Résoudre un problème inverse combinant mélange de bruit et modèle direct boîte noire avec des méthodes d'échantillonnage - ANR RICOCHET -

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financed by CNRS via 80|Prime

Outline

Outline

~~Mostly empty?~~

Mostly Interstellar Medium!

Observations of GMC: Orion B in visible frequencies

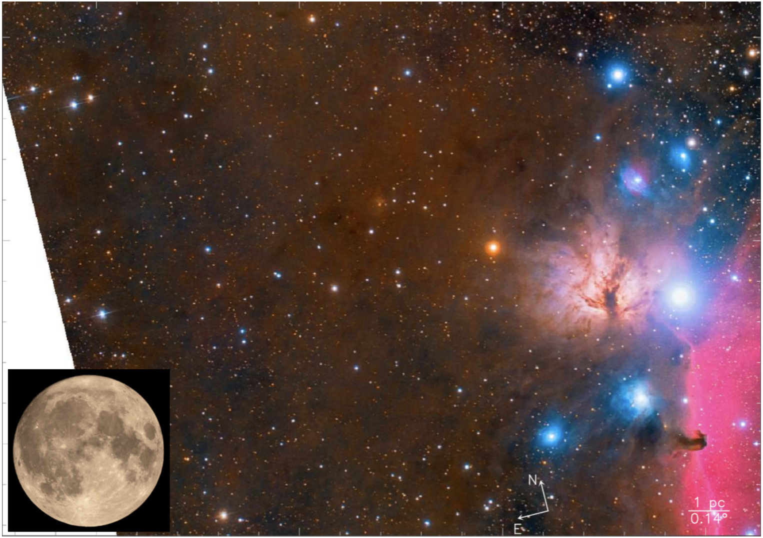

Figure: Image from Pety et al. [2016]

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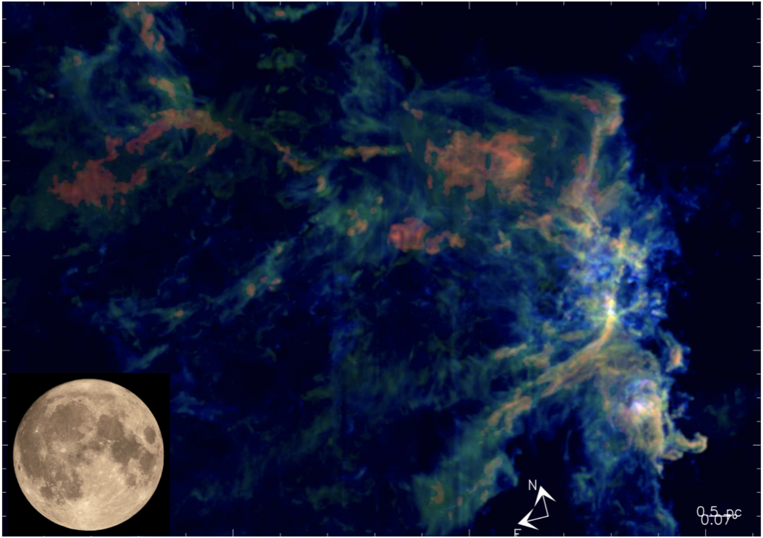


Figure: Image from Pety et al. [2016]
blue: ^{12}CO, green: ^{13}CO, red: $C^{18}O$

Photo-Dissociation Region (PDR)

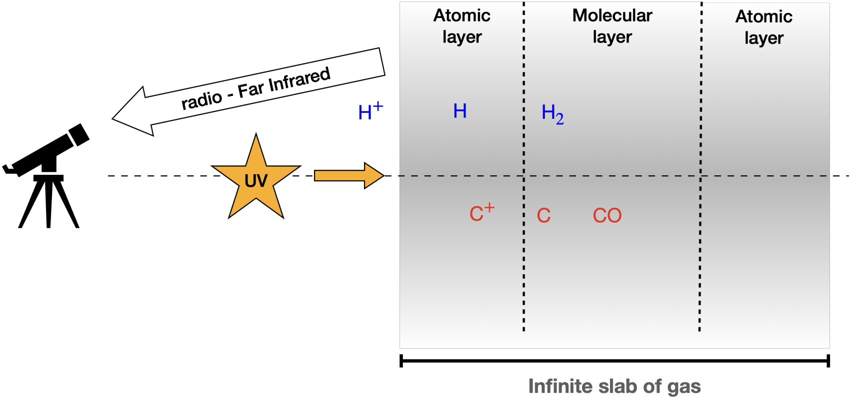


Figure: Structure of a PDR

Photo-Dissociation Region (PDR)

