A simulation to test the property of mean of exponential distribution

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Synopsis:

The central limit theory shows that for any distribution, the sample mean of large number will converge to the normal distribution. In this analysis, we will investigate the exponential distribution and compare it to the central limit theory. In this analysis, we set lambda = 0.2 for all of the simulations.

Data Processing

First, we use 1000 sample of mean of 40 exponentials as:

```
mns = NULL
for (i in 1 : 5000) mns = c(mns, mean(rexp(40,0.2)))
```

Now let us answer the questions one by one. First, we calculate the theory mean as:

```
theo_mean<-1/0.2
theo_mean
```

```
## [1] 5
```

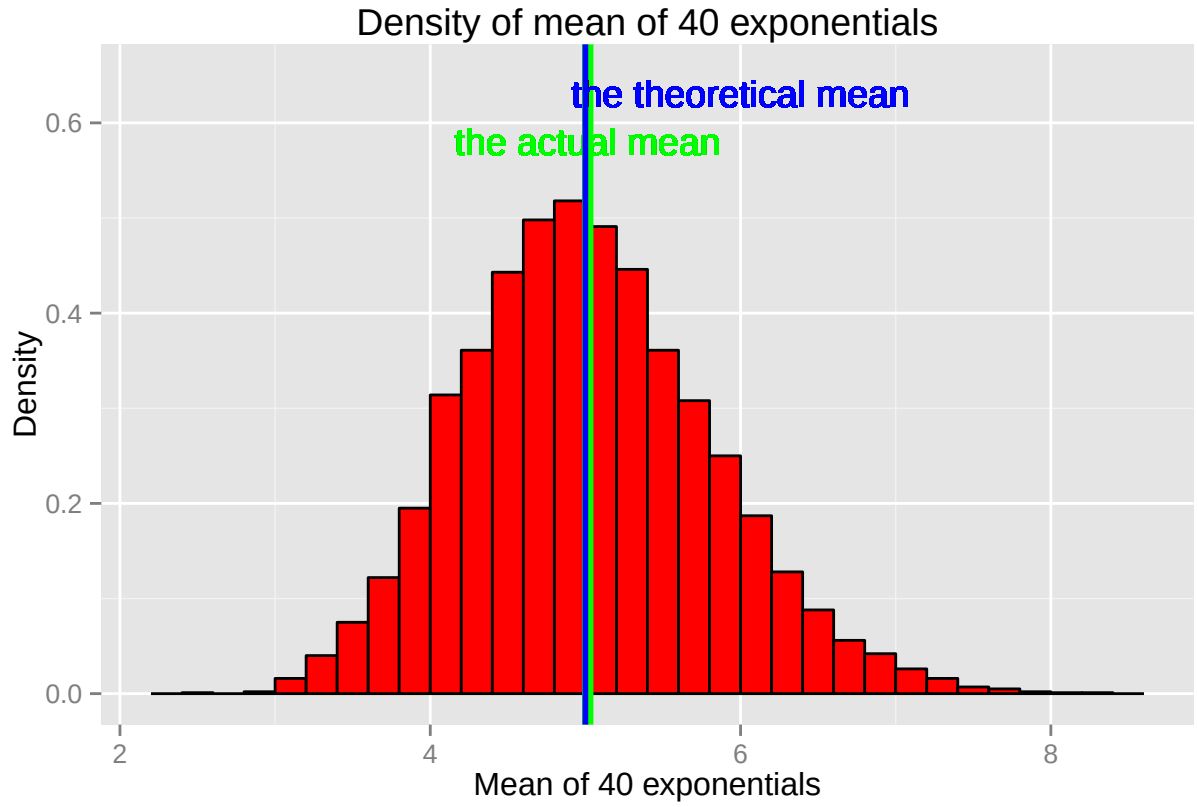
Then the actual mean:

```
actual_mean<-mean(mns)
actual_mean
```

```
## [1] 5.015109
```

Then, we show you by the figure:

```
library("ggplot2")
mns<-data.frame(mns)
plot1<-ggplot(mns,aes(x=mns))
plot1<-plot1+geom_histogram(binwidth = 0.2,fill="red",color="black",aes(y = ..density..))
plot1<-plot1 + labs(title="Density of mean of 40 exponentials", x="Mean of 40 exponentials", y="Density
plot1<-plot1+geom_vline(xintercept=actual_mean,size=2.0, color="green")
plot1<-plot1+geom_text(aes(x=actual_mean, label="the actual mean", y=0.6), colour="green", angle=0, vju
plot1<-plot1+geom_vline(xintercept=theo_mean,size=1.0, color="blue")
plot1<-plot1+geom_text(aes(x=theo_mean+1, label="the theoretical mean", y=0.65), colour="blue", angle=0
print(plot1)
```



We can see the two means coincide with each other quite well.

