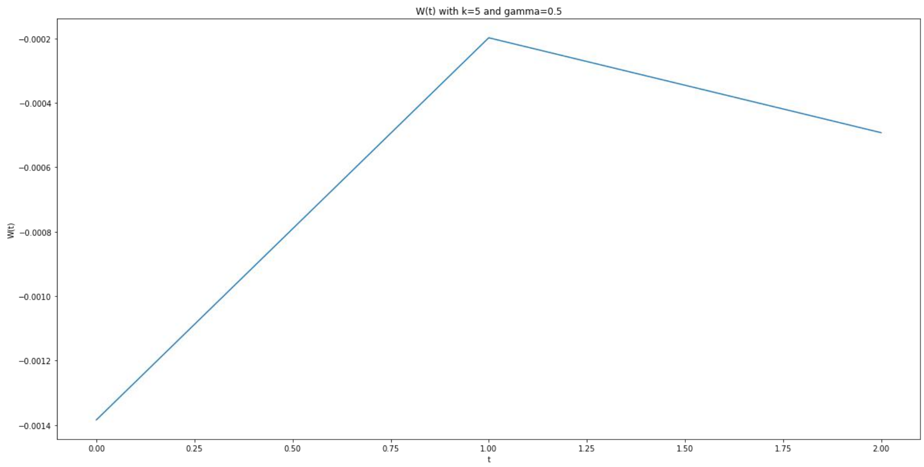

Fig. 62. . wi = ln[Wγ(ti)/Wγ(ti–1)] based on 5 days rules with γ = 0.5

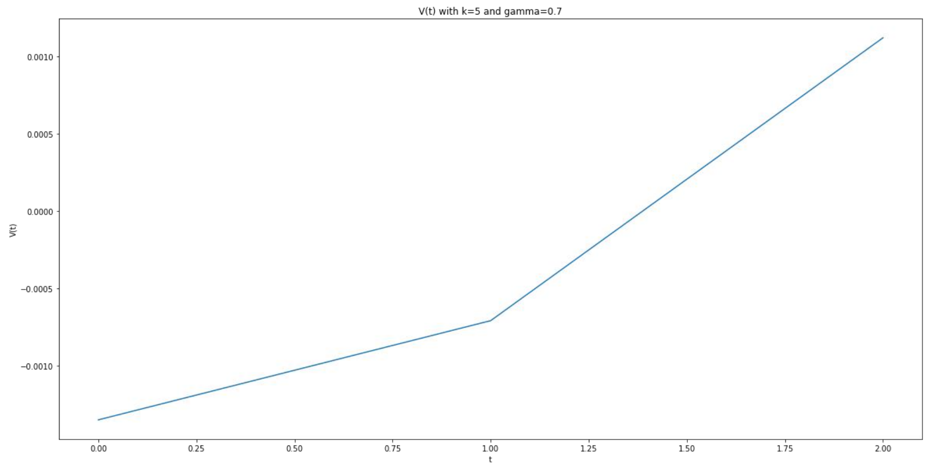

Fig. 63. . vi = ln[Vγ(ti)/Vγ(ti–1)]based on 5 days rules with γ = 0.7

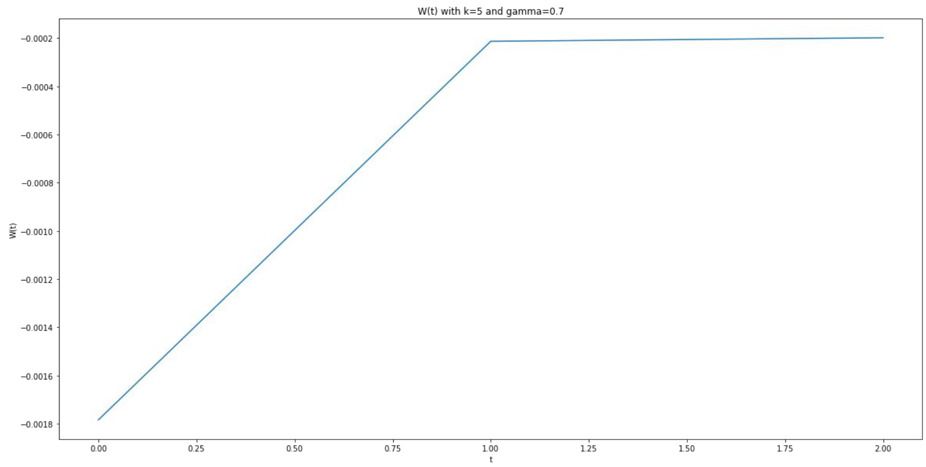

Fig. 64. wi = ln[Wγ(ti)/Wγ(ti–1)] based on 5 days rules with γ = 0.7

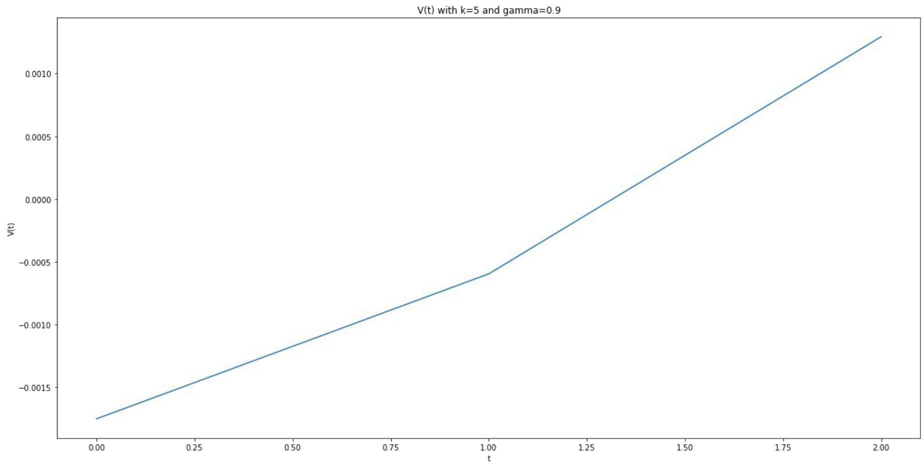

Fig. 65 . vi = ln[Vγ(ti)/Vγ(ti–1)]based on 5 days rules with γ = 0.7

