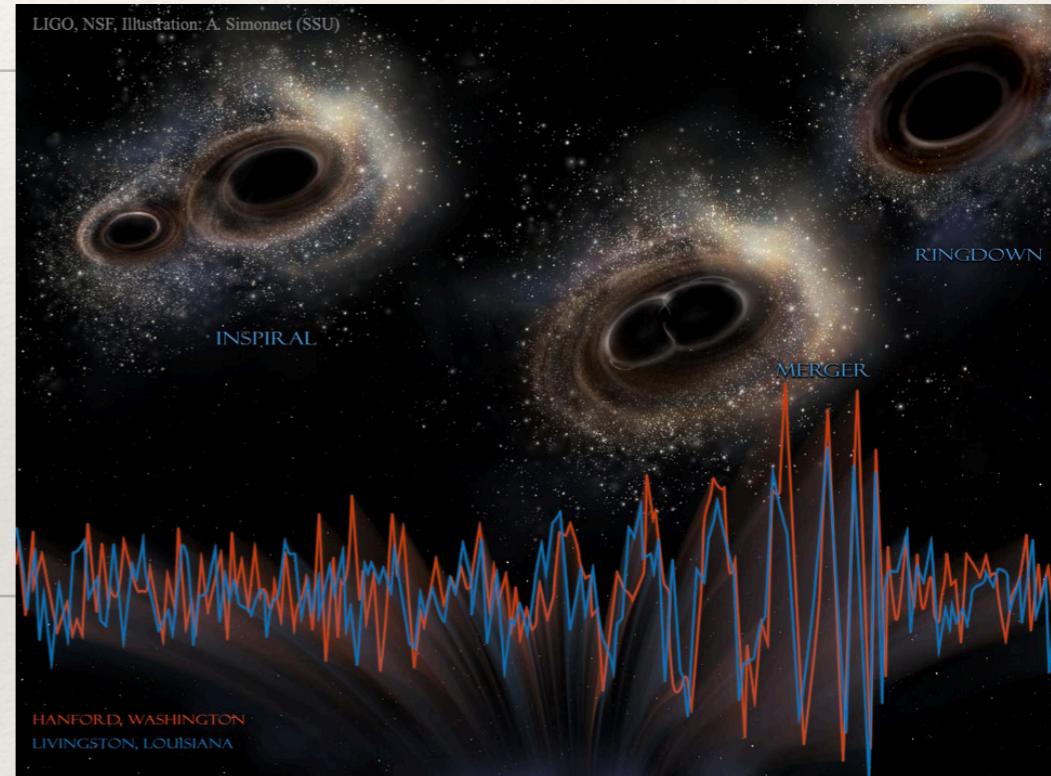


*Stanislav (Stas) Babak.*  
*AstroParticule et Cosmologie, CNRS (Paris)*

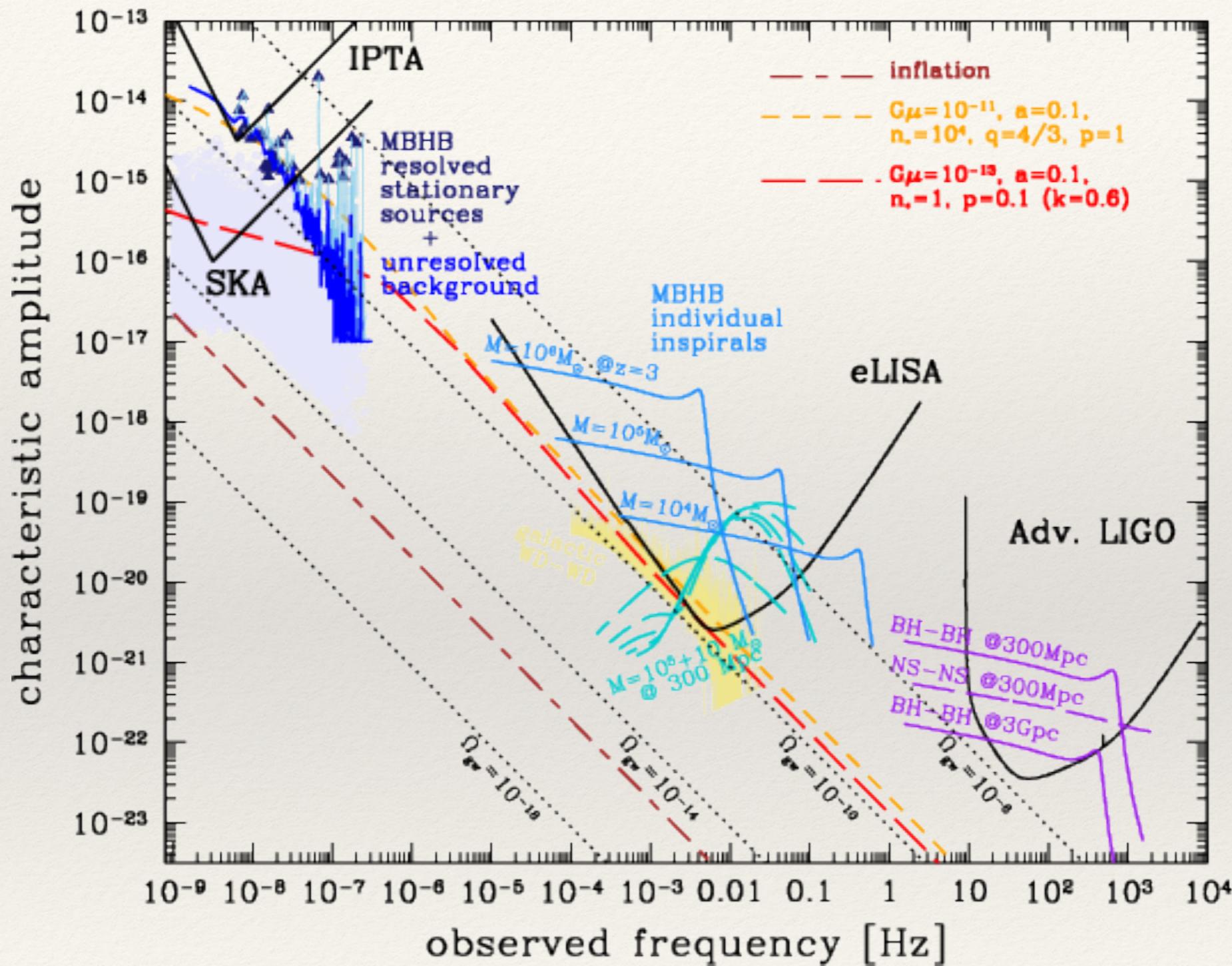


# LISA: data analysis challenges



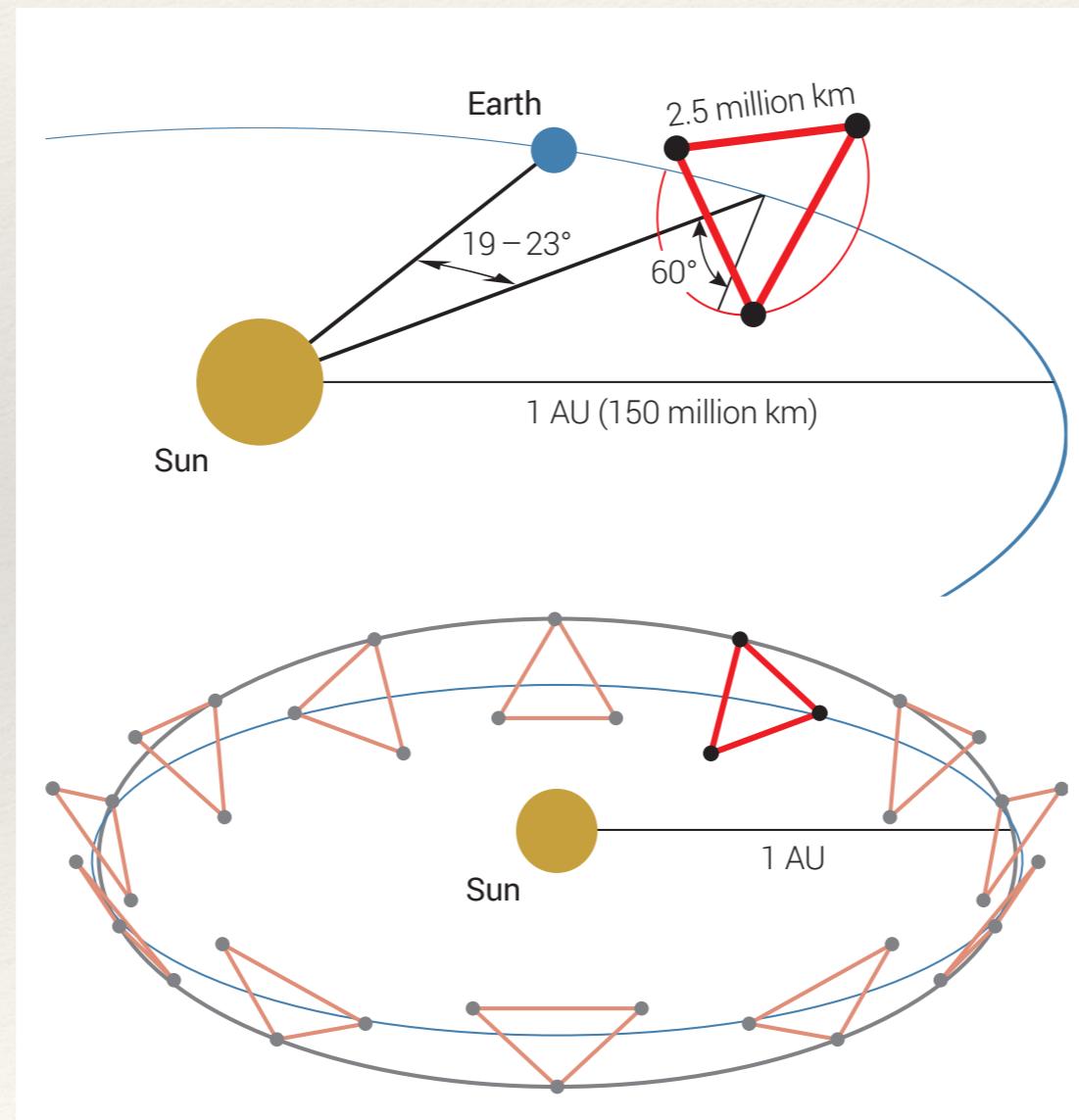
Grenoble, 23-25 March 2022

# GW landscape



# LISA: Laser Interferometric Space Antenna

- LISA: GW observatory in space: **The launch date 2034**. Leading by European Space Agency.
- LISAPathfinder - Technological mission to prove the technical readiness of LISA - fantastic results, order of magnitude better than minimum requirement

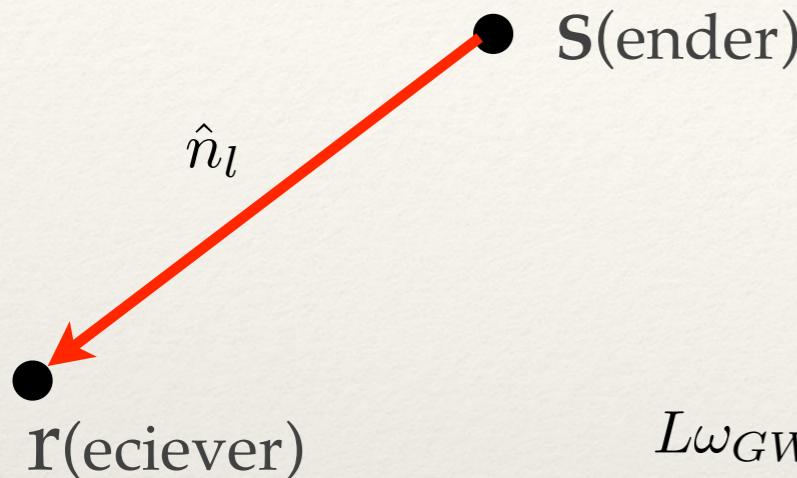


# LISA (cartoon) - LISA as it was before 2010



# LISA

## Principle of measurement



LISA: three satellites in free falling orbits around the sun, constellation forms equilateral triangle

$L_1 \sim L_2 \sim L_3 = 2.5 \text{ mln. km.}$

Operates in freq. range  $0.1 \text{ mHz} - 0.1 \text{ Hz}$ .

Exchange laser light - measurement of the proper distance between satellites .

$$L\omega_{GW} = 1 \rightarrow f_{GW} \approx 20 \text{ mHz} \quad \text{Long wavelength is not applicable}$$

We cannot cover the detector by LIF, use “TT” frame: in this frame GW can be seen as affecting the phase (or frequency) of the laser light.

Change in laser freq.  
due to GWs

unperturbed freq.  
of a laser

$$\frac{\Delta\nu}{\nu_0} = \frac{n_l^i n_l^j \Delta h_{ij}}{2(1 - \hat{k} \cdot \hat{n}_l)}$$

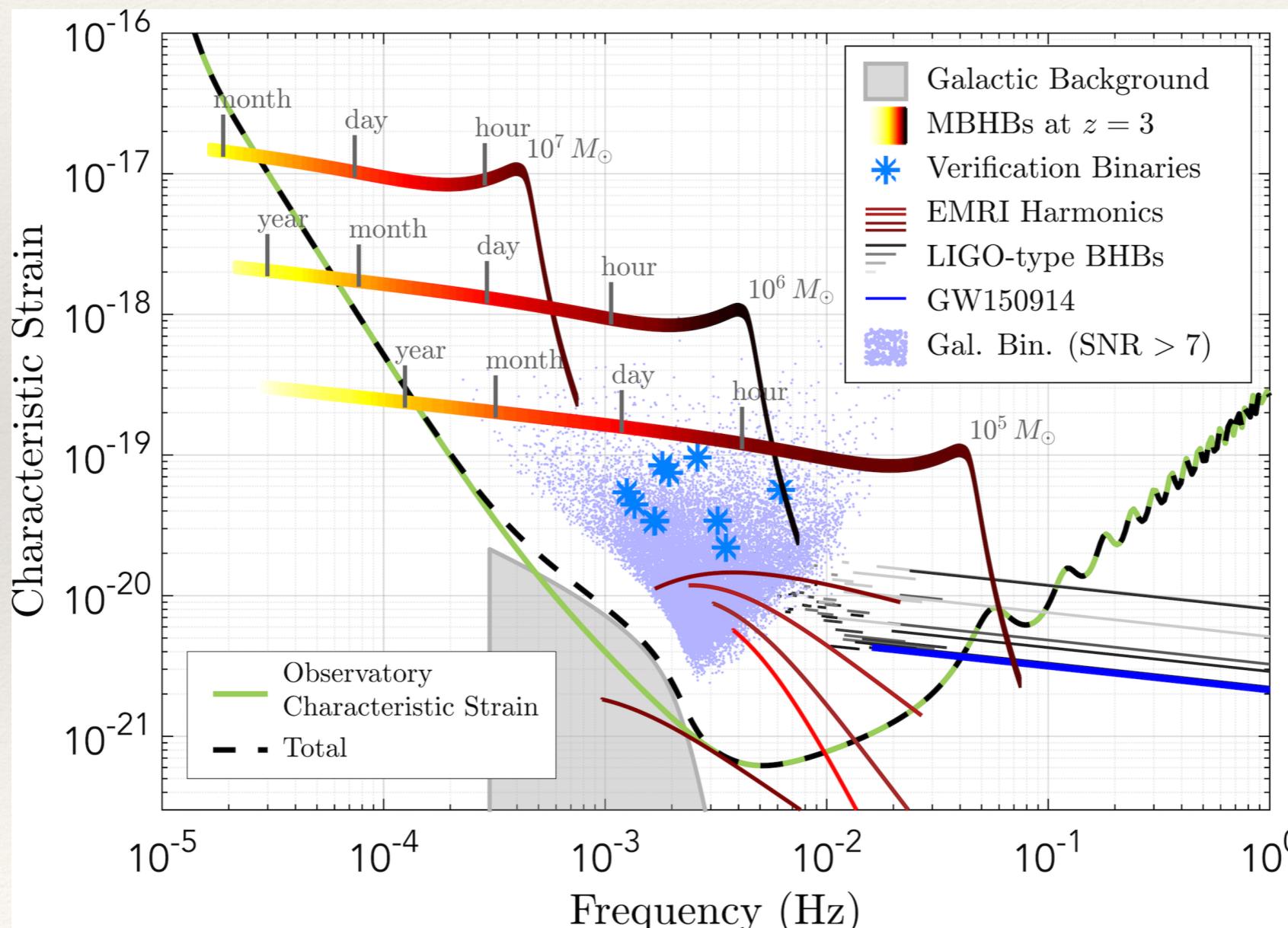
$\Delta h^{ij} = h^{ij}(t_s) - h^{ij}(t_r)$

direction of GW  
propagation

$$t = t_r, \quad t_s = t - |\vec{R}_r(t) - \vec{R}_s(t_s)| \approx t - |\vec{R}_r(t) - \vec{R}_s(t)| \approx L_l$$

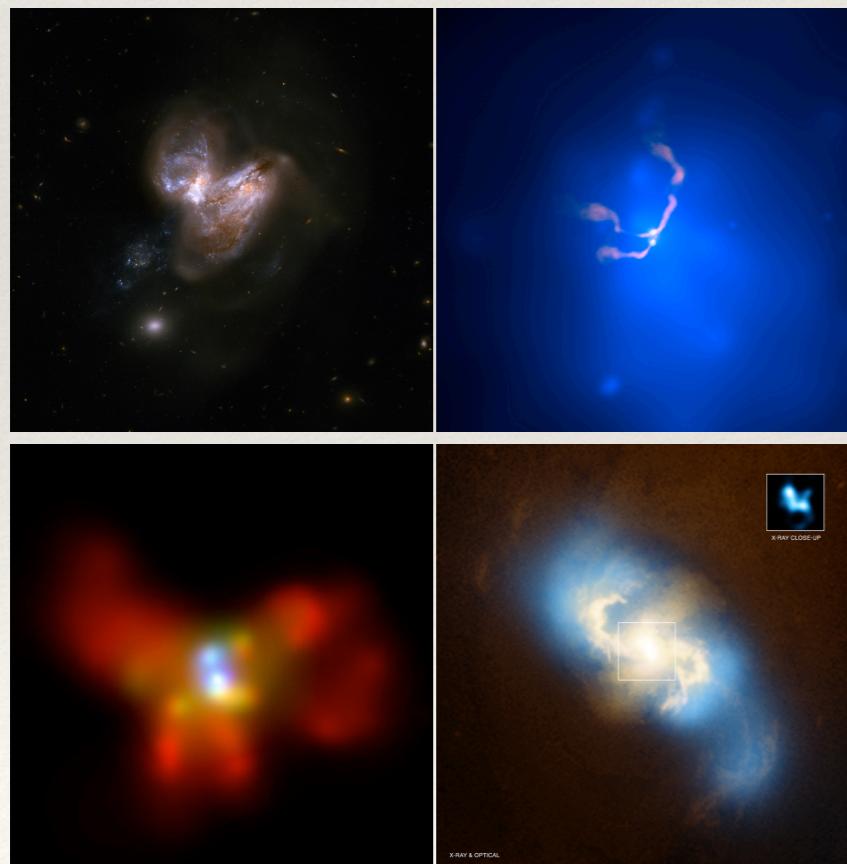
# GW sources in LISA band

- GW signals in LISA are strong and long-lived.
- LISA data will contain thousands of GW signals simultaneously: need to separate and characterize them
- Non-stationary noise



# LISA sources

- We believe that all galactic nuclei host Massive Black Holes: Milky Way has 4 mln. solar mass BH
- Galaxies merge: we can form Massive Black Hole Binary (MBHB) system
- We need stars and gas to bring MBHs close together for GW to be efficient (binary is merging within Hubble time)

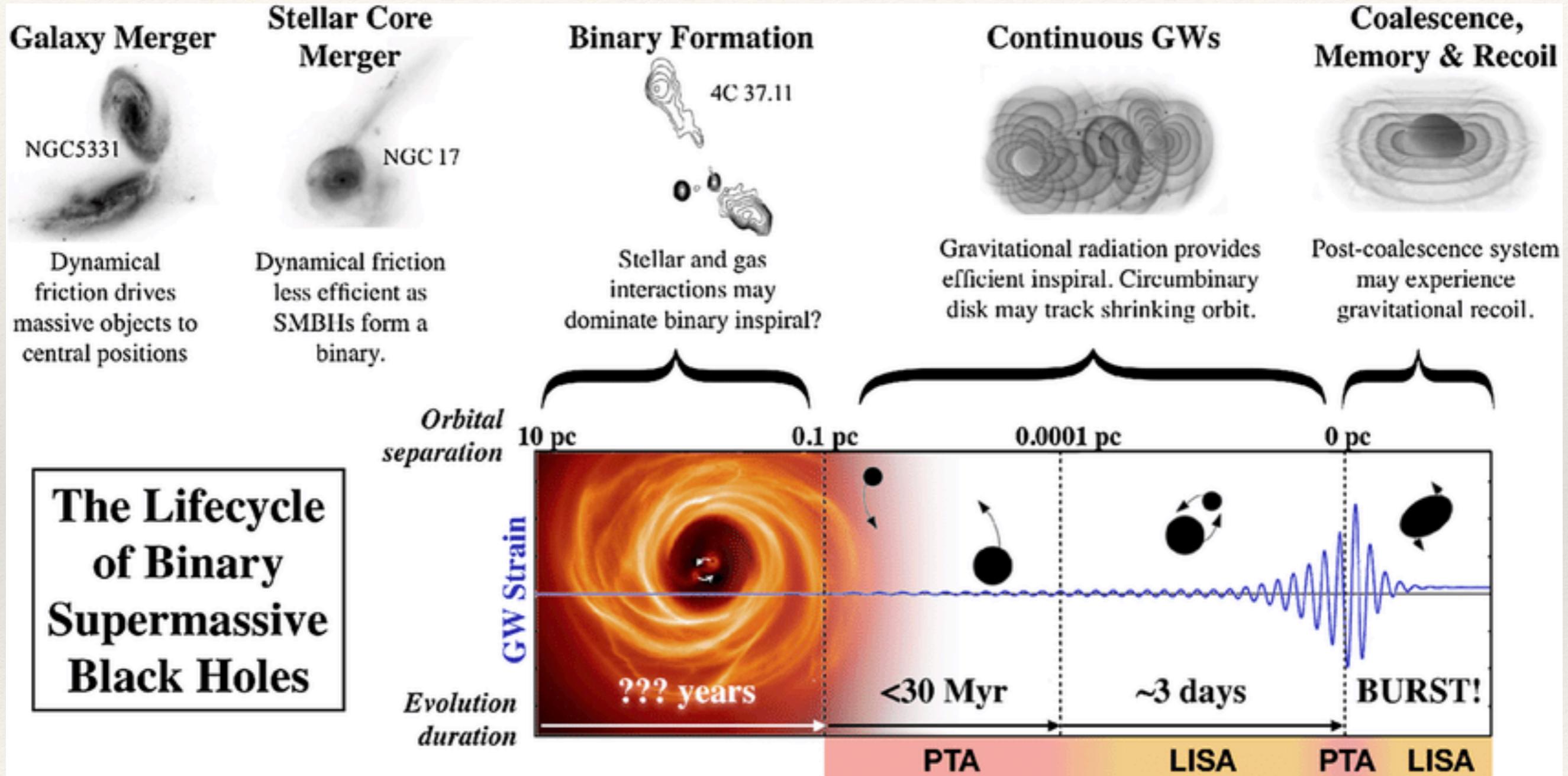


[Credits: Hassinger+, VLA, Chandra , NASA]



[Image: Hubble telescope]

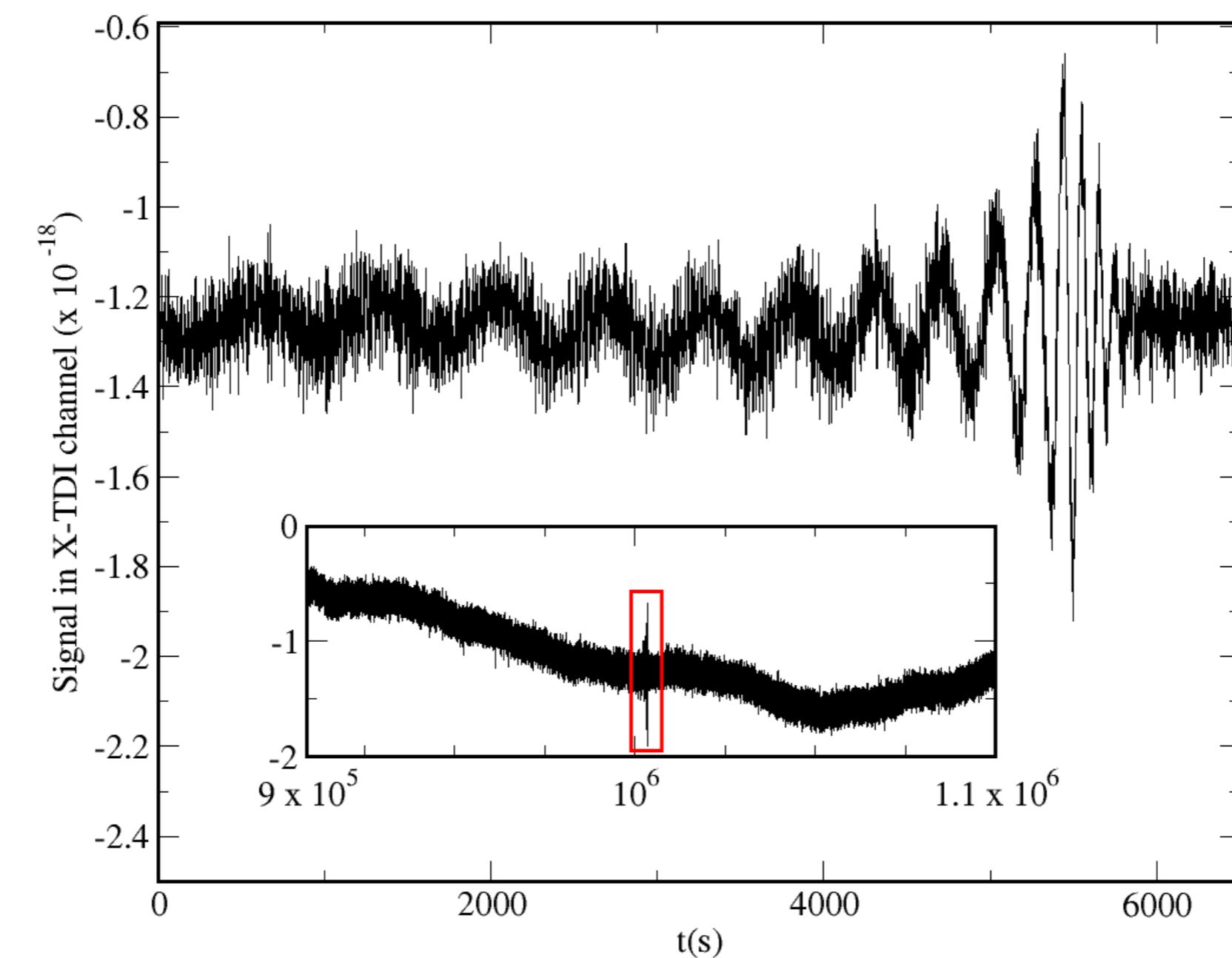
# Massive black hole binaries



[S. Burke-Spolaor A&A review (2019)]

# LISA: GW signal from MBHB

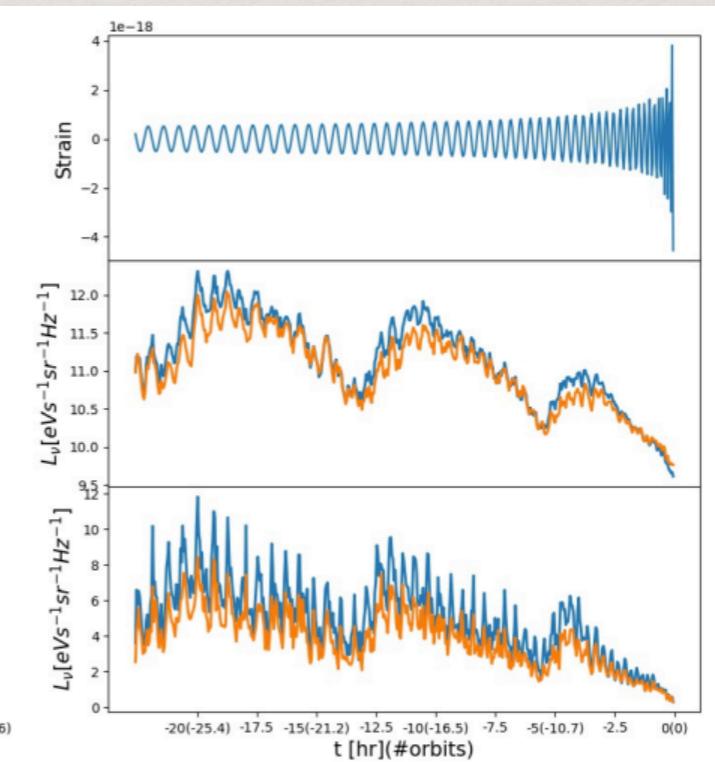
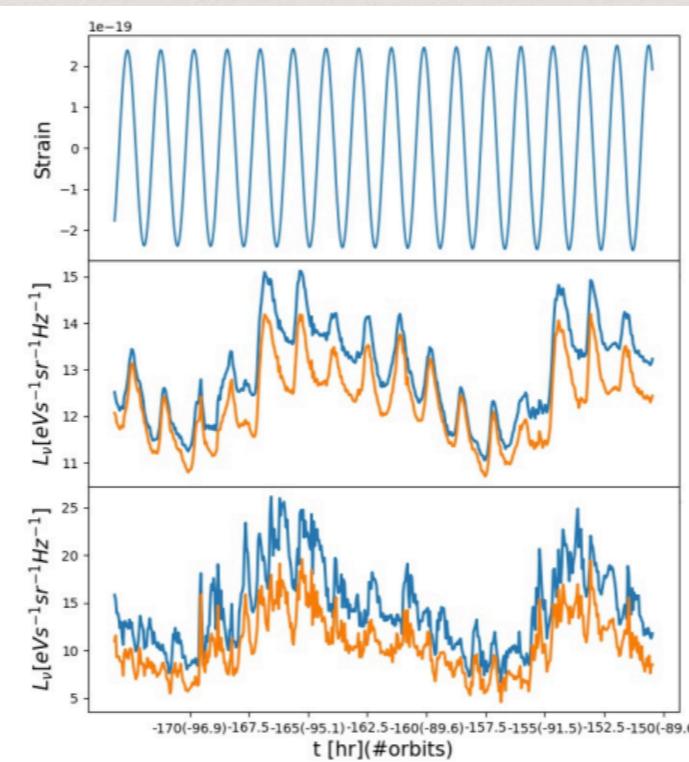
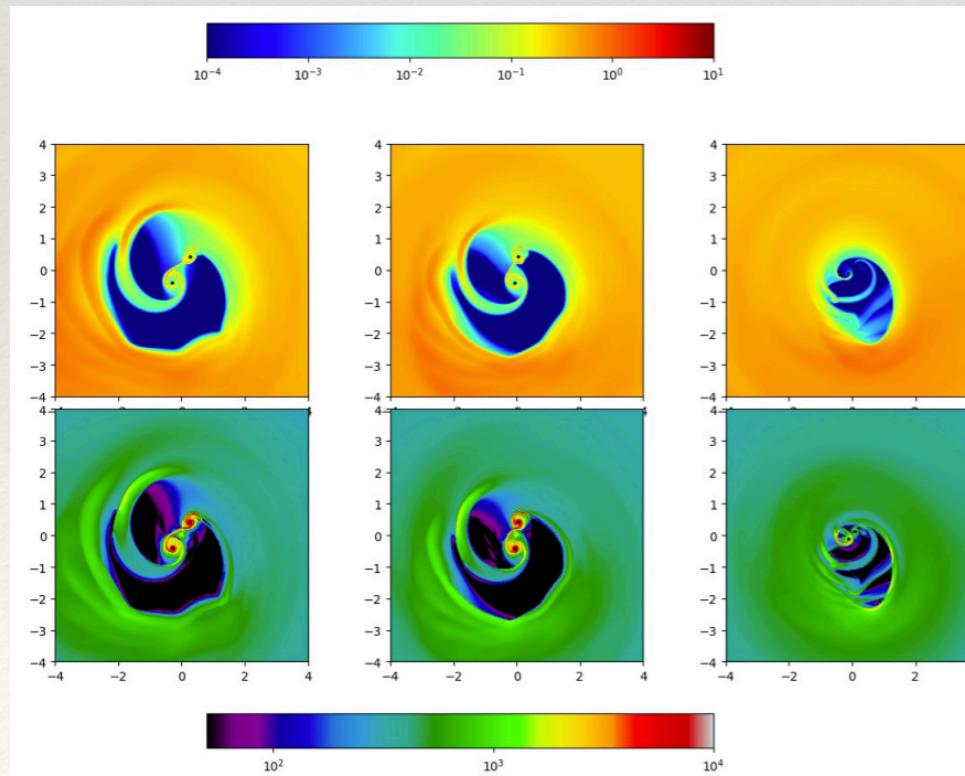
The signal from MBHB is similar to what we have observed in LIGO (scaled up in the amplitude and stretched in time). GW signal from MBHB is expected to be the strongest signal (seen by eye in the simulated data). Imposes stringent demands on the accuracy of GW signal modelling



# Pre-merger e/m signal

X-ray emission during the late stages of the inspiral (days to hours before final merger) comes from:

- Circumbinary disc:
  - X-ray emission in soft x-rays ( $\leq 1\text{keV}$ )
- Mini-discs around black holes
  - Hard x-ray emission ( $\geq 10\text{keV}$ ) from accretion of minidiscs individually onto each black hole
- Interaction of circumbinary and mini discs:
  - Accretion of circumbinary disc onto mini-discs via optically thick streams
  - Thermal radiation dominated by the inner edge of the circumbinary disc, producing soft x-rays ( $\sim 2\text{keV}$ )
- X-ray emission shows clear modulation on timescales as short as a few hours



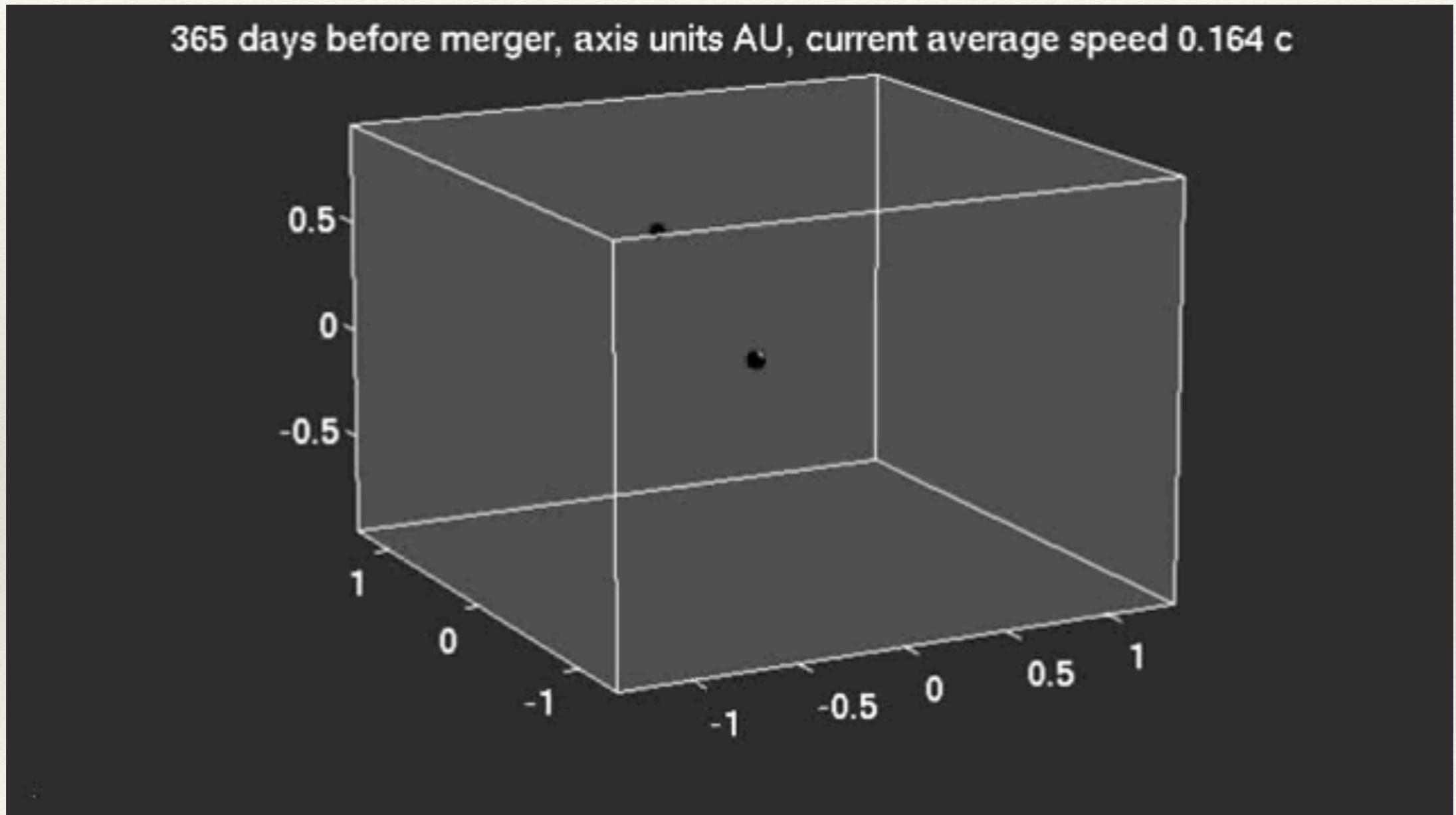
# EMRIs (extreme mass ratio inspirals)

---

- Massive BHs in galactic nuclei surrounded by stars and gas with quite high density
- MBH could capture a compact object (BH, NS, WD) which is thrown on a very eccentric orbit (due to N-body interaction). The orbit shrinks and circularizes due to grav. radiation.
- EMRI: binary system with extreme mass ratio of component  $10^{-7}$  -  $10^{-5}$
- Compact object revolves  $10^6$  orbits in the proximity of MBH before the plunge.