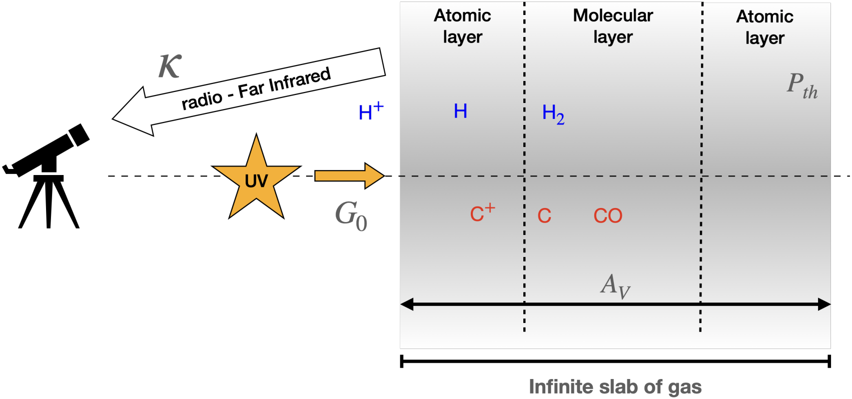
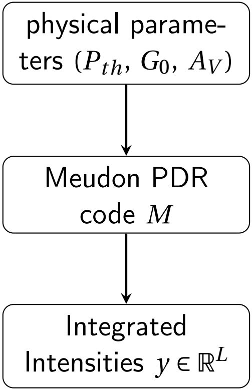


Figure: Structure of a PDR

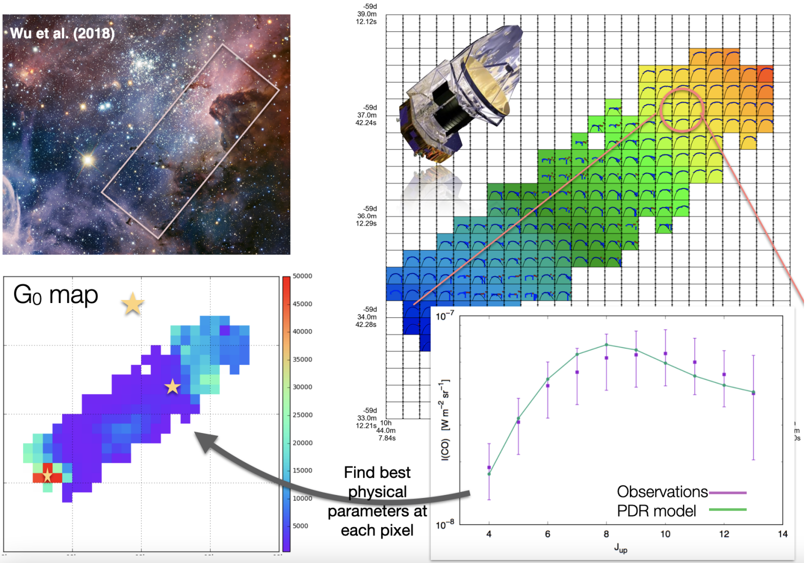
- Introduced in Le Petit et al. [2006].
- for stationary 1D slab of gas, solves:
  1. radiative transfer
  2. chemistry
  3. thermal balance

  that are all **coupled**!



Can we infer (P_{th}, G_0, A_V) from y and M?
With **no ground truth** → with **credibility intervals**

Current state of the art in astrophysics



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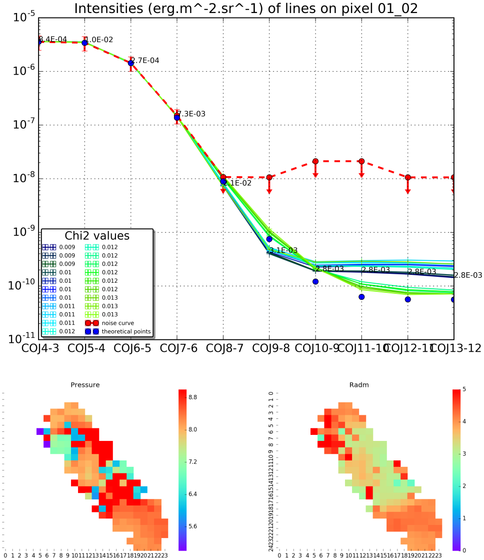