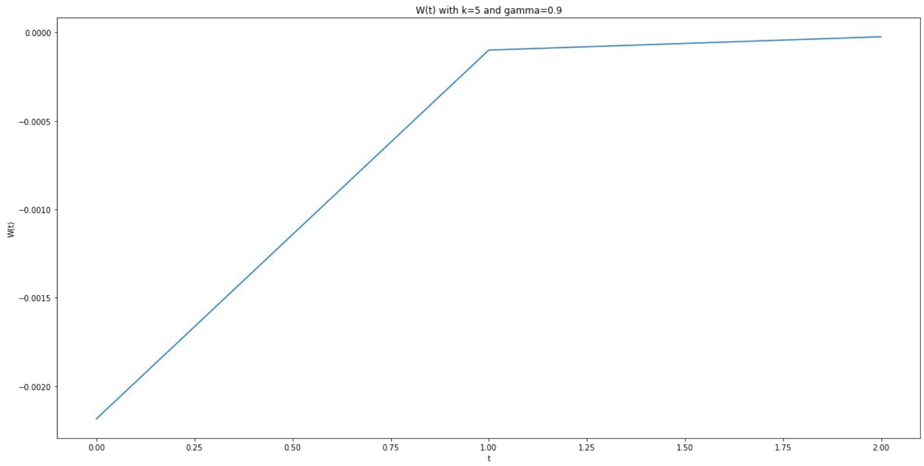

Fig. 66. wi = ln[Wγ(ti)/Wγ(ti–1)] based on 5 days rules with γ = 0.9

After the simulation, we calculate their value and show out that v_i and w_i don't have the same sign in the same γ. It means that they don't synchronous.

10.4 Extra: Summation of v_i and w_i

Based on our selected stocks, there are only three transaction happened and all of them are buy decision. Both the summation of $v(t_j)$ and $w(t_j)$ are plotting as follow.