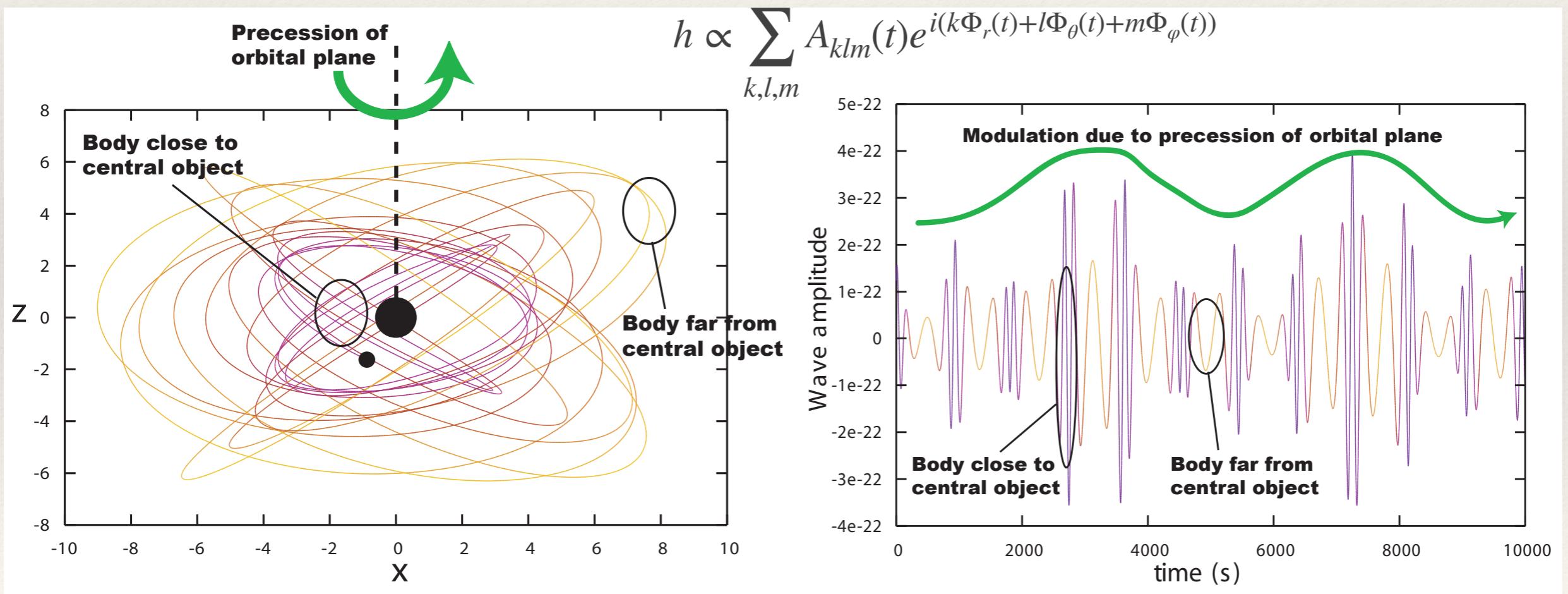
# EMRI



[Credits: S Draco, CalTech]

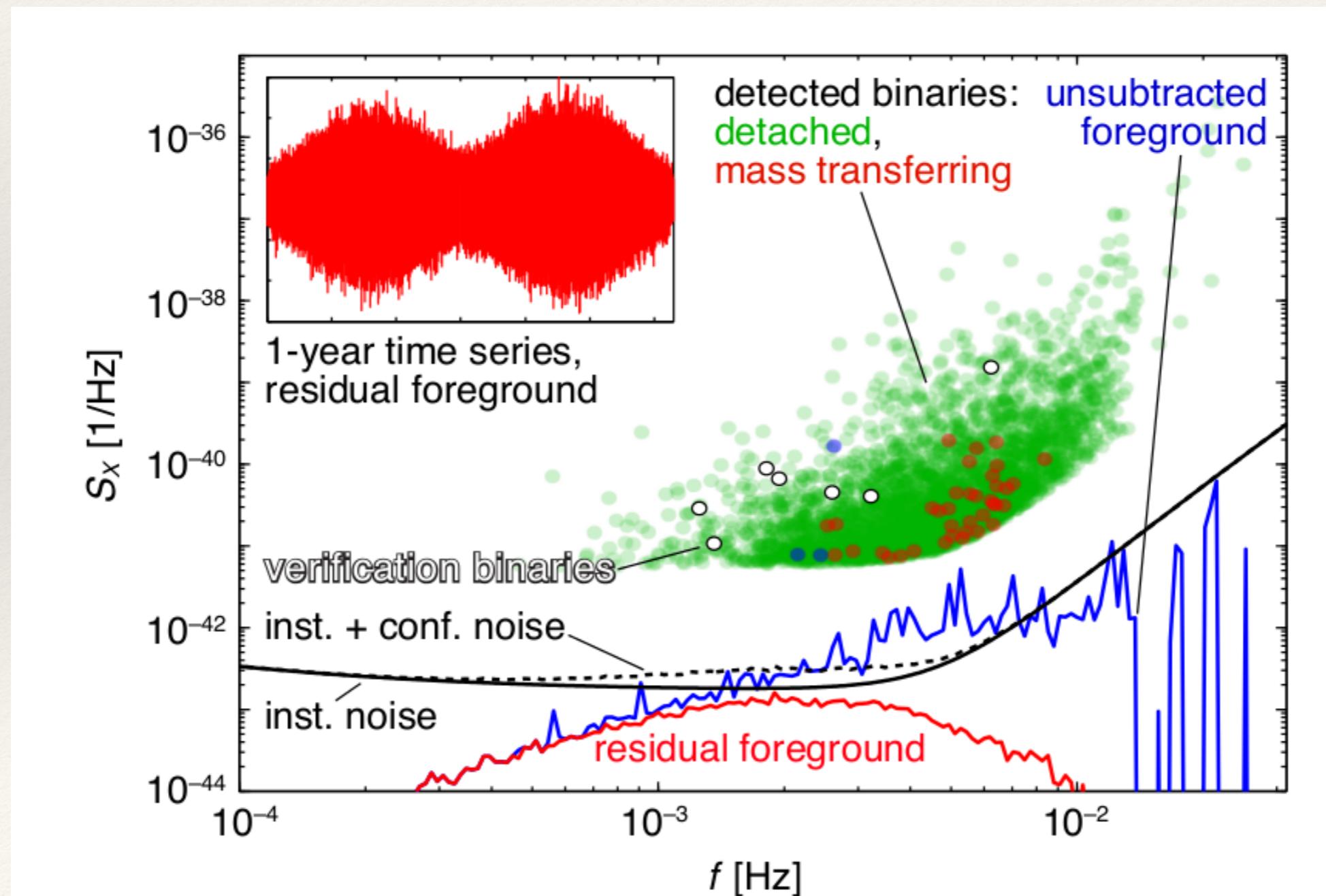
# EMRI

- Orbital motion: (almost) elliptical with a strong relativistic precession + orbital precession due to spin-orbital coupling
- Signal is very rich in structure (hard to detect but gives a lot of information).
- Ultra-precise parameter determination (if detected). Can map spacetime of a heavy object: holiodesy



# Galactic white dwarf binaries

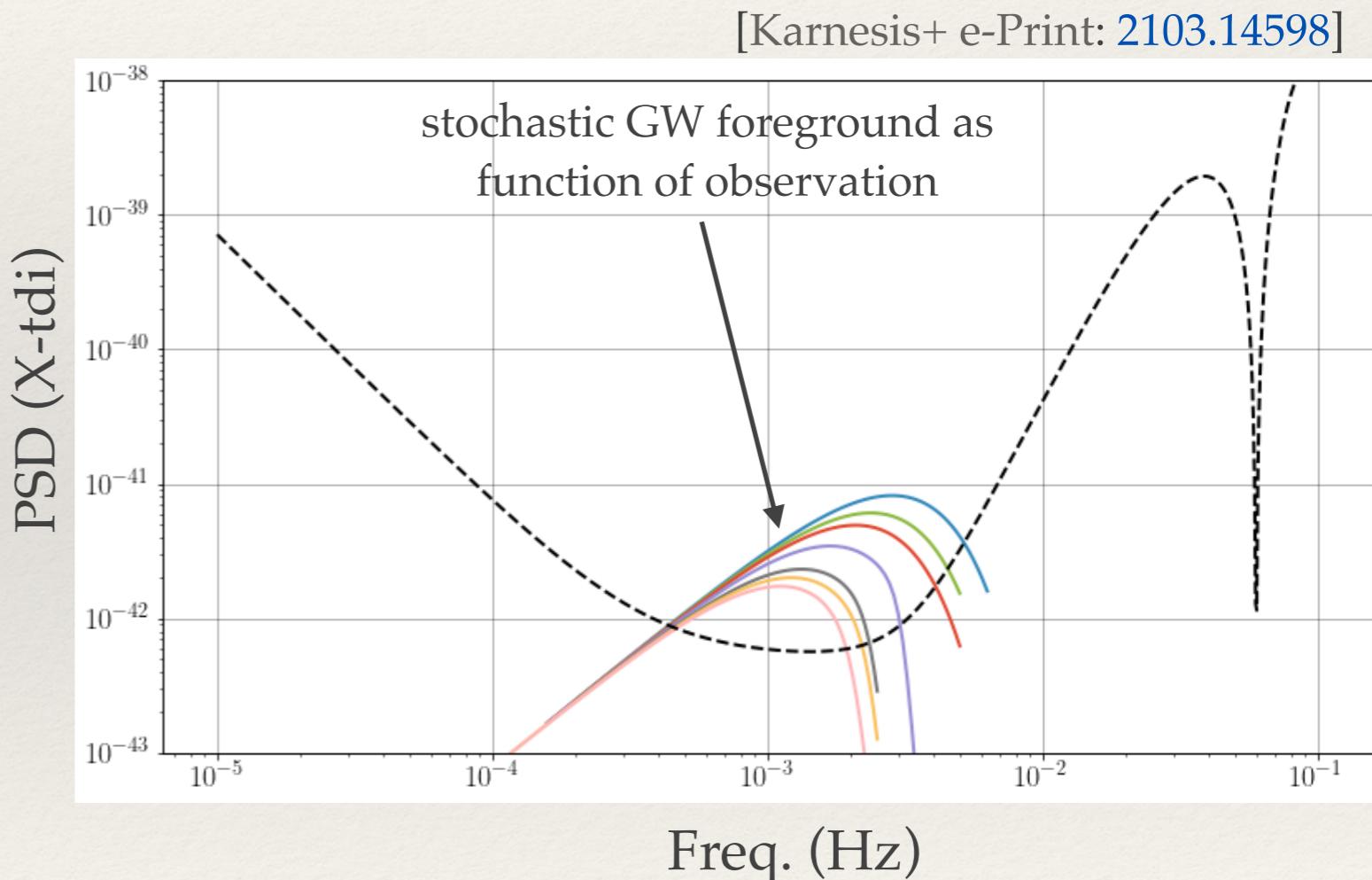
- We expect to have  $10^7$  WD binaries all emitting GWs in the LISA band, only  $10^4$  can be resolved individually, other form stochastic GW signal (foreground)
- GW signal is almost monochromatic
- Verification binaries: known from current e / m observations (+GAIA,+ LSST)



# Detecting GBs as function of time

Data comes continuously (daily): the signal-to-noise grows as  $\sqrt{T}$  we start resolving previously unresolved signals

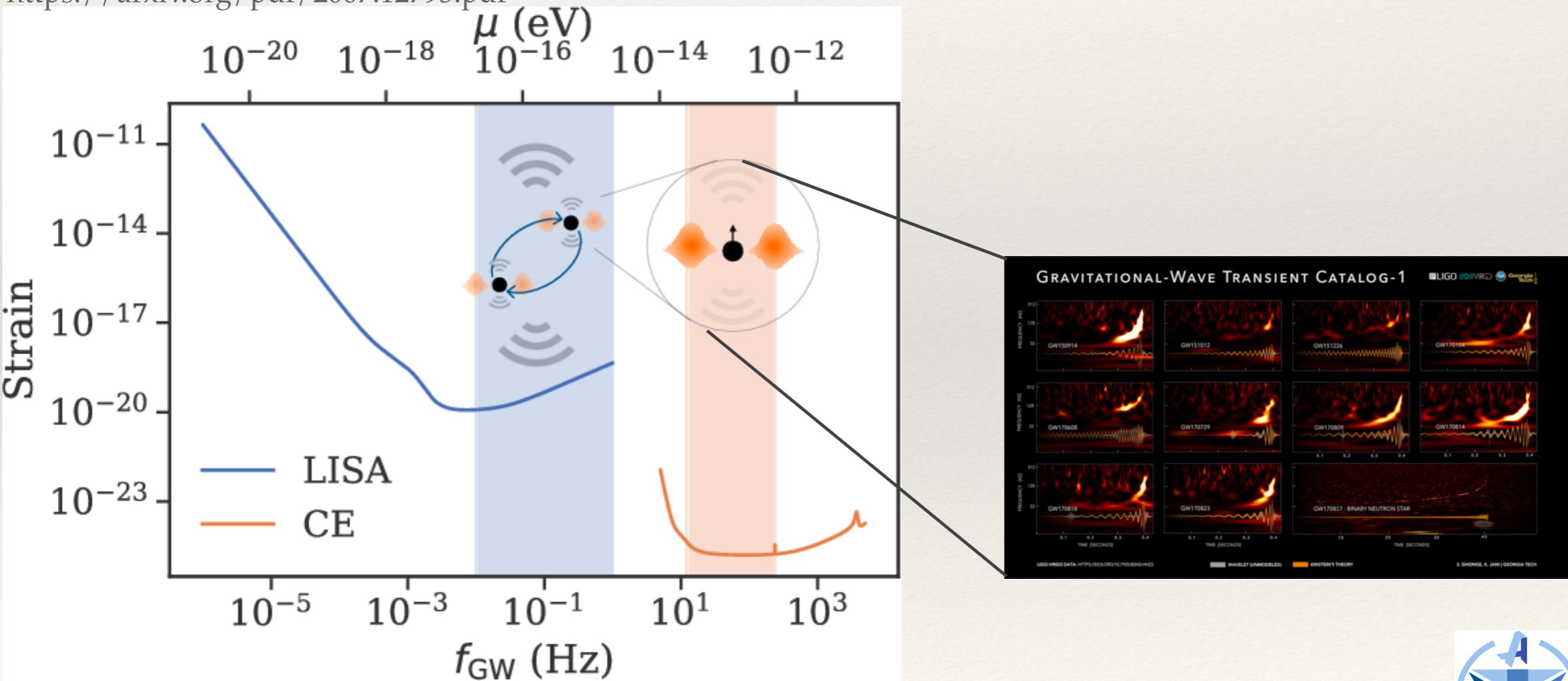
Observation duration	Max number of detectable source
10 days	156
3 months	1984
6 months	3818
1 year	7116
2 years	13103
5 years	25488
7 years	31150
10 years	40023



# Stellar mass black hole binaries

- Stellar mass black hole binaries: SBBH — the same black holes which merger we observe with LIGO-Virgo. If consider those binaries 5-50 years in the past (slow inspiral) they emit in the LISA's band

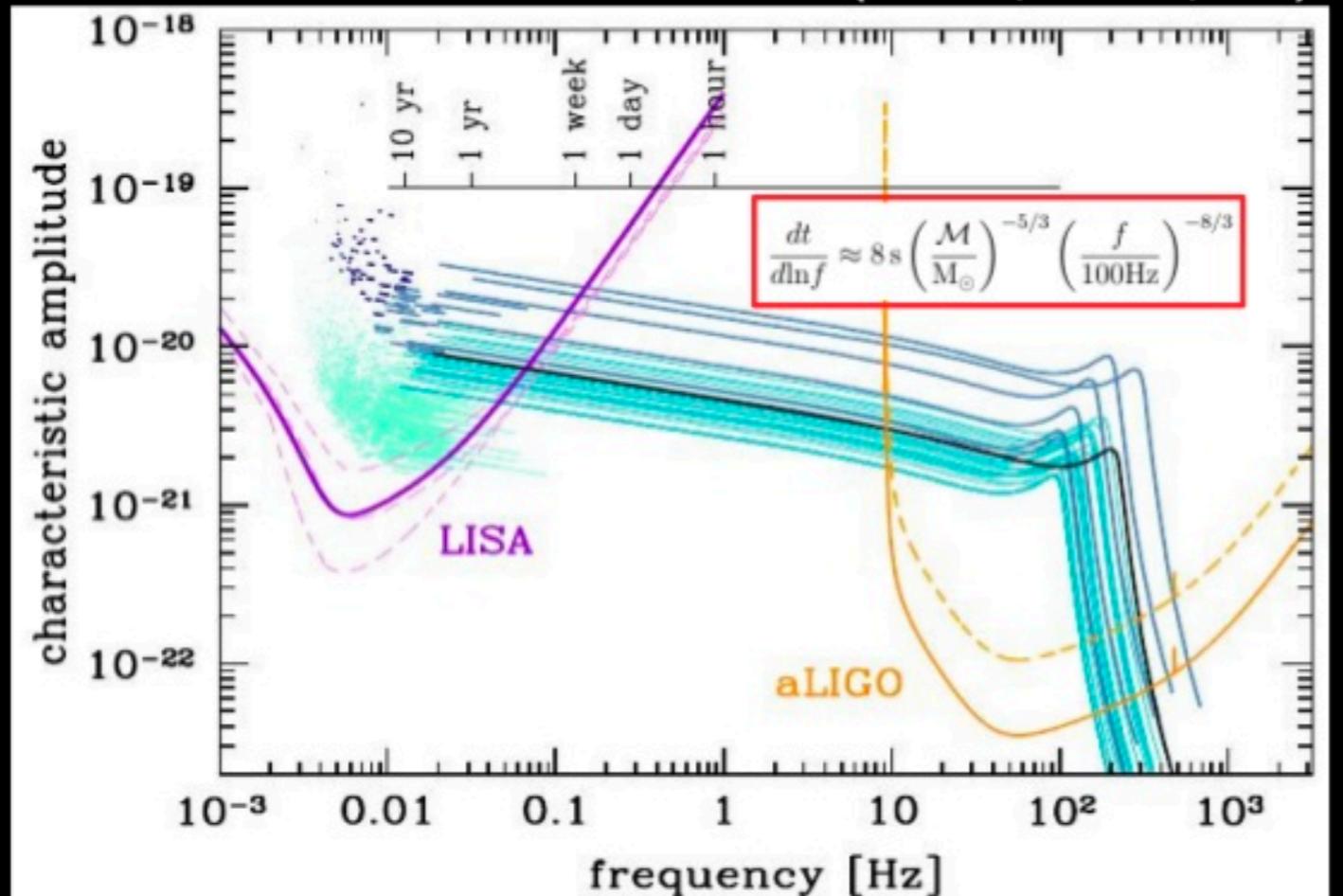
<https://arxiv.org/pdf/2007.12793.pdf>



# Stellar mass black hole binaries

**An unexpected scenario: multi-band GW astronomy**

(AS 2016, PRL 116, 1102)



BHB will be detected by eLISA and cross to the LIGO band,  
assuming a 5 year operation of eLISA.

Multi-band observations: some binaries will be first observed/detected by LISA and then 5-10 years later re-appear and merge in the band of ground-based detectors (Einstein Telescope, Cosmic Explorer)

A. Sesana PRL 2016



# Stellar mass black hole binaries

A. Toubiana+2020

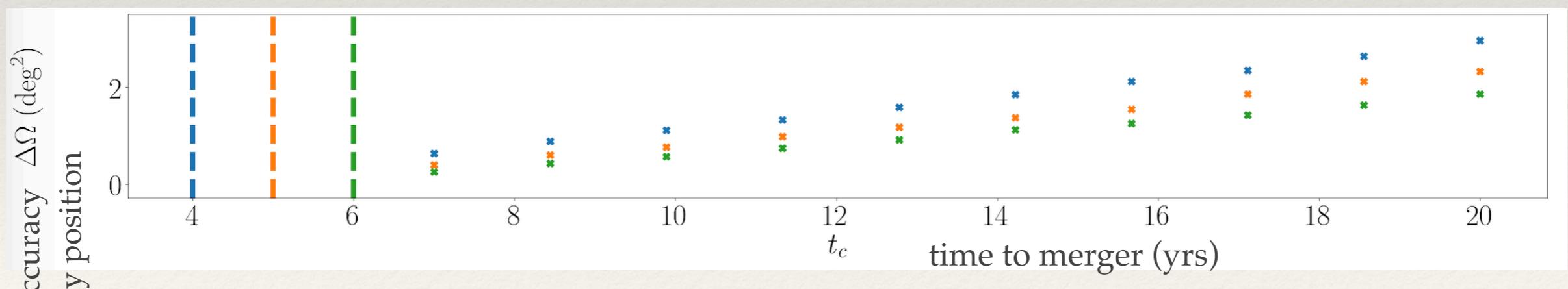
	$B$	$m_g$
System 1	$< 2 \cdot 10^{-11}$	$< 6 \cdot 10^{-25}$
System 2	$< 9 \cdot 10^{-11}$	$< 4 \cdot 10^{-25}$
System 3	$< 1 \cdot 10^{-10}$	$< 5 \cdot 10^{-25}$
Current constraints	$< 4 \times 10^{-2}$	$< 8 \times 10^{-23}$

Constraint on  
dipolar radiation

Constraint  
on graviton mass

- SBBH - multiband observations:  
amazing laboratories for testing  
General Relativity theory

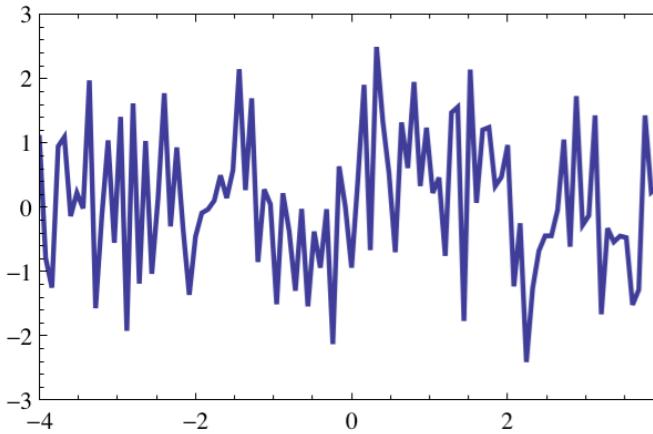
- We can detect and estimate sky position of those sources years before they merge:  
pre-merger multimessenger observations



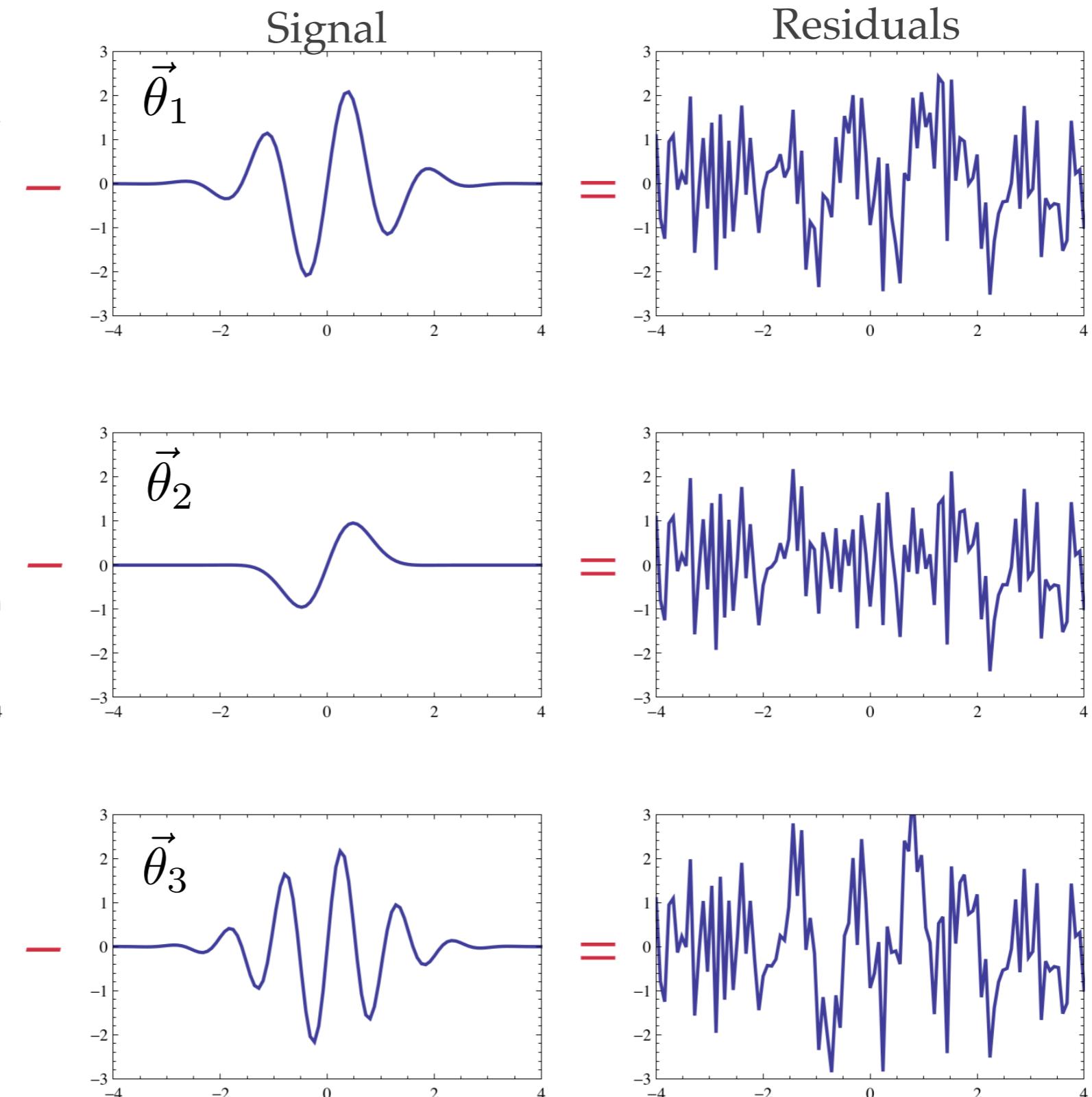
# Matched filtering and parameter estimation

noise = data - signal

Data



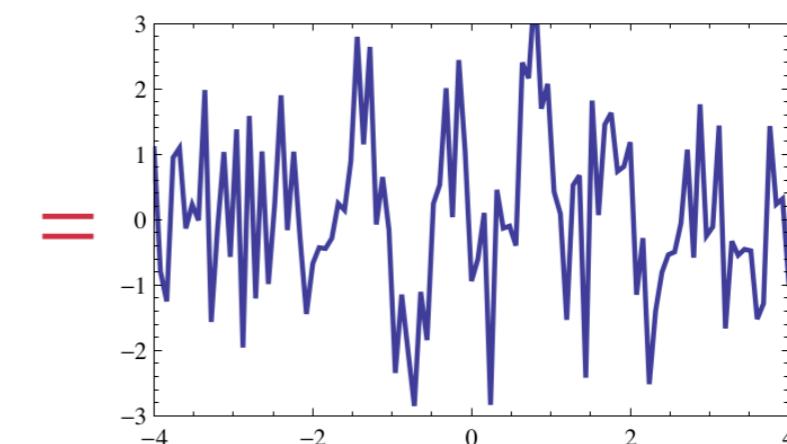
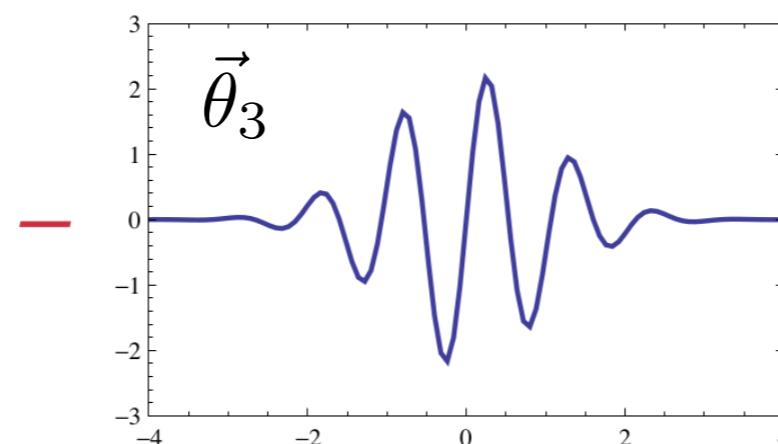
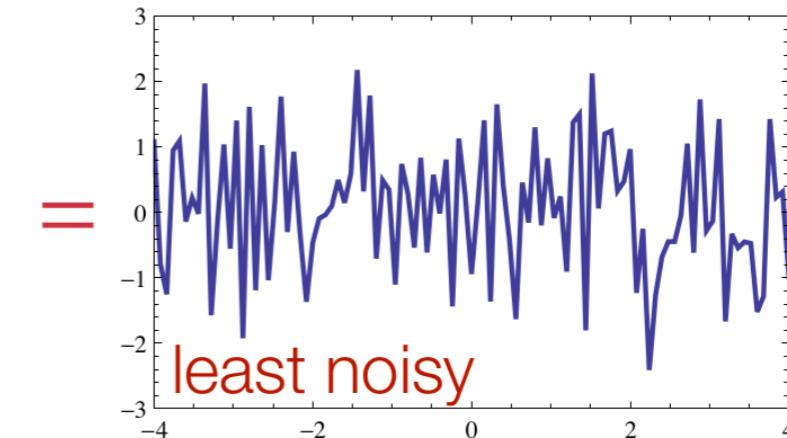
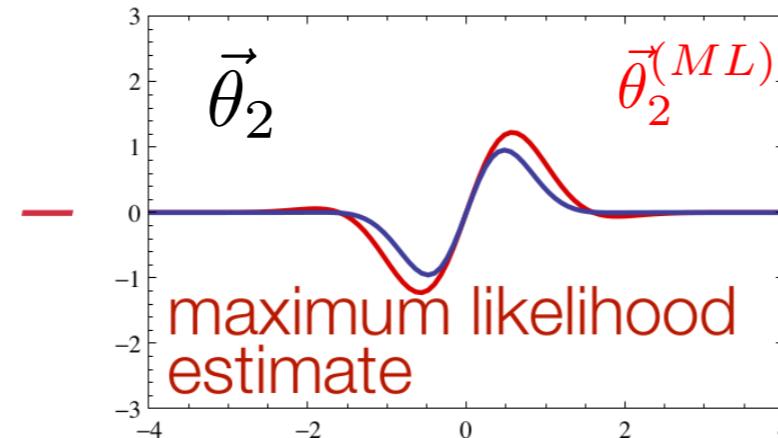
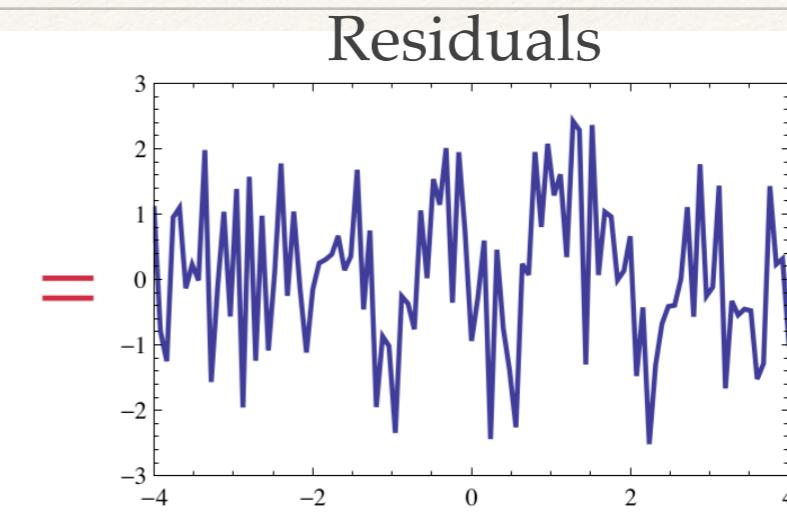
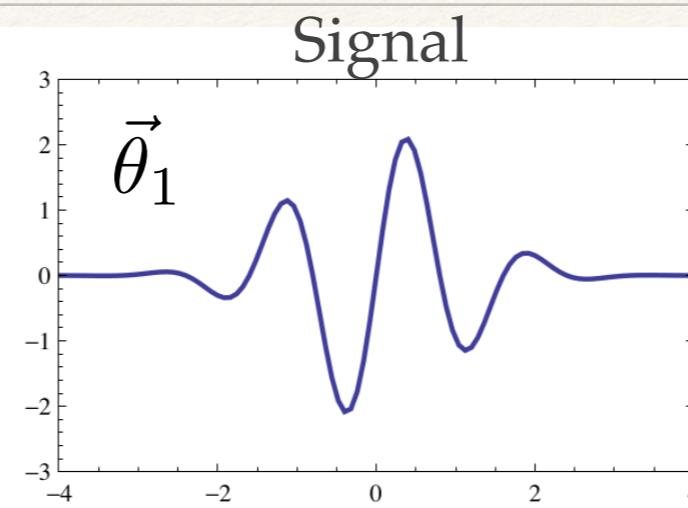
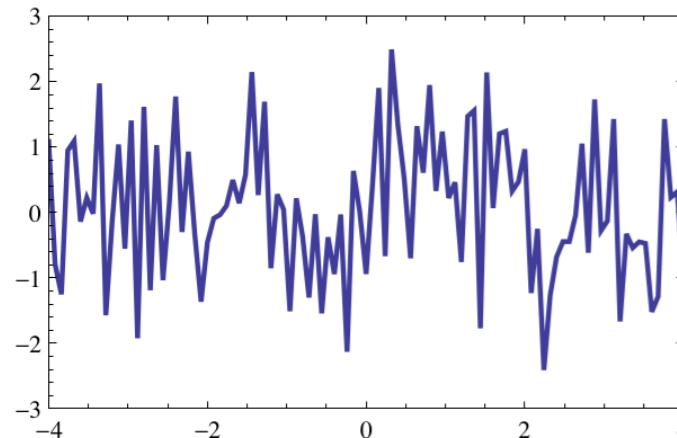
(Credits: M. Vallisneri)