Figure: Images from Nicolas Chabalier internship report

Outline

Bayesian map inversion

inference with credibility interval

$$\Downarrow$$

a posteriori probability distribution $\mathbb{P}[(\kappa, P_{th}, G_0, A_V) \mid y]$

$$\underbrace{\mathbb{P}[(\kappa, P_{th}, G_0, A_V) \mid y]}_{\text{a posteriori}} \propto \underbrace{\mathbb{P}[y \mid (\kappa, P_{th}, G_0, A_V)]}_{\text{likelihood}} \times \underbrace{\mathbb{P}[(\kappa, P_{th}, G_0, A_V)]}_{\text{a priori}}$$

Complex distribution

$\implies$ impossible to manipulate as is

$\implies$ sampling with MCMC

Observation model

Observation: $\mathbf{y} = (y_{n,\ell})$
Parameters: $X = (x_n)$, with $x_n = (\kappa, P_{th}, G_0, A_V)$
Forward Model : $\forall \ell, \quad f_\ell : x \in \mathbb{R}^D \mapsto \kappa M_\ell(P_{th}, G_0, A_V) \in \mathbb{R}_+^*$

Observation Model:

$$\forall n, \ell,\ y_{n,\ell} = \max\left\{\omega,\ \epsilon^{(m)}_{n,\ell} f_\ell(x_n) + \epsilon^{(a)}_{n,\ell}\right\}$$

with

- M_ℓ : Meudon PDR code ⚠non explicit ⚠
- κ_n : scaling factor (includes dilution factor, etc.)
- $\epsilon^{(a)}_{n,\ell}$: additive noise (thermal, instruments)
- $\epsilon^{(m)}_{n,\ell}$: multiplicative noise (calibration error)
- ω : minimum detectable value by telescope

How to deal with **non explicit** forward map f ?
How to deal with **both** additive and multiplicative noises?

Approximation of the Meudon PDR code

Forward map f :

1. long to evaluate (~2h)
2. non-explicit $\implies$ non accessible gradients

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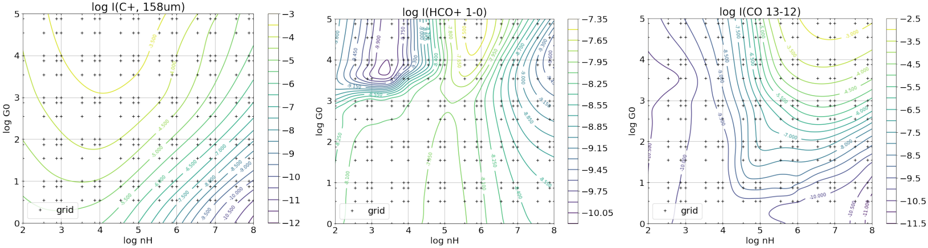

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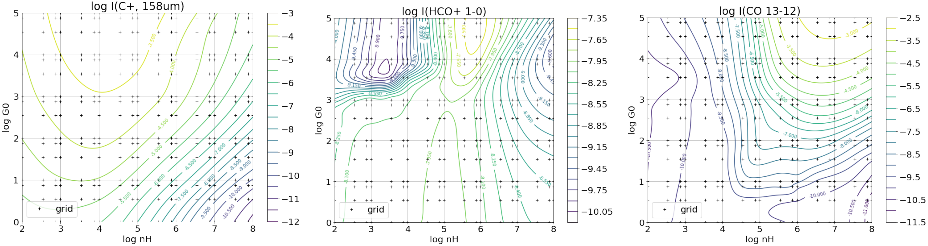

model reduction : polynomial regression in the log space on some set $\mathscr{C}$

$$\forall \ell, \quad f_\ell(x) = \exp\left[\log \kappa + P_\ell(P_{th},\, G_0,\, A_V)\right]$$