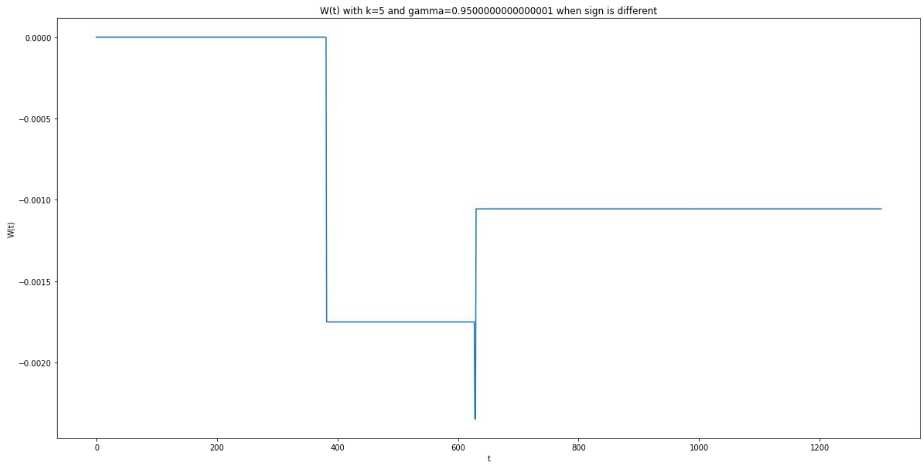

Fig. 67. $v(t_j) = \sum_{i=1}^{j} v_j$ based on the ideal gamma configuration

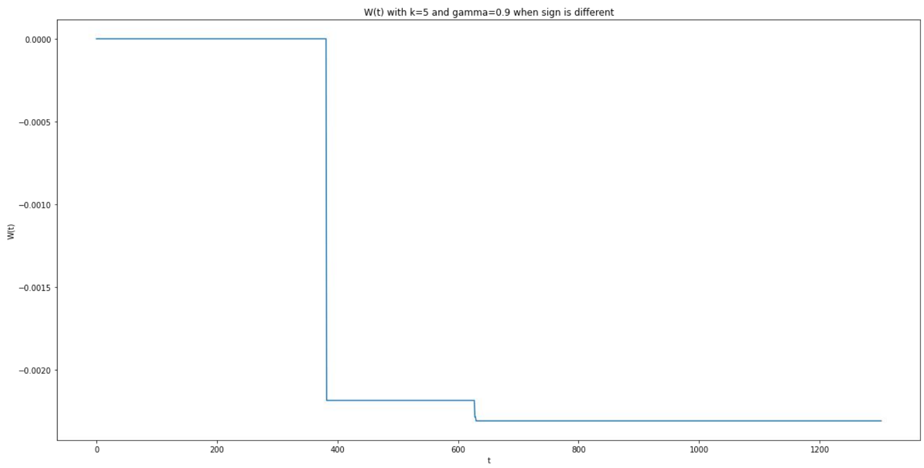

Fig. 68. $w(t_j) = \sum_{i=1}^{j} w_j$ based on the ideal gamma configuration

In fact, both the summation of $\mathrm{v}(t_j)$ and $\mathrm{w}(t_j)$ is negative during the whole testing period which means they are the same sign. Therefore, we fail to draw a conclusion on whether we will have a gain or loss when signs of $v(t)$ and $w(t)$ are different.

Generally, the summation of $\mathrm{v}(t_j)$ increase when the stock price s(t) increase and vice versa. The definition of v(t) and w(t) show as follow.

$$W(t) = N(t) + M(t) / s(t)$$

$$V(t) = M(t) + N(t)\, s(t)$$

Similarly, when the s(t) increase, the W(t) tends to decrease in value and the V(t) tends to increase in value and vice versa.

$$Log[W(t)/W(t\text{-}1)] = Log[(N(t) + M(t) / s(t)) / (N(t\text{-}1) + M(t\text{-}1) / s(t\text{-}1))]$$

$$Log[V(t)/V(t\text{-}1)] = Log[\ (M(t) + N(t)\, s(t)) / (M(t\text{-}1) + N(t\text{-}1)\, s(t\text{-}1))]$$

When it comes to the log ratio of value between t and t-1. It will have the same insight like what we analysis above. So the trade will earn when the summation of $\mathrm{v}(t_j)$ is positive and lost when the summation of $\mathrm{v}(t_j)$ is negative.

11 Efficient Frontier

In this section, we calculate the express of different notation and the result will show as mathematics form for the first several question.

11.1 Express A_{u} and A_D in terms of $\{\gamma_{\mathrm{U}}, \gamma_{\mathrm{D}}, \mathrm{M(t)}, \mathrm{N(t)}, \mathrm{s(t)}, \xi\}$

$$A_U = \frac{M(t) - \gamma_u M(t)}{M(t) - \gamma_u M(t) + (1-\xi)\gamma_u M(t) + N(t)s(t)}$$
$$= \frac{M(t)(1-\gamma_u)}{M(t)(1-\xi\gamma_u) + N(t)s(t)}$$

$$A_D = \frac{M(t) + (1-\xi)\gamma_D N(t)s(t)}{M(t) + (1-\xi)\gamma_D N(t)s(t) + N(t)(1-\gamma_D)s(t)}$$

$$= \frac{M(t) + (1-\xi)\gamma_D N(t)s(t)}{M(t) + N(t)(1-\xi\gamma_D)s(t)}$$

11.2 Express A, A_u and A_D in terms of $\{u, u_U, u_D, u_1, u_2\}$, and then A_U / A and $(1-A_D)/(1-A)$ in term of γ

$$u = Au_1 + (1-A)u_2$$

$$A = \frac{u-u_2}{u_1-u_2}$$

$$A_U = \frac{u_U-u_2}{u_1-u_2}$$

$$A_D = \frac{u_D-u_2}{u_1-u_2}$$

$$\frac{A_U}{A} = \frac{u_U-u_2}{u-u_2}$$

$$\frac{1-A_D}{1-A} = \frac{u_1-u_D}{u_1-u}$$

$$\frac{A_U}{A} = \frac{u_U-u_2}{u-u_2} = \frac{u_U-u+u-u_2}{u-u_2} = \frac{u_U-u}{u-u_2} + 1 = \frac{u_U-u}{-(u_2-u)} + 1 = 1-\gamma$$

$$\frac{1-A_D}{1-A} = \frac{u_1-u_D}{u_1-u} = \frac{u_1-u+u-u_D}{u_1-u} = 1 + \frac{u-u_D}{u_1-u} = 1 - \frac{u_D-u}{u_1-u} = 1-\gamma$$

