Fast generation of time domain gravitational waveforms

Kick-off Ricochet
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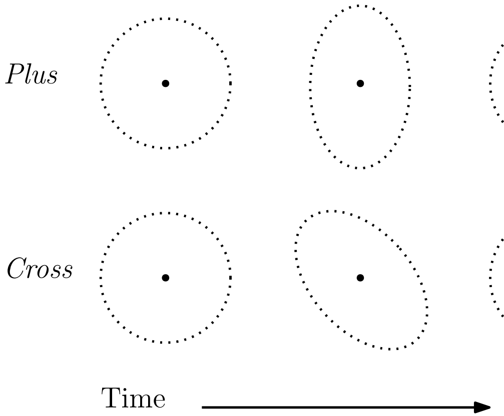
Cyril Cano (Gipsa-lab)
Nicolas Le Bihan (Gipsa-lab)
Éric Chassande-Mottin (APC)

Introduction



Each detector :

$$h_+(t)F_+^D(\Theta) + h_\times(t)F_\times^D(\Theta) \in \mathbb{R}$$

Detector network :

$$\begin{bmatrix} x_1(t-\tau_{1,\Theta}) \\ x_2(t-\tau_{2,\Theta}) \\ x_3(t-\tau_{3,\Theta}) \end{bmatrix} = \begin{bmatrix} F_+^1(\Theta) & F_\times^1(\Theta) \\ F_+^2(\Theta) & F_\times^2(\Theta) \\ F_+^3(\Theta) & F_\times^3(\Theta) \end{bmatrix} \begin{bmatrix} h_+(t) \\ h_\times(t) \end{bmatrix} + \begin{bmatrix} n_1(t) \\ n_2(t) \\ n_3(t) \end{bmatrix}$$

Gravitational wave observations

Each detector : $F_+^D(\Theta)h_+(t) + F_\times^D(\Theta)h_\times(t) + n(t)$

Line of sight
and
polarization angle

Observations

Polarisations

Network : $x_\Theta(t) = F(\Theta)h(t) + n(t)$

Mixing matrix

Colored noise

$$x_\Theta(t) = \begin{pmatrix} x^L(t-\tau_{L,\Theta}) \\ x^H(t-\tau_{H,\Theta}) \\ x^V(t-\tau_{V,\Theta}) \end{pmatrix}$$

$$F(\Theta) = \begin{pmatrix} \underbrace{\begin{pmatrix} F_+^L(\Theta) \\ F_+^H(\Theta) \\ F_+^V(\Theta) \end{pmatrix}}_{\mathrm{F}_+(\Theta)} & \underbrace{\begin{pmatrix} F_\times^L(\Theta) \\ F_\times^H(\Theta) \\ F_\times^V(\Theta) \end{pmatrix}}_{\mathrm{F}_\times(\Theta)} \end{pmatrix}$$

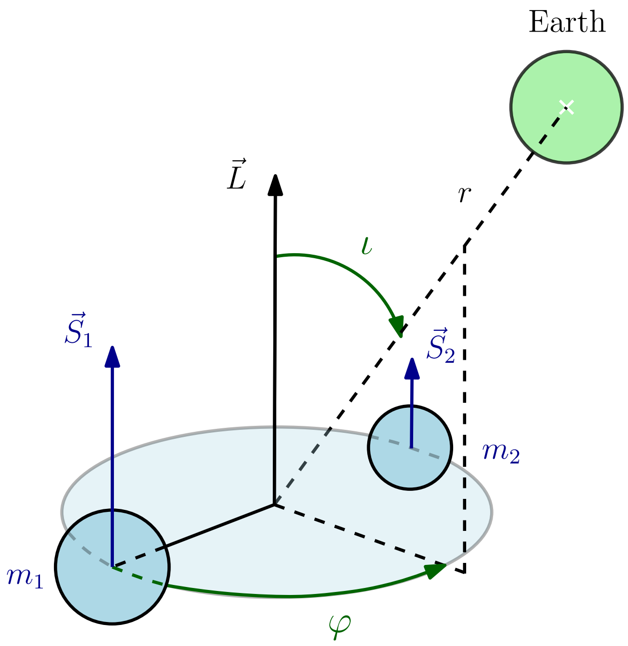
$$h(t) = \begin{pmatrix} h_+(t) \\ h_\times(t) \end{pmatrix}$$