Observation Model

$$u_{n,\ell} = \epsilon^{(m)}_{n,\ell} f_\ell(x_n) + \epsilon^{(a)}_{n,\ell}$$

Additive approximation

$$u_{n,\ell} \simeq f_\ell(x_n) + e^{(a)}_{n,\ell}$$

$$e^{(a)}_{n,\ell} \sim \mathcal{N}(m_{a,n,\ell},\, s^2_{a,n,\ell})$$

Multiplicative approximation

$$u_{n,\ell} \simeq e^{(m)}_{n,\ell} f_\ell(x_n)$$

$$e^{(m)}_{n,\ell} \sim \log\mathcal{N}(m_{m,n,\ell},\, s^2_{m,n,\ell})$$

Final Likelihood

$$\forall n, \ell,\ y_{n,\ell} = \begin{cases} \omega & \text{if } \epsilon^{(m)}_{n,\ell} f_\ell(x_n) + \epsilon^{(a)}_{n,\ell} \le \omega \quad \text{(censored)} \\ \epsilon^{(m)}_{n,\ell} f_\ell(x_n) + \epsilon^{(a)}_{n,\ell} & \text{else} \quad \text{(uncensored)} \end{cases}$$

$$\Downarrow$$

$$\pi_{\text{mix}}(y_{n,\ell}|x_n) = \left\{ \pi^{(c)}_a (y_{n,\ell}|x_n)^{\lambda_{n,\ell}}\ \pi^{(c)}_m (y_{n,\ell}|x_n)^{1-\lambda_{n,\ell}} \right\}^{[y_{n,\ell}=\omega]} \times \left\{ \pi^{(u)}_a (y_{n,\ell}|x_n)^{\lambda_{n,\ell}}\ \pi^{(u)}_m (y_{n,\ell}|x_n)^{1-\lambda_{n,\ell}} \right\}^{[y_{n,\ell}>\omega]}$$

- $y_{n,\ell} = \omega$: if true, censored data
- $\pi^{(c)}_a$, $\pi^{(u)}_a$: additive approximation
- $\pi^{(c)}_m$, $\pi^{(u)}_m$: multiplicative approximation
- $\lambda_{n,\ell}$: **controls the mixing of the two approximation models**

Choice of mixing parameter λ

How to define λ ? We have that

- When $\sigma_m \to 0$, observation model $\rightsquigarrow$ additive approx
- When $f_\ell(x) \gg \sigma_a$, observation model $\rightsquigarrow$ multiplicative approx

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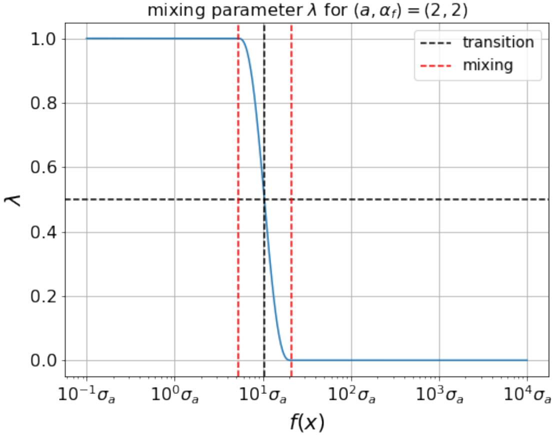
$\Longrightarrow$ λ parameteric function :



How relevant is our approximation?

$$\forall \ell,\ D^{(\ell)}_{\text{K-S}}\left(\pi, \tilde{\pi}_{(a,\alpha_f)}\right) : x \mapsto \sup_{y \in \mathbb{R}} \left| F^{(\ell)}(y|x) - \tilde{F}^{(\ell)}_{(a,\alpha_f)}(y|x) \right|$$

Assuming $X \sim \text{Unif}(\mathscr{C})$:

$$(a, \hat{\alpha}_f) \in \underset{a>0, \alpha_f>1}{\arg\min} \frac{1}{L} \sum_{\ell=1}^{L} \mathbb{E}\left[D^{(\ell)}_{\text{K-S}}\left(\pi, \tilde{\pi}_{(a,\alpha_f)}\right)(X) \right]$$

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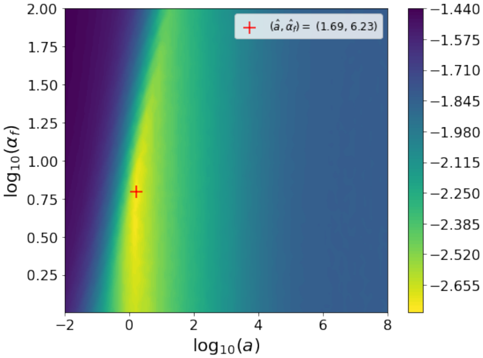