11.3 Express γ_u and γ_D in terms of $\{\gamma_U, \gamma_D, M(t), N(t), s(t), \xi\}$

We have this formula:

$$\frac{(1-\gamma_u)(M(t)+N(t)s(t))}{M(t)(1-\xi\gamma_u)+N(t)s(t)} = 1-\gamma$$

Then we can transform this formula step by step:

$$\begin{aligned} &M(t) + N(t)s(t) - \gamma_u M(t) - \gamma_u N(t)s(t) \\ &\quad = M(t)(1-\xi\gamma_u) + N(t)s(t) - \gamma M(t)(1-\xi\gamma_u) - \gamma\, N(t)s(t) \end{aligned}$$

$$\begin{aligned} &M(t) + N(t)s(t) - \gamma_u M(t) - \gamma_u N(t)s(t) \\ &\quad = M(t) - \xi\gamma_u M(t) + N(t)s(t) - \gamma M(t) + \gamma M(t)\xi\gamma_u \\ &\quad - \gamma\, N(t)s(t) \end{aligned}$$

$$-\gamma_u M(t) - \gamma_u N(t)s(t) =- \xi\gamma_u M(t) - \gamma M(t) + \gamma M(t)\xi\gamma_u - \gamma\, N(t)s(t)$$

$$-\gamma_u M(t) - \gamma_u N(t)s(t) + \xi\gamma_u M(t) - \gamma M(t)\xi\gamma_u =- \gamma M(t) - \gamma\, N(t)s(t)$$

$$\gamma_u M(t) + \gamma_u N(t)s(t) - \xi\gamma_u M(t) + \gamma M(t)\xi\gamma_u = \gamma M(t) + \gamma\, N(t)s(t)$$

$$\gamma_u(M(t) + N(t)s(t) - \xi M(t) + \gamma M(t)\xi) = \gamma(M(t) + N(t)s(t))$$

So we got:

$$\gamma_u = \frac{\gamma(M(t) + N(t)s(t))}{(M(t) + N(t)s(t) - \xi M(t) + \gamma M(t)\xi)}$$

Similarity, we have :

$$1 - \frac{M(t)+(1-\xi)\gamma_D N(t)s(t)}{M(t)+N(t)(1-\xi\gamma_D)s(t)} = (1-\gamma)(1 - \frac{M(t)}{M(t)+N(t)s(t)})$$

$$\frac{N(t)(1-\xi\gamma_D)s(t)-(1-\xi)\gamma_D N(t)s(t)}{M(t)+N(t)(1-\xi\gamma_D)s(t)} = (1-\gamma)(\frac{N(t)s(t)}{M(t)+N(t)s(t)})$$

$$\frac{(1-\xi\gamma_D)-(1-\xi)\gamma_D}{M(t)+N(t)(1-\xi\gamma_D)s(t)} = (\frac{1-\gamma}{M(t)+N(t)s(t)})$$

$$((1-\xi\gamma_D) - (1-\xi)\gamma_D) * (M(t) + N(t)s(t)) = (1-\gamma)(M(t) + N(t)(1 - \xi\gamma_D)s(t))$$

$$M(t)(1-\xi\gamma_D) - M(t)(1-\xi)\gamma_D + N(t)s(t)(1-\xi\gamma_D) - N(t)s(t)\,(1-\xi)\gamma_D = M(t) + N(t)(1-\xi\gamma_D)s(t) - \gamma\, M(t) - \gamma\, N(t)(1-\xi\gamma_D)s(t)$$

$$M(t)(1-\xi\gamma_D) - M(t)(1-\xi)\gamma_D - N(t)s(t)\,(1-\xi)\gamma_D = M(t) - \gamma\, M(t) - \gamma\, N(t)(1-\xi\gamma_D)s(t)$$

$$M(t) - \xi\gamma_D M(t) - M(t)\gamma_D + \xi\gamma_D M(t) - N(t)s(t)\gamma_D + N(t)s(t)\gamma_D\xi = M(t) - \gamma\, M(t) - \gamma\, N(t)s(t) + \gamma\, N(t)s(t)\xi\gamma_D$$

$$-M(t)\gamma_D - N(t)s(t)\gamma_D + N(t)s(t)\gamma_D\xi - \gamma\, N(t)s(t)\xi\gamma_D =- \gamma\, M(t) - \gamma\, N(t)s(t)$$

$$M(t)\gamma_D + N(t)s(t)\gamma_D - N(t)s(t)\gamma_D\xi + \gamma\, N(t)s(t)\xi\gamma_D =- \gamma\, M(t) + \gamma\, N(t)s(t)$$