# Matched filtering and parameter estimation

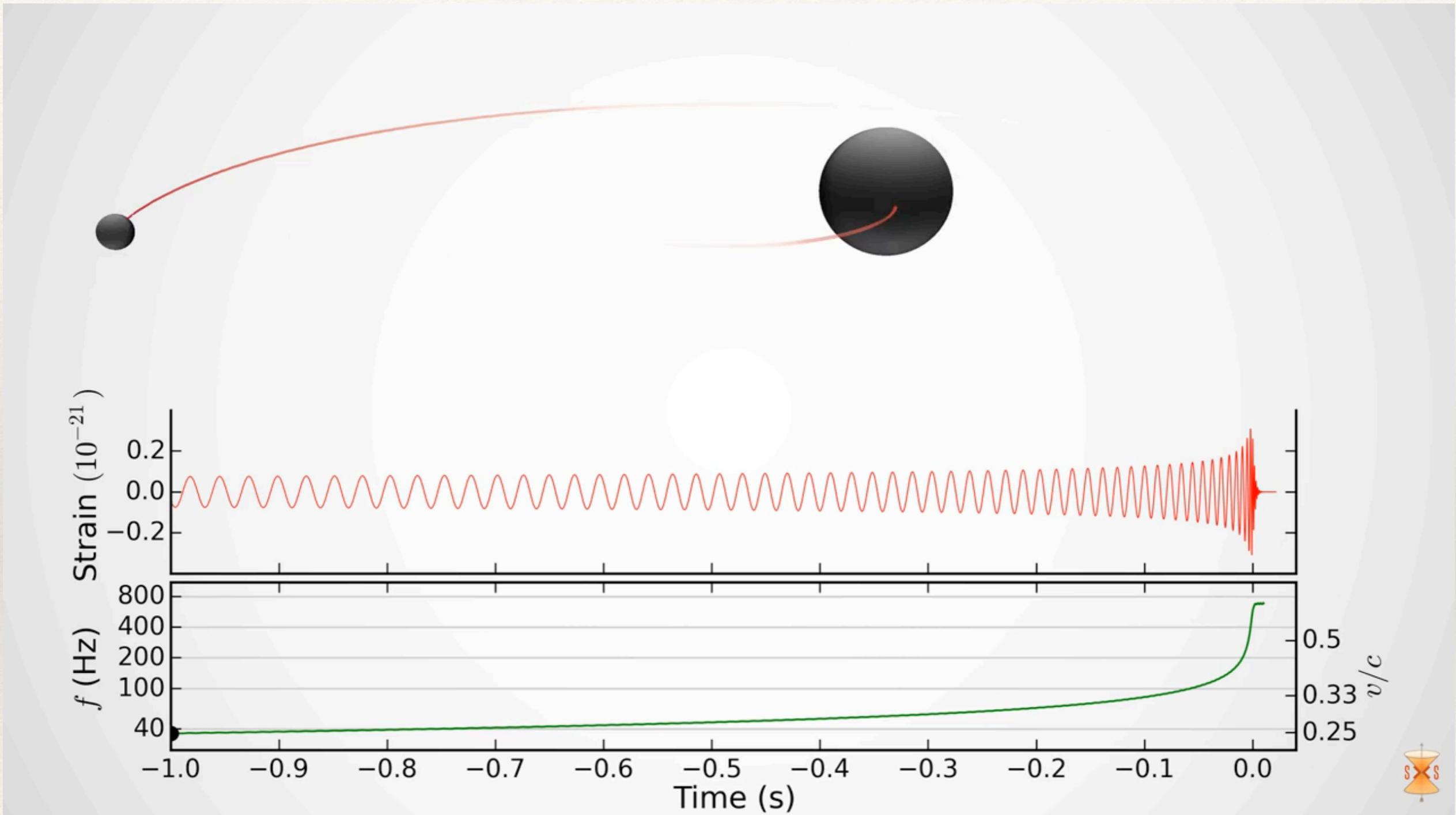
noise = data - signal

Data



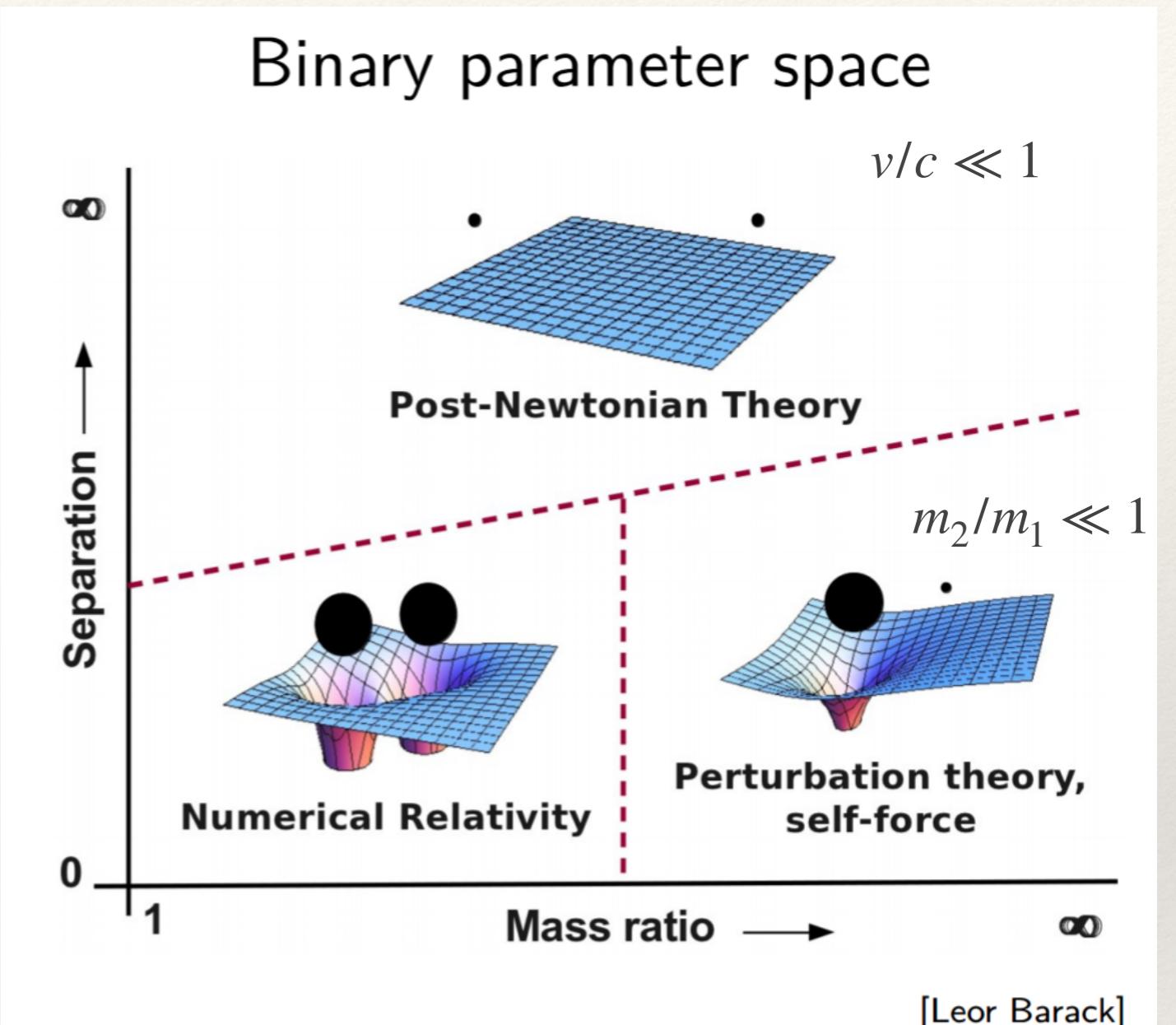
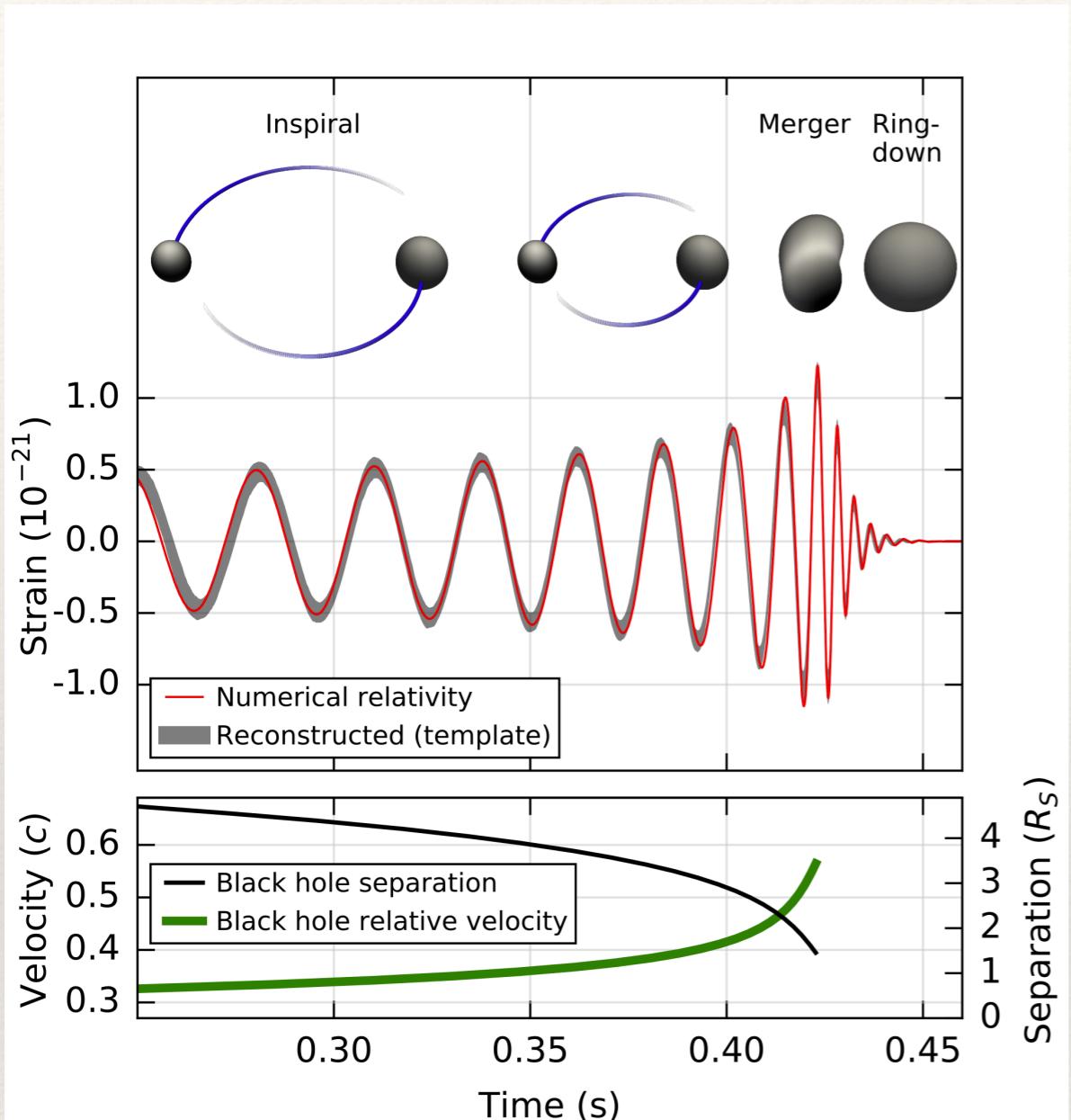
(Credits: M. Vallisneri)

# GW signal



[Credits: SXS collaboration]

# Modelling GW signal



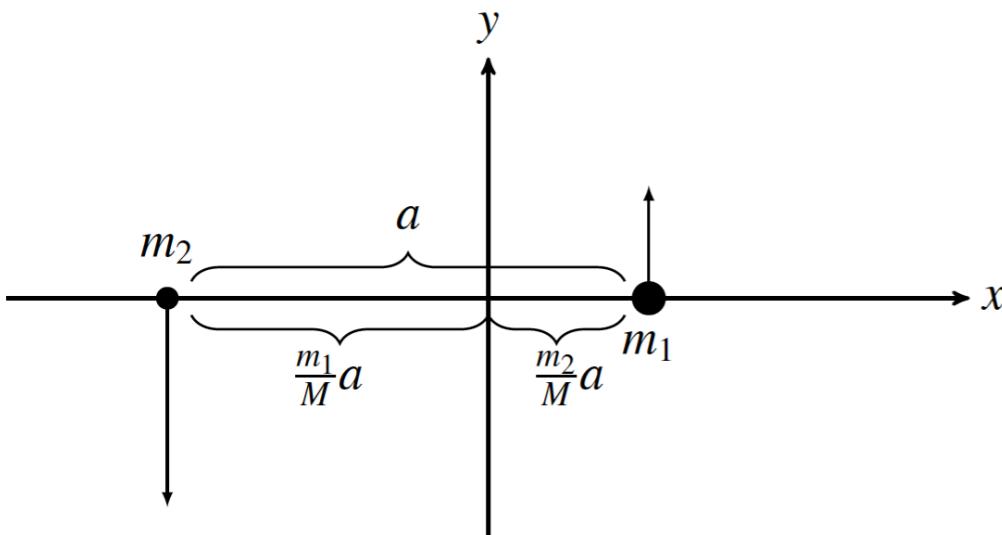
GW signal from binary system can be conventionally split into three parts: inspiral (Post-Newtonian decomposition), merger (numerical relativity), ringdown (BH perturbation)



# Binary system

$$[\bar{h}^{jk}]^{TT} = \frac{2}{R} \partial_t^2 [I_{jk}^{TT}(t - R)]$$

quadrupole moment of a system



Consider binary system on a circular orbit.

$$m_1 > m_2, \quad M = m_1 + m_2 \quad \mu = m_1 m_2 / M$$

Use Kepler's law

$$\omega = \sqrt{\frac{M}{a^3}}$$

$$x_1 = \frac{m_2}{M} a \cos \omega t, \quad y_1 = \frac{m_2}{M} a \sin \omega t$$

$$x_2 = -\frac{m_1}{M} a \cos \omega t, \quad y_2 = -\frac{m_1}{M} a \sin \omega t$$

$$I_{jk} = \int T^{00} x_j x_k d^3x = \int [\delta(\vec{x} - \vec{x}_1)m_1 + \delta(\vec{x} - \vec{x}_2)m_2] x_j x_k d^3x = m_1 x_1^j x_1^k + m_2 x_2^j x_2^k$$

polarization basis

$$\ddot{I}_{xx} = -\ddot{I}_{yy} = -2\mu (M\omega)^{2/3} \cos 2\omega t, \quad \hat{e}_\theta = \hat{e}_x \cos \theta - \hat{e}_z \sin \theta, \quad \hat{e}_\phi = \hat{e}_y.$$

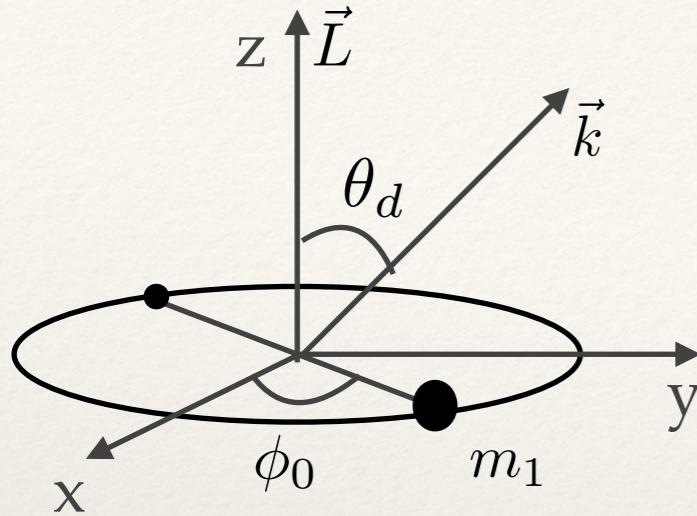
$$\ddot{I}_{xy} = \ddot{I}_{yx} = -2\mu (M\omega)^{2/3} \sin 2\omega t, \quad I_{\theta\theta} = I_{xx} \cos^2 \theta, \quad I_{\phi\phi} = I_{yy}, \quad I_{\theta\phi} = I_{xy} \cos \theta.$$

$$h_+ = h_{\theta\theta} = -2(1 + \cos^2 \theta) \frac{\mu}{R} (M\omega)^{2/3} \cos [2\omega(t - R) - \phi_0]$$

$$h_\times = h_{\theta\phi} = -4 \cos \theta \frac{\mu}{R} (M\omega)^{2/3} \sin [2\omega(t - R) - \phi_0]$$



# Binary system

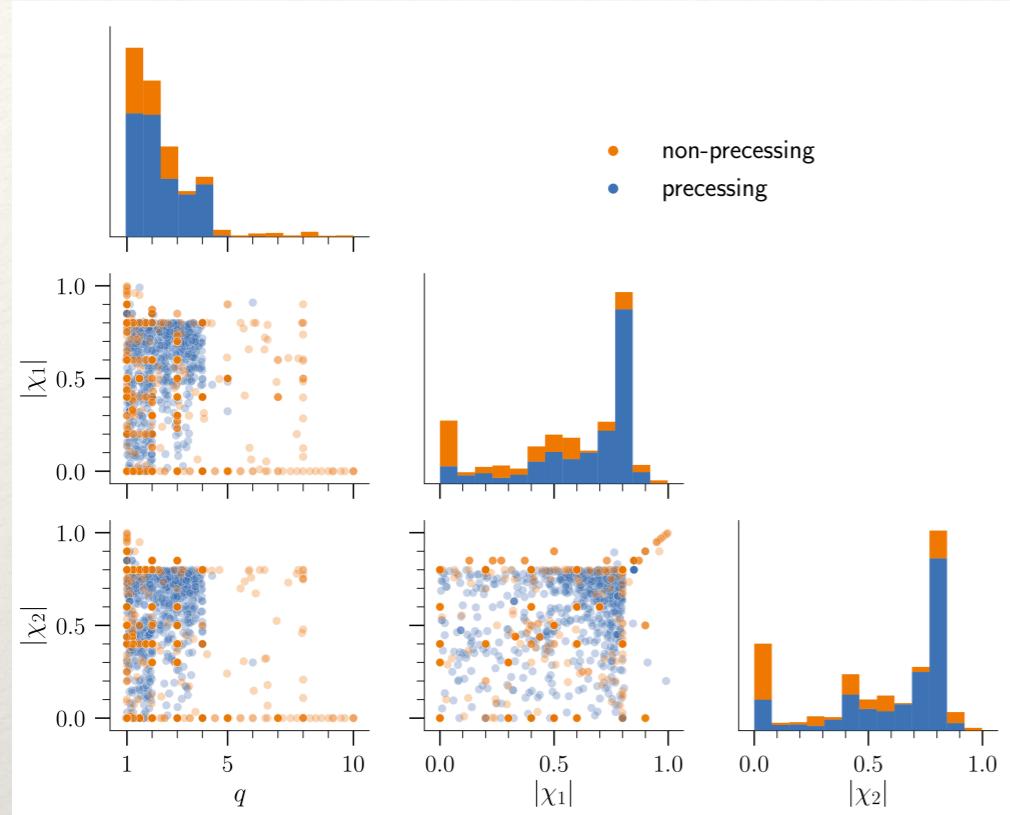


$$h_+ = h_{\theta\theta} = -2 (1 + \cos^2 \theta_d) \frac{\mu}{R} (M\omega)^{2/3} \cos [2\omega(t - R) - \phi_0]$$

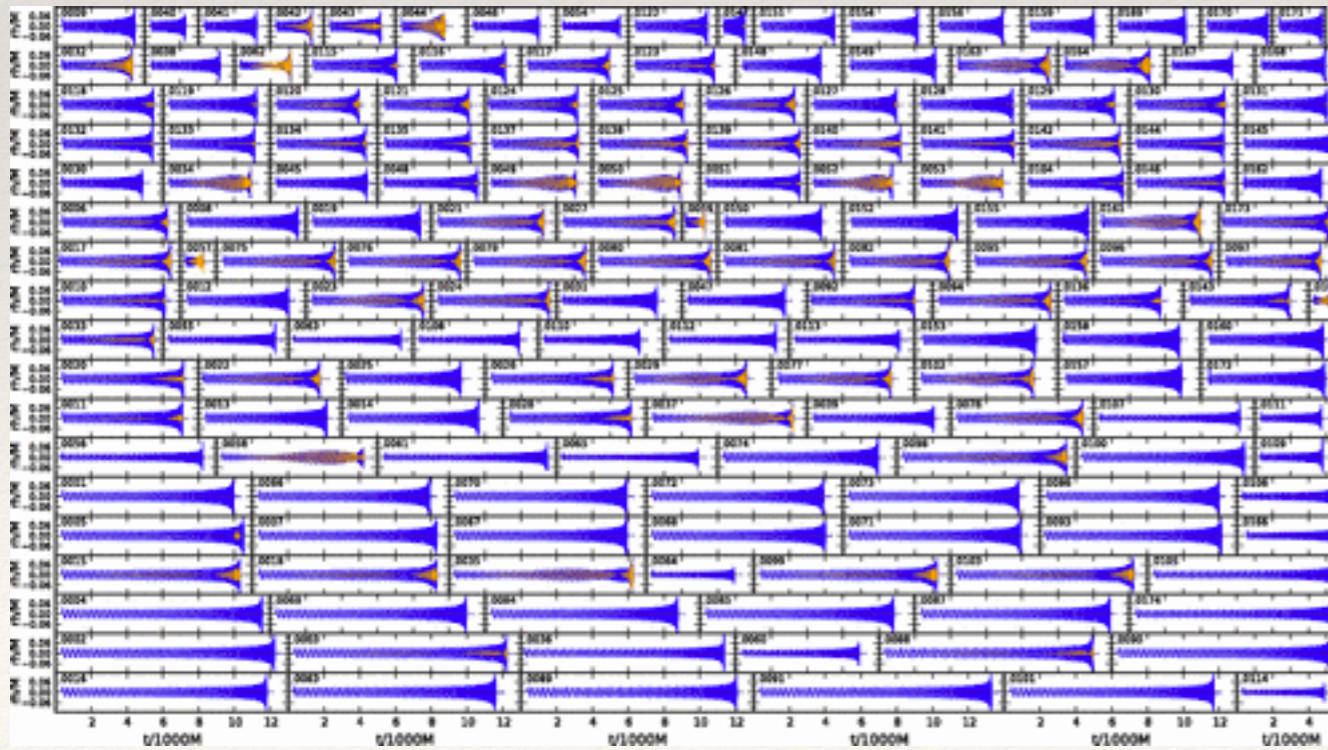
$$h_x = h_{\theta\phi} = -4 \cos \theta_d \frac{\mu}{R} (M\omega)^{2/3} \sin [2\omega(t - R) - \phi_0]$$

- Inclination: angle between orb. angular momentum and propagation direction ( $\theta_d$ ), alternatively  $\iota = \pi - \theta_d$  angle between L and direction *to* the source.
- Distance to the source: luminosity distance  $R = D_L$
- GW emission is strongest if the source is face on/off, weakest - edge-on
- Dominant harmonic:  $2 \times$  orbital freq. (circular), there are harmonics  $1, 3, 4, \dots \times$  orbital freq. but lower in amplitude
- GW carry away energy (orbit shrinks) and orbital momentum (orbit circularizes)

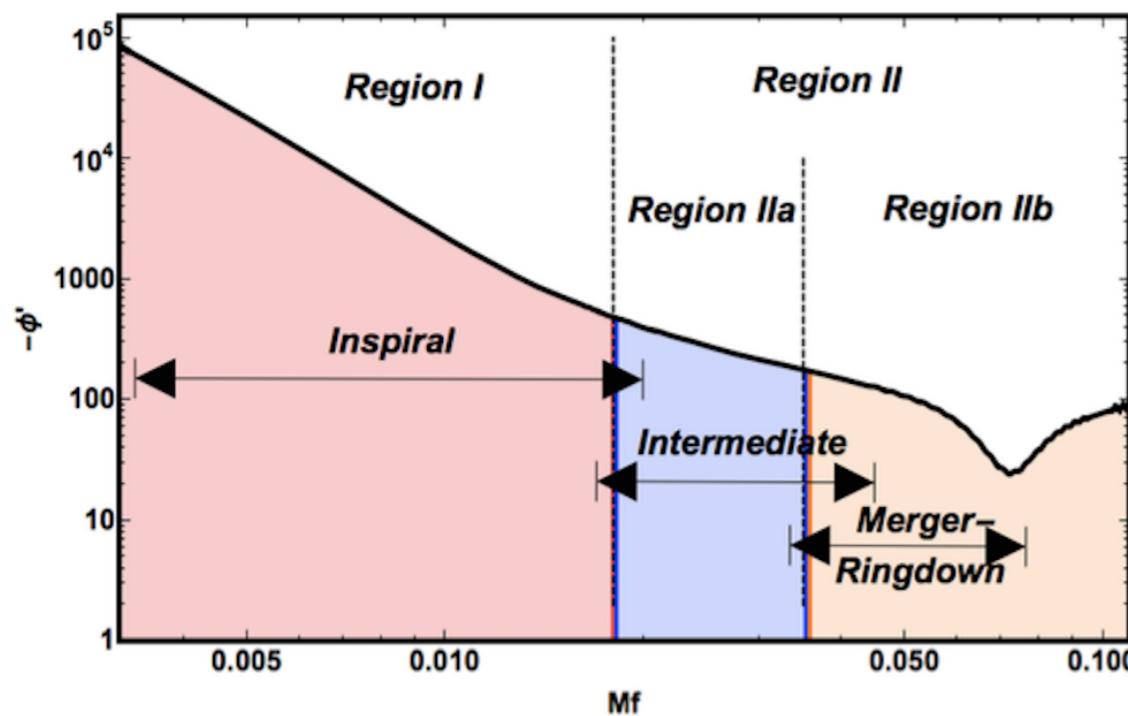
# NR surrogate waveforms



- Using a large number of numerical waveforms  
(solving Einstein equations numerically: very short - about 20 orbits before the merger.)
- Using them as a basis for waveform decomposition
- Interpolating across parameters space
- The most accurate to-date model, but limited in the parameter space

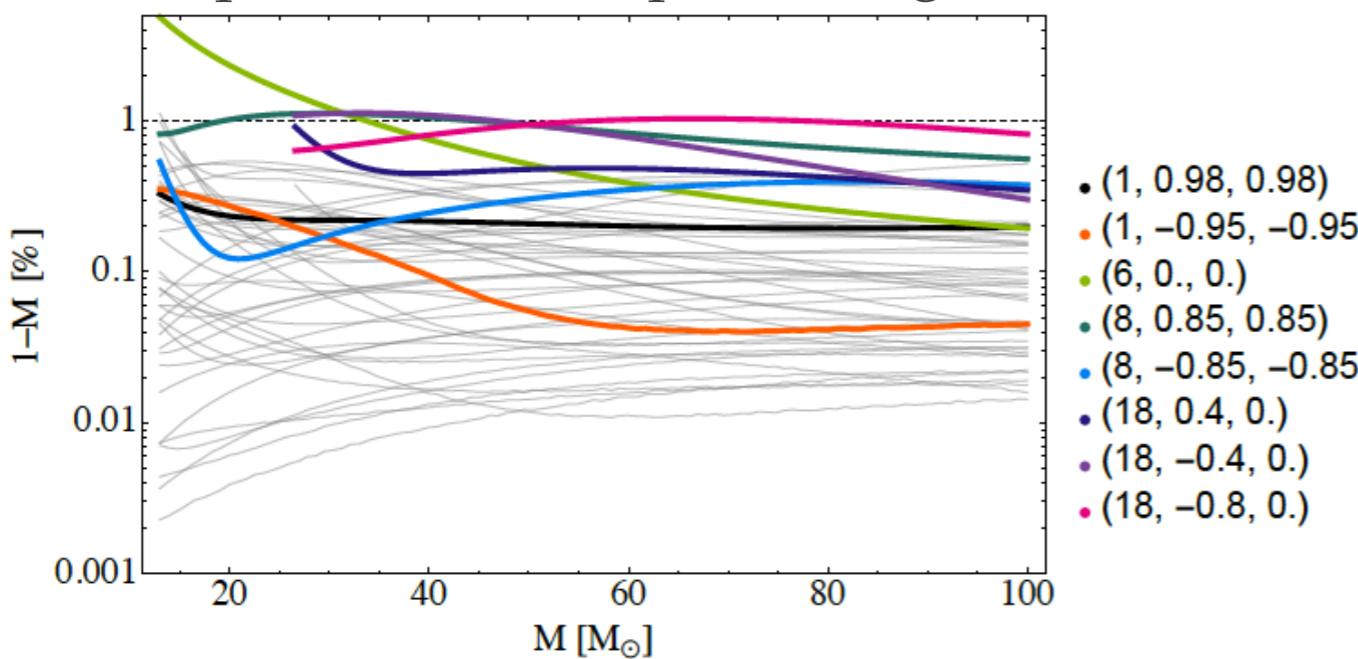


# Phenomenological template family



$$\begin{aligned}\phi_{\text{Ins}} &= \phi_{\text{TF2}}(Mf; \Xi) \\ &\quad + \frac{1}{\eta} \left( \sigma_0 + \sigma_1 f + \frac{3}{4} \sigma_2 f^{4/3} + \frac{3}{5} \sigma_3 f^{5/3} + \frac{1}{2} \sigma_4 f^2 \right) \\ \phi_{\text{Int}} &= \frac{1}{\eta} \left( \beta_0 + \beta_1 f + \beta_2 \log(f) - \frac{\beta_3}{3} f^{-3} \right) \\ \phi_{\text{MR}} &= \frac{1}{\eta} \left\{ \alpha_0 + \alpha_1 f - \alpha_2 f^{-1} + \frac{4}{3} \alpha_3 f^{3/4} \right. \\ &\quad \left. + \alpha_4 \tan^{-1} \left( \frac{f - \alpha_5 f_{\text{RD}}}{f_{\text{damp}}} \right) \right\}.\end{aligned}$$

Comparison of non-precessing waveforms

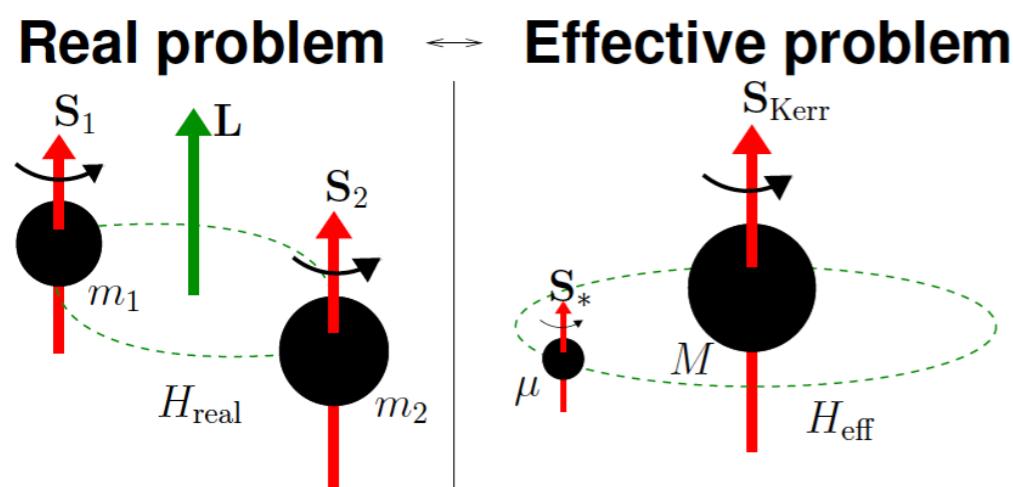


- Waveform constructed in the frequency domain
- Uses Post-Newtonian results for the early evolution (inspiral) of a binary
- For merger-ringdown part: there is an analytical expression with free parameters which are calibrated to fit the NR data
- Precession is added by rotation taken from the Post-Newtonian evolution
- Very fast to generate

# Modelling GW signal using Effective-one-body (EOB) approach

Effective-one-body approach [Buonanno & Damour 1999]:

The main idea is to map the (real) dynamic of two comparable mass binary to an effective problem of motion of test mass in an effective spacetime



$$M = m_1 + m_2, \quad \mu = \frac{m_1 m_2}{M}$$

EOB has three essential components

- Map conservative two-body dynamics to a motion of a test mass  $\mathbf{m}$  in the field of a central body  $\mathbf{M}$ .
- Add the radiation reaction force
- Construct the waveform based on the computed dynamics with attached the ring-down RD part of the signal

See review: [Damour 2012]



# EOB: Dynamics

## Equations of motion

$$\frac{d\phi}{dt} = \Omega = \frac{\partial H}{\partial p_\phi}, \quad \frac{dr}{dt} = \frac{\partial H}{\partial p_r},$$

$$\frac{dp_\phi}{dt} = F_\phi, \quad \frac{dp_r}{dt} = -\frac{\partial H}{\partial r}$$

[Buonanno, Damour 2000]

Radiation reaction force  
(quasi-circular motion)

$$\left( \frac{dH}{dt} \right) \approx \Omega F_\phi,$$

Dissipation of energy from the system =  
flux of energy carried by GWs

$$F_\phi = -\frac{1}{\Omega} \left( \frac{2}{16\pi} \sum_{\ell} \sum_{m=-\ell}^{\ell} (m\Omega)^2 |D_L h_{\ell m}|^2 \right)$$

Luminosity distance

$$h_+ - i h_\times = \sum_{\ell} \sum_{m=-\ell}^{\ell} h_{\ell m} Y_{\ell m}^{(-2)}(\theta, \phi)$$

$$h_{\ell m} = h_{\ell m}^N \quad \hat{h}_{\ell m}^{\text{PN}}$$

“Newtonian” part

$$x = (M\Omega)^{2/3}$$

$$h_{\ell m}^N = \frac{M\eta}{D_L} n_{lm}^{(\epsilon)} c_{\ell+\epsilon}(\eta) x^{(\ell+\epsilon)/2} Y^{\ell-\epsilon, -m} \left( \frac{\pi}{2}, \phi(t) \right)$$

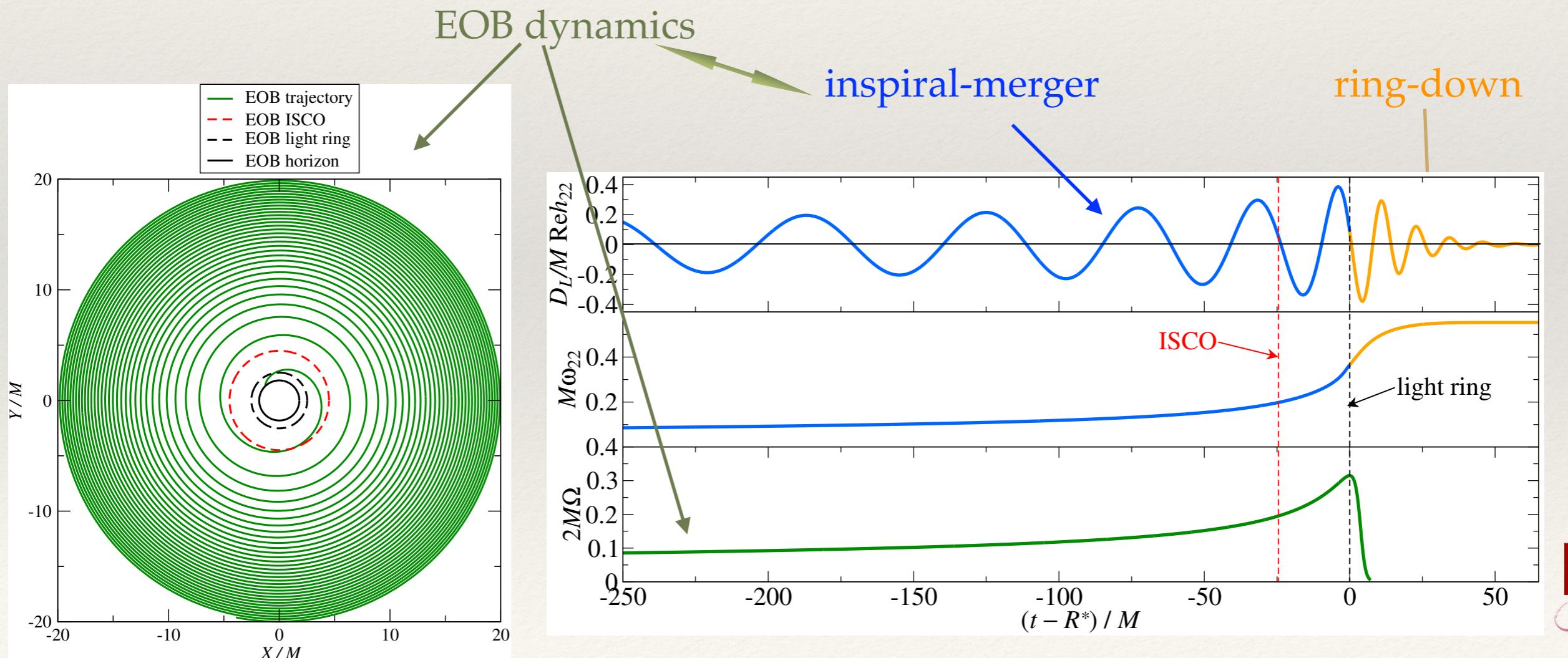
numerical coefficients (f-ns of  $\ell, m, \eta$ )



# EOB: ring-down (RD) signal

- Identify RD attachment time: maximum of orb. freq.  $\sim$  light ring
- Generate the RD signal as superposition of damped eigen frequencies of final BH
- Define the amplitude of each QNM by demanding continuity of matching to inspiral-merger part of the signal

$$\left(\frac{D_L}{M}\right) h_{\ell m}^{\text{RD}}(t > t_{\text{match}}) = \sum_{m',n} A_{\ell,m',n} e^{-(i\omega_{\ell m' n} + 1/\tau_{\ell m' n})(t - t_{\text{match}})}$$