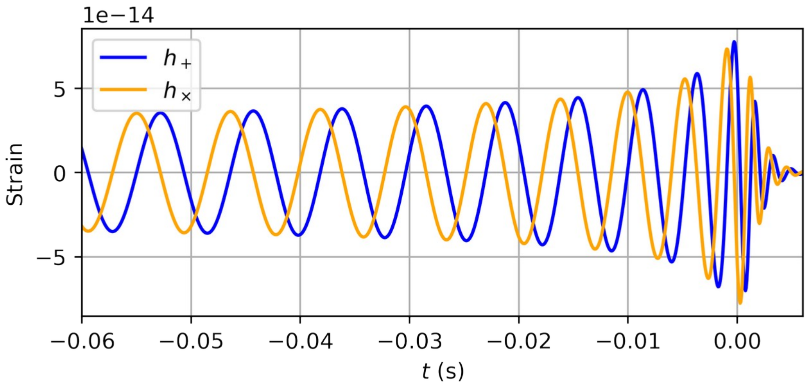
Motivations

- Bayesian inference **needs large amount of waveforms** (~10^5)
- Time domain GW generation is **computationally expensive**
- **Need for fast and accurate generative model**
- Reduced Order Models [Pürrer 2016]
- ML with Mixture of Experts (med. mismatch ~10^{-4}) [Schmidt et al. 2020]

- Proposed model: **principal component regression**

BBH parameters





$m_1,\ m_2,\ S_{1z},\ S_{2z}$: binary parameters

ι, φ : line of sight

r : luminosity distance

Generative model

SEOBNRv4

Only (2,2) mode

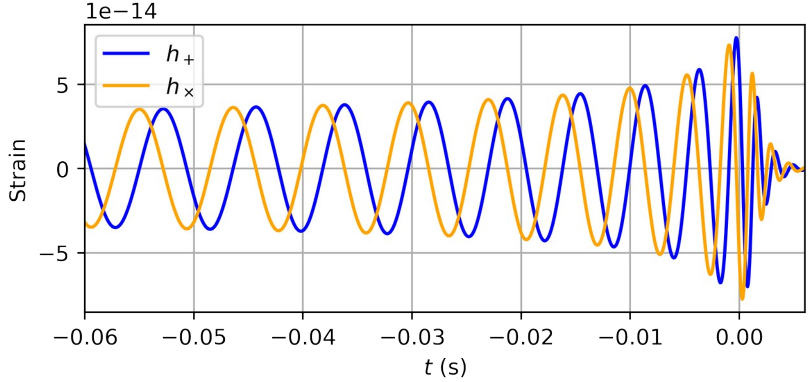
$$\mathrm{h}(\mathrm{t};\mathrm{m}_1,\ m_2,\ S_{1z},\ S_{2z},\ \iota,\ \varphi,\ r)$$

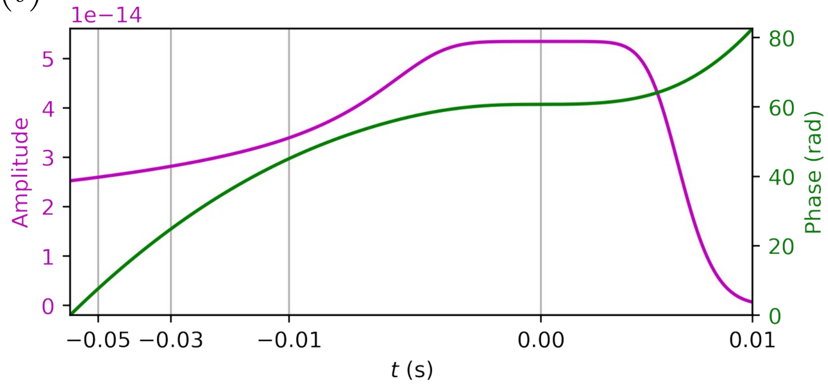
$$\mathrm{h}(\mathrm{t}) = \mathrm{h}_+ - ih_\times(t)$$