Including a priori information

a priori information on $(\kappa, P_{th}, G_0, A_V)$:

- spatial regularization with unknown parameter (Pereyra et al. [2015])

- f_ℓ: estimated from a grid on $\mathscr{C}$
  → **constraint of belonging to a cube**
  ⚠non smooth prior ⚠but can be tempered with a smooth penalty

Sampler

smooth prior + **smooth** likelihood $\implies$ **smooth** posterior
classic MCMC algorithm (e.g., MALA) OK

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Other difficulties :

- f and ∇f cover **various decades**
  $\implies$ classic methods **inefficient**
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Other difficulties :

- f and ∇f cover **various decades**
  $\implies$ classic methods **inefficient**
  $\implies$ Preconditioned MALA kernel with RMSProp preconditioner
- **non-convex** negative log-posterior (because of Meudon PDR code)
  $\implies$ need to avoid being trapped in **local minima**
  $\implies$ Multiple-Try Metropolis (MTM) kernel

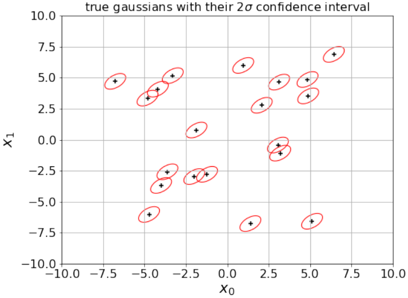
Final sampler : combination of these two kernels

Outline

Mixture of gaussians in a square

Illustration that our algorithm explores interesting **local minima** :

- mixture of 20 gaussians
- with constraint $X \in \mathscr{C} = [-10, 10] \times [-10, 10]$



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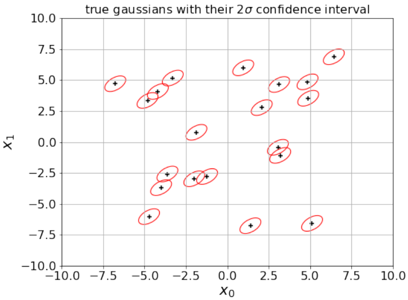
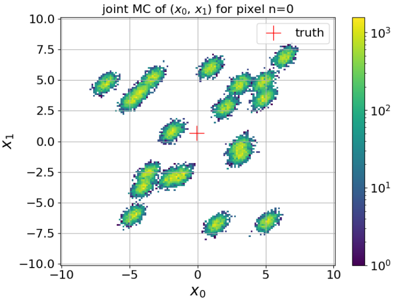
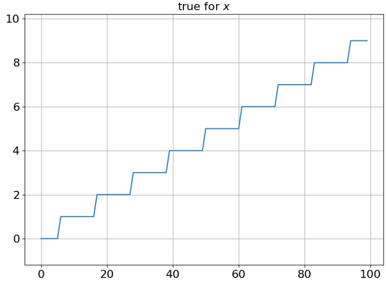
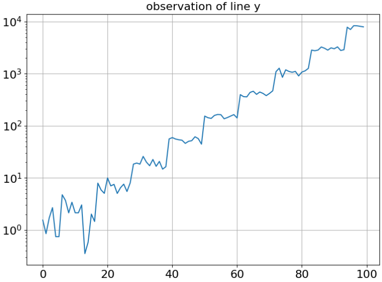




Toy case : Time Series Inversion

$$y_{n,\ell} = \epsilon^{(m)}_{n,\ell} f(x_n) + \epsilon^{(a)}_{n,\ell} \text{ with } f : x \in \mathbb{R} \mapsto e^x,\ \sigma_a = 1,\ \sigma_m \sim 10\%$$