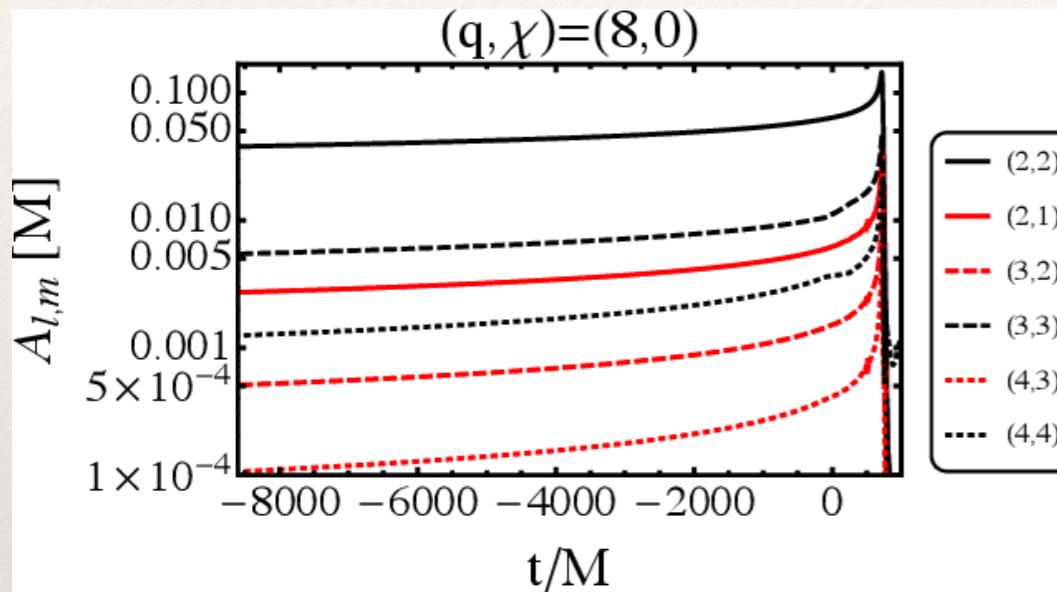


# Higher order modes

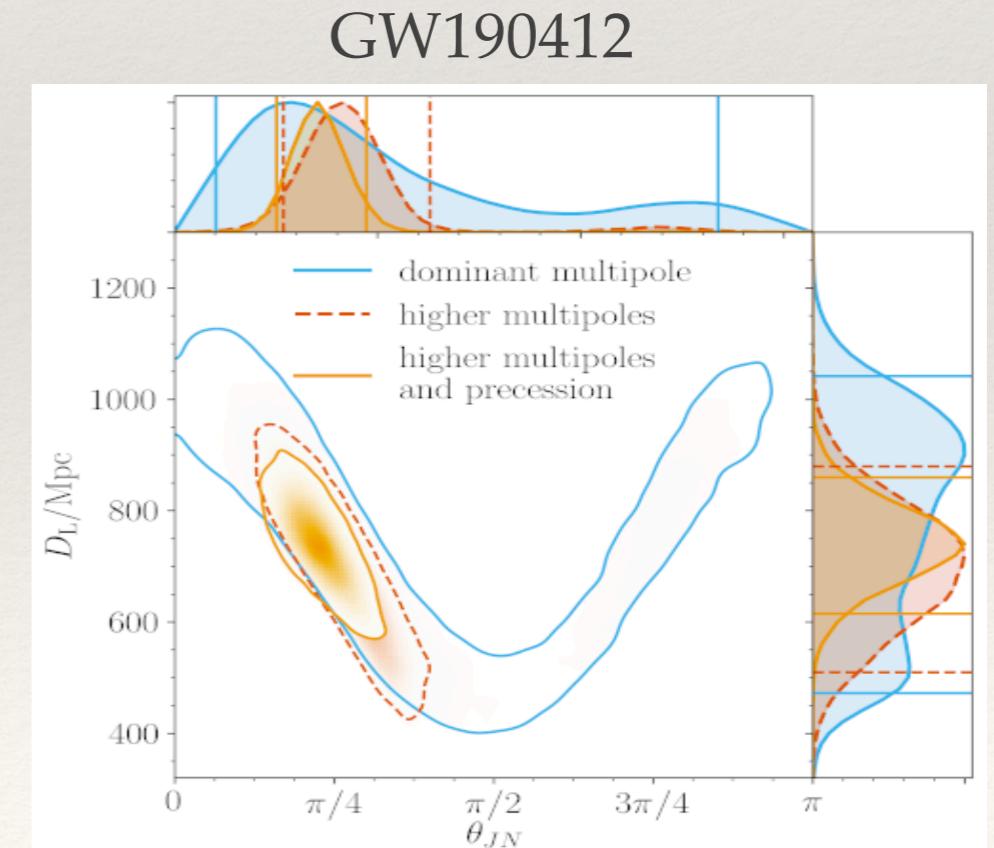
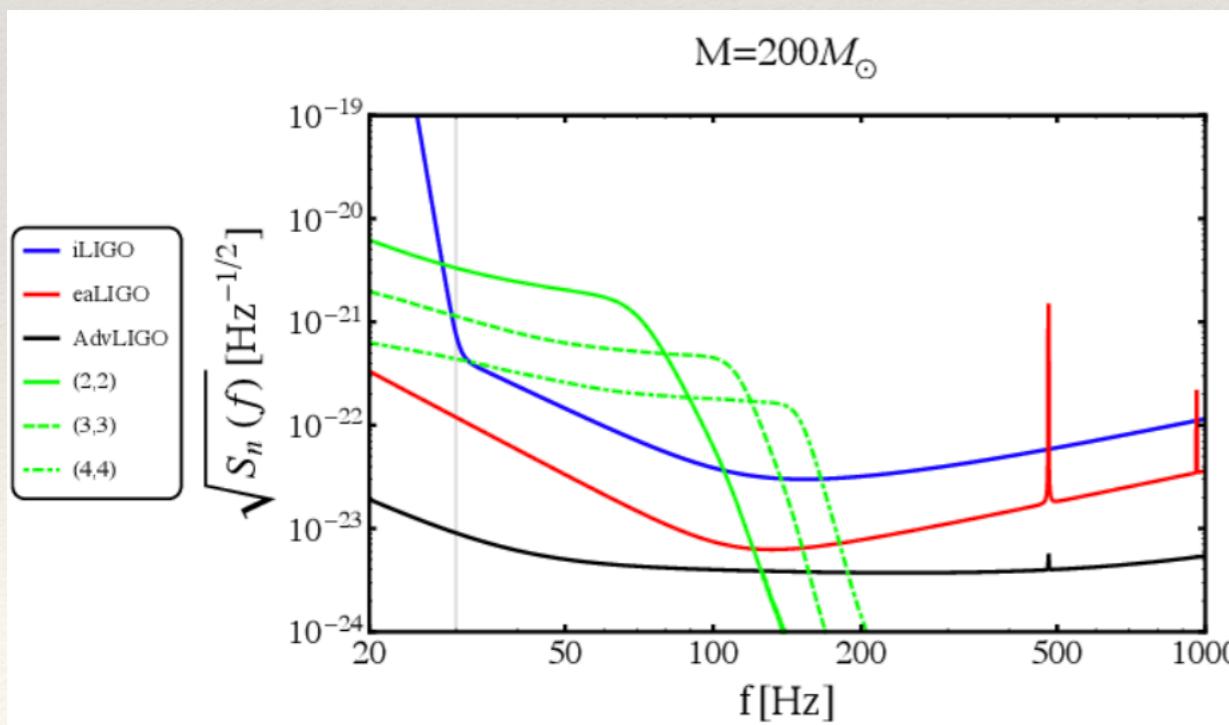
Decomposing the waveform in spherical harmonics

$$h_+ - ih_x = \sum_{l \geq 2} \sum_{m=-l}^l h_{lm}^{(-2)} Y_{lm}(\theta, \phi)$$

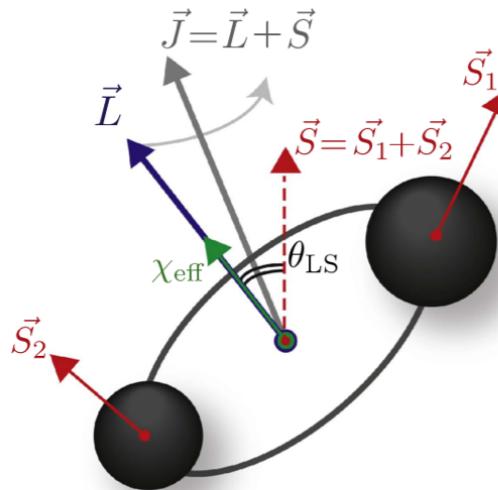
For (quasi)circular binaries, the dominant mode is  $l = 2, m = \pm 2$  (twice orbital frequency)



- For inspiral: odd harmonics suppressed by  $(v/c)$  and  $(m_1 - m_2)/(m_1 + m_2)$
- Coupled differently to inclination: breaking degeneracy in parameter space



# Orbital precession



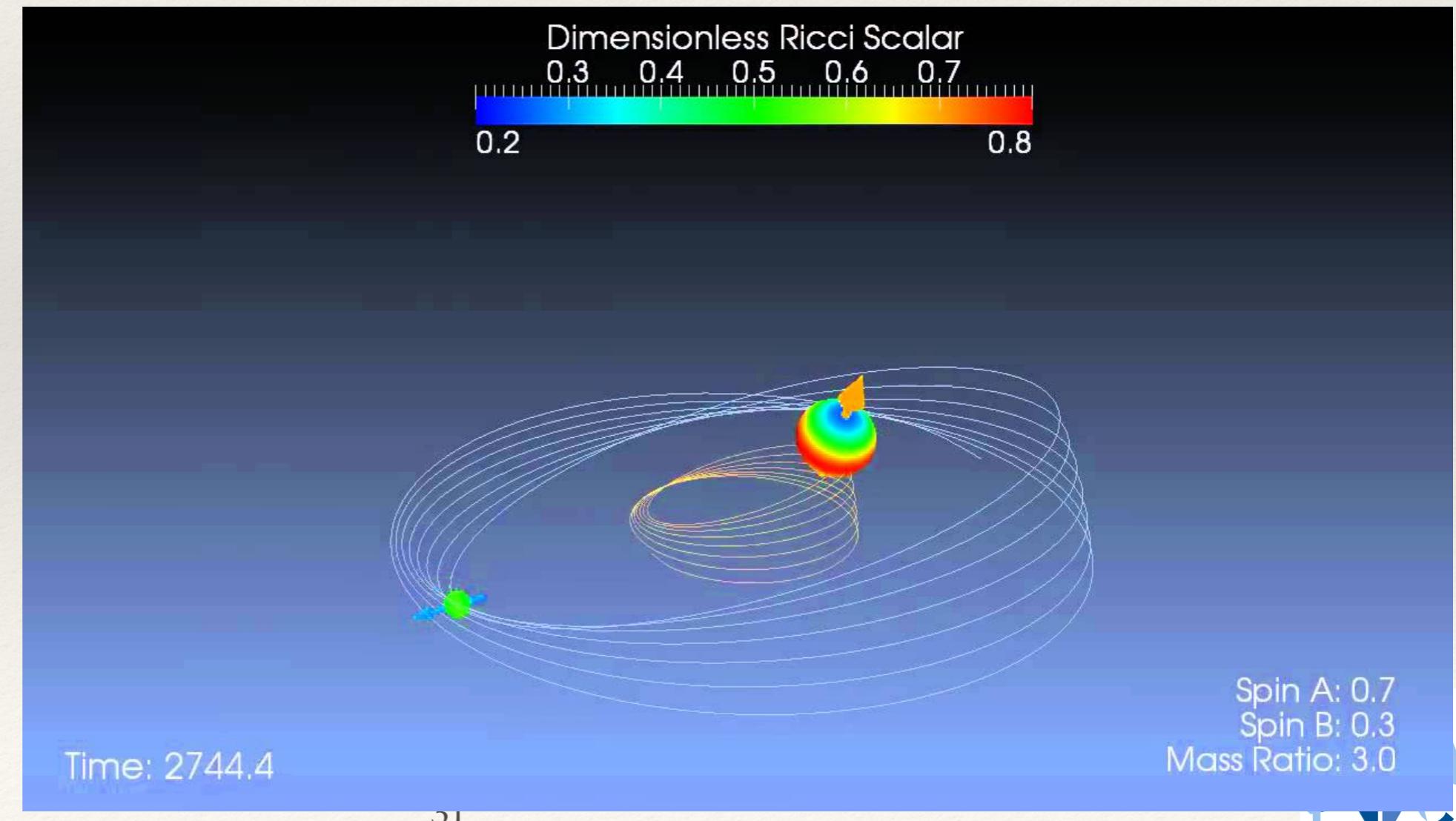
Credit: Carl Rodriguez

- If the spins of BHs are not parallel to the orbital angular momentum: spin and orbital precession around total momentum of the binary

$$\vec{J} = \vec{L} + \vec{S}_1 + \vec{S}_2$$

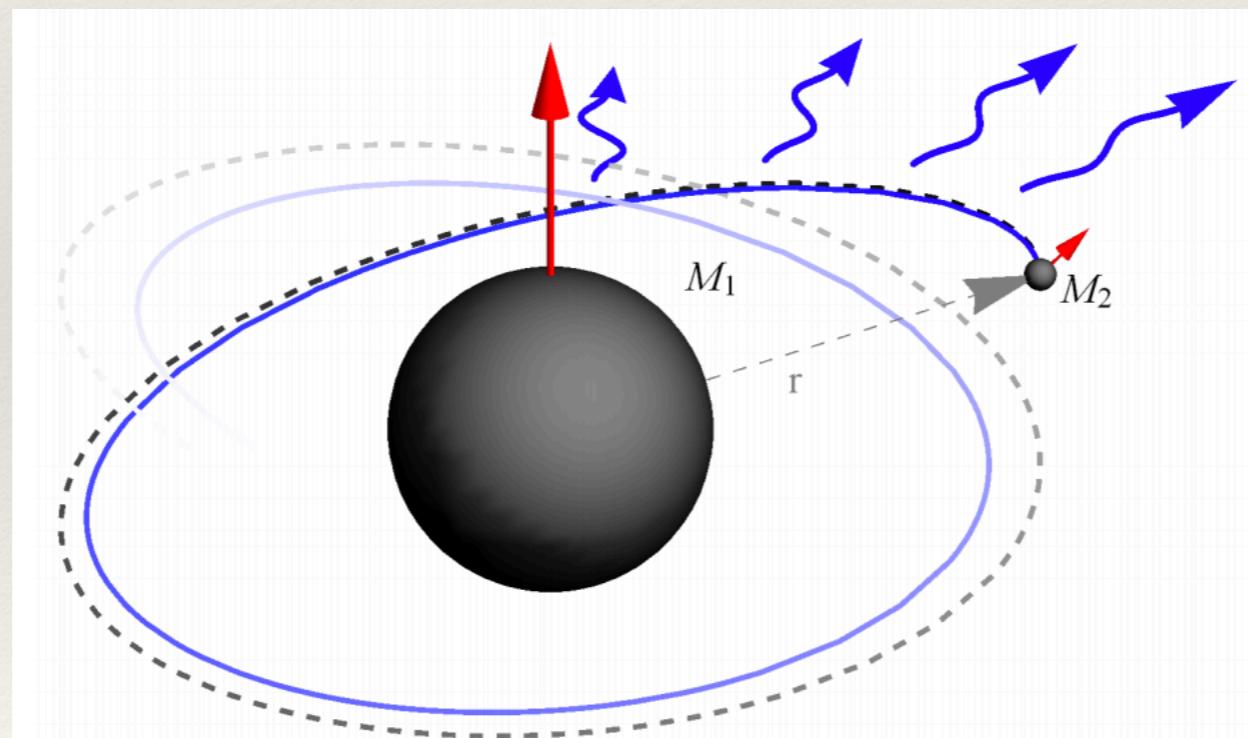
Modelling:

- Waveform is produced in the co-precessing frame
- Rotated to inertial frame of the observer
- Problems after merger...



# Extreme mass ratio inspirals

- Capture of a small compact object (CO): stellar mass BH, NS, WD by a massive black hole in the galactic nuclei (LISA sources)
- Extremely larg mass ratio: spend long time in vicinity of MBH before they plunge: ( $v/c$ ) is not small: PN theory is not accurate. NR — not efficient.
- The mass ratio ( $m_2/m_1 \equiv m/M \ll 1$ ) is now a small parameter
- Geodesic motion in Kerr + adiabatic evolution: osculating elements



Credits: Maarten van de Meent

# EMRIs: long story

- Geodesic motion of a test mass in Kerr spacetime (Boyer-Lindquist coordinates):  
derived from super-Hamiltonian:  $\mathcal{H} = \frac{1}{2}g_{\mu\nu}p^\mu p^\nu = -\frac{1}{2}m^2$

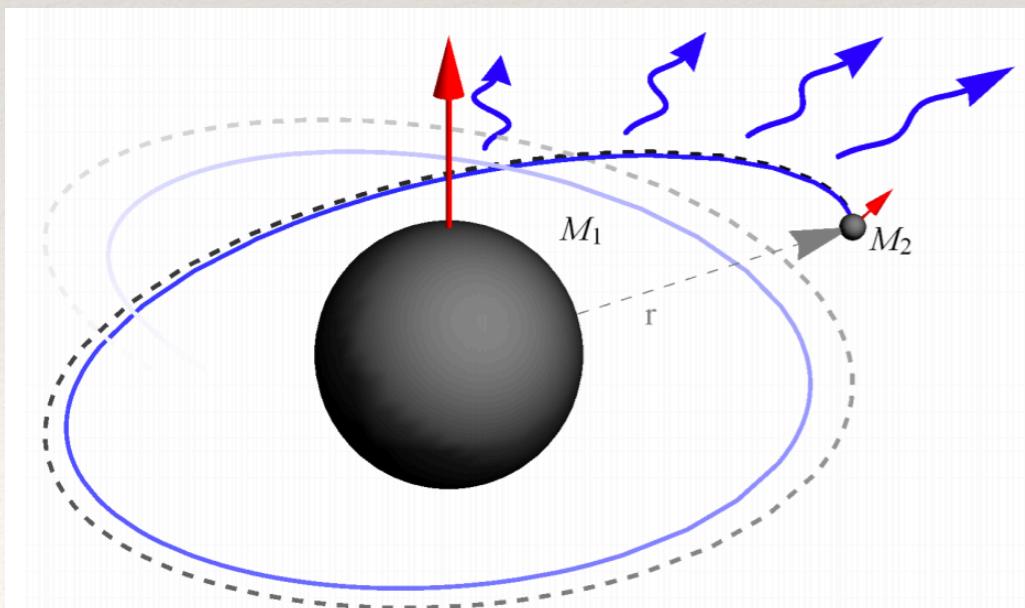
$$\begin{aligned}\Sigma \frac{dr}{d\lambda} &= \pm \sqrt{R(r, L_z, E, Q)}, & \Sigma \frac{d\phi}{d\lambda} &= \Phi(r, \theta, L_z, E, Q) \\ \Sigma \frac{d\theta}{d\lambda} &= \pm \sqrt{\Theta(r, L_z, E, Q)}, & \Sigma \frac{dt}{d\lambda} &= \mathcal{T}(r, \theta, L_z, E, Q)\end{aligned}$$

Turning points

$$\Sigma = r^2 + a^2 \cos^2 \theta$$

↑  
spin of MBH

- Geodesic is parametrized by affine parameter  $\lambda$ , and there are 8 constants of motion:  
4 initial coordinates ( $t_0, r_0, \theta_0, \phi_0$ ) and 4 1st integrals ( $E$  - energy,  $L_z$  projection of orbital momentum on the spin of MBH,  $Q$  - Carter constant,  $m$ )



Credits: Maarten van de Meent

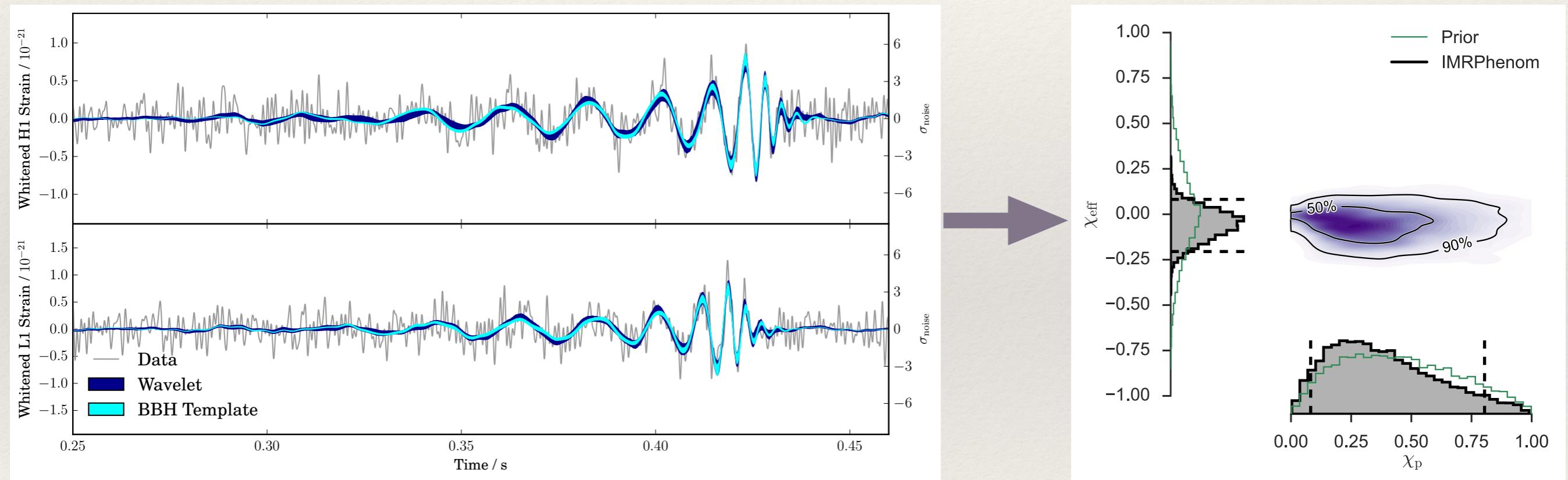
- GW carry energy and angular momentum away
- Adiabatic approximation: Orbit evolves slowly
- Osculating approach: at each instance fit a geodesic: slow change from one geodesic to another:  $\dot{E}, \dot{L}_z, \dot{Q}$  — slowly changing functions to be plugged into eq. of motion

# Bayesian approach: parameter estimation

$$p(\theta|d) = \frac{p(d|\theta) p(\theta)}{p(d)}$$

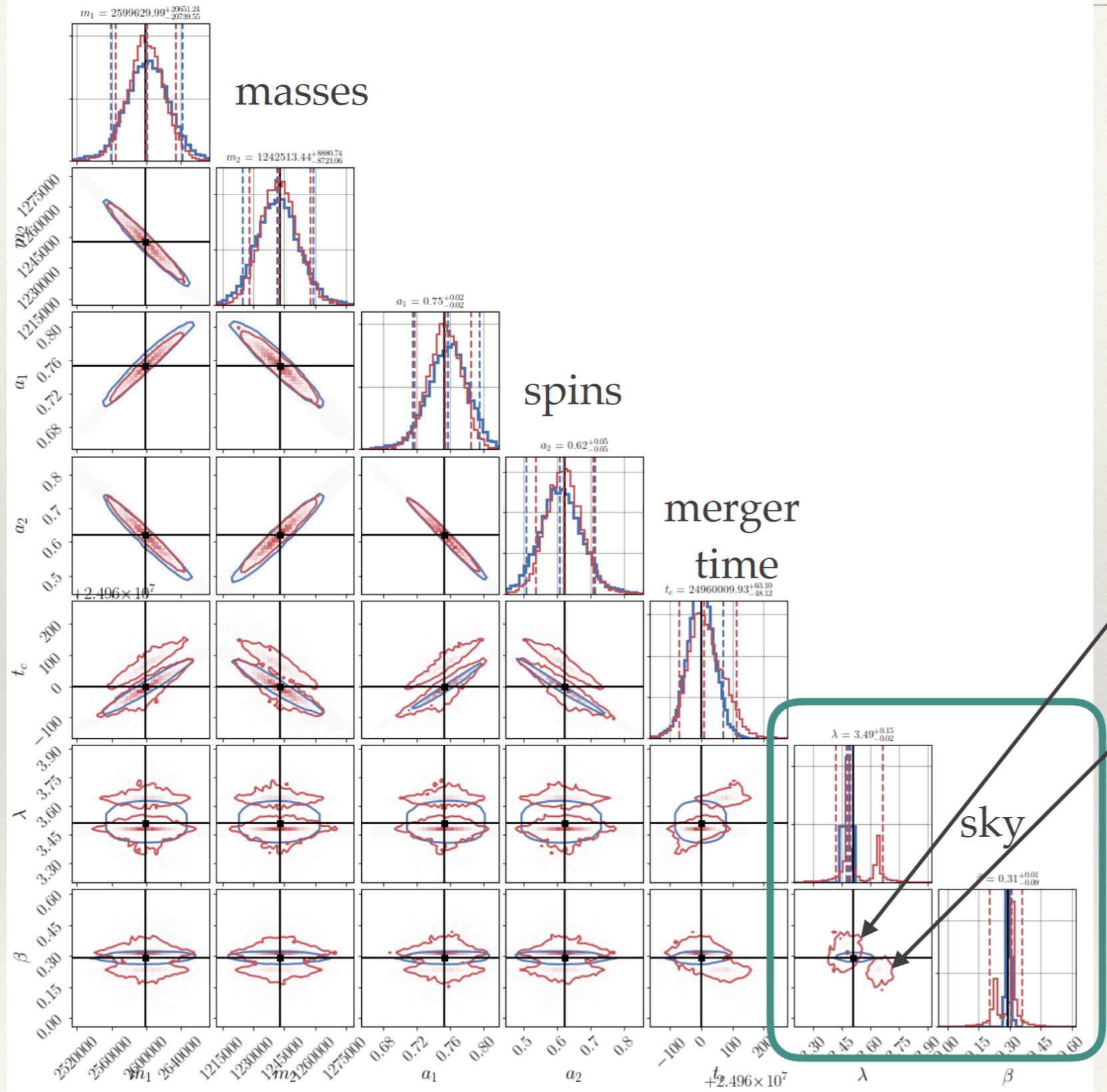
Diagram illustrating the Bayesian formula for parameter estimation:

- Likelihood:  $p(d|\theta)$  (green circle)
- Prior:  $p(\theta)$  (blue circle)
- Posterior:  $p(\theta|d)$  (brown circle)
- Evidence:  $p(d)$  (black arrow)



[GW150914, LSC+VIRGO PRL (2016)]

# Search for MBHBs



Bimodality in the position  
of source in the sky

# Bayesian approach: model selection

- Sometimes we have several competing models: Are BHs spinning? Is GR or an alternative theory? Is there one GW signal in the data or more?

$$P(M_i|d) = \frac{P(d|M_i)\pi(M_i)}{p(d)}$$

Probability of model  $M_i$  given observational data  $d$

likelihood      prior

$$P(\vec{\theta}_i|M_i, d) = \frac{P(d|\vec{\theta}_i, M_i)\pi(\vec{\theta}_i)}{p(d|M_i)}$$

posterior      evidence

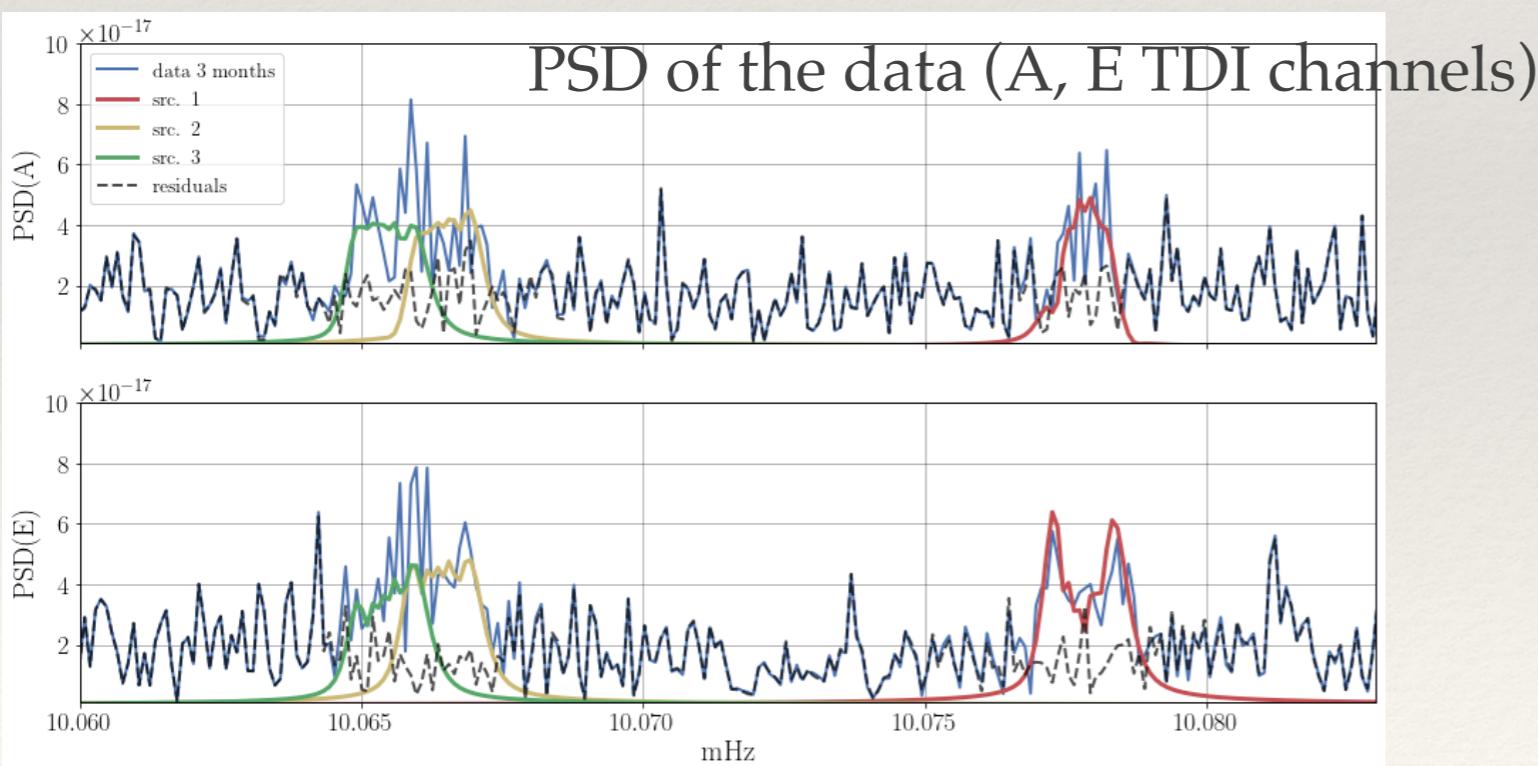
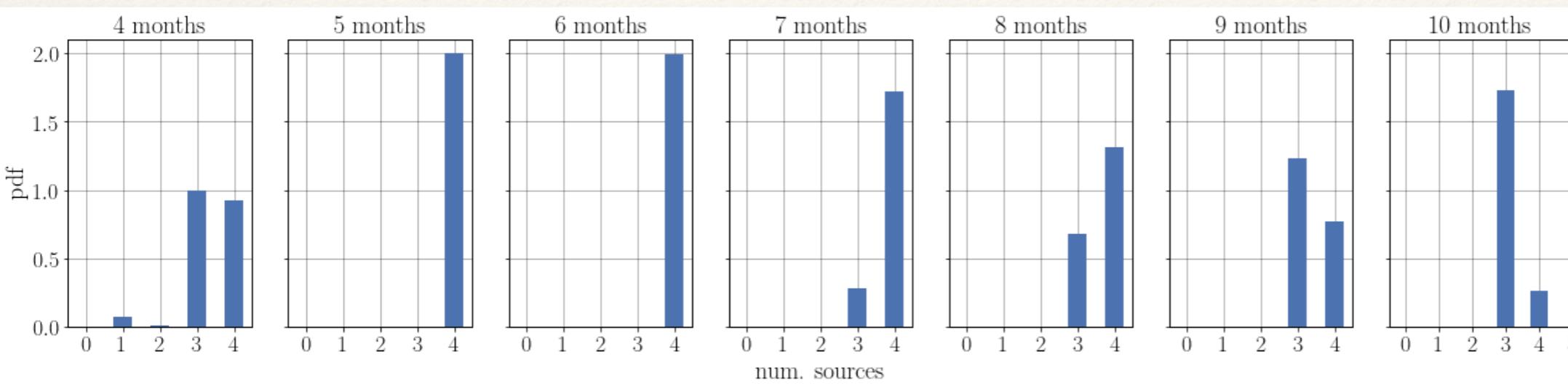
$$p(d|M_i) = \int d\vec{\theta}_i p(d|\vec{\theta}_i, M_i)\pi(\vec{\theta}_i)$$

Odds ratio: which model is preferred

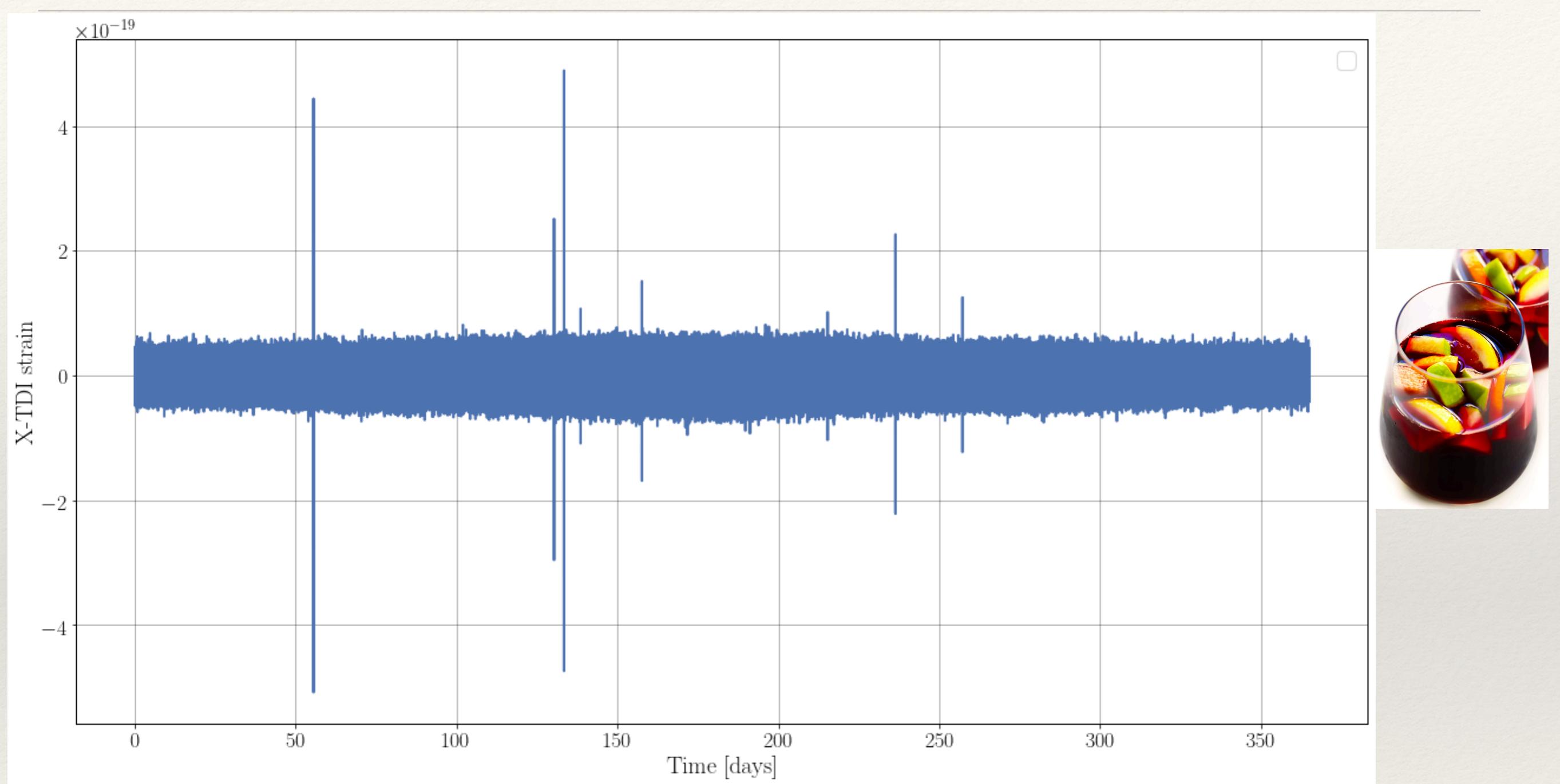
$$O_{a,b} = \frac{p(M_a|d)}{p(M_b|d)} = \frac{p(d|M_a)}{p(d|M_b)} \frac{\pi(M_a)}{\pi(M_b)}$$

# Identifying number of sources

- Take a narrow band next to 10mHz: 3 GW signals
- Perform time-data adaptive (4 months, 5 months, ... 10 months) search
- Consider 4 models: 1 GW source, 2 GW sources, 3 GW source, 4 GW sources