Waveform attributes

$$a(t) = |h(t)|$$

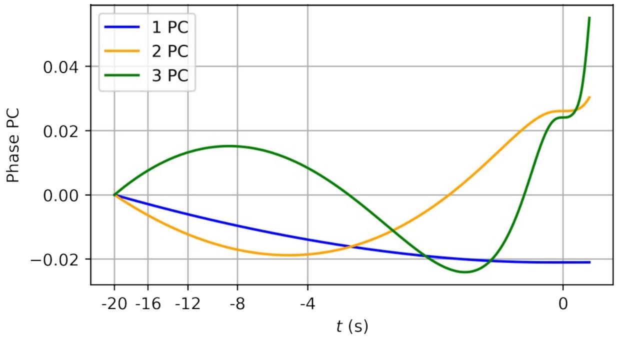
$$\Phi(t) = \arctan \frac{h_+(t)}{h_\times(t)}$$





$$h(t) = a(t)e^{-i\Phi(t)}$$

- ML model **generates amplitude and phase**
- Non uniform time grid $\mathrm{sign}(t)|t|^{\frac{1}{\alpha}}$
- Needs dimension reduction: **truncated PCA**

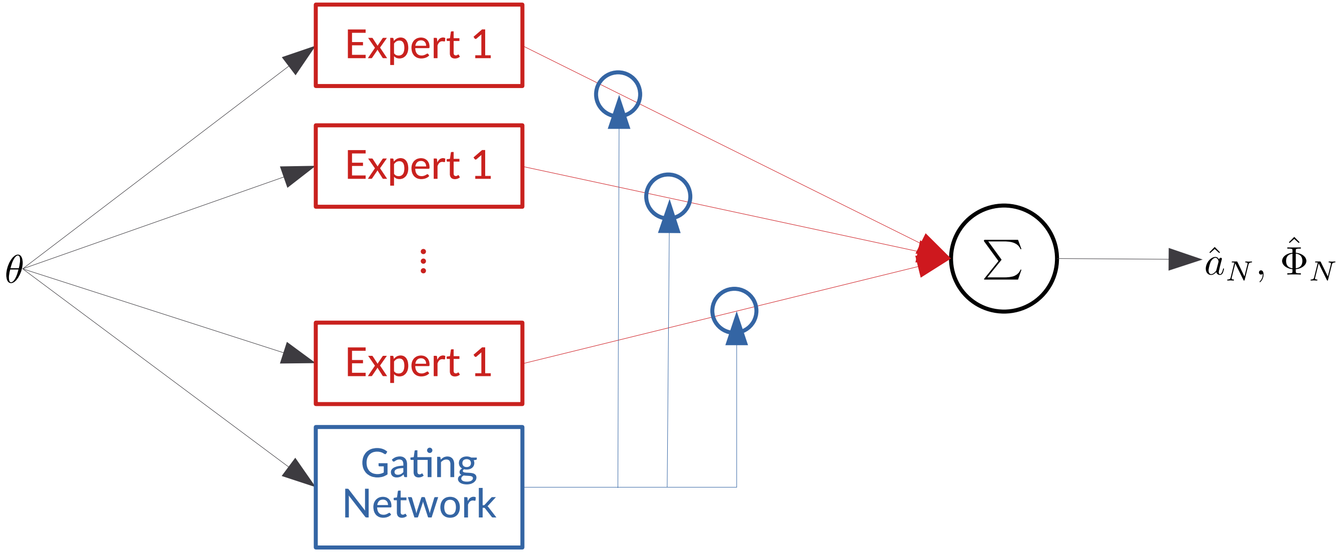


Schmidt's model [Schmidt et al. 2020]

Input — **Output**

Binary parameters → Model features → Expanded features → Reduced coeff. → Attributes → Waveform

$$\{m_1,\, m_2,\, \chi_{1z},\, \chi_{2z}\} \longrightarrow \{q,\, \chi_{1z},\, \chi_{2z}\} \xrightarrow{\text{Polynomial expansion}} \theta \xrightarrow{\text{Mixture of Expert}} \hat{a}_N,\, \hat{\Phi}_N \xrightarrow{\text{PCA invertion}} \hat{a},\, \hat{\Phi} \xrightarrow{\text{Post-processing}} \hat{h}$$



Overview of our model

Generation

Input → **Output**

Binary parameters	Model features	Expanded features	Reduced coeff.	Attributes	Waveform
$\{m_1,\, m_2,\, \chi_{1z},\, \chi_{2z}\}$	$\{q,\, m_2,\, \chi_{2z},\, \chi_{\text{eff}}\}$	θ	$\hat{a}_N,\, \hat{\Phi}_N$	$\hat{a},\, \hat{\Phi}$	$\hat{h}$

$$\{m_1,\, m_2,\, \chi_{1z},\, \chi_{2z}\} \longrightarrow \{q,\, m_2,\, \chi_{2z},\, \chi_{\text{eff}}\} \xrightarrow{\text{Polynomial expansion}} \theta \xrightarrow{\text{Linear regression}} \hat{a}_N,\, \hat{\Phi}_N \xrightarrow{\text{PCA invertion}} \hat{a},\, \hat{\Phi} \xrightarrow{\text{Post-processing}} \hat{h}$$