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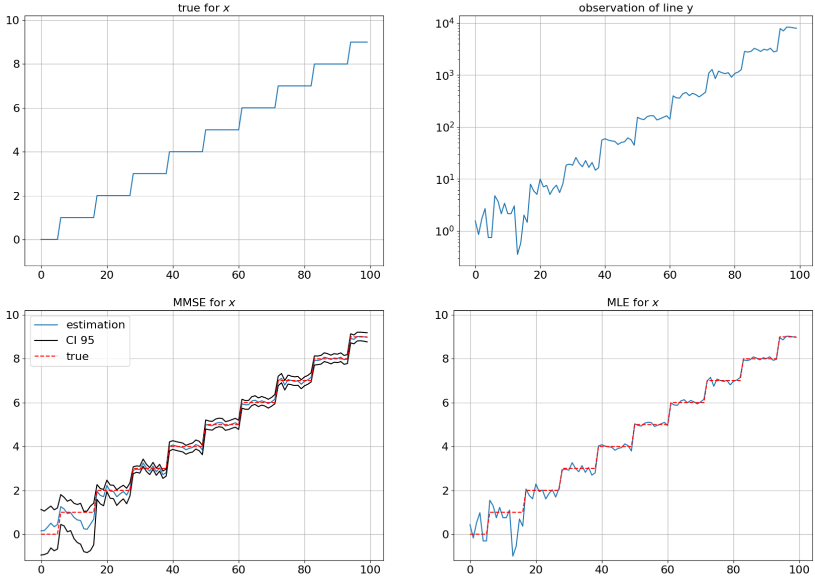

Toy case 1: Time Series Inversion

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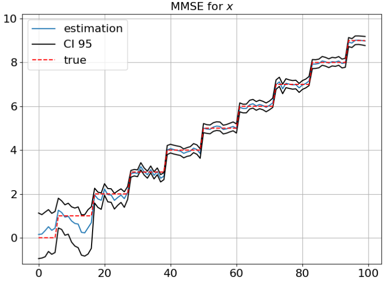


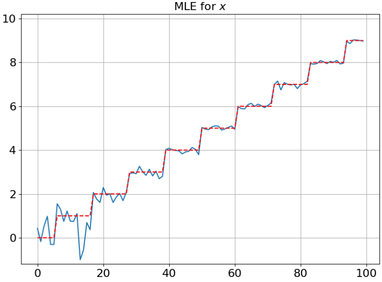
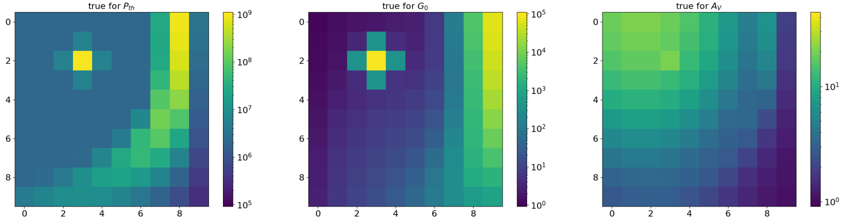


Table: Estimation Summary

estimator	MSE	SNR
MMSE	3.6	28.8
MLE	10.5	24.1

Astrophysical Toy case: Map Inversion

$$y_{n,\ell} = \max\left\{\omega,\ \epsilon_{n,\ell}^{(m)} f_\ell(x_n) + \epsilon_{n,\ell}^{(a)}\right\}$$



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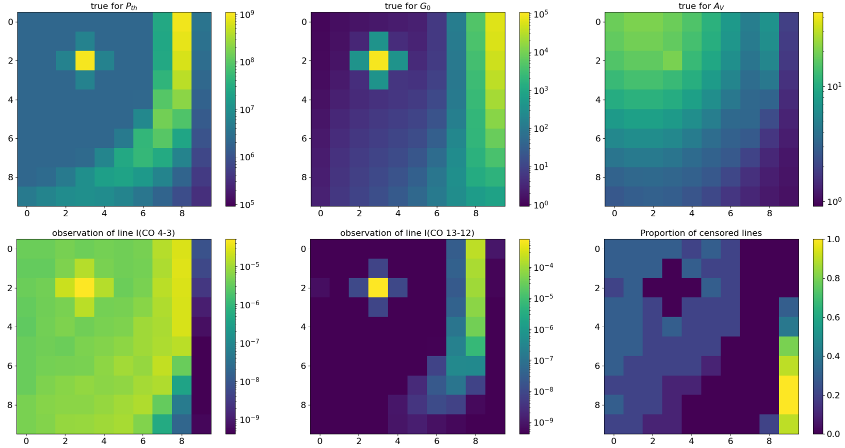