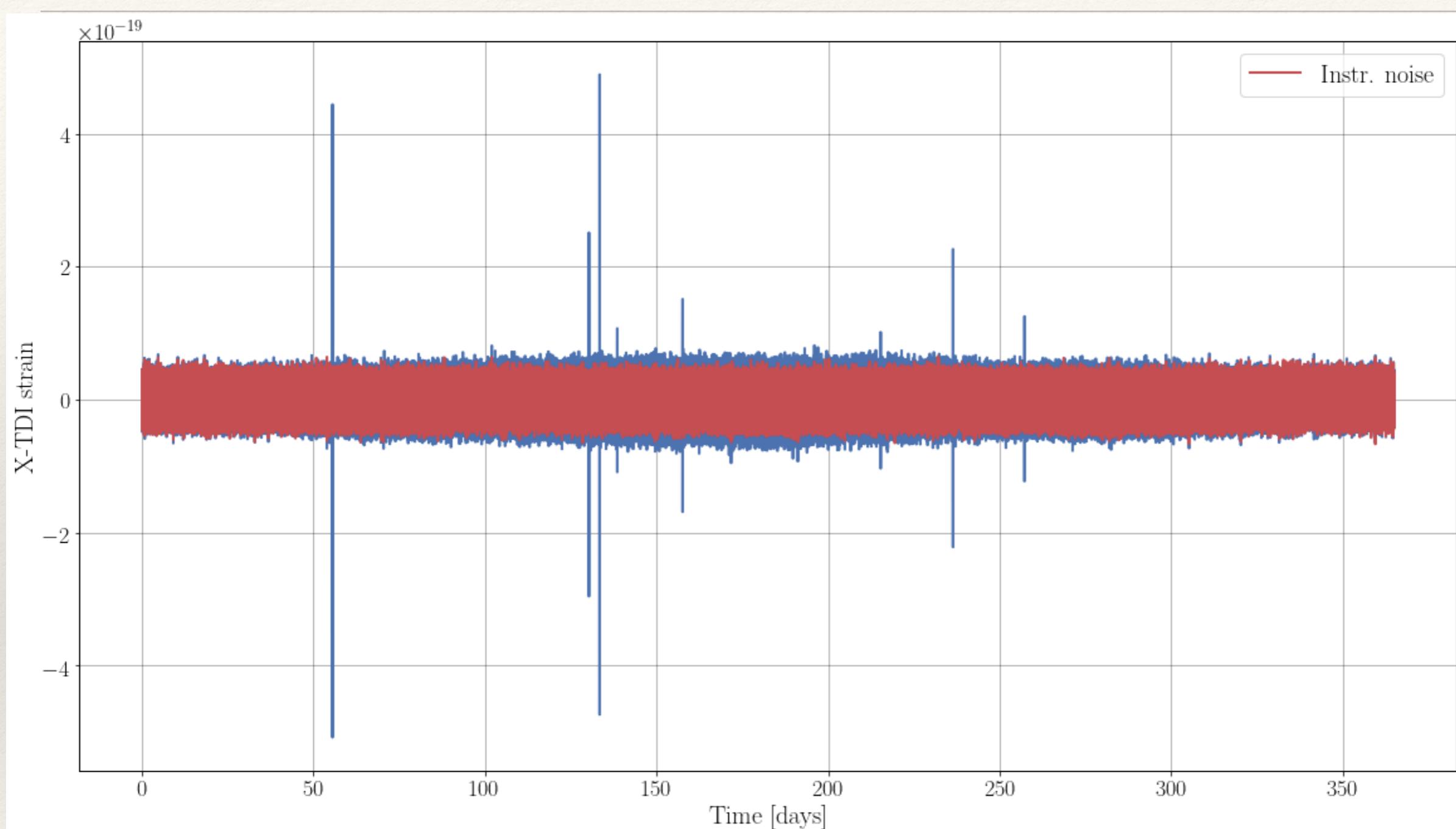


# Simulated LISA data in time domain

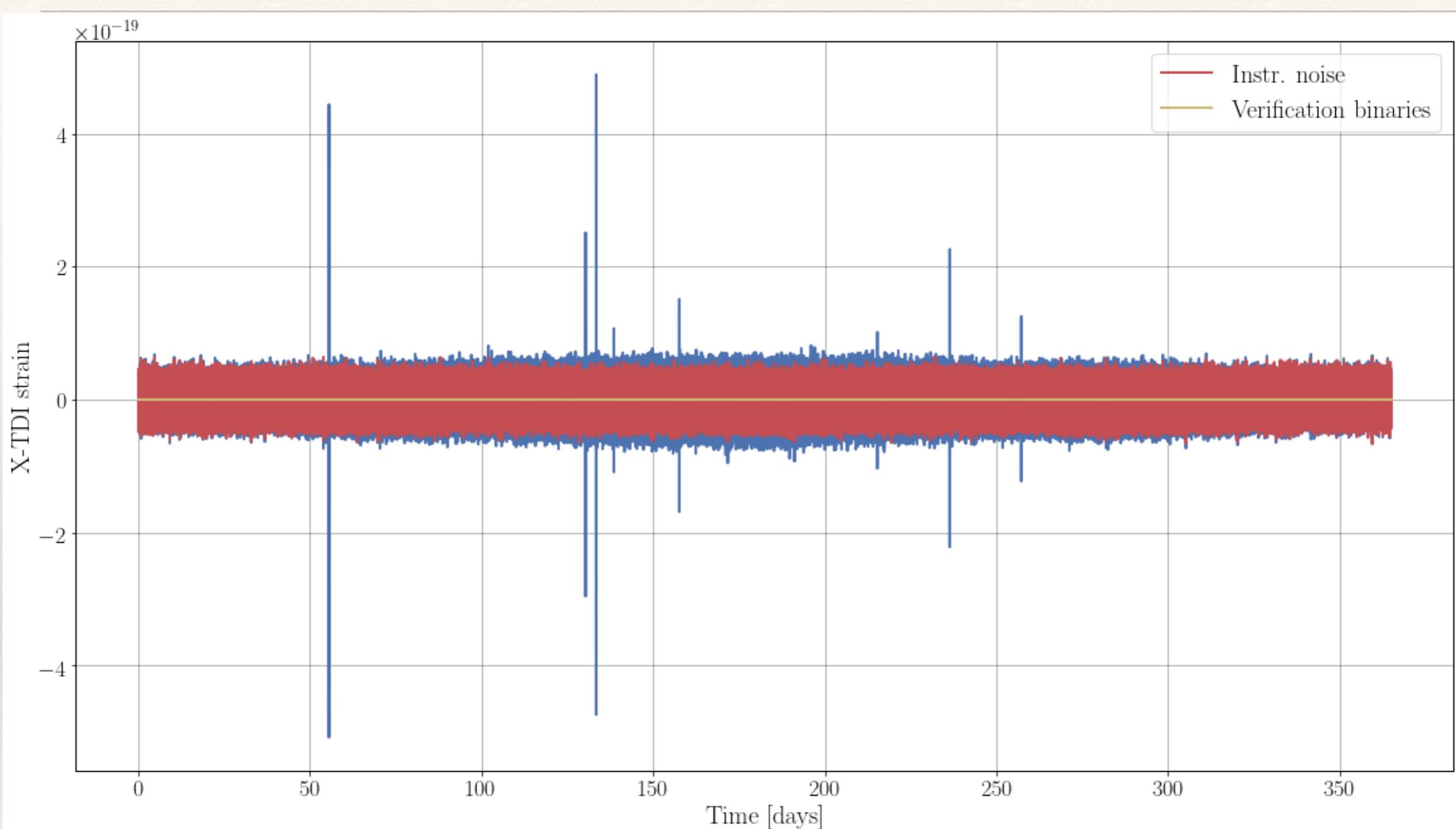


LISA data challenge 2a (**Sangria**): 20-30 mln Galactic binaries and some merging MBHB

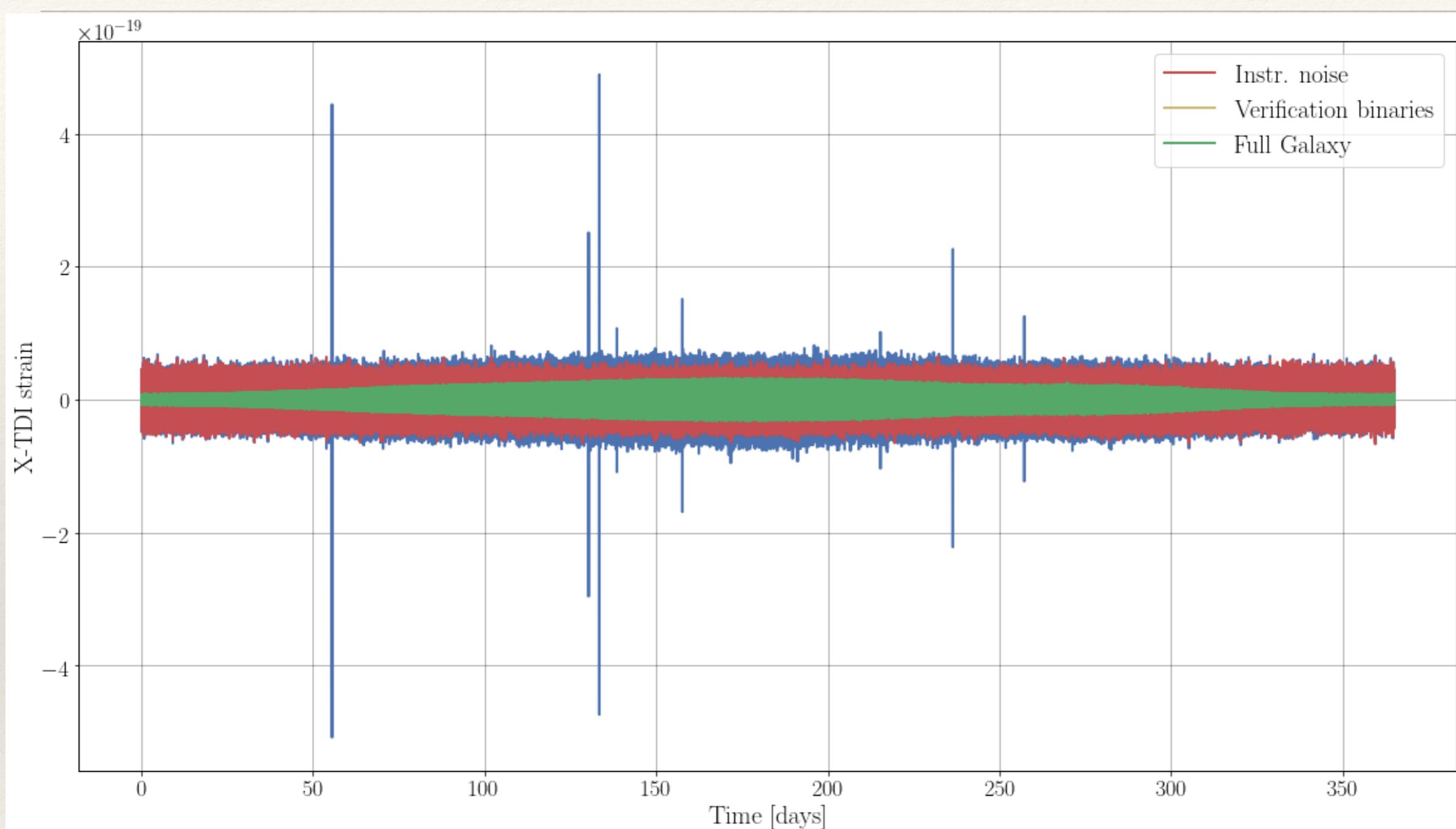
# Simulated LISA data in time domain



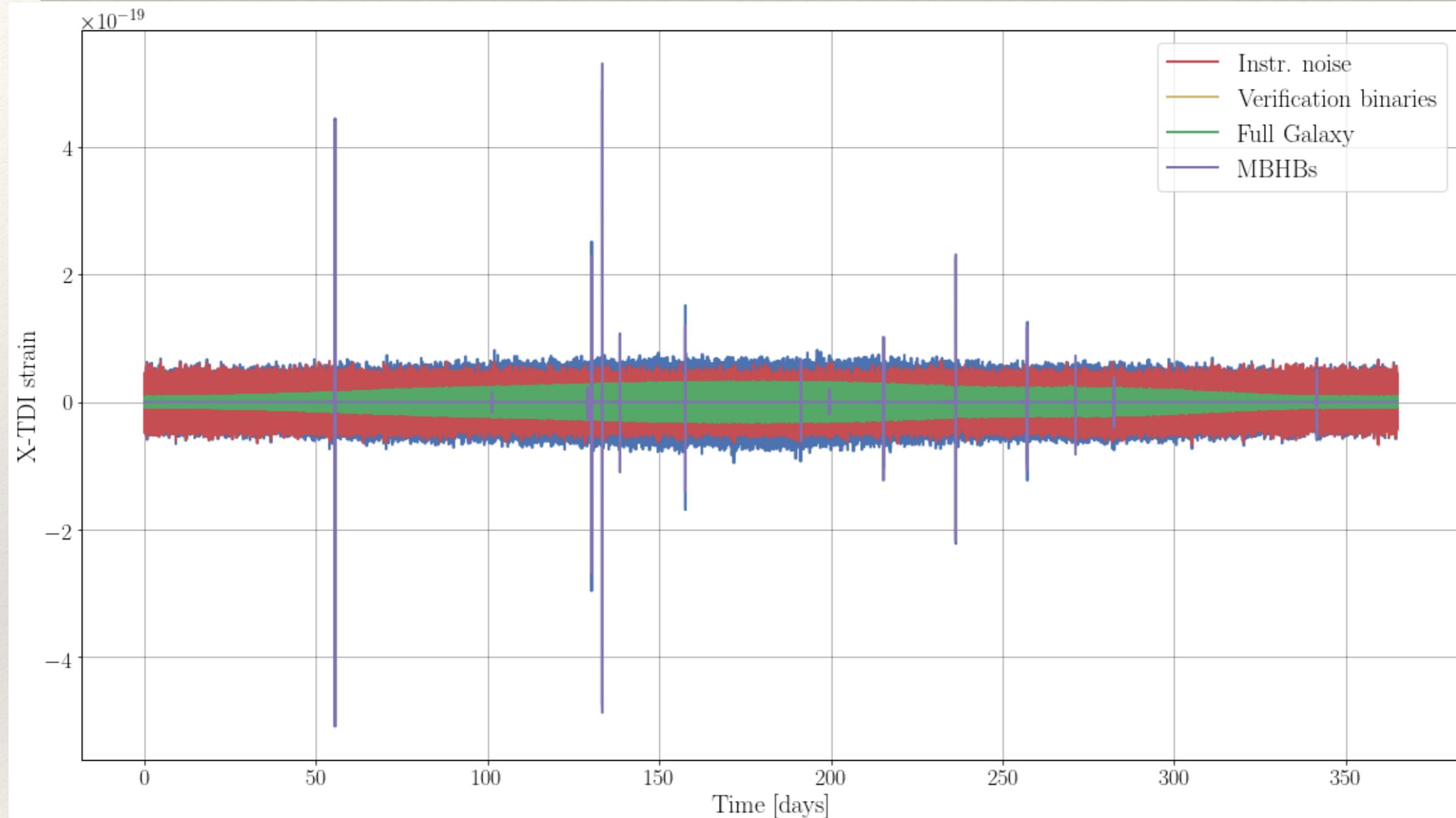
# Simulated LISA data in time domain



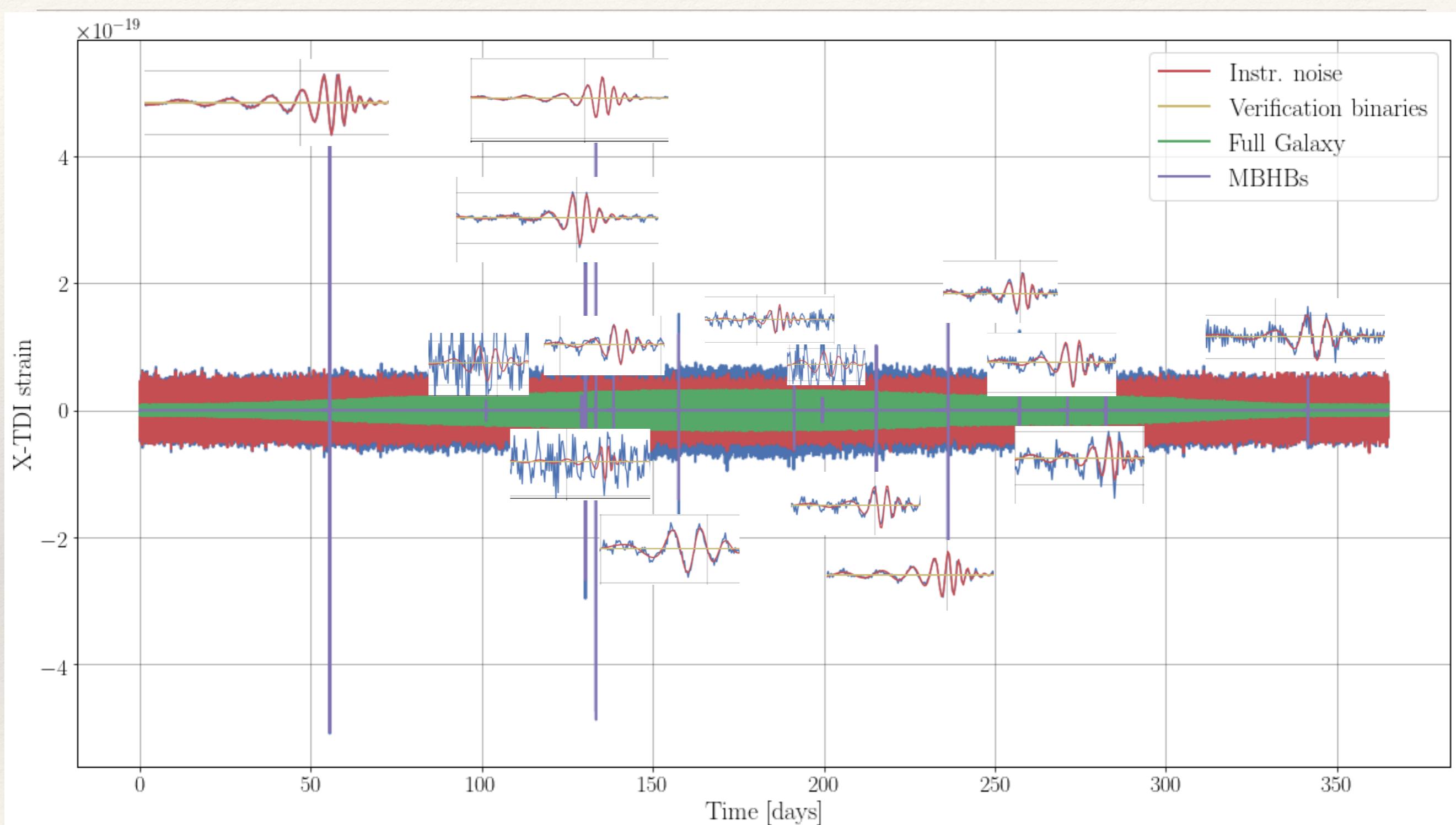
# Simulated LISA data in time domain



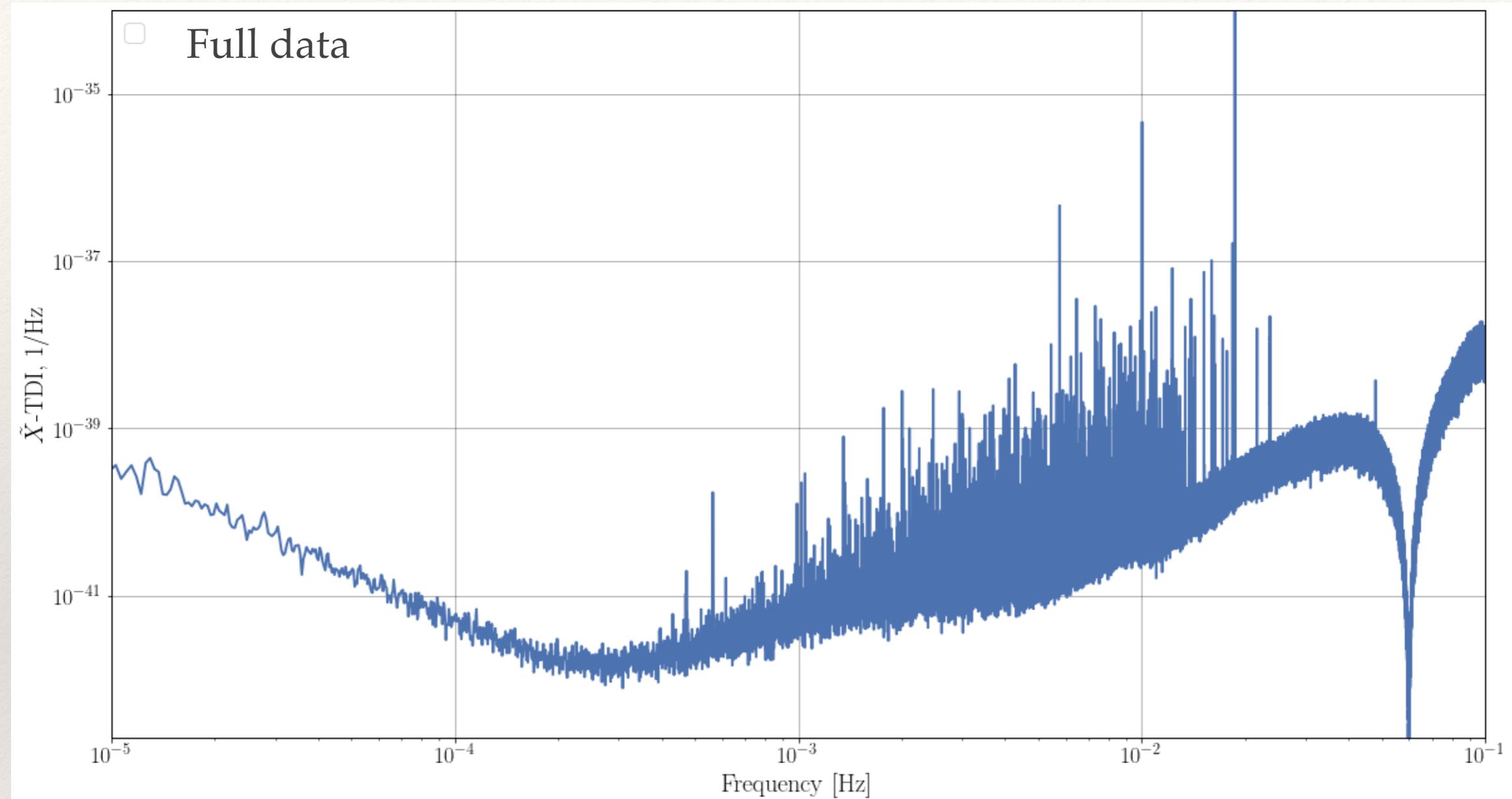
# Simulated LISA data in time domain



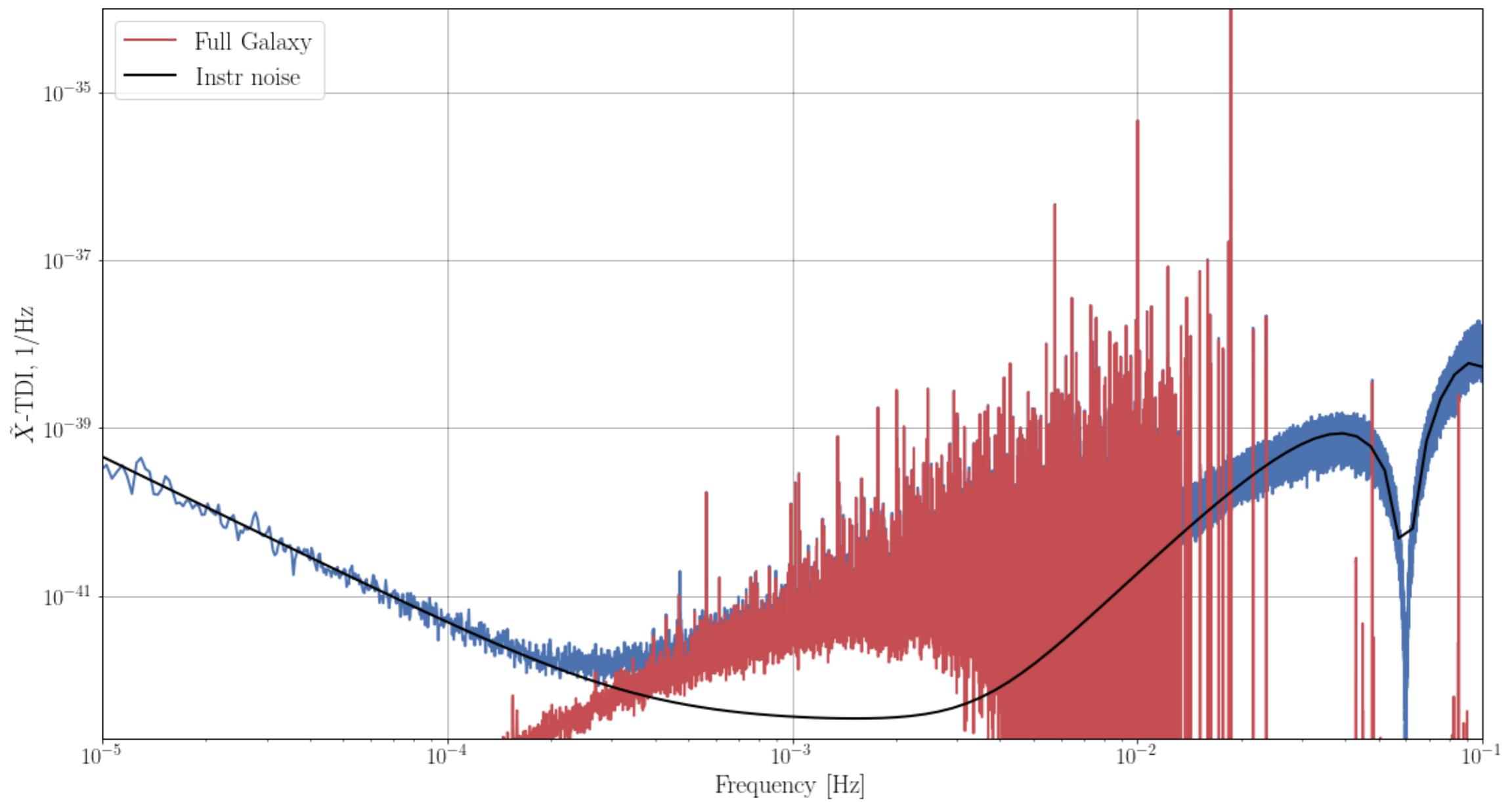
# Simulated LISA data in time domain



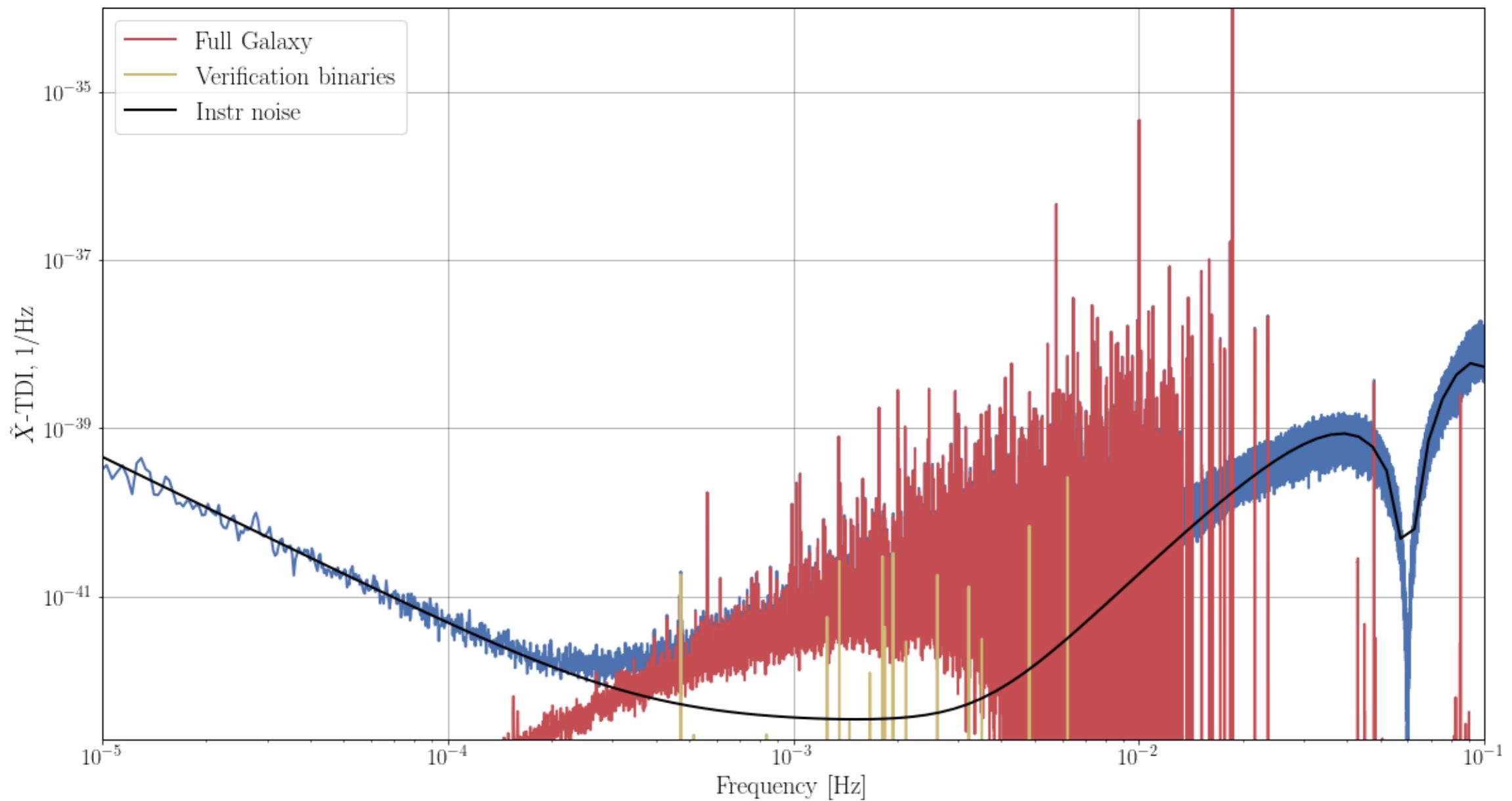
# Simulated LISA data in frequency domain