Polynomial expansion

Linear regression: $L\theta$

PCA invertion: $\hat{a}_N V_N,\, \hat{\Phi}_N W_N$

Post-processing

Goodness-of-fit metric:

$$\text{mismatch}\,(h,g) = \min_{\tau \in \mathbb{R}} \left[1 - \frac{|\langle h_\tau, g \rangle|}{||h_\tau||\,||g||} \right] \qquad \text{with} \qquad \langle f, g \rangle = \int \frac{h(f) g^*(f)}{S(f)}\, df$$

Overview of our model

Fitting

Training binary parameters	Model features		Expanded features		Reduced coeff.		Attributes	Training waveform
$\{m_1,\, m_2,\, \chi_{1z},\, \chi_{2z}\}$	$\{q,\, m_2,\, \chi_{2z},\, \chi_{\rm eff}\}$	$\longrightarrow$	θ	$\longrightarrow$	$a_N,\, \Phi_N$	$\longleftarrow$	$a,\, \Phi$	$\hat{h}$
		Polynomial expansion		Linear regression		PCA reduction		
				$L\theta$		$a_N V_N^T,\, \Phi_N W_N^T$		

Goodness-of-fit metric:

$$\text{mismatch}\,(h,g) = \min_{\tau\in\mathbb{R}}\left[1 - \frac{|\langle h_\tau, g\rangle|}{||h_\tau||\,||g||}\right] \qquad \text{with} \qquad \langle f,g\rangle = \int \frac{h(f)g^*(f)}{S(f)}\,df$$

Hyperparameters tuning

Feature set

- **Tested features:**

$$m_1,\ m_2,\ \chi_{1z},\ \chi_{2z},\ q,\ \mathcal{M},\ \chi_{\rm eff},\ {m_1}^{-1},\ {m_2}^{-1}$$

- **Tested feature sets:**

$$\begin{array}{cccc} \{\mathrm{m}_1\} & \{\mathrm{m}_1,\ m_2\} & \{\mathrm{m}_1,\ m_2,\ \chi_{1z}\} & \ldots \\ \{\mathrm{m}_2\} & \{\mathrm{m}_1,\ \chi_{1z}\} & \{\mathrm{m}_1,\ m_2,\ q\} & \ldots \\ \vdots & \vdots & \vdots & \end{array}$$

- **Several good choices:**

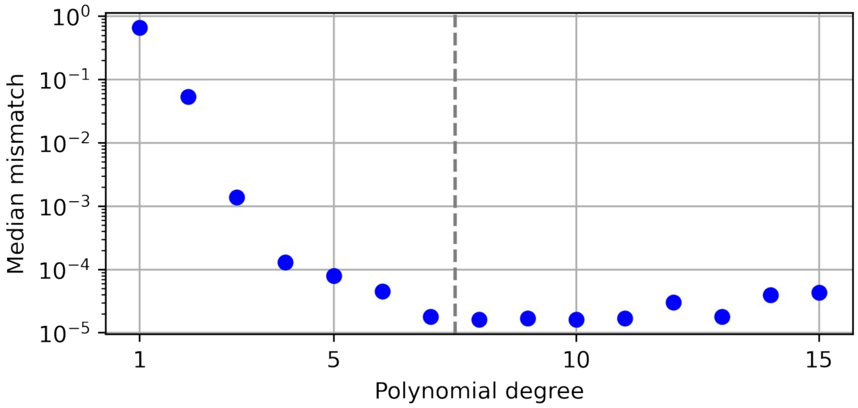
$$\begin{array}{cc} \{\chi_{2z},\ \chi_{\rm eff},\ \mathcal{M}\} & \{\mathrm{q},\chi_{2z},\ \chi_{\rm eff},\ \mathcal{M}\} \\ \underline{\{\mathrm{q},\mathrm{m}_2,\ \chi_{2z},\ \chi_{\rm eff}\}} & \{\chi_{2z},\ \chi_{\rm eff},\ m_1,\ {m_2}^{-1}\} \end{array}$$