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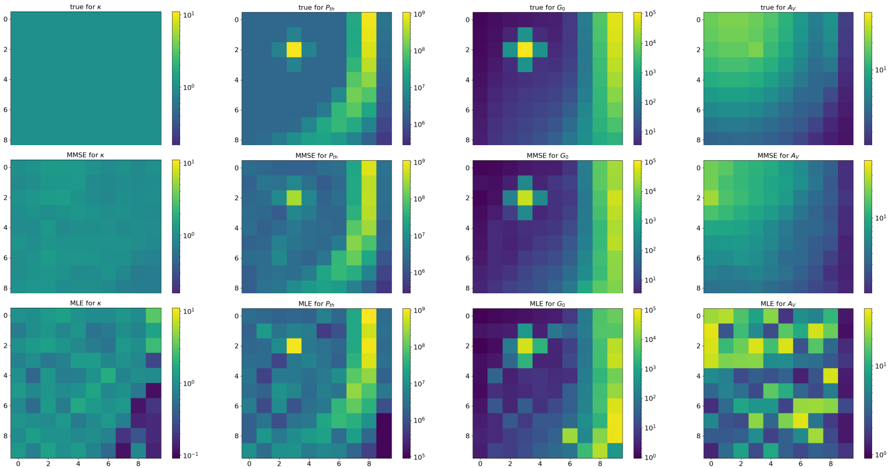
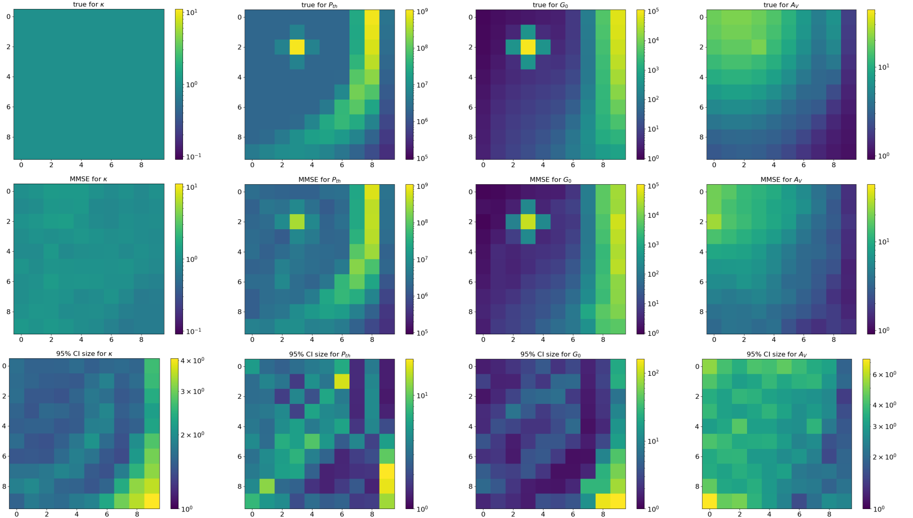


estimator	MSE	SNR
MMSE	13.9	14.6
MLE	146.2	4.4

Astrophysical Toy case: Map Inversion



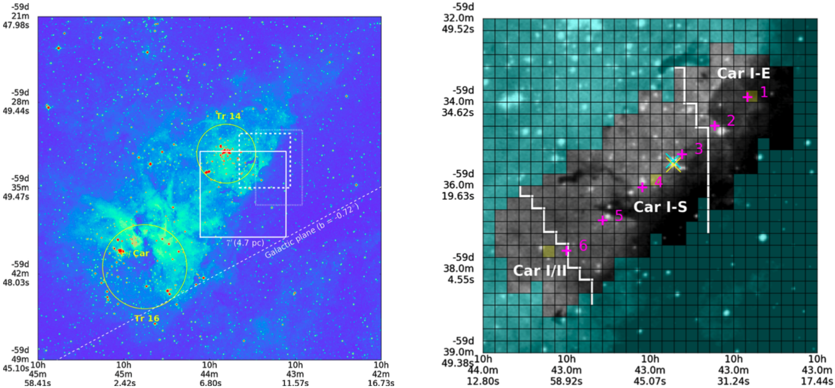


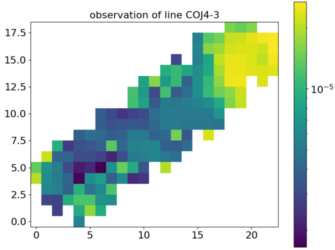
Figure: images from Wu et al. [2018]

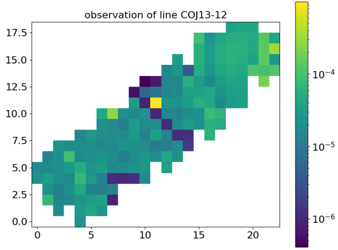
Application to Carina cloud (Car-I)