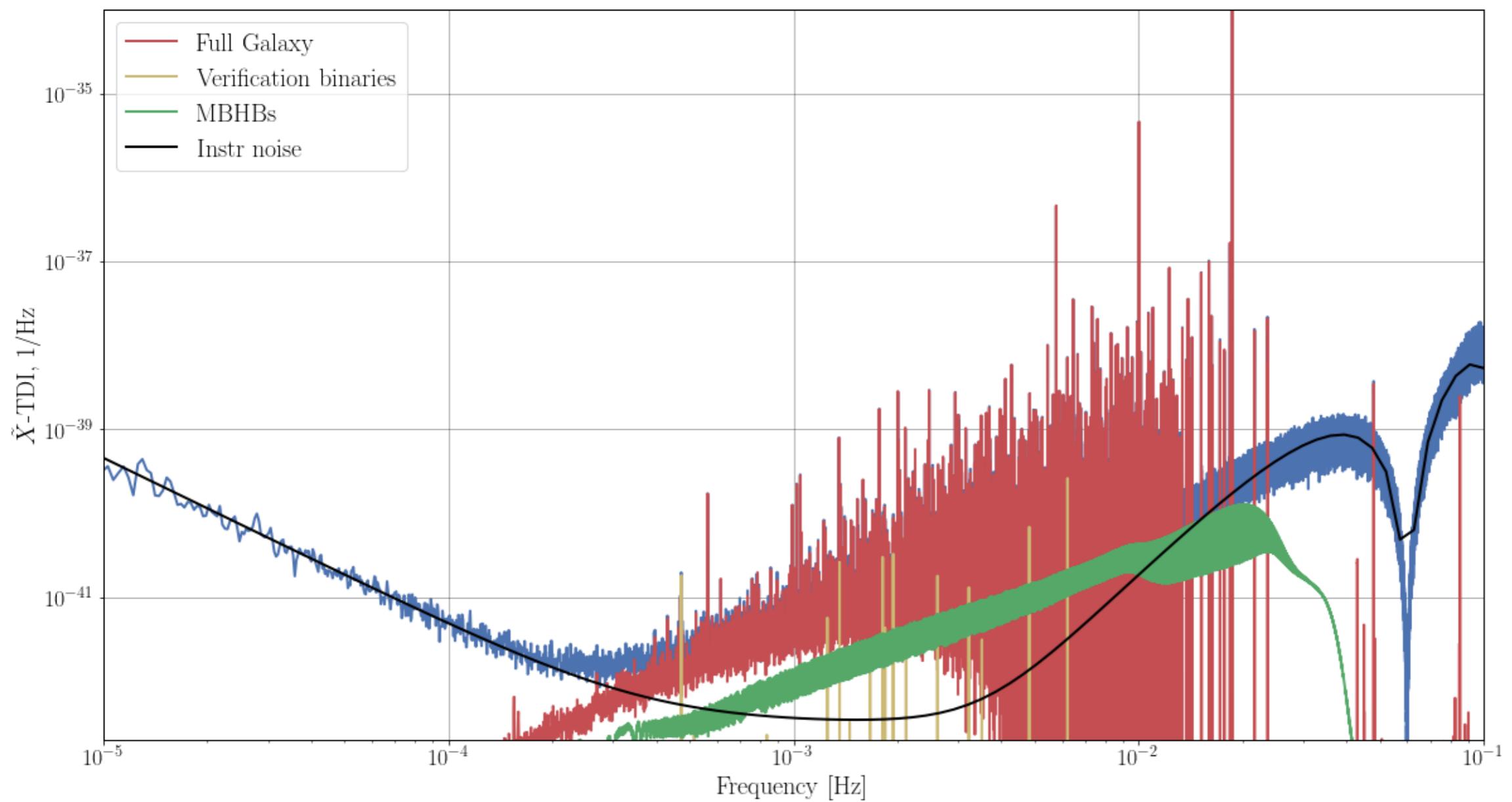
# “Sangria” in frequency domain



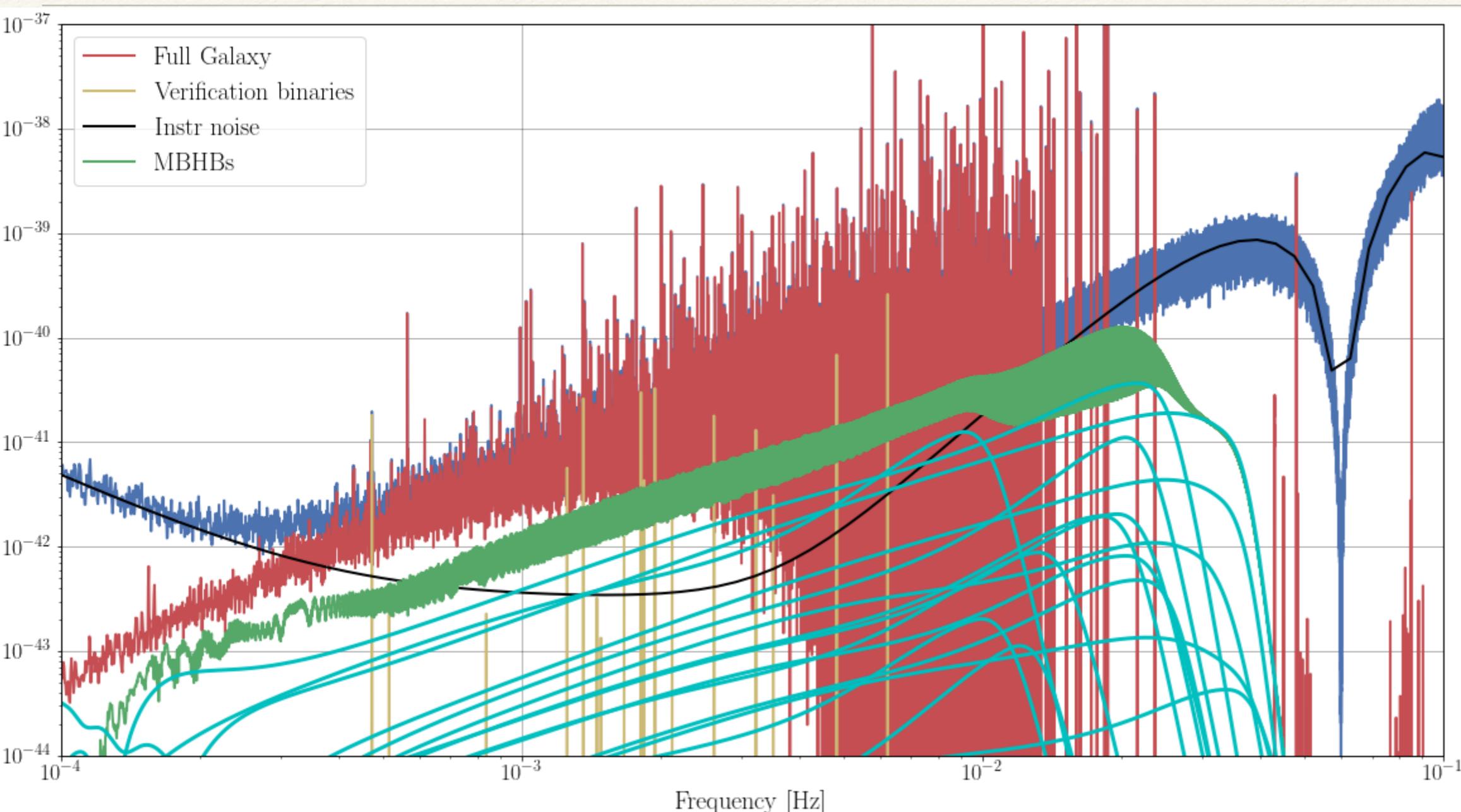
# “Sangria” in frequency domain



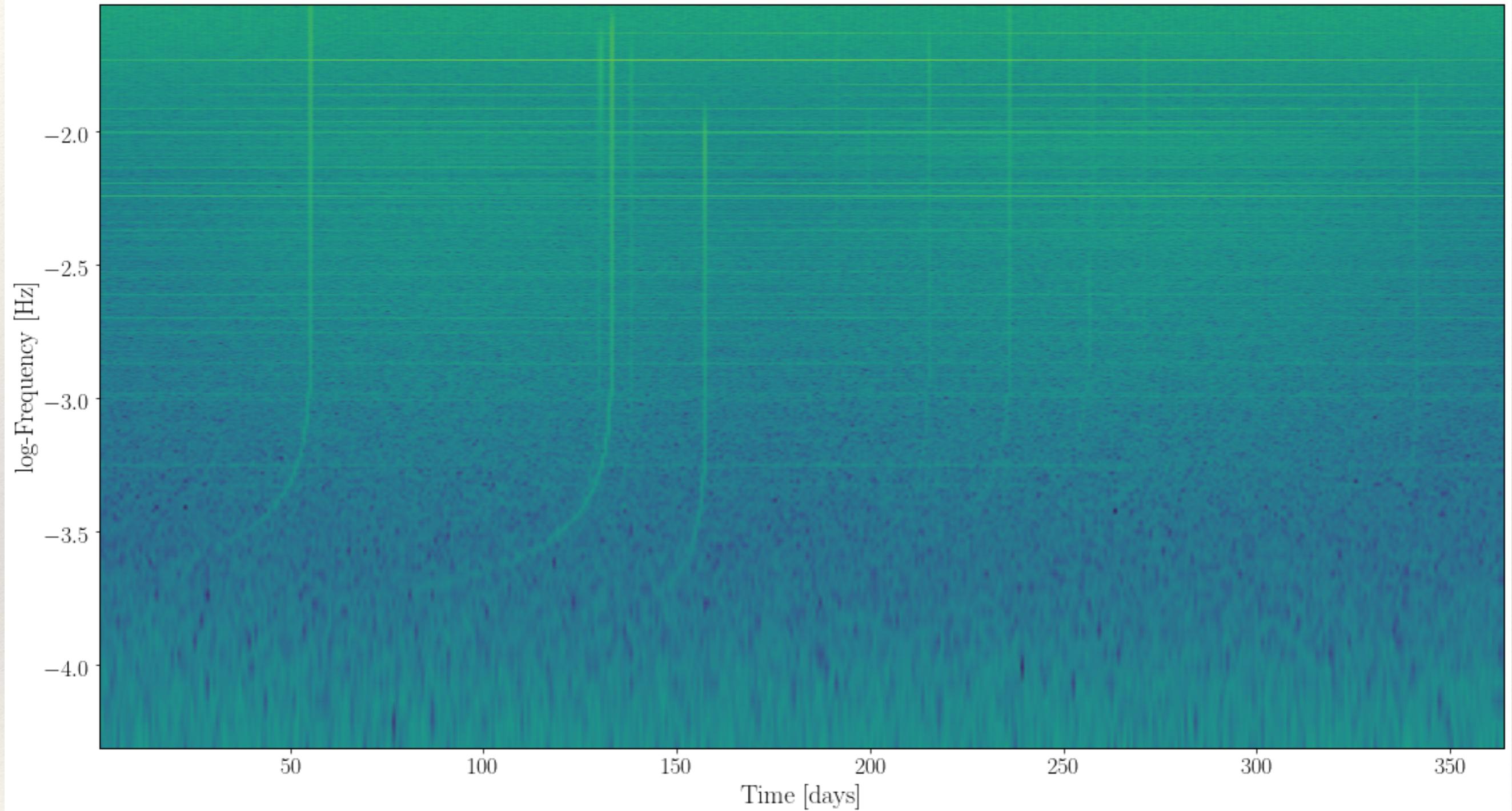
# “Sangria” in frequency domain



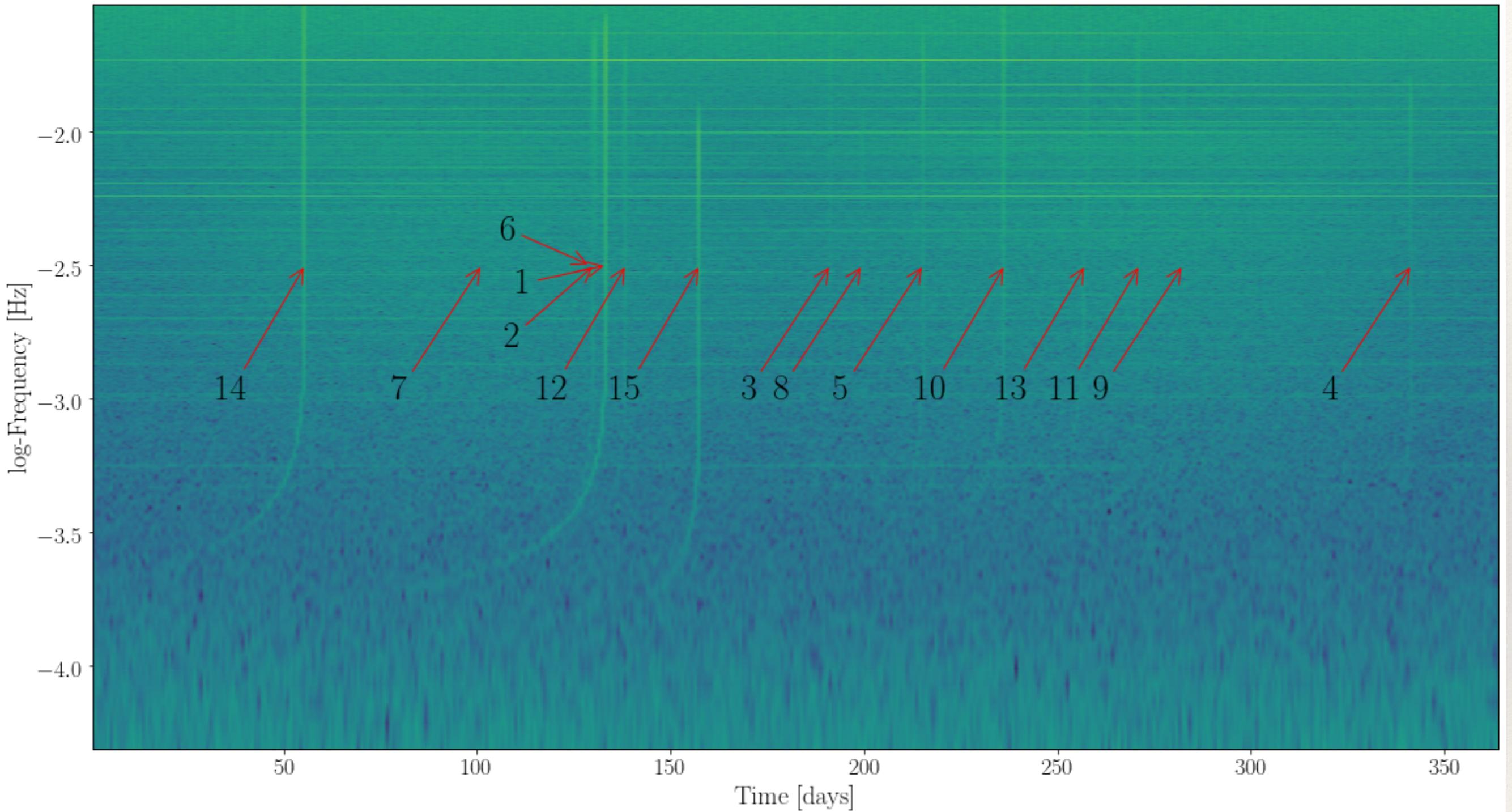
# “Sangria” in frequency domain



# “Sangria” in time-frequency



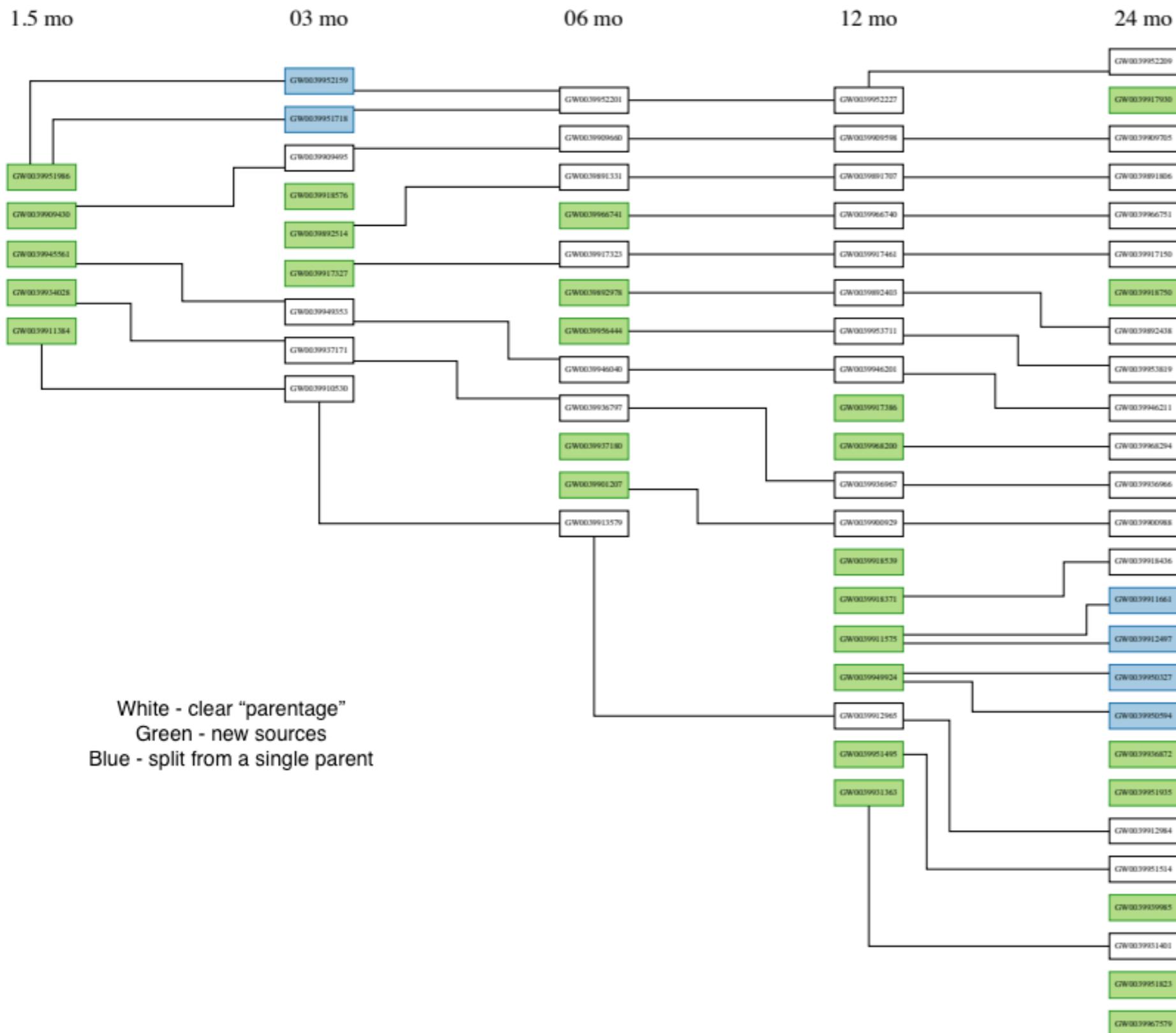
# “Sangria” in time-frequency







# Time-evolving catalogue building



[Littenberg+ PRD, 2020]

Number of sources is  
a random variable:  
building catalogue of  
sources?



# Detection of GWs in TT-frame

$$\lambda^{GW} \approx L, \quad \omega_{GW} L \approx 1$$

We cannot set the LIF covering the whole detector, but we can introduce LIF for the background curvature . This is the case for LISA and PTA. We use TT-gauge:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}^{TT} + O(h^2)$$

Consider the wave propagating in z-direction

$$ds^2 = dt^2 + [1 + h_+(t - z)]dx^2 + [1 - h_+(t - z)]dy^2 + dz^2$$

Mirrors are moving along geodesics. In TT frame the coordinate distance does not change (but the proper distance does)  $d\hat{x}^2 = g_{xx}dx^2$

Need to consider equation of propagation of e/m signal  $g^{\alpha\beta}\phi_{,\alpha}^l\phi_{,\beta}^l = 0$

The phase difference for the round trip is

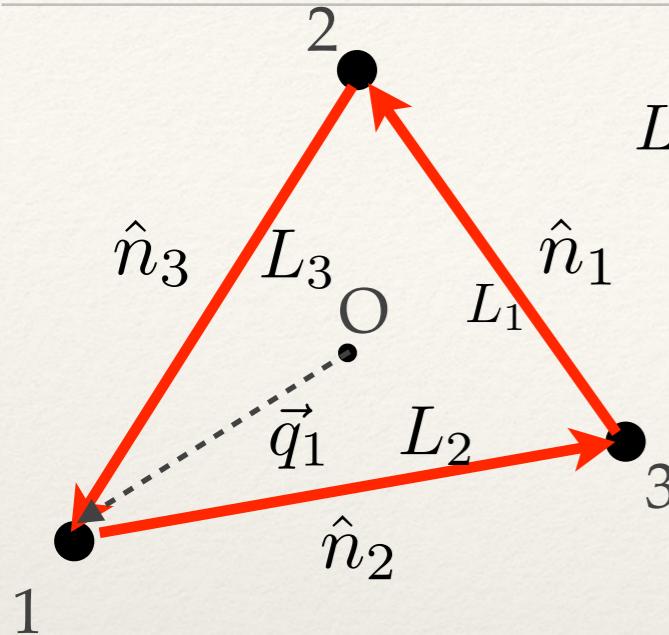
$$\Delta\phi_l = \phi_l|_x - \phi_l|_y = -\nu_l \left[ -2(L_x^{(0)} - L_y^{(0)}) + \frac{1}{2}H(t - 2L_x^{(0)}) + \frac{1}{2}H(t - 2L_y^{(0)}) - H(t) \right]$$

$$H(t) = \int_0^t h(t')dt'$$

$$\omega_{GW} L \ll 1 \longrightarrow \Delta\phi^l \approx 2\nu_l(L_x^{(0)} - L_y^{(0)} + L^{(0)}h_+(t))$$



# LISA



$L_1 \approx L_2 \approx L_3 \approx L$  — but not exactly!

$$\frac{\Delta\nu}{\nu_0} \rightarrow y_{slr} \quad \text{— for a single link}$$

Consider GW signal from a binary system

$$y_{slr}^{GW} = -i \frac{\omega L}{2} A_l(\iota, \psi, \theta_s, \phi_s) e^{i\Phi(t - \hat{k} \cdot \vec{R}_0)} e^{-i\omega \hat{k} \cdot \vec{q}_r} \text{Sinc} \left[ \frac{\omega L}{2} (1 - \hat{k} \cdot \hat{n}_l) \right] e^{-i\frac{\omega L}{2} (1 - \hat{k} \cdot \hat{n}_l)}$$

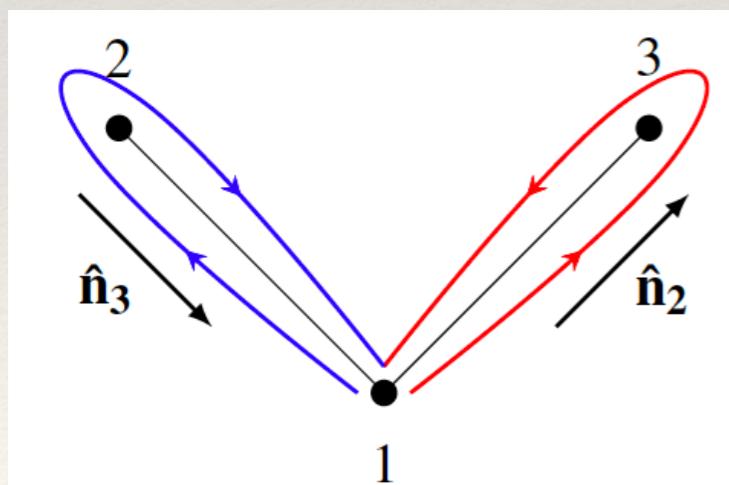
- $A_l$  Amplitude of GW times antenna beam function
- Sinc — zero of sinc function gives freq. of GW signal which cannot be measured (f-n of sky position) - wiggles in the sensitivity at high frequencies
- Phase:  $t - \hat{k} \cdot \vec{R}_0(t)$  Doppler modulation (dominant) due to relative motion of the detector and the source



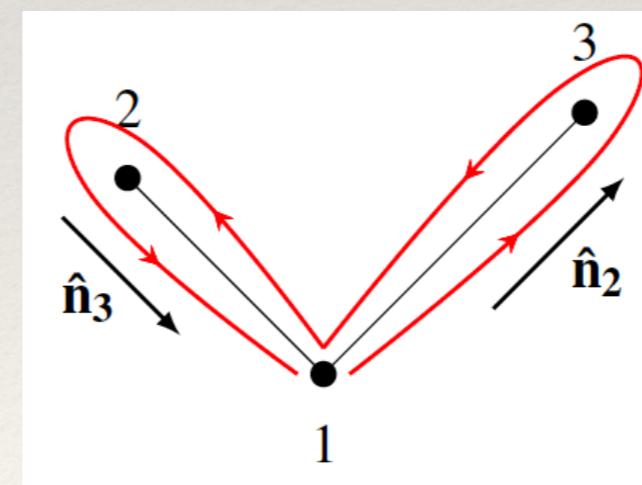
# LISA

- The localization of the sources in the sky comes from the Doppler modulation of the phase and from the amplitude modulation (time dependent antenna beam f-n)
- The term(s) dependent on the position of each spacecraft explicitly (q-vectors): important at very high frequency: constellation “feels” GW propagation
- The biggest problem is the laser frequency noise: orders of magnitude higher than other noise sources

TDI — Time Delay Interferometry: technique which we apply to cancel the laser noise (in post-processing the data)



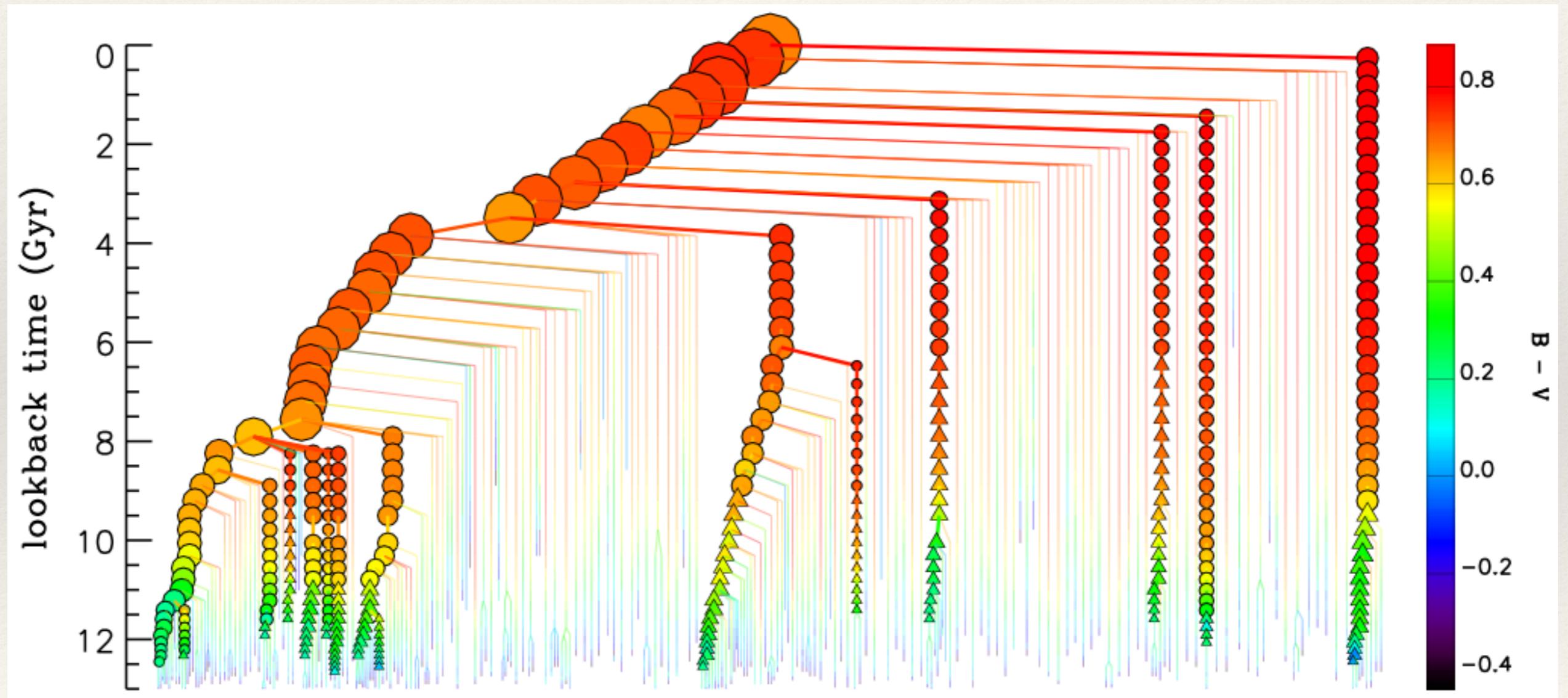
Equal arm Michelson



Unequal arm Michelson

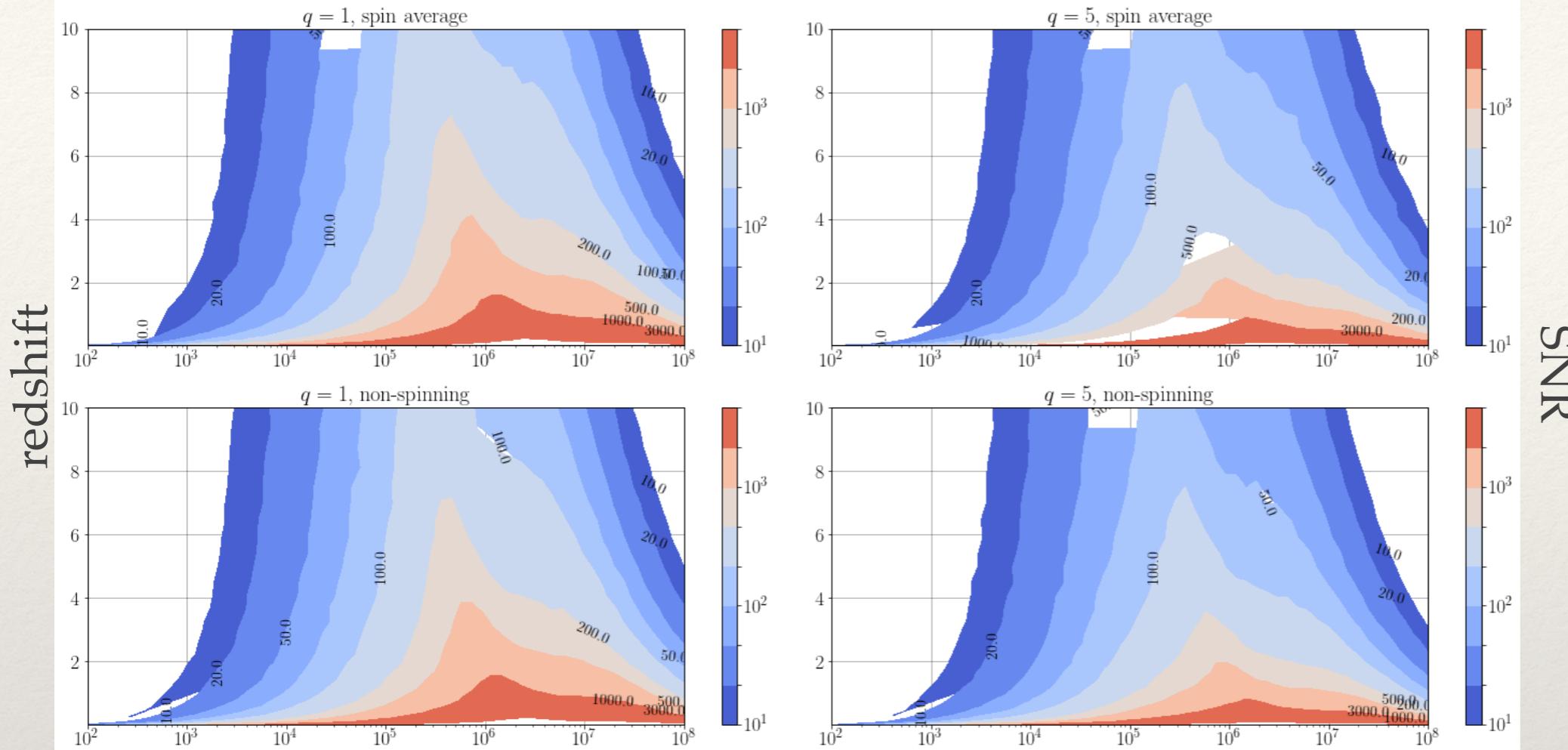
# LISA sources: MBHB

MBHs are formed from the initial BH seed. Those seeds could be “light” remnant of the first generation of stars or “heavy” from the direct collapse of a giant gas cloud. BHs accumulated the mass through gas accretion and merging.

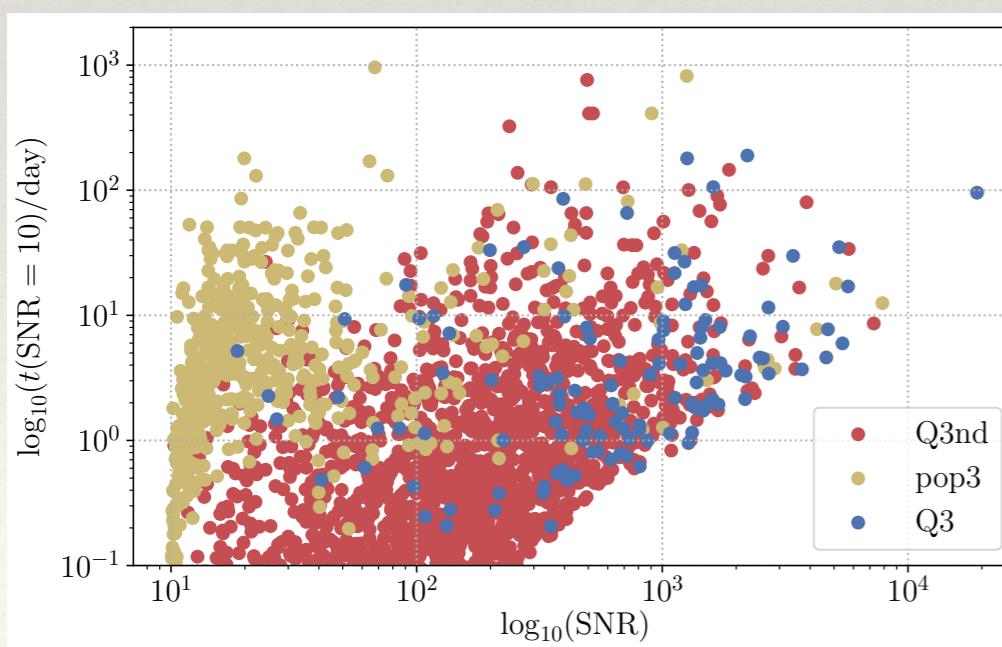


[Credits: Gabriella De Lucia]

# Detecting GW signal from MBHBs



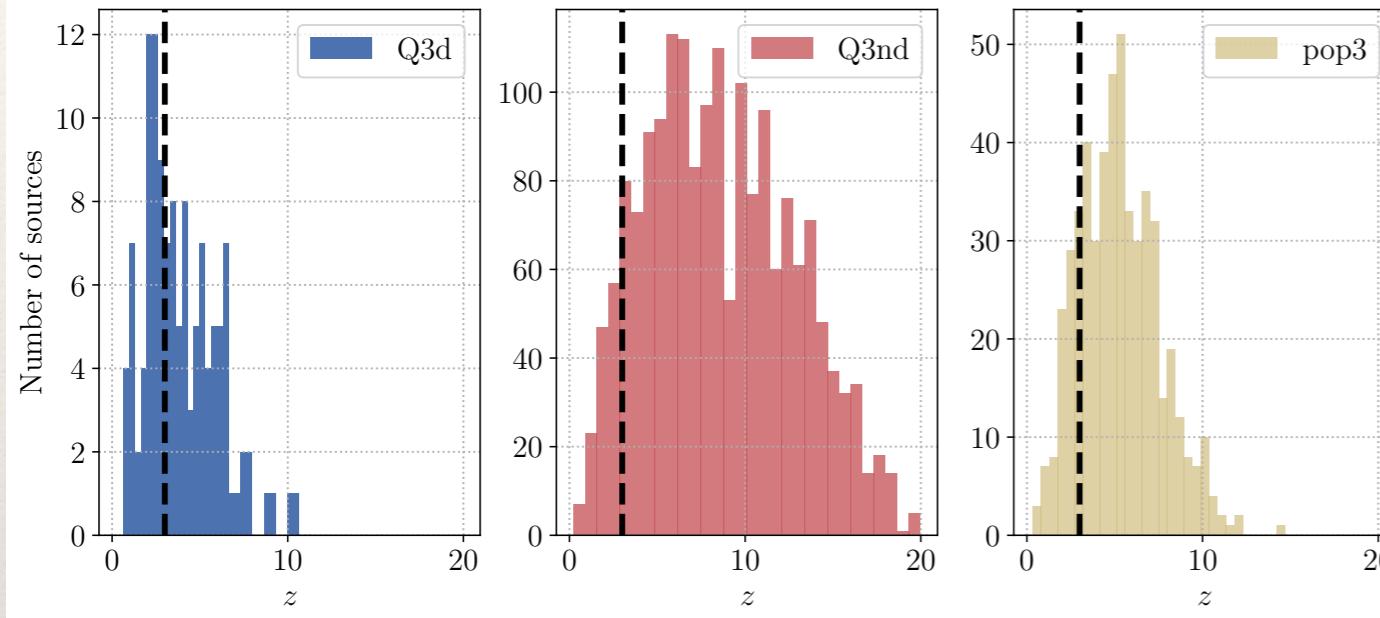
SNR



- Merger is very strong: detectable throughout the universe in mass range  $M = 10^4 - 10^7 M_\odot$
- Detectable part of the signal lasts few hours - few months
  - pop3 - light seed from first stellar population
  - Q - heavy seed from direct collapse of gaseous cloud
    - Q3d - delayed MBHBs merger from galactic collisions
    - Q3nd - no delay between galaxy collisions and MBHBs merger

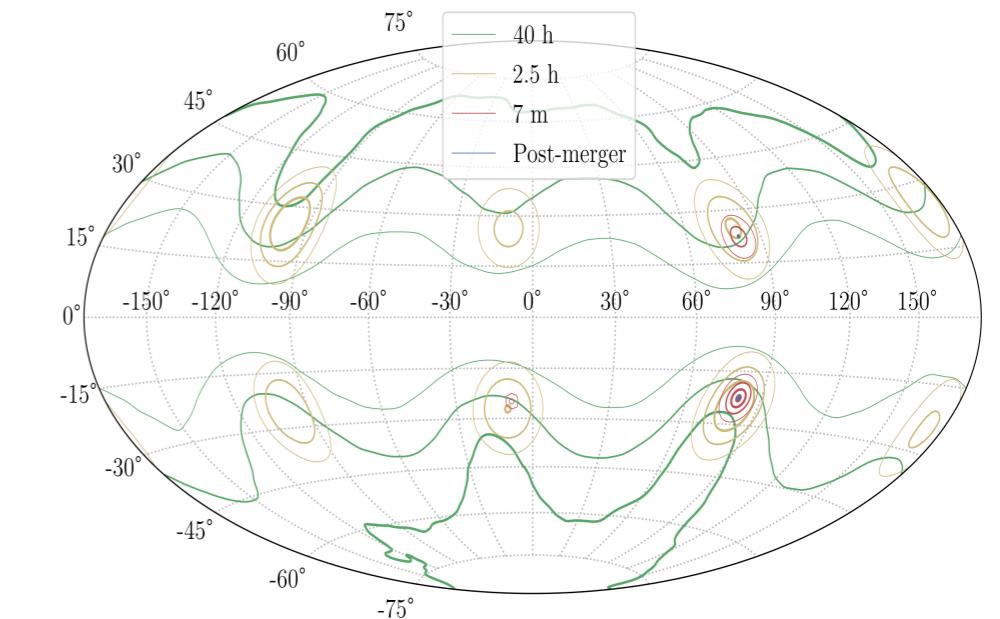
# Observing MBHBs

- Different populations have different characteristic: help to disentangle and solve inverse problem: given the LISA observation what can we infer about the evolution of MBHBs: how they were born? how they accumulated its mass



Simultaneous (multimessenger) observations of MBHB: GW + e/m. Need to localize the source in the sky to point telescopes: pre-merger is weak signal, not enough info - up to 8 modes on the sky - improvement as we approach merger. Necessity to follow the signal in the real time

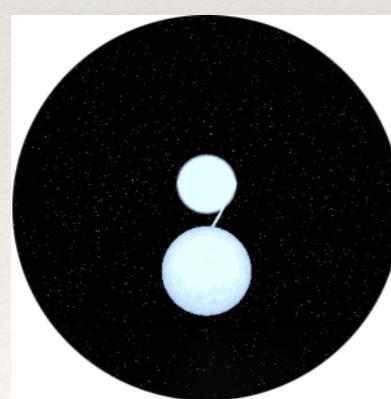
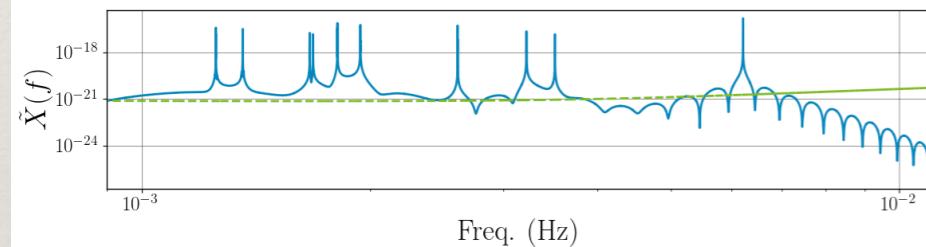
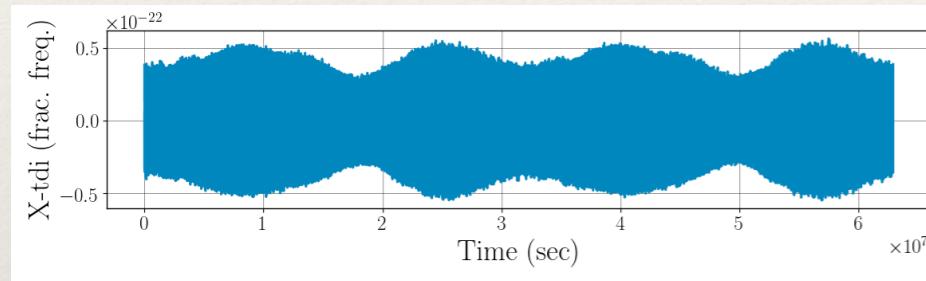
Sky map as a function of time: multimodality



Marsat+2020

# Verification Galactic binaries

- There are few dozen of GBs observed in e/m.
- Can measure orb. period, sky position, distance: guaranteed sources in the LISA's band
- More verification binaries are being discovered (ZTF, GAIA, LSST)



credits: G. Nelemans

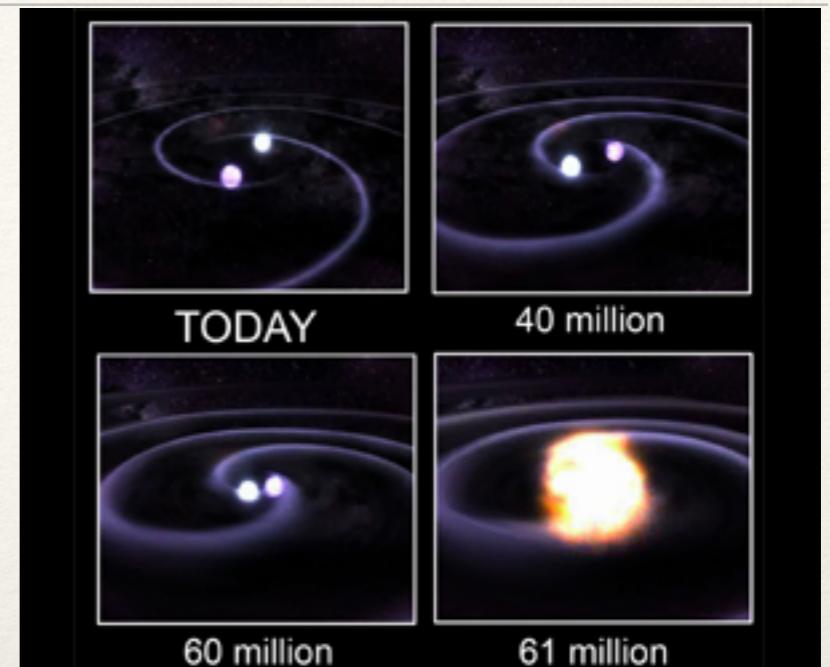
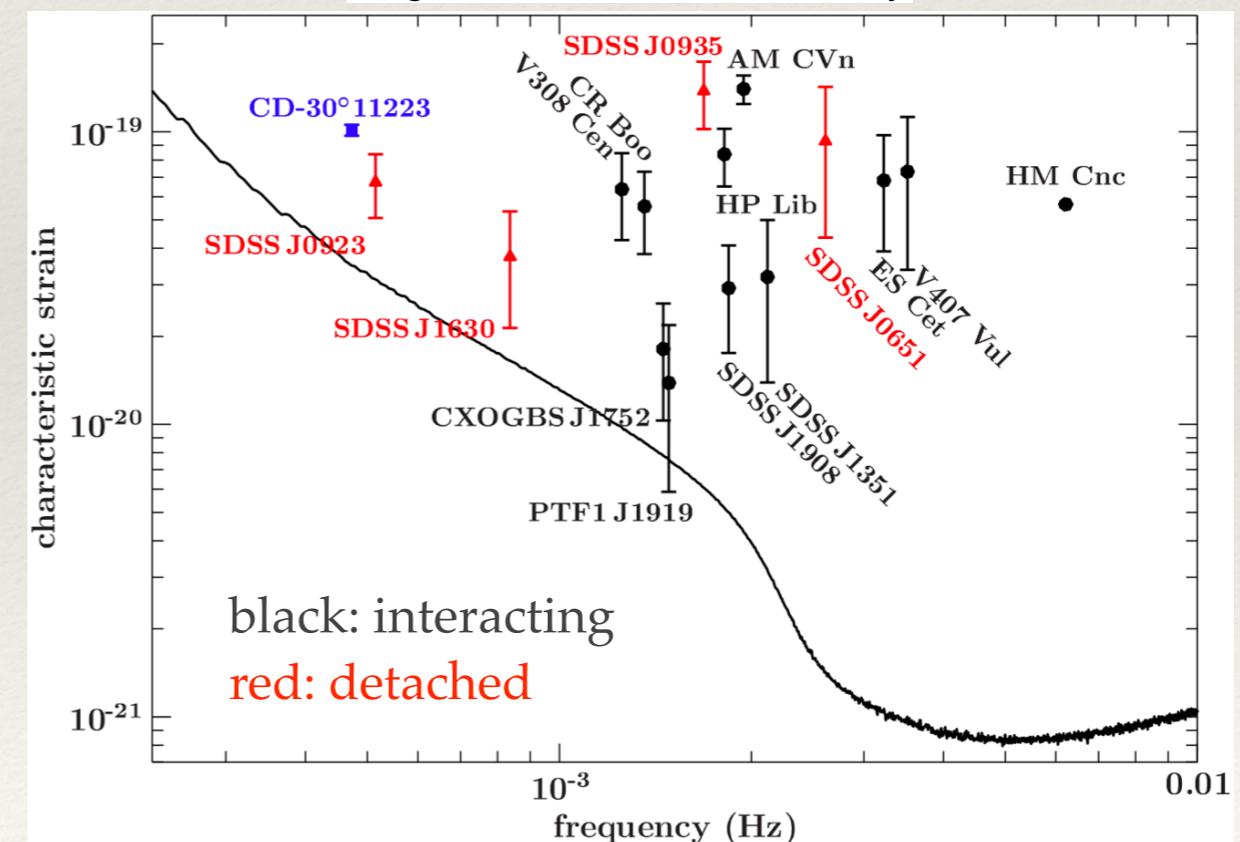


Image Credit: NASA/GSFC/D.Berry.



