Ricardo L. Morales Crespo

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Dr. López

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Making Simple Decisions

1. Juan considers four used cars before buying one with maximum expected utility. Jose considers ten cars and does the same. All other things being equal, which one is more likely to have the better car? Which is more likely to be disappointed with their car's quality?
   a. Jose is more likely to have the better car due to the larger selection, while Juan is more likely to be disappointed with his car's quality due to the limited options.
2. You have the opportunity to play a game in which a fair coin is tossed repeatedly until it comes up heads. If the first heads appear on the *n*th toss, you win 2^n dollars.
   a. Show that the expected monetary value of this game is infinite.
      1. The probability of getting first heads on the nth toss is $P(n) = \left(\frac{1}{2}\right)^n$. We get $n - 1$ tails and followed by one head.
      2. If the first heads appear on the $n - th$ toss, we will win 2^n dollars.
      3. Then, the expected value of E is: $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n * 2^n$, which can be simplified to $\sum_{n=1}^{\infty} 1 = \infty$, with this we can say that the value of the game is **infinite.**
   b. How much would you, personally, pay as an entry fee to play the game?
      1. I will not pay much, since the risk is 50% of losing, maybe dollars or tens of dollars.

c. It has been suggested that the utility of money is measured on a logarithmic scale (i.e. $U(S_n) = a\log_2 n + b$, where S_n is the state of having \$$n$). What is the expected utility of the game under this assumption?

1. With the probability of winning being $P(n) = \left(\frac{1}{2}\right)^n$ we can estimate the expected utility of the game: $\sum_{n=1}^{\infty} P(n) * U(2^n) = \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n (an + b)$ which is : $a\sum_{n=1}^{\infty} n \left(\frac{1}{2}\right)^n + b\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n$. We can calculate that the second sum will be $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n = 1$ and the first one is $\sum_{n=1}^{\infty} n \left(\frac{1}{2}\right)^n = \frac{1}{\left(1-\frac{1}{2}\right)^2} = 4$. Which is $EU = a * 4 + b * 1 = 4a + b$

3. Tickets to a lottery cost \$1. There are two possible prizes: a \$10 payoff with probability 1/50, and a \$1,000,000 payoff with probability 1/2,000,000. What is the expected monetary value of a lottery ticket? When (if ever) is it rational to buy a ticket?

a. For each prize, the expected value for the \$10 dollar prize is $EV_{10} = 10 * \frac{1}{50} = \frac{10}{50} = 0.20$ for the million it is 0.50. We add these two expected values, which gives us 0.70, we subtract the price of the ticket and that will give us -0.30, therefore, the expected monetary value is -0.30.

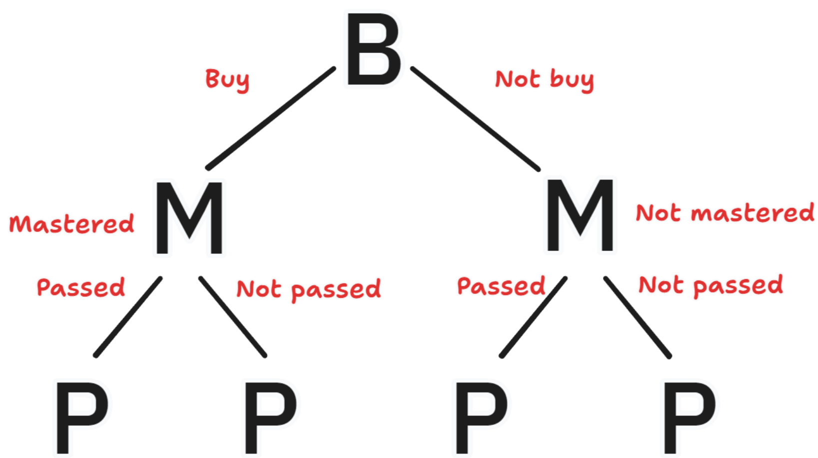
4. Consider a student who has the choice to buy or not buy a textbook for a course. We'll model this as a decision problem with one Boolean decision node, *B*, indicating whether the agent chooses to buy the book, and two Boolean chance nodes, *M*, indicating whether the student has mastered the material in the book, and *P*, indicating whether the student passes the course. Of course, there is also a utility node, *U*. A certain student, Juan, has an additive utility function: 0 for not buying the book and -\$100 for buying it; and \$2000 for passing the course and 0 for not passing. Juan's conditional probability estimates are as follows:

- $P(p|b,m) = 0.9$
- $P(p|b, \neg m) = 0.5$
- $P(p|\neg b, m) = 0.8$

- $P(p|\neg b, \neg m) = 0.3$
- $P(m|b) = 0.9$
- $P(m|\neg b) = 0.7$

You might think that *P* would be independent of *B* given *M*, but this course has an open-book final — so having the book helps.

g. Draw the decision network for this problem.



h. Compute the expected utility of buying the book and of not buying it.

If Juan masters the material (M is true)

Given that the probability of passing is 90%, we can calculate that the utility $U(B) + U(P) = -100 + 2000 = 1900$ and the contribution to the expected utility is $P(m|b) * P(p|b,m) * (U(B)) + I(P)) = 0.9 * 0.9 * 1900 = 0.81 * 1900 = 1539$

If juan does not master the material (M is False

The probability of mastering is 0.5, and its utility contribution is $P(m \mid b) \cdot P(p \mid b, \neg m) \cdot (U(B) + U(P)) = 0.9 \cdot 0.1 \cdot 1900 = 0.09 \cdot 1900 = 171$

i. What should Juan do?

Juan should buy the book since it will make it more likely that he will pass the class final.