So we got:

$$\gamma_D = \frac{\gamma(M(t)+N(t)s(t))}{M(t)+N(t)s(t)-N(t)s(t)\xi+\gamma\, N(t)s(t)\xi}$$

11.4 Trade with m = γM(t) and n = γN(t). Plot the portfolio's value V(t).

It turns out that $\gamma = \gamma U = \gamma D$ for a small tax ξ. This justifies trading with $m = \gamma M(t)$ and $n = \gamma N(t)$. We consider a heavy tax $\xi = 20\%$ with $r = 0.001\%$ and $\gamma = \gamma 0$. In simulation, we trade with $m = \gamma M(t)$ and $n = \gamma N(t)$ and the portfolio's value $V(t)$ show as follow.

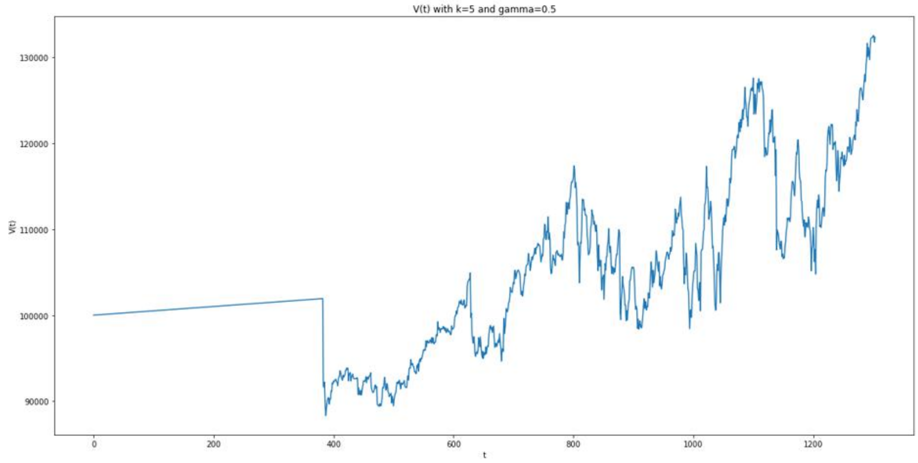

Fig.69. portfolio's value $V(t)$ when $m = \gamma M(t)$ and $n = \gamma N(t)$

The final portfolio value is 129446.98366300965

11.5 Trade with $\tilde{m} = \gamma_u M(t)$ and $\tilde{n} = \gamma_D N(t)$. Plot the portfolio's value $\tilde{V}(t)$

we simulate $\tilde{m}$ = γU$M(t)$ and $\tilde{n}$ = γD$N(t)$ and the portfolio's value $V(t)$ show as below:

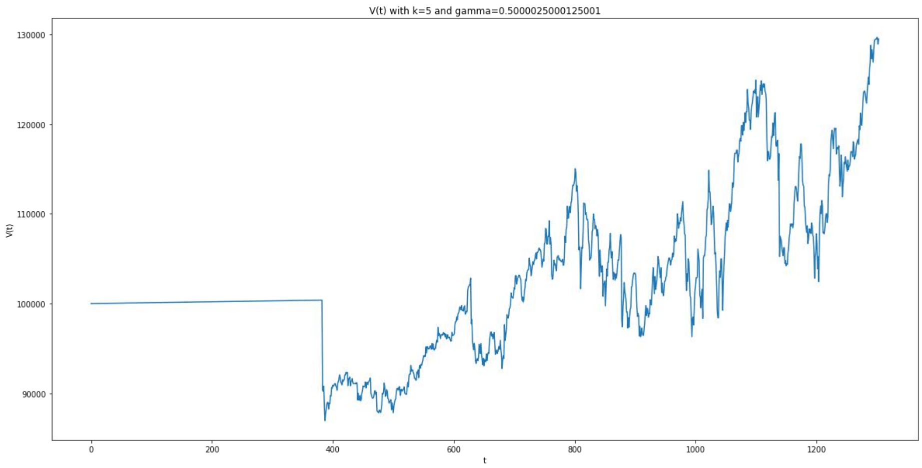

Fig.70. portfolio's value $V(t)$ when $\tilde{m} = \gamma UM(t)$ and $\tilde{n} = \gamma DN(t)$

The final portfolio value is 129447.03947227128

Base on 11.4 and 11.5, with new $\tilde{m}$ and $\tilde{n}$ of V(t), the final value is slightly higher than that with m and n. Therefore, such adjustment with $\tilde{m}$ and $\tilde{n}$ is useful.