As for the second question, first let us calculate the actual variance:

```
theo_var<-((1/0.2) * (1/sqrt(40)))^2
theo_var
```

```
## [1] 0.625
```

```
actual_var<-var(mns)
actual_var
```

```
##           mns
## mns 0.6280624
```

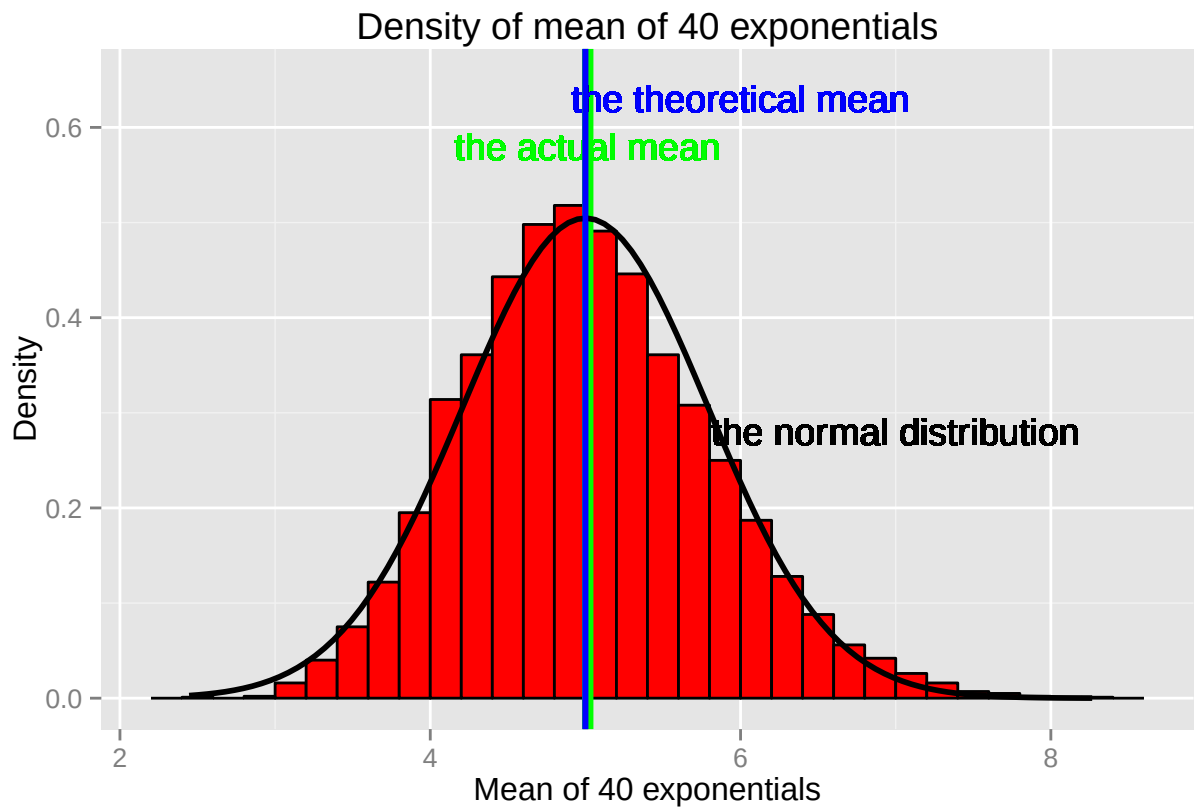
Now we can see that actual_var and theo_var match with each other. It is hard to show just this point using a singular figure. We will see it in the next figure to show that the distribution of the average converge to the normal distribution.

According to the central limit theory, the corresponding normal distribution should have a mean of theo_mean and variance of theo_var.

Hence, let us figure them out as:

```
library("ggplot2")
plot1<-plot1 + stat_function(fun=dnorm,args=list(mean=theo_mean, sd=sqrt(theo_var)),color = "black", si
```

```
plot1<-plot1+geom_vline(xintercept=theo_mean,size=1.0, color="blue")
plot1<-plot1+geom_text(aes(x=theo_mean+2, label="the normal distribution", y=0.3), colour="black", angl
print(plot1)
```



This figure shows that the two distribution is close and the variance is close too.