Selected

Polynomial degree

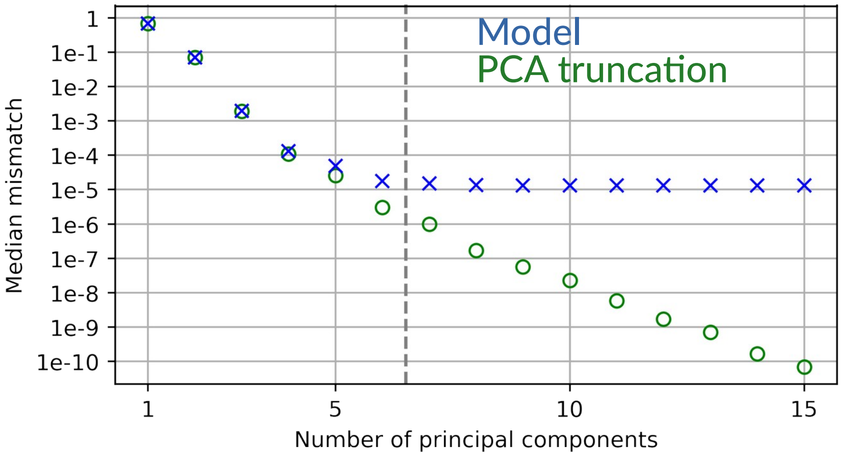
First order	$\{\mathrm{a},\mathrm{b}\}$
Second order	$\{\mathrm{a},\mathrm{b},\mathrm{ab},\mathrm{a}^2,\ b^2\}$
Third order	$\{\mathrm{a},\mathrm{b},\mathrm{ab},\mathrm{a}^2,\ b^2,\ a^2b,\ ab^2,\ a^3,\ b^3\}$
	$\vdots$