12 Adaptive Greed

12.11 Posterior Analysis

The simulation use different level of gamma for getting the best greed value for trading time. We set 5 gamma values ranging from 0.1 to 0.9, and the portfolio's value $V\gamma(t)$ are shown in the table below.

Gamma	Portfolio's value $V\gamma(t)$.

0.1	[100001. 100002.00001 100003.00003 ... 116617.61350839, 116358.03395984, 116564.02333212]
0.3	[100001. 100002.00001 100003.00003 ... 140962.30907008140316.07896259, 140826.22803232]
0.5	[100001. 100002.00001 100003.00003 ... 158538.35297931 157650.45853763, 158350.75564381]
0.7	[100001. 100002.00001 100003.00003 ... 171254.593323 170228.7486443 ,171037.63663945]
0.9	: [100001. 100002.00001 100003.00003 ... 181019.87818803, 179918.52524194 180786.90149211]
Ideal Gamma	[100001. 100002.00001 100003.00003 ... 143176.21281626 142354.65554111 143002.56861889]

Besides, we compute $V_{\mathrm{r}}(t)$ base on the optimal gamma.

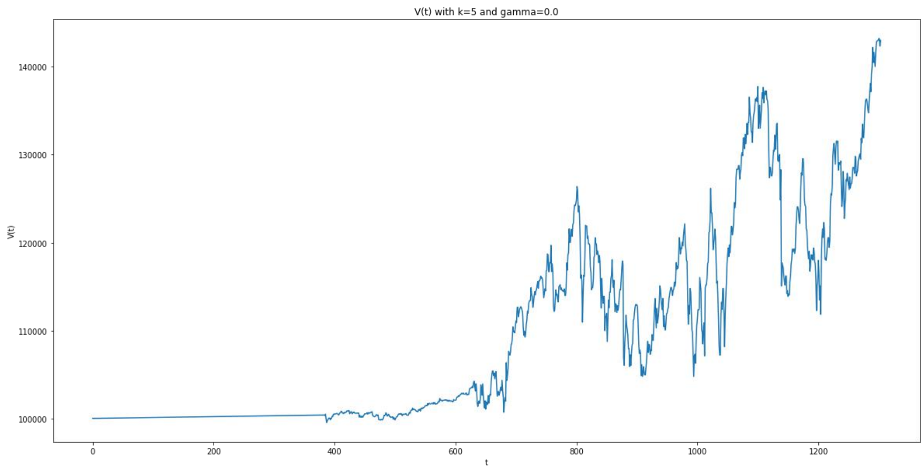

Fig.71. Portfolio's value $V(t)$ based on the optimal gamma after 5 gamma trials

12.12 Plot γ* = argmaxγ Vγ(ti)/Vγ(ti–1) against i

The optimal greed value set as $V\gamma(t_i)/V\gamma(t_{i-1})$ in each trading step. Base on this change, we simulate the

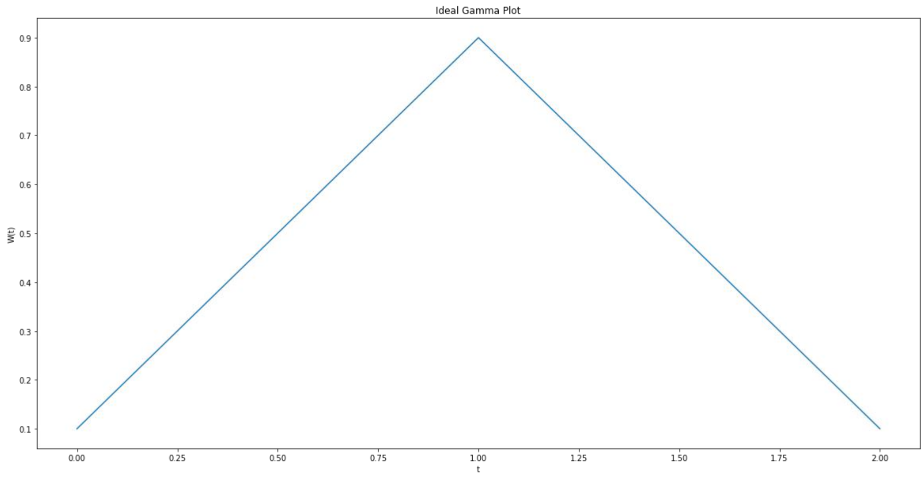

Fig.72. Optimal gamma value based on 5 gamma trials {0.1, 0.3, 0.5, 0.7, 0.9}

From the figure we know that when γ^* =1, we get the largest w(t)

12.13 (Extra) Run the simulation with fifteen more samples of γ.

The above process is repeated with fifteen more gamma values to obtain the best gamma values.