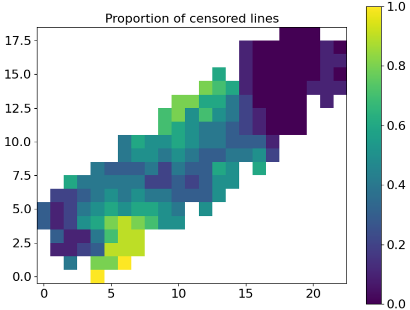
1. one of the most studied PDR
2. $\sim$2kpc ($\sim 5\times$ further to us than Orion B)
3. close to Trumpler-14 star cluster of O and B stars
4. previous estimations of G_0 :
   - $\simeq 10^4$ (Brook et al., 2003) (estimation from stellar composition)
   - 1390 (Oberst et al., 2011) (estimation from PDR model)
   - 3200 (Kramer et al., 2011) (estimation from PDR model)

Note : There is apparently an inconsistency between estimations of G_0

Application to Carina cloud (Car-I)







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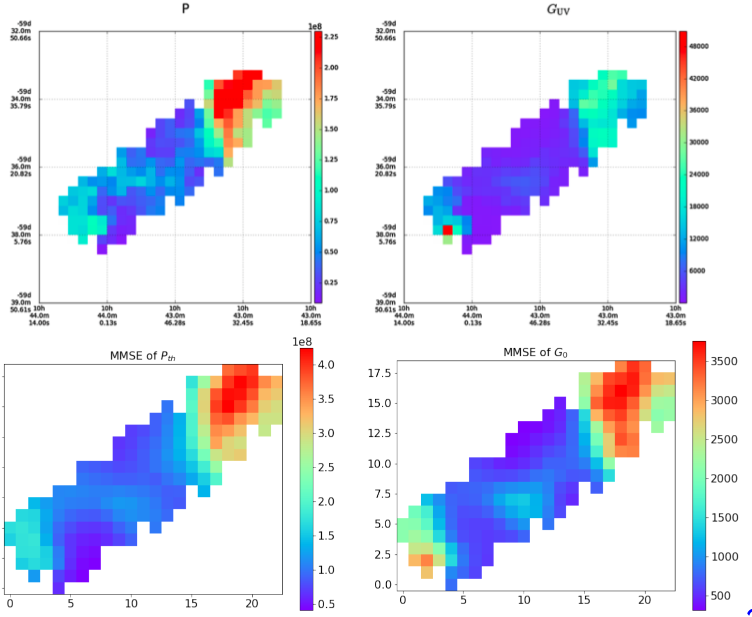
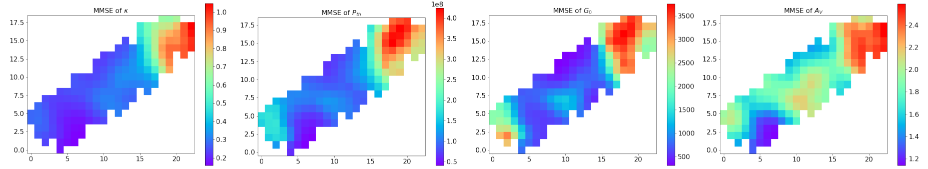


Application to Carina cloud (Car-I)

Figure: MMSE estimation



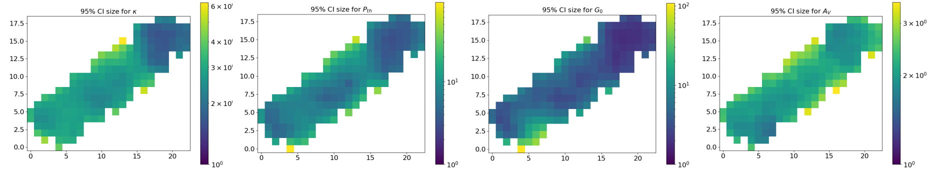


Figure: size of 95% confidence interval $u_{97.5\%}/u_{2.5\%}$

Outline

Summary

- Definition of a likelihood that handles mixture of noises
- Definition of a method of physical parameters inference with
  1. Model reduction to handle « black box » forward model
  2. P-MALA kernel to tackle regularity issues
  3. MTM kernel to tackle the non-log-concavity of the posterior
- Evaluation of the method on toy data
- Application to real world data

Thank you for your attention!

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