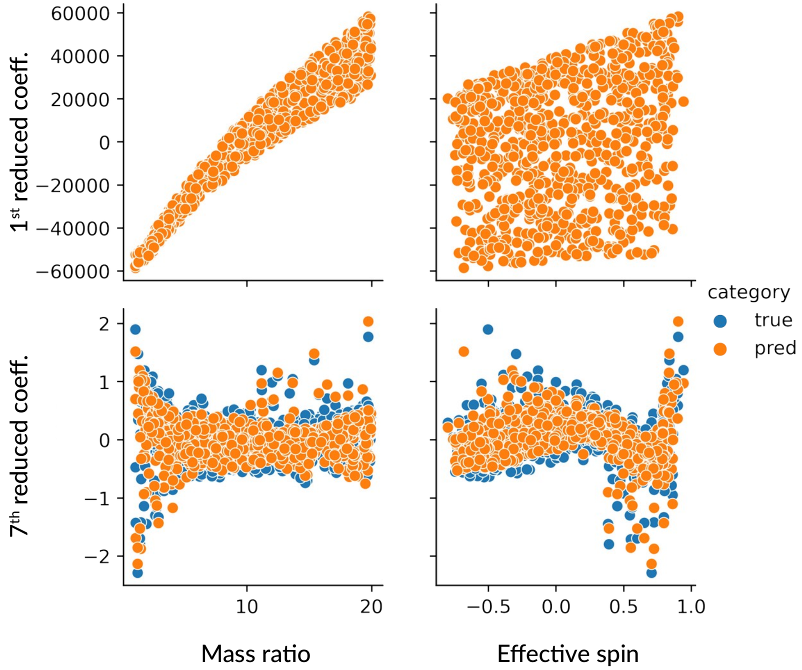
Seventh order used for regression



Hyperparameters tuning

Number of PC for the **phase**



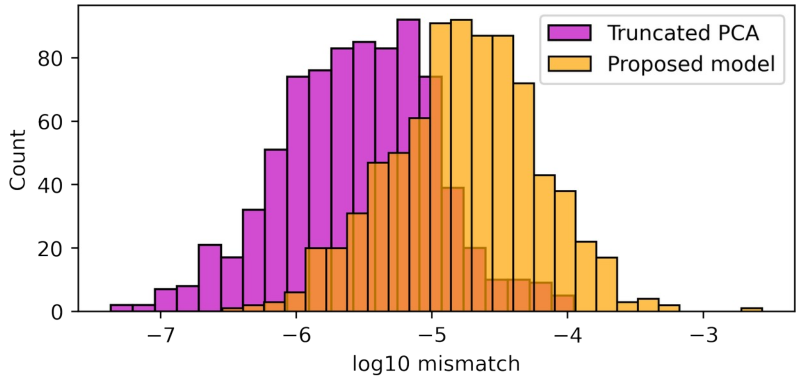


Six PC used for dimension reduction

Results on SEOBNRv4

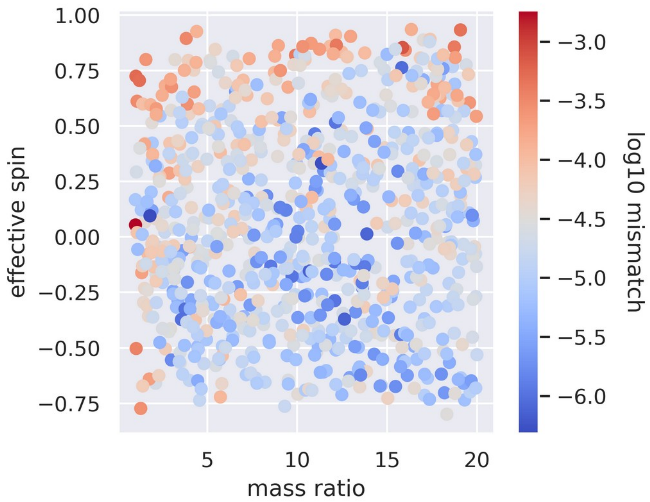
Dataset properties

- Training size: 3200 Testing size: 800
- Mass ratio: U([1, 20])
- Dimensionless spins: U([-0.8, 0.95])



Accurate

- $q_{50\%}$ = 2.10^{-5}
- $q_{5\%}$ = 2.10^{-6}
- $q_{95\%}$ = $1.5.10^{-4}$



Fast

- **~100 times faster than SEOBNRv4**
- **Can be faster** without interpolation (from non-uniform to regular time grid)

Conclusion/perspectives

Take home messages :

- **Fast** and **accurate** GW generation with **principal component regression**
- **Applicability up to SNR ~ 225** (18 in the worst case) *: $\text{mismatch} < \frac{N}{2\text{SNR}^2}$
- **Non conventional features** lead to better results
- Simple method with off-the-shelf algorithms from scikit-learn

Perspectives :

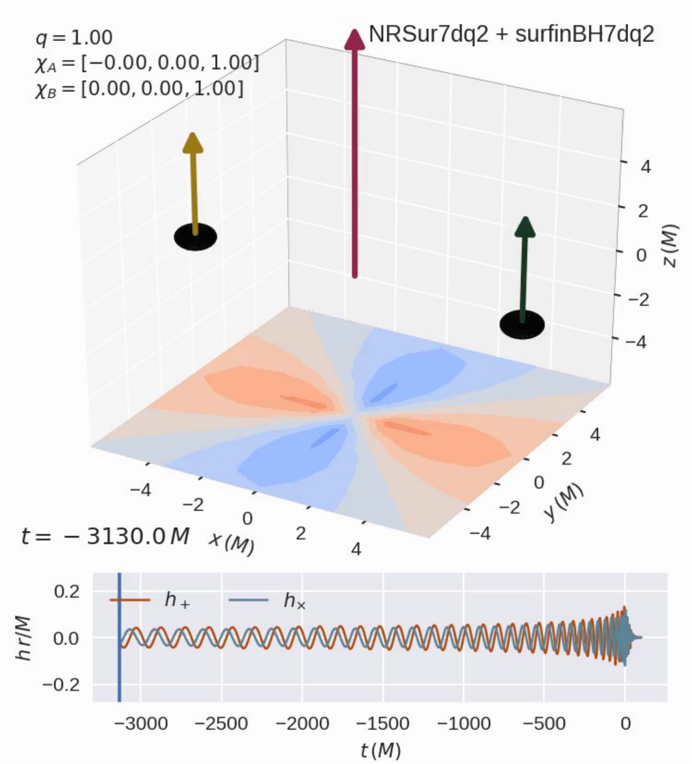
- Subdominant modes
- Comparison with other ML state of the art algorithms
- **Precessing BBH**