Gamma	Portfolio's value *V*γ(*t*).
0.05	[100001. 100002.00001 100003.00003 ... 109250.14466209, 109114.71842834 109223.05029092]
0.1	[100001. 100002.00001 100003.00003 ... 116617.613508 116358.03395984 116564.02333212]
0.15	[100001. 100002.00001 100003.00003 ... 123442.738620 123070.64690465 123365.13965072]
0.2	[100001. 100002.00001 100003.00003 ... 129755.34575122

	129281.73813714 129656.08722287]
0.25	[100001. 100002.00001 100003.00003 ... 135585.260650 135020.48853167 135466.55402469]
0.3	[100001. 100002.00001 100003.00003 ... 140962.309070 08 140316.07896259 140826.22803232]
0.35	[100001. 100002.00001 100003.00003 ... 145916.316761 41 145197.69030429 145764.79722192]
0.4	[100001. 100002.00001 100003.00003 ... 150477.109475 82 149694.50343112 150311.9495696]
0.45	[100001. 100002.00001 100003.00003 ... 154674.512964 67 153835.69921744 154497.37305152]
0.5	[100001. 100002.00001 100003.00003 ... 158538.352979 31 157650.45853763 158350.75564381]
0.55	[100001. 100002.00001 100003.00003 ... 162098.455271 11 161167.96226605 161901.78532261]
0.6	[100001. 100002.00001 100003.00003 ... 165384.645591 42 164417.39127705 165180.15006405]
0.65	[100001. 100002.00001 100003.00003 ... 168426.749691 6 167427.92644502 168215.53784429]
0.7	[100001. 100002.00001 100003.00003 ... 171254.593323 170228.7486443 171037.63663945]
0.75	[100001. 100002.00001 100003.00003 ... 173898.002236 98 172849.03874927 173676.13442567]
0.8	[100001. 100002.00001 100003.00003 ... 176386.802184

	91 175317.97763429 176160.7191791]
0.85	[100001. 100002.00001 100003.00003 ... 178750.818918 14 177664.74617372 178521.07887586]
0.9	[100001. 100002.00001 100003.00003 ... 181019.878188 03 179918.52524194 180786.90149211]
0.95	[100001. 100002.00001 100003.00003 ... 183223.805745 93 182108.4957133 182987.87500398]
Ideal Gamma	[100001. 100002.00001 100003.00003 ... 140131.031145 04 139308.8614417 139957.20122635]

Base on the optimal gamma, we can simulate the V(t) again:

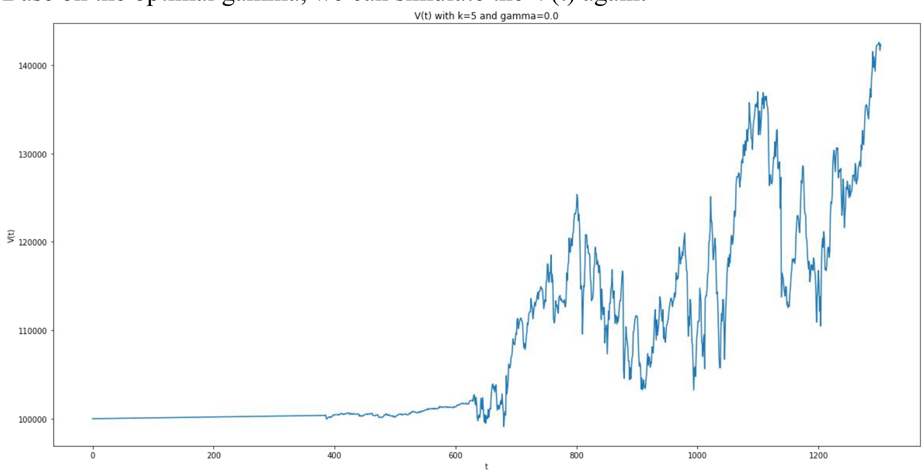

Fig.73. Portfolio's value $V(t)$ based on the optimal gamma after 20 gamma trials [0.05, 0.1,….0.95]