# Expected event rate in LISA

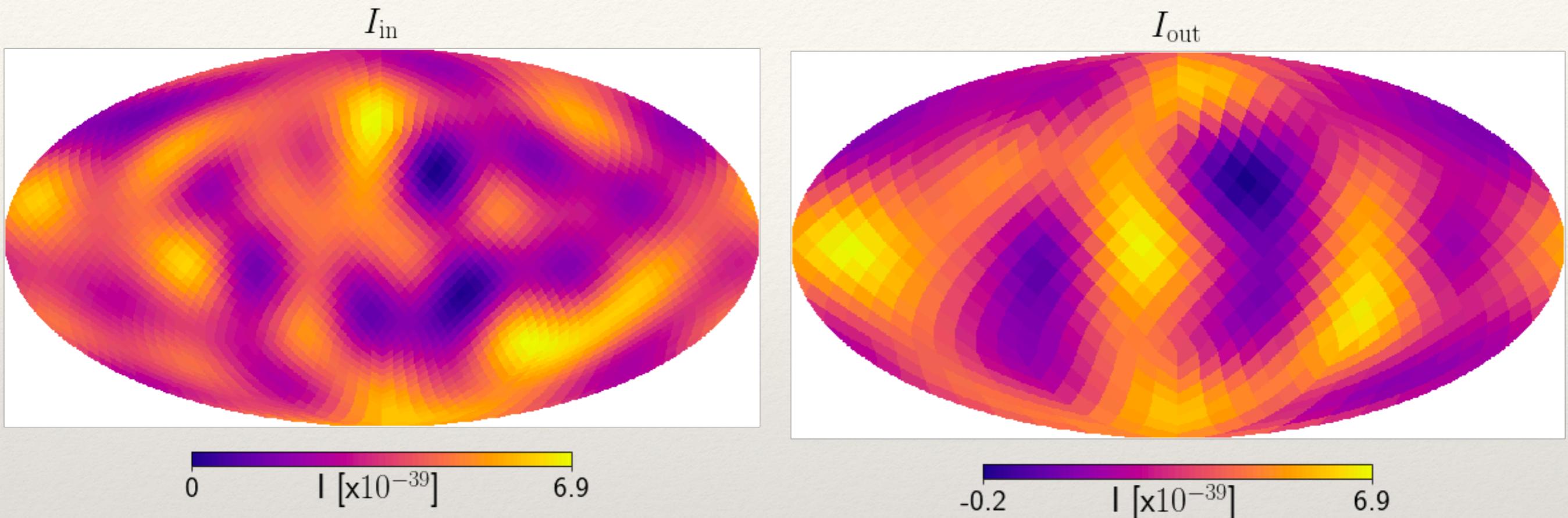
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- LISA's minimum mission time is 6 years (expected at 75% duty cycle), consumables up to 10 years
- MBHB : high uncertainties in the event rate - from few to few hundreds per year
- EMRIs: even more uncertain - from few to few thousands of detectable GW signals per year.
- GW signal from solar mass BBH (LIGO/VIRGO sources). We expect to observe about 10 sources: GW signal first observed in LISA and then 5-10 years later with the ground based detectors.
- Possible detection of the stochastic GW signal from energetic processes in the early Universe.



# Stochastic GW signal

Reconstruction of Anysotropy



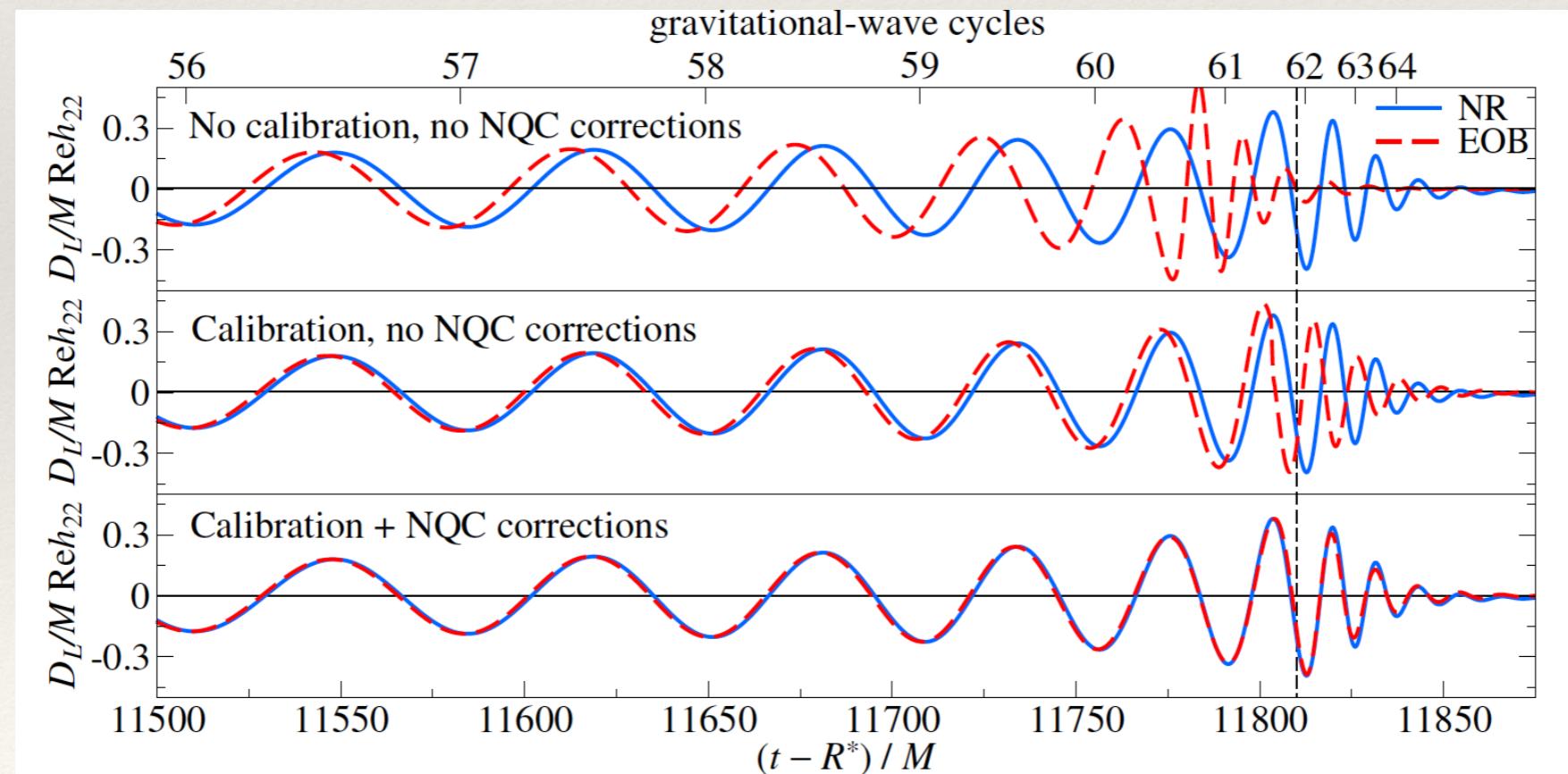
*Smoothing on small scales (complete loss of sensitivity at  $l \sim 15$ )*

[Contaldi+ (2006)]

# EOBNR: calibration

- It was somewhat simplified description of EOB model (reality is a bit ugly)
- Adiabatic transition from circular-to-circular breaks: non-quasi-circular (NQC) corrections
- Missing high PN-terms important close to the merger
- The RD part is taken from the *linear* perturbation of a single BH: two merging BHs pass through a highly non-linear regime: requires extra (pseudo) QNMs or phenomenological RD part [Damour & Nagar 2014].

**NR waveforms used to extend and to improve EOB-> EOBNR which also makes them partially phenomenological model**



Plot: courtesy of A. Taracchini



# Bayesian approach: parameter estimation

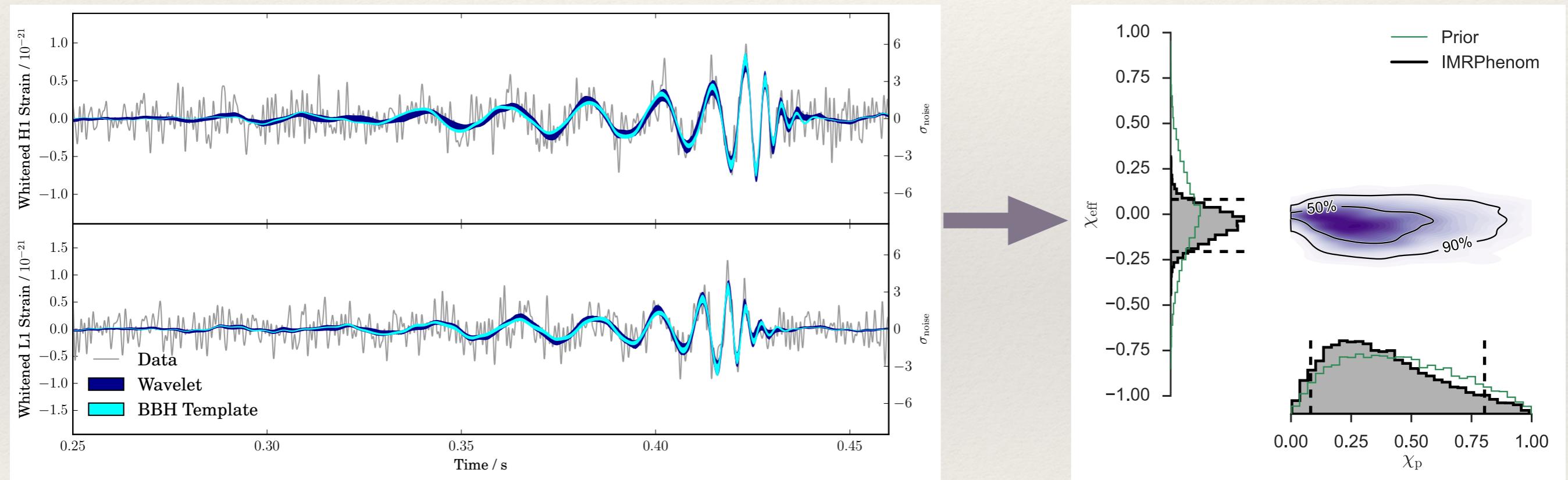
$$p(\theta|d) = \frac{p(d|\theta)p(\theta)}{p(d)}$$

Posterior

Likelihood

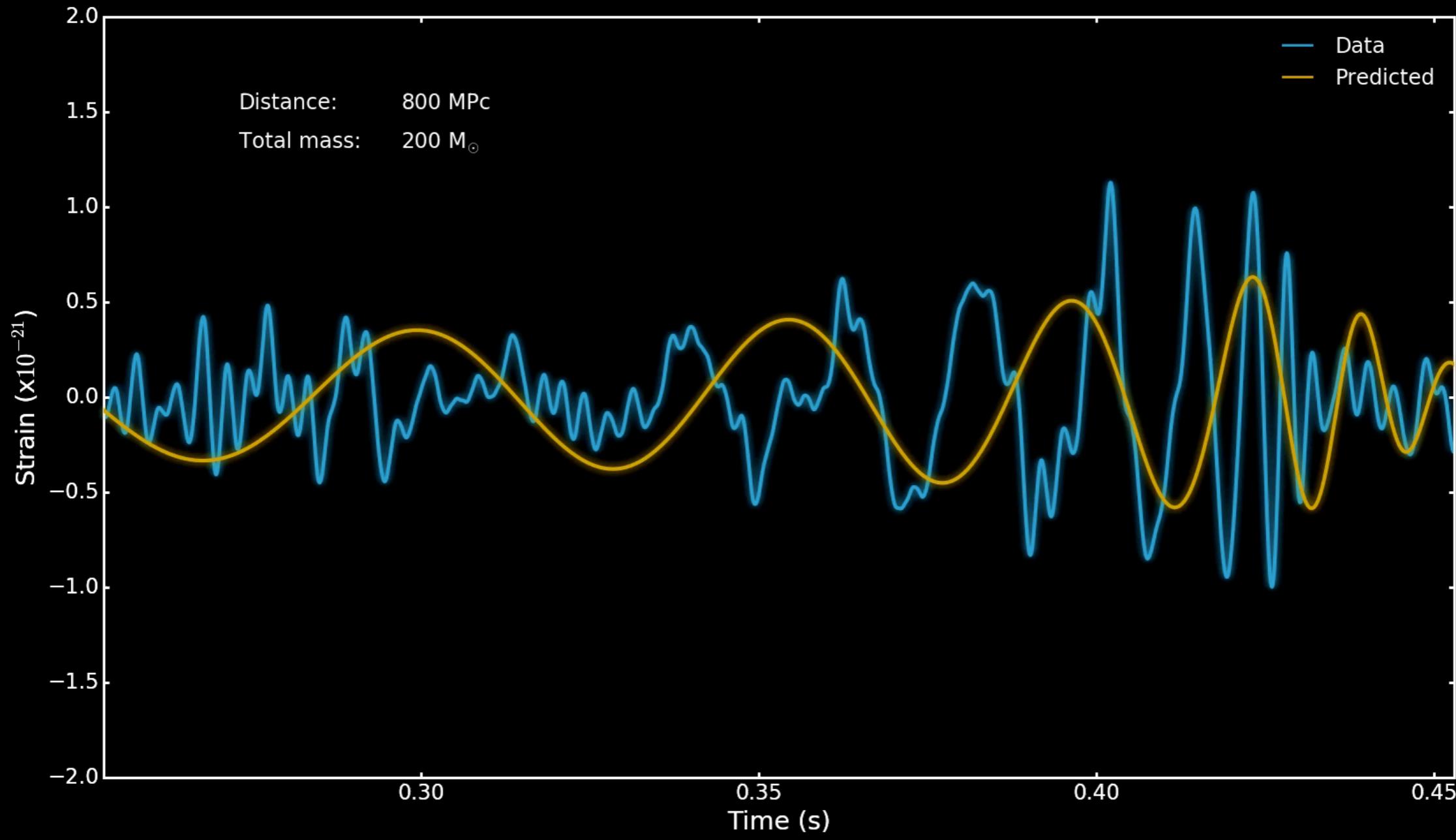
Prior

Evidence



[GW150914, LSC+VIRGO PRL (2016)]

# GW data analysis



# Bayesian approach: model selection

- Sometimes we have several competing models: Are BHs spinning? Is GR or an alternative theory? Is there one GW signal in the data or more?

$$P(M_i|d) = \frac{P(d|M_i)\pi(M_i)}{p(d)}$$

Probability of model  $M_i$  given observational data  $d$

likelihood      prior

$$P(\vec{\theta}_i|M_i, d) = \frac{P(d|\vec{\theta}_i, M_i)\pi(\vec{\theta}_i)}{p(d|M_i)}$$

posterior      evidence

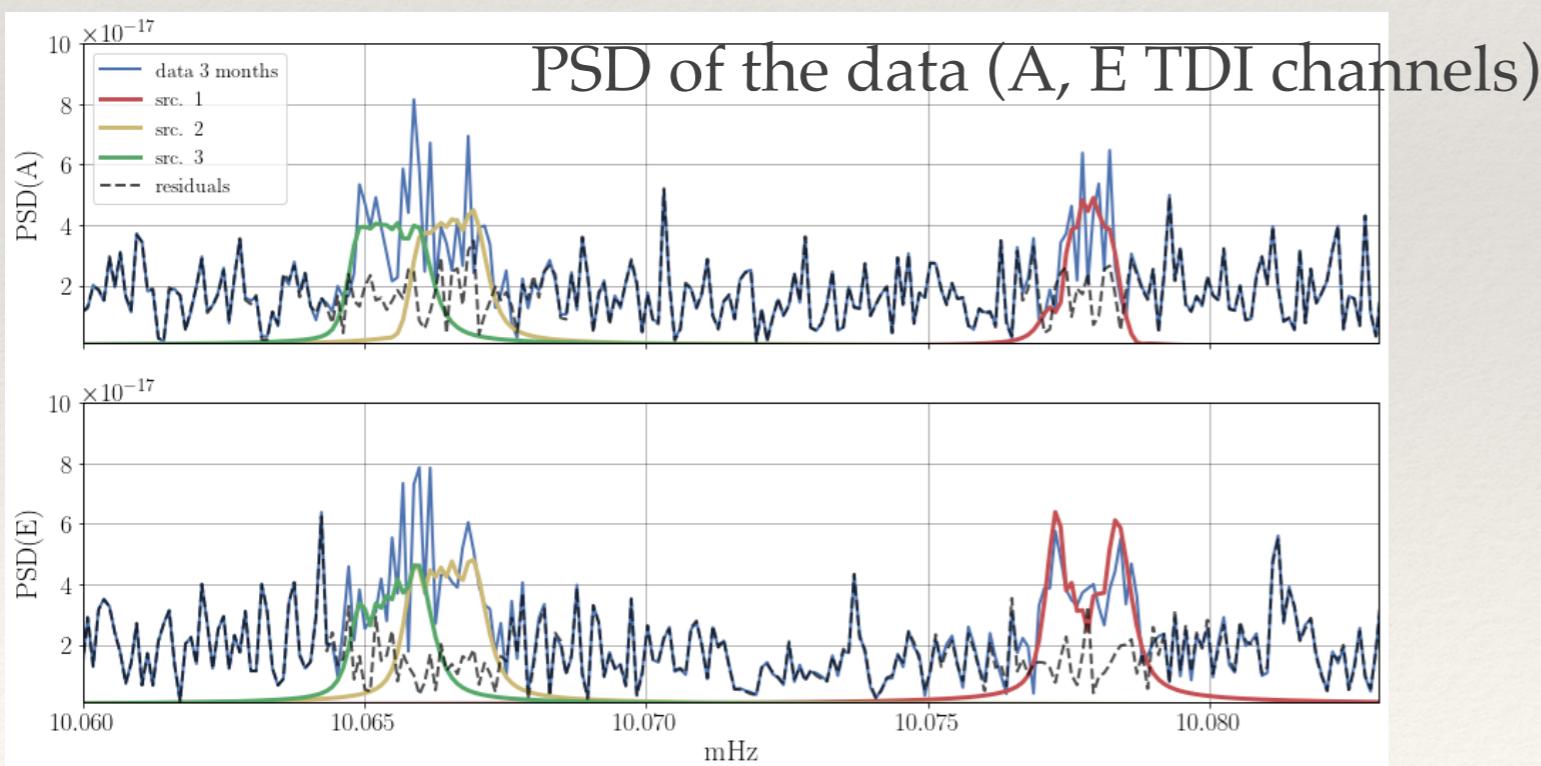
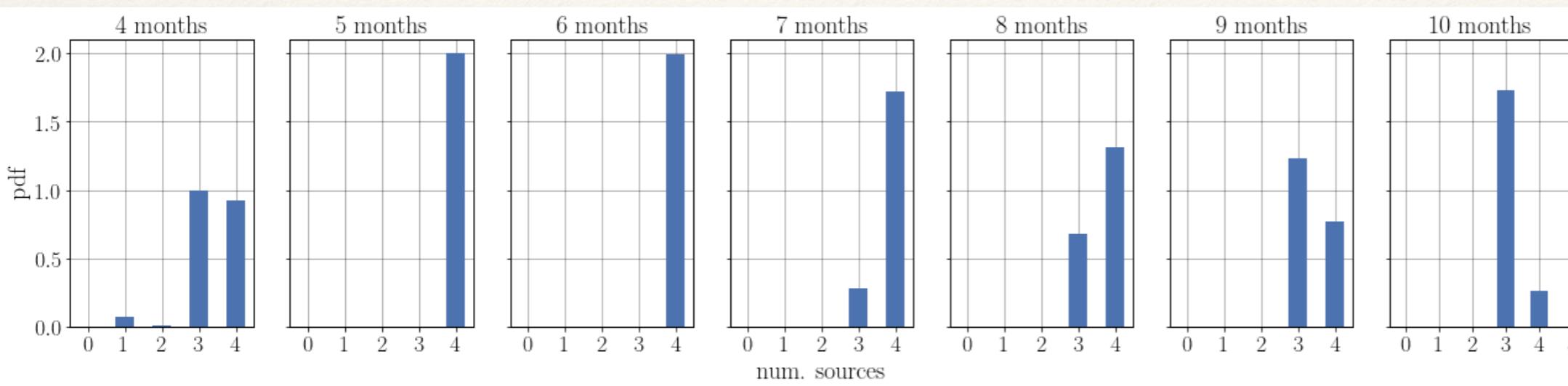
$$p(d|M_i) = \int d\vec{\theta}_i p(d|\vec{\theta}_i, M_i)\pi(\vec{\theta}_i)$$

Odds ratio: which model is preferred

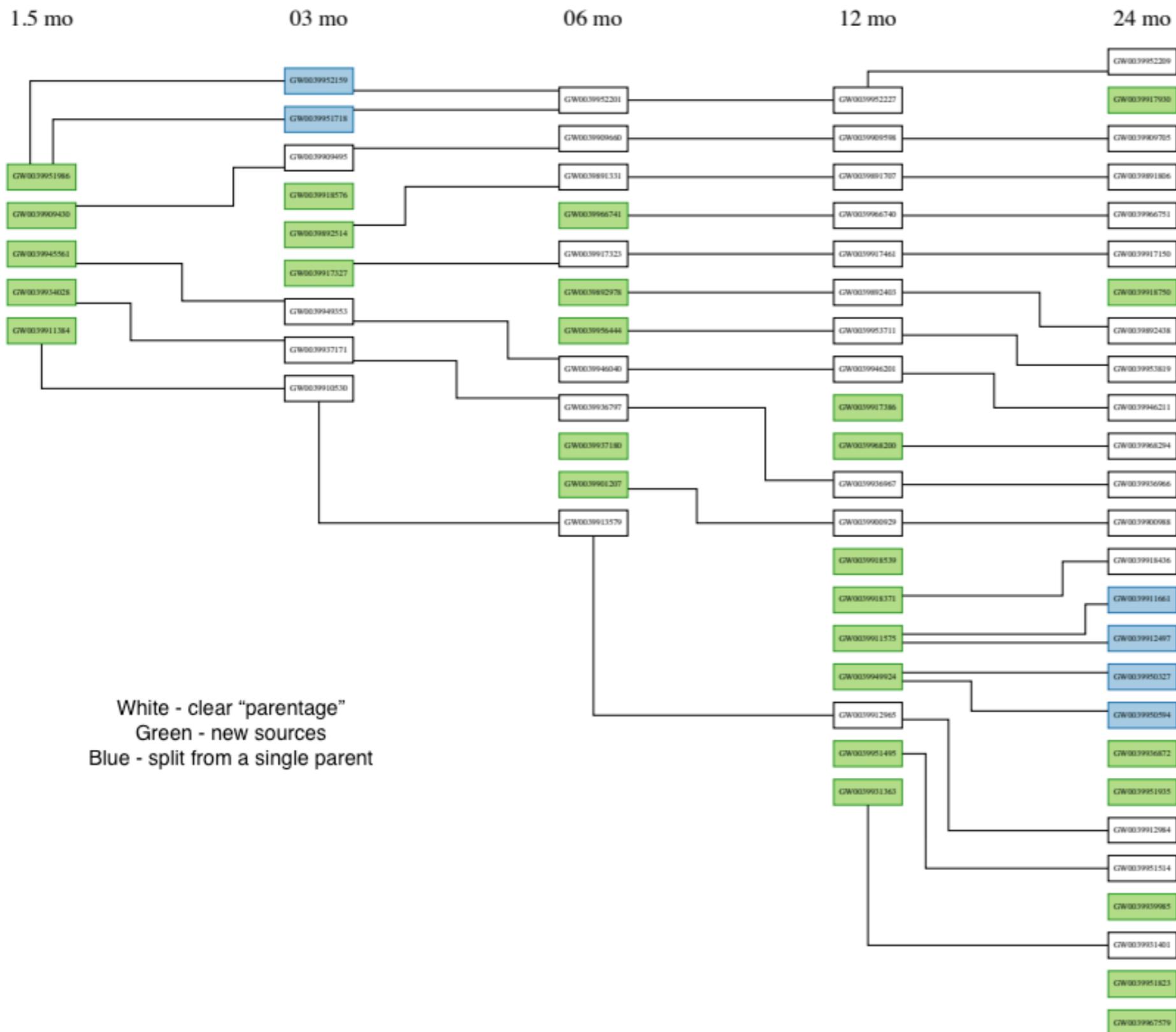
$$O_{a,b} = \frac{p(M_a|d)}{p(M_b|d)} = \frac{p(d|M_a)}{p(d|M_b)} \frac{\pi(M_a)}{\pi(M_b)}$$

# Identifying number of sources

- Take a narrow band next to 10mHz: 3 GW signals
- Perform time-data adaptive (4 months, 5 months, ... 10 months) search
- Consider 4 models: 1 GW source, 2 GW sources, 3 GW source, 4 GW sources



# Time-evolving catalogue building

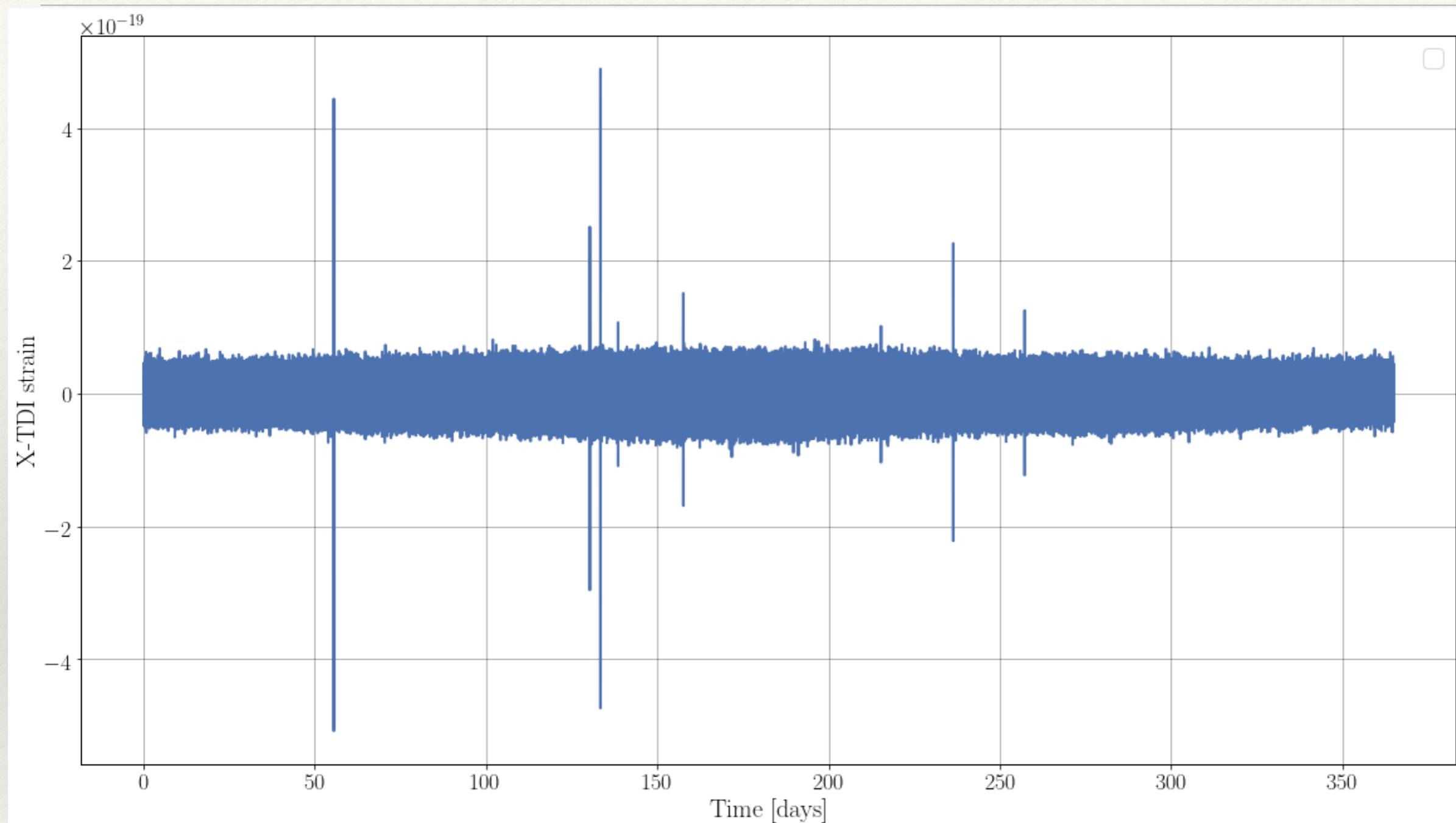


[Littenberg+ PRD, 2020]

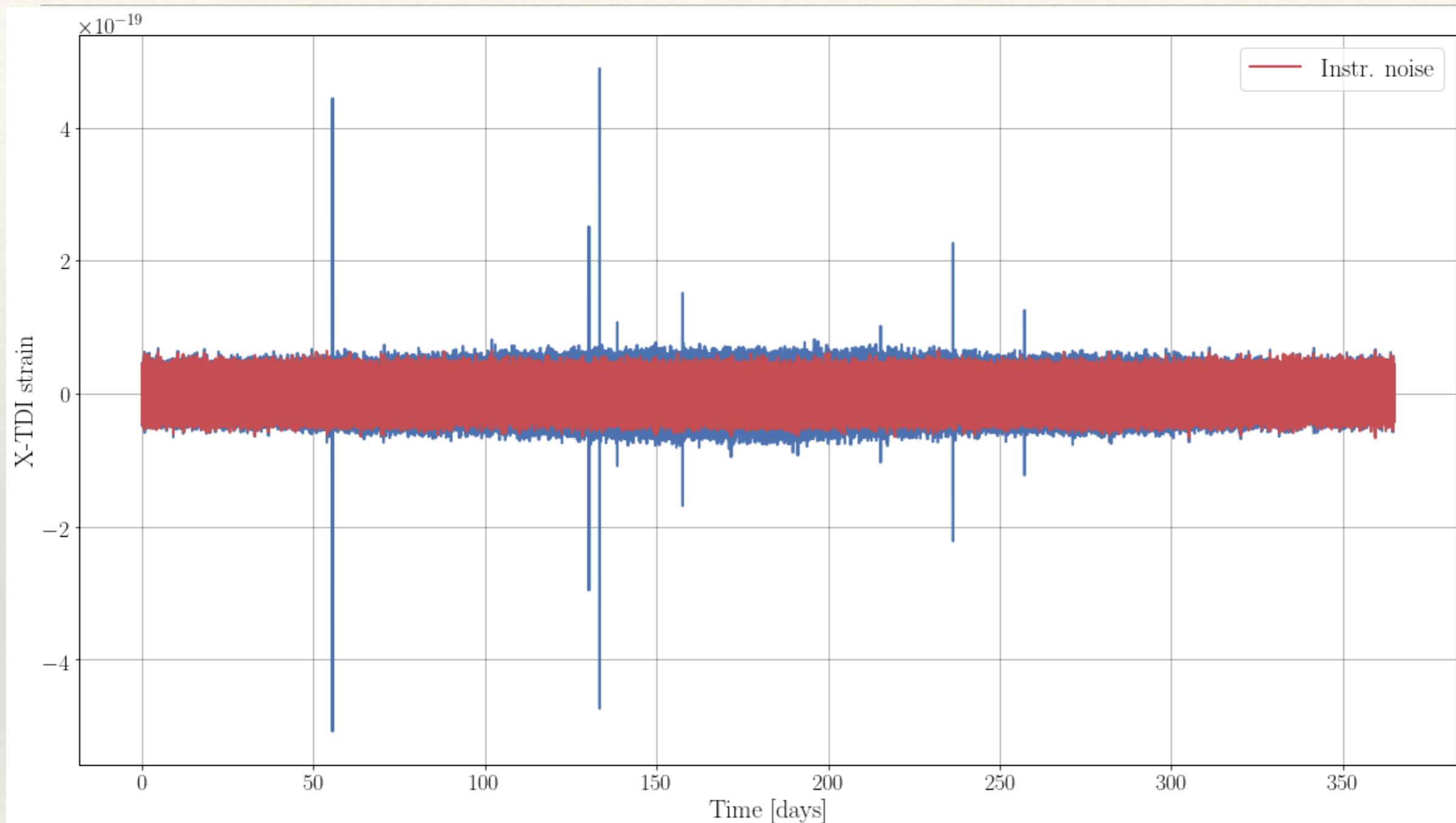
Number of sources is  
a random variable:  
building catalogue of  
sources?