12.14 calculate different *V*γ(*t*) with γ^*

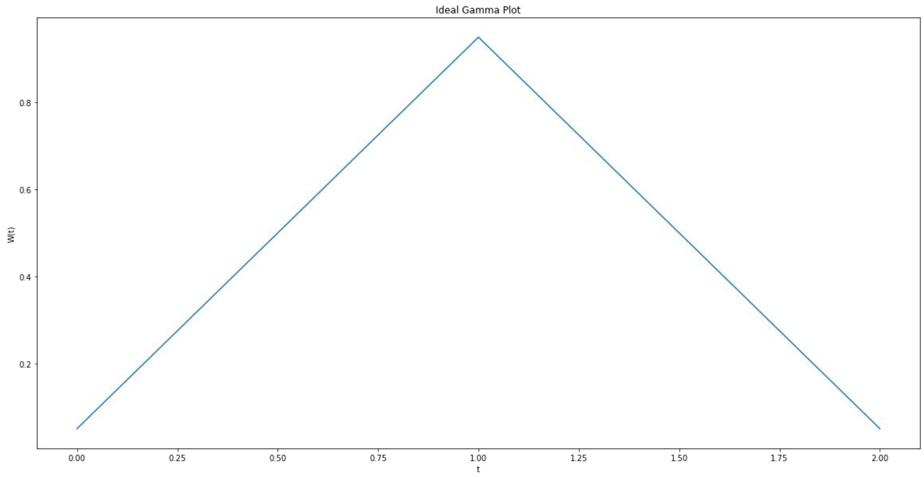

Fig.74. Optimal gamma value based on 20 gamma trials [0.05, 0.1,….0.95]

From the figure we know that when γ^* =1, we get the largest w(t). While this w(t) is slightly lower than the w(t) in 12.12

12.2 (Extra) Prior Analysis

12.21 Bob’ choice

Depending on the risk appetite of Bob, he can apply different strategy to maximum the profit based on his acceptable risk. For example, a aggressive strategy, Bob can decided to choose a higher gamma value (0.7) when decide to buy and using a lower gamma value (0.3) when decide to sell. If Bob is a more risk averted strategy, Bob can choose 0.3 as gamma value for buy decision and 0.7 as sell decision.

12.22 Simulate of Bob’ choice

Under this two possible choice, we simulate it respectively and show as below:

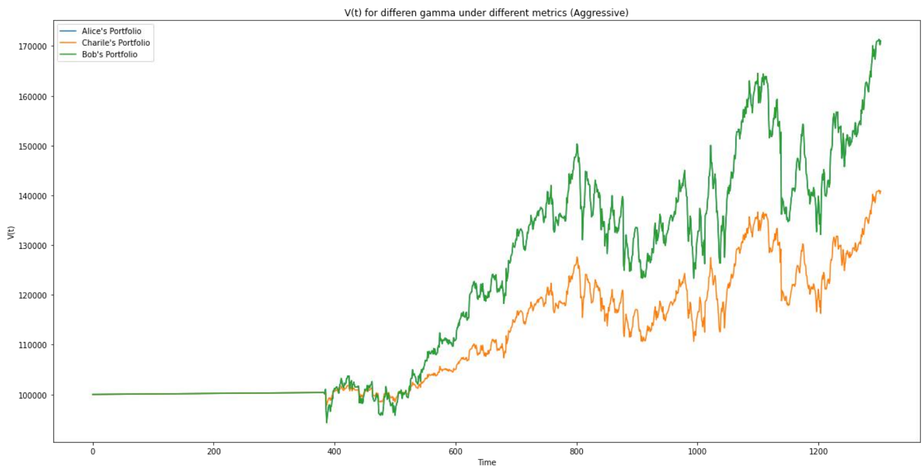

Fig.75. Portfolio's value *V*(*t*) using the aggressive strategy

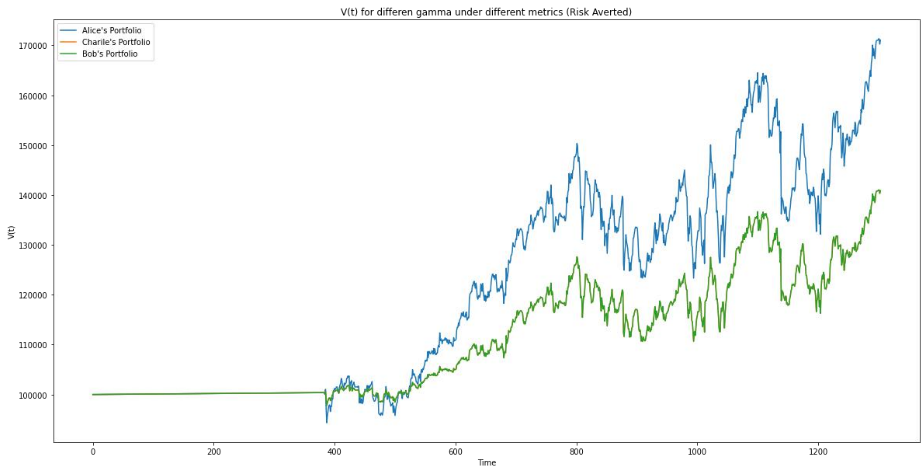

Fig.76. Portfolio's value *V*(*t*) using the risk-averse strategy

According to the result above, Bob does not higher nor lower than Alice or Charlie in this two figure(we see the green line does not higher than any other line obviously). Because there are only 3 buy decision in the testing period. Secondly, our strategy for Bob maybe not good enough, since the best gamma may between 0.3 to 0.7.

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