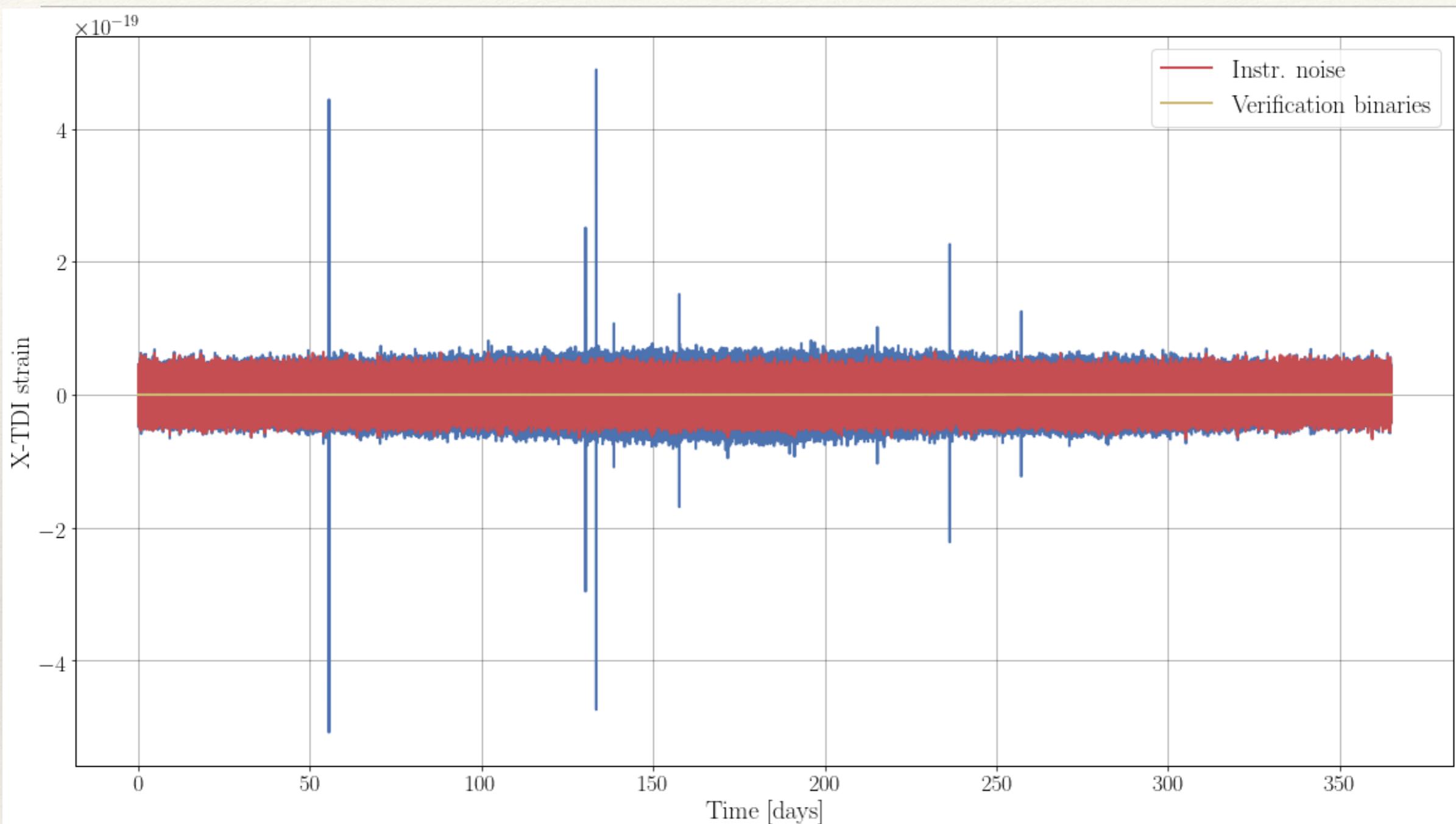
# Simulated LISA data in time domain



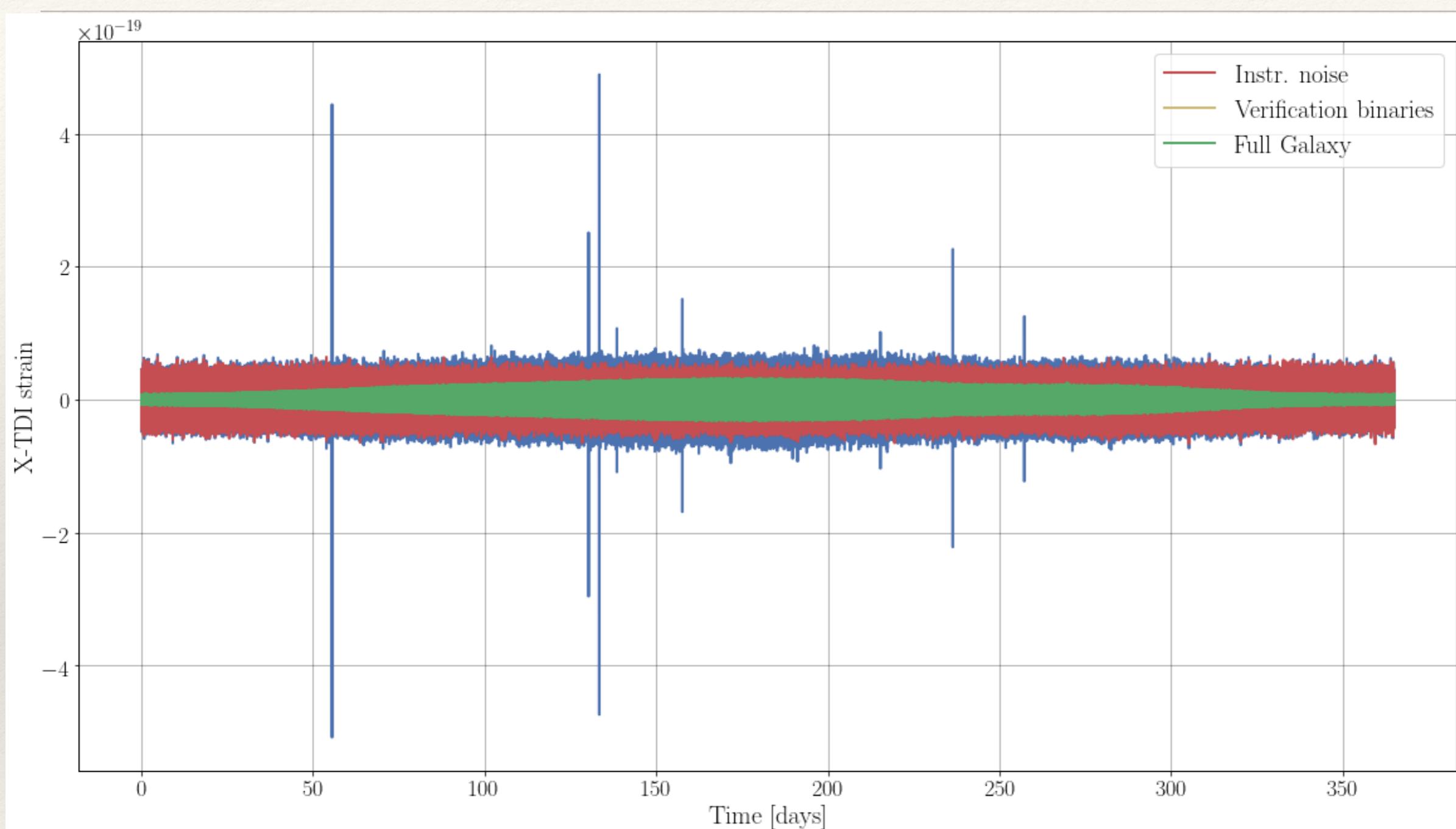
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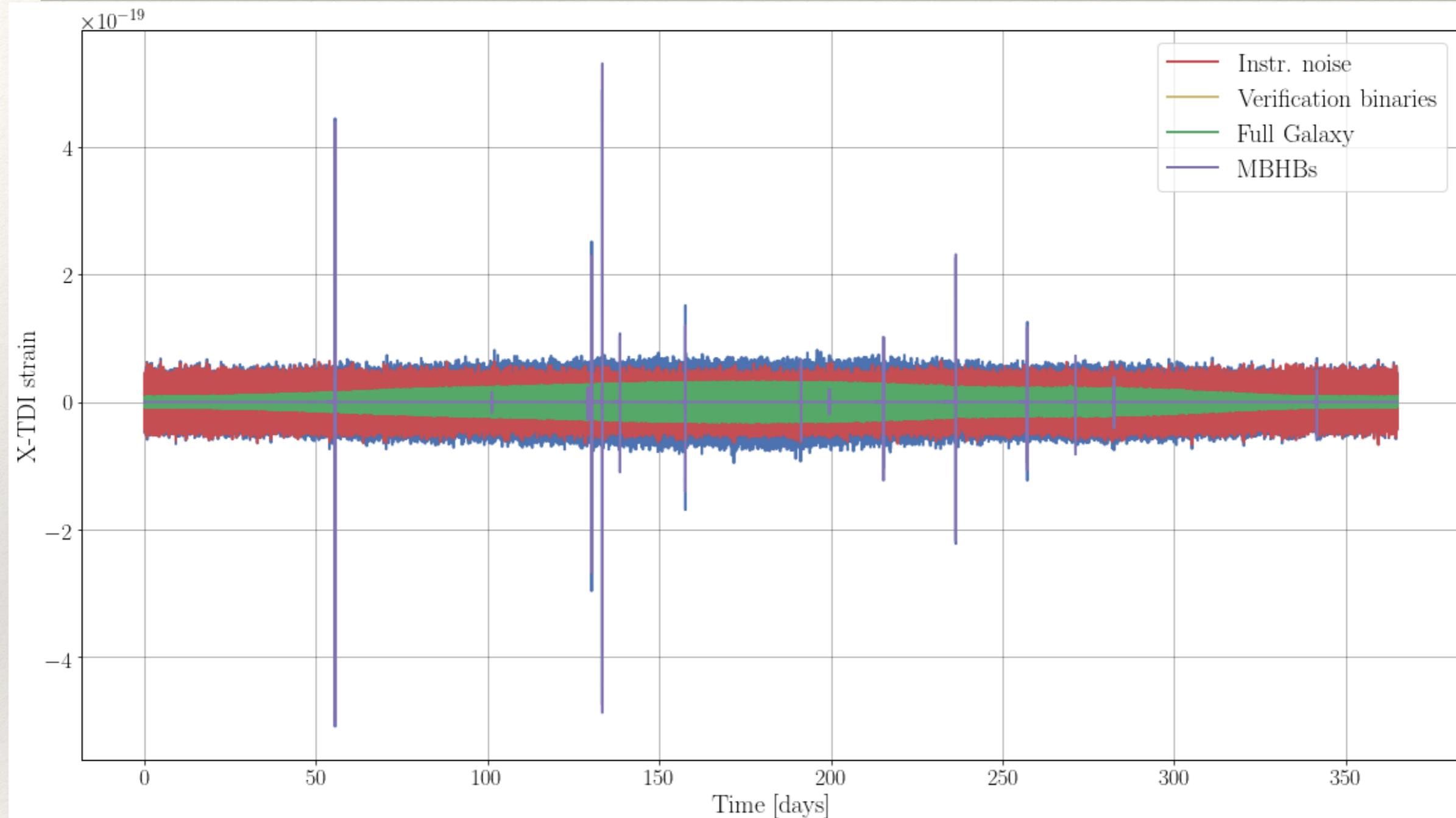
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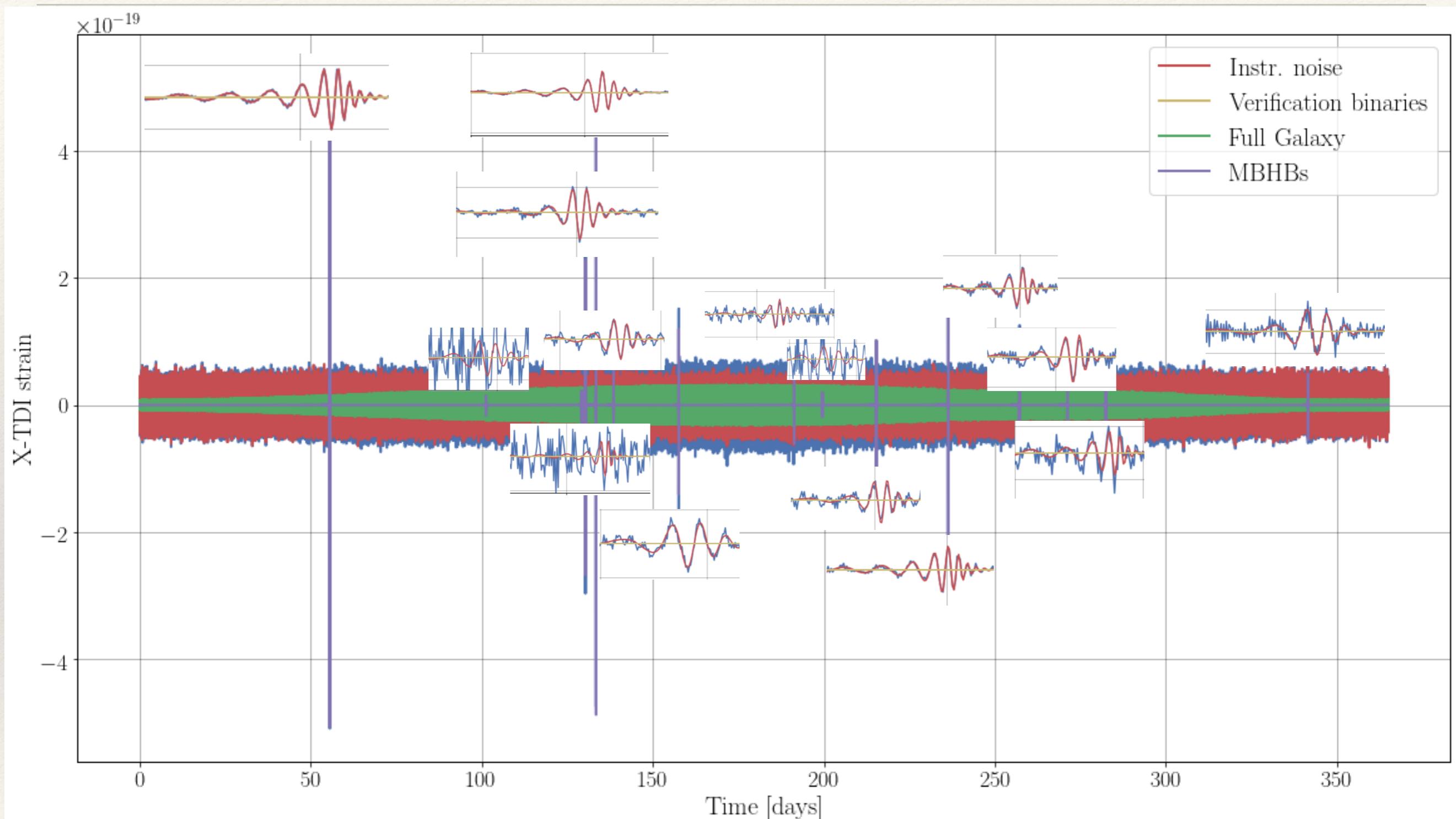
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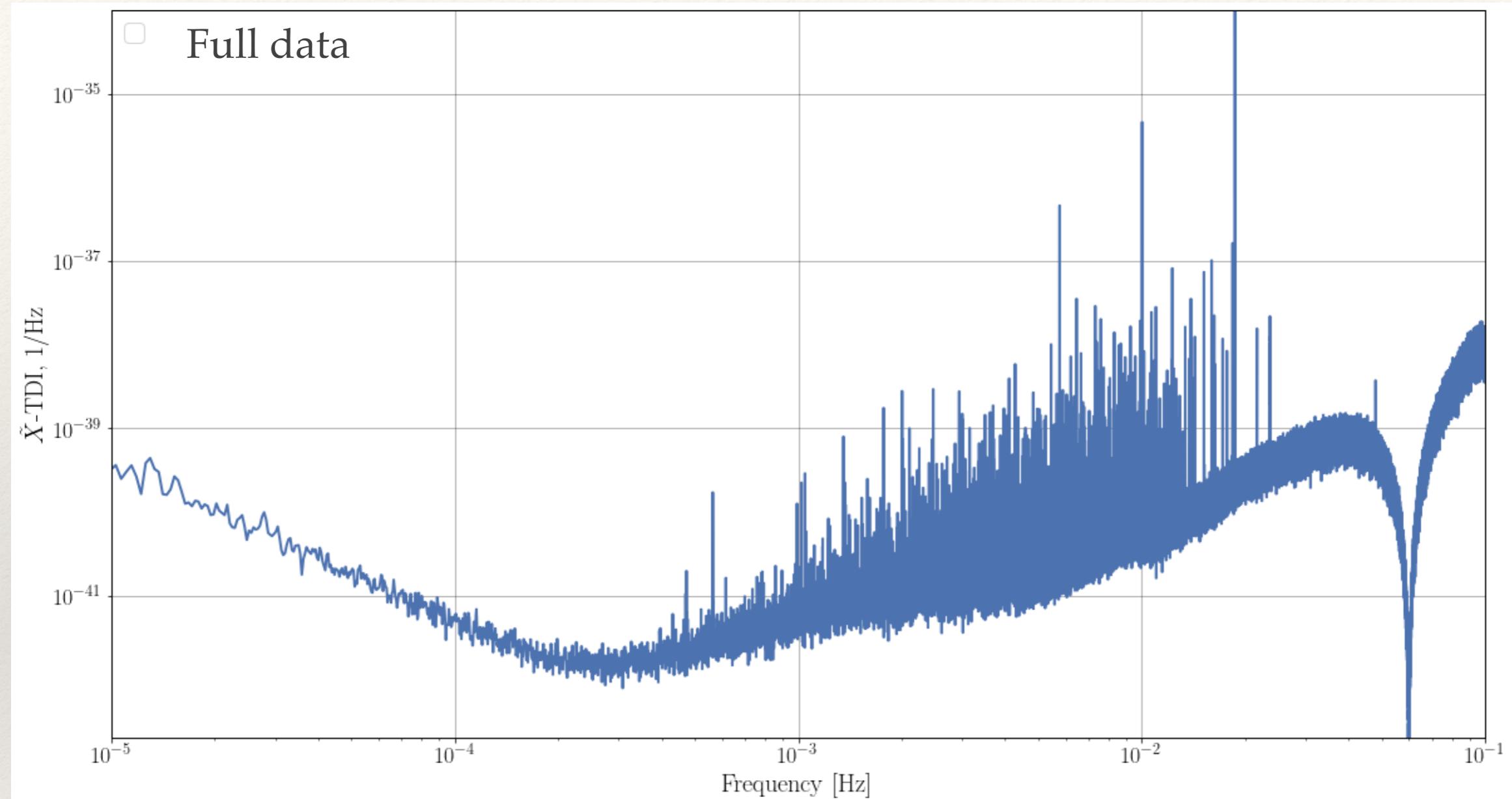
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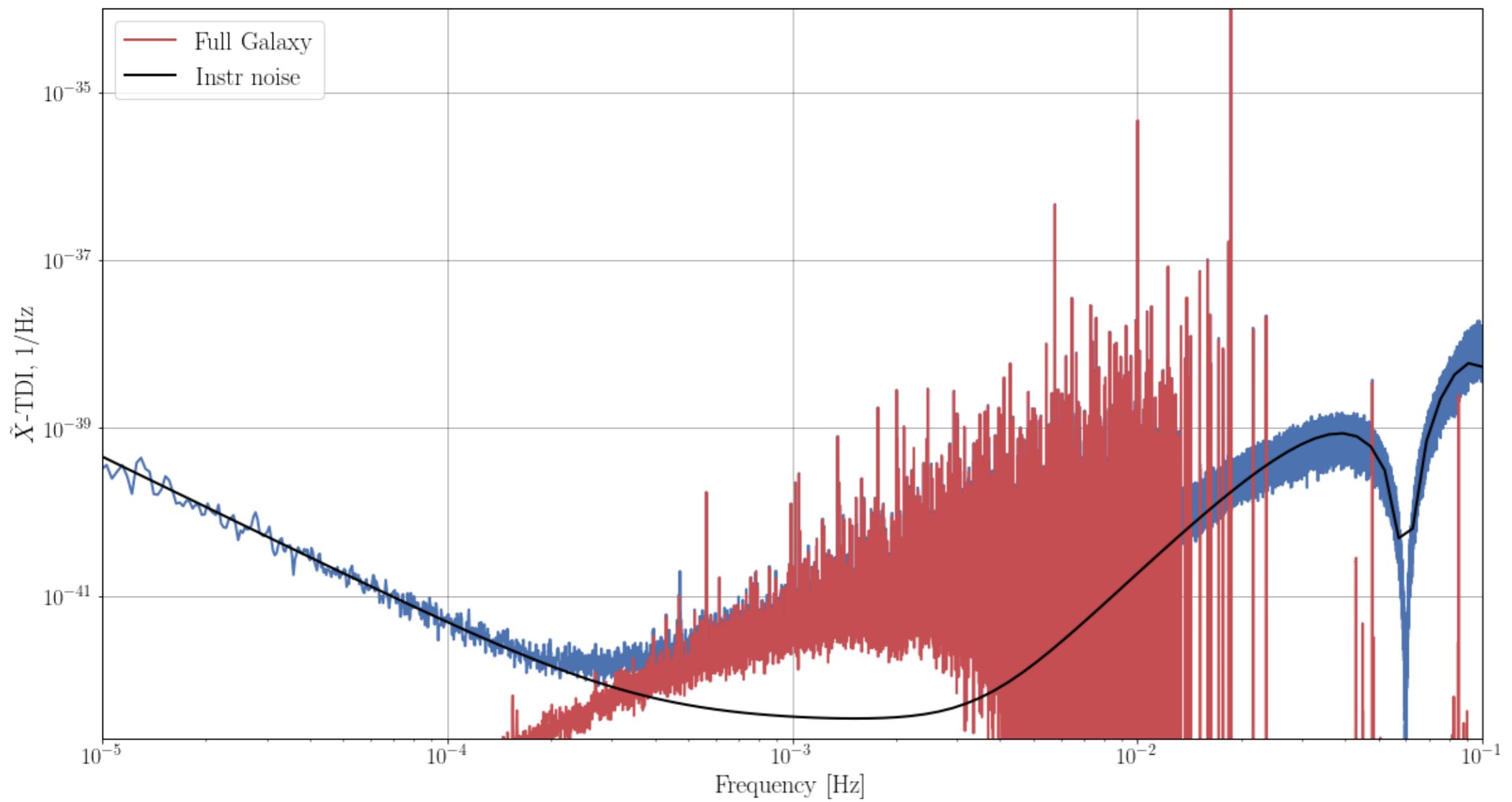
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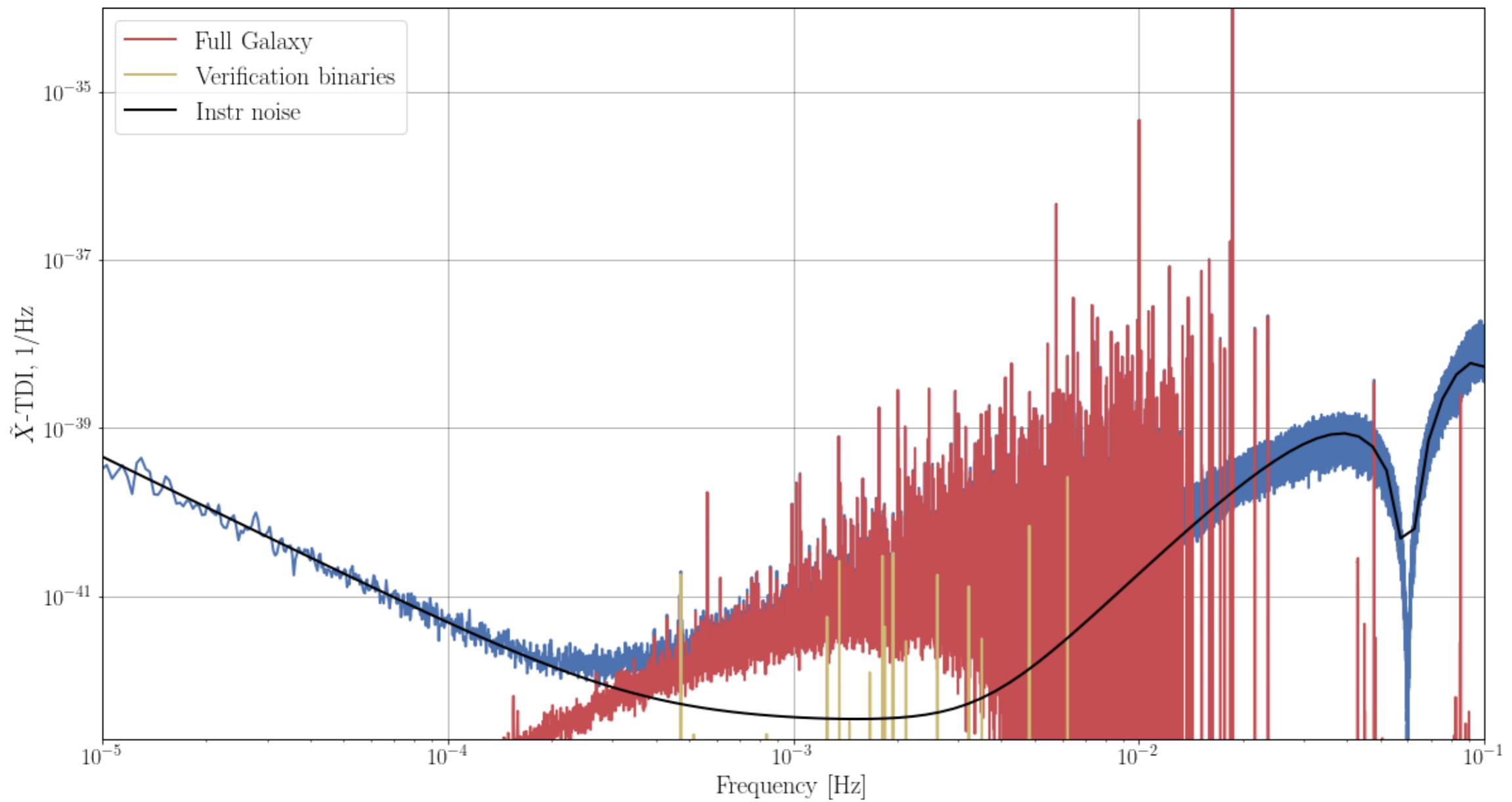
# Simulated LISA data in frequency domain



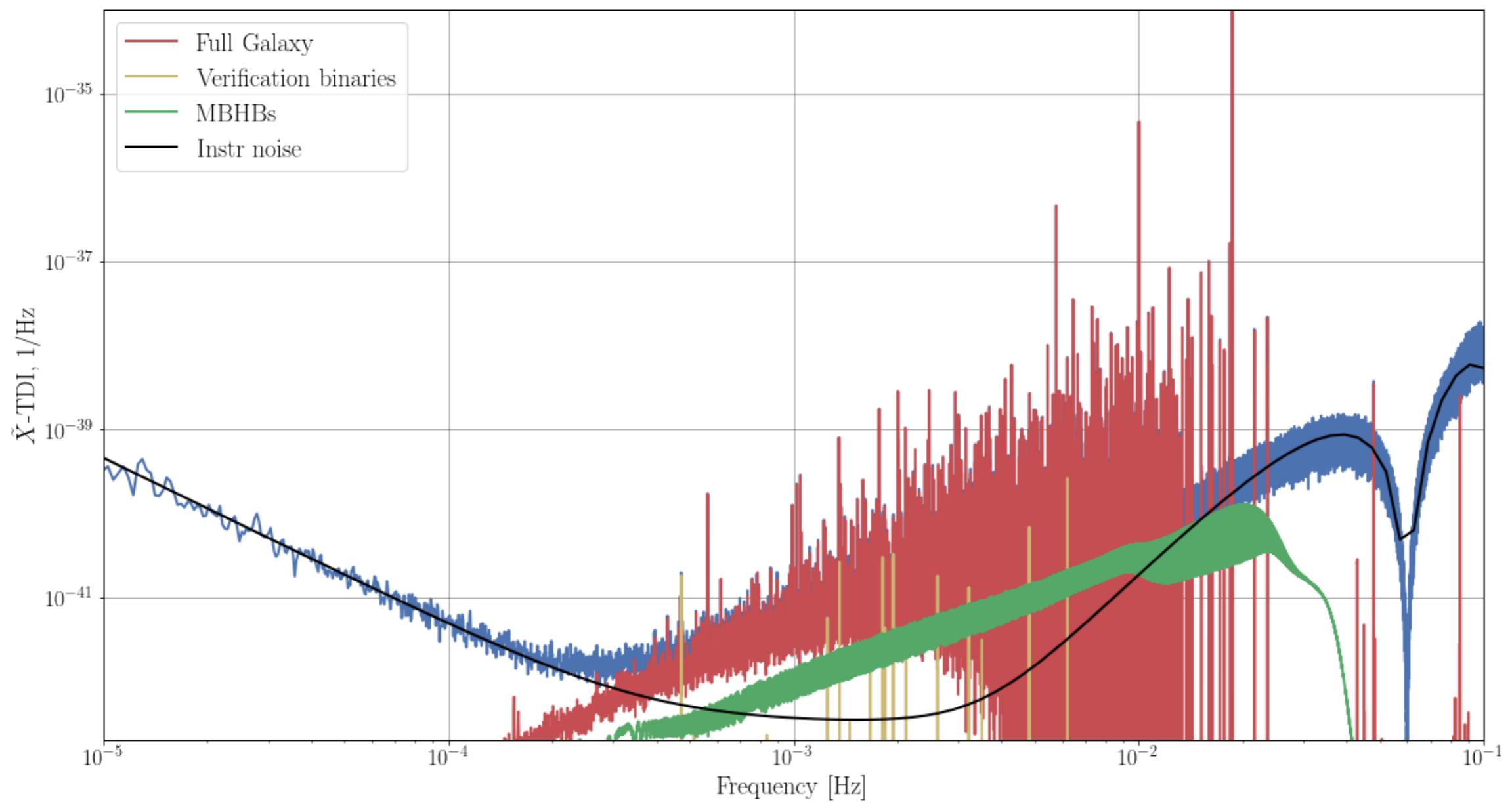
# “Sangria” in frequency domain



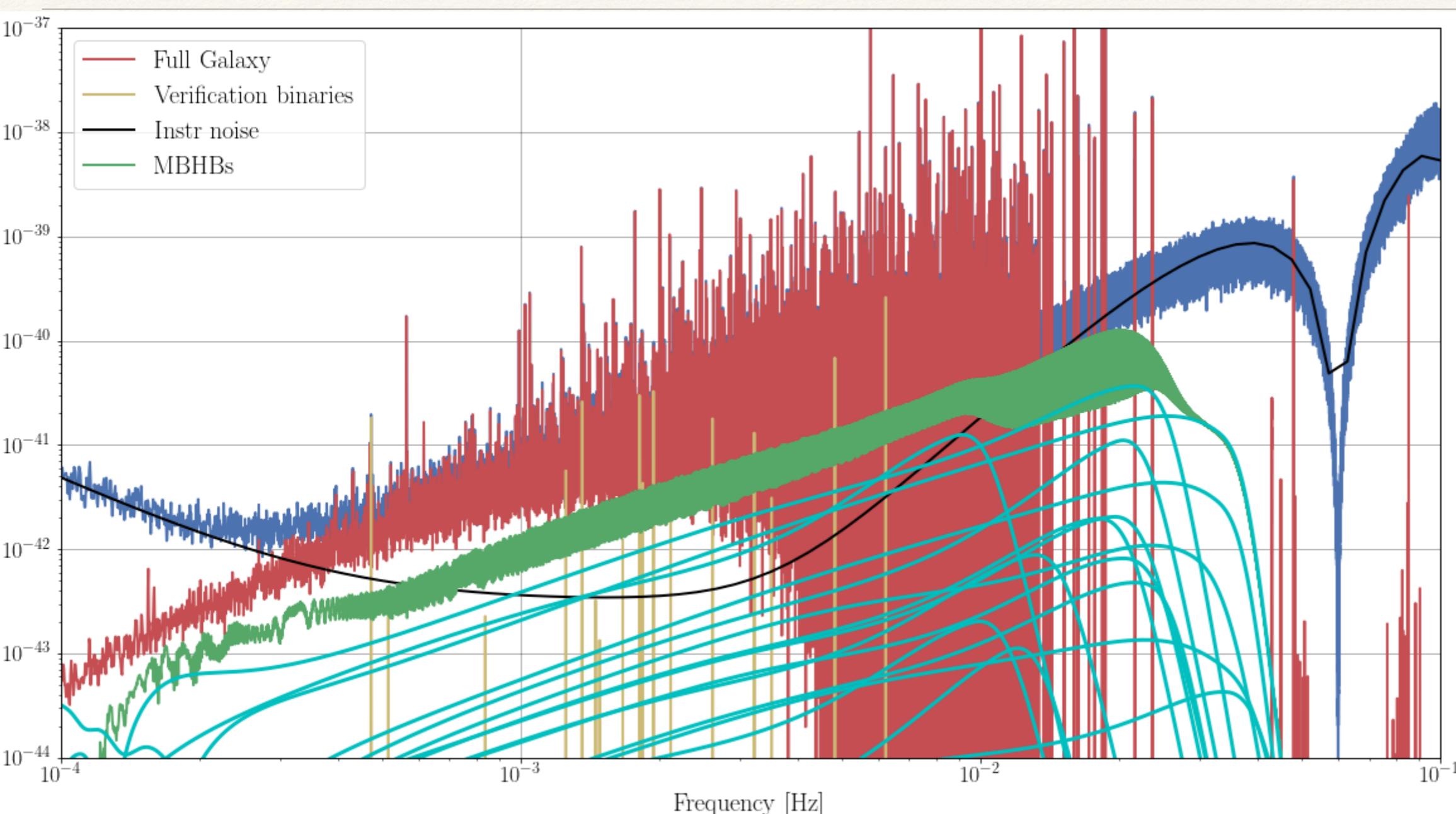
# “Sangria” in frequency domain



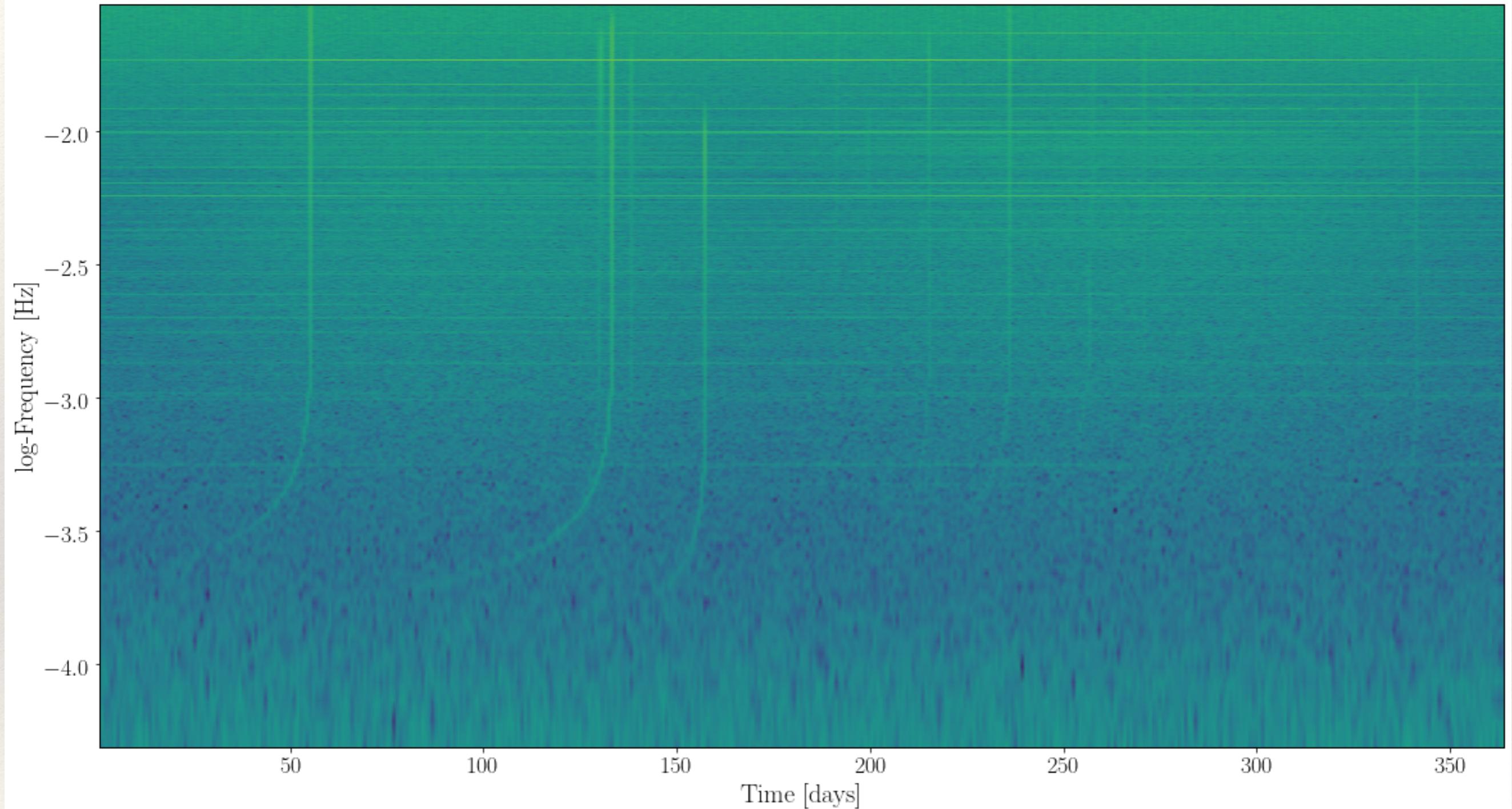
# “Sangria” in frequency domain



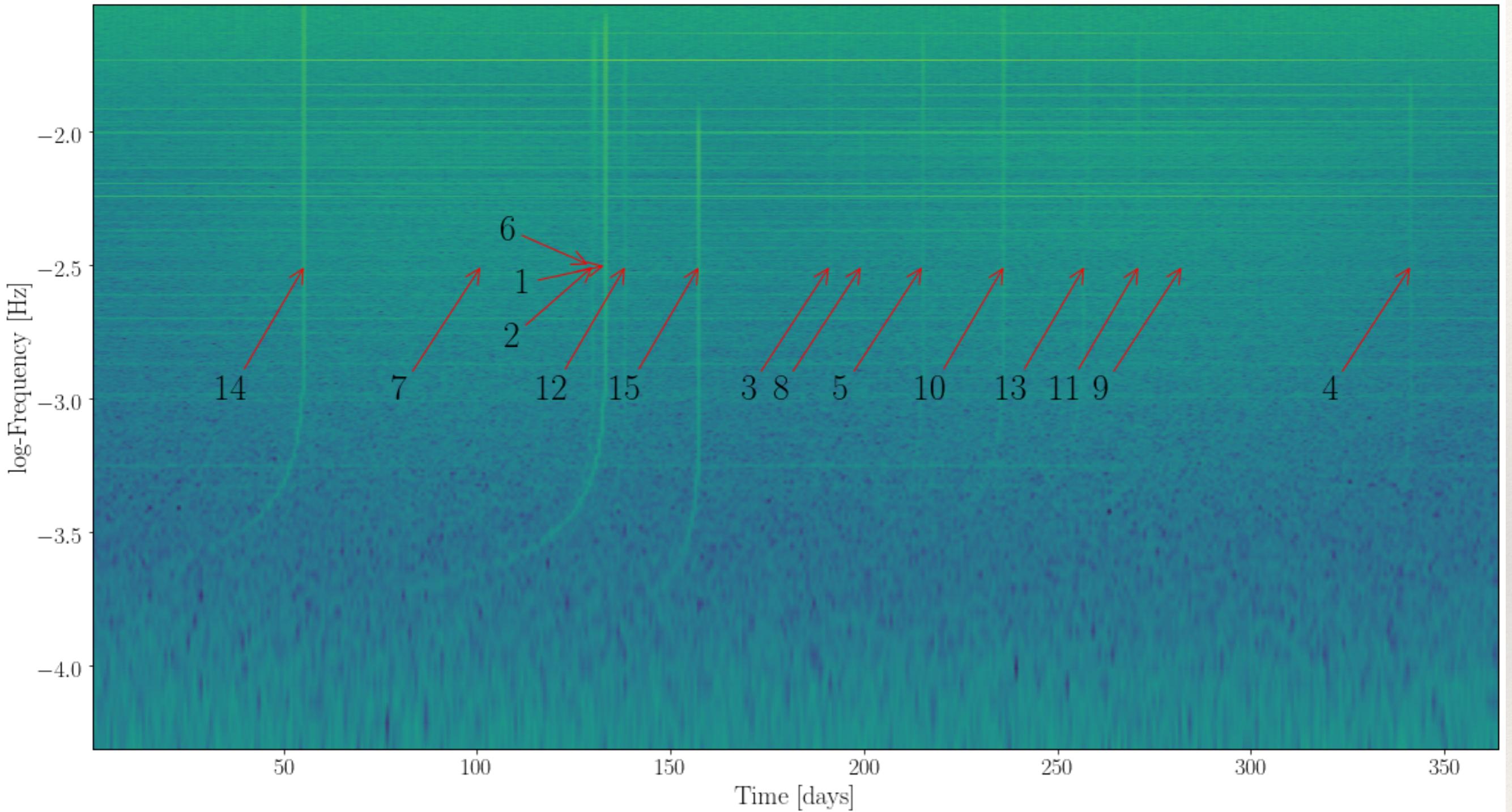
# “Sangria” in frequency domain



# “Sangria” in time-frequency



# “Sangria” in time-frequency



# LISA: summary

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- LISA: ESA lead mission to be launched in 2034 — your mission, your data
- Data is source dominated, GW signals are strong and long lived
- Possibility of simultaneous observations with ATHENA (X-Ray)
- Multi-band observations of stellar mass black holes
- Testing GR with amazing accuracy
- Looking at the hearts of galaxies far-far away
- Side science: exoplanets in white dwarf binaries, small celestial bodies near Earth orbit, helioseismology (g-modes of sun)