Git : https://git.ligo.org/cyril.cano/gw-generation

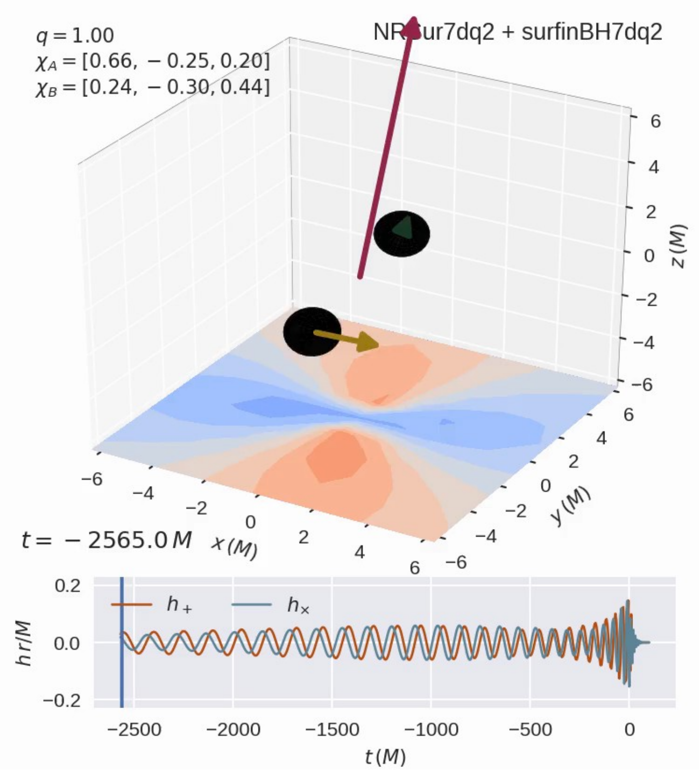
* see [Chatziioannou 2017]

Precessing and non-precessing BBH

Without precession

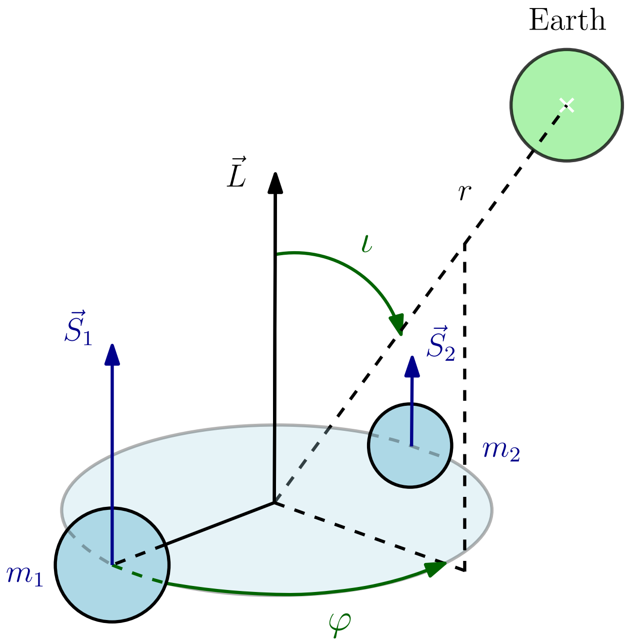


With precession

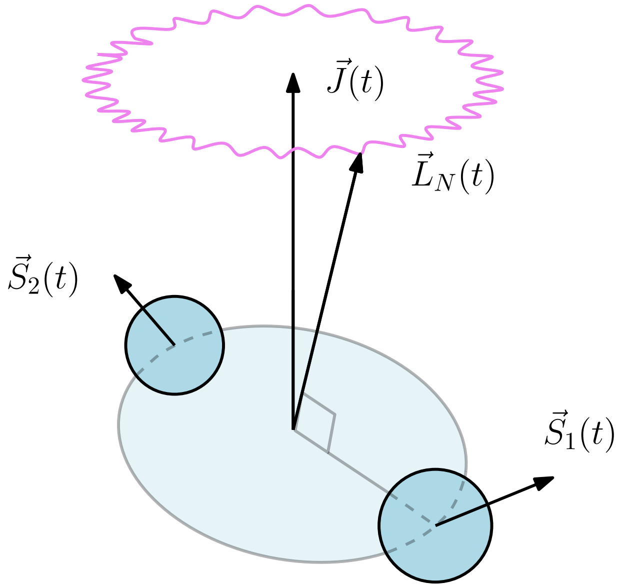


Credit : [arXiv:1811.06552], vijayvarma392.github.io/binaryBHexp

Precession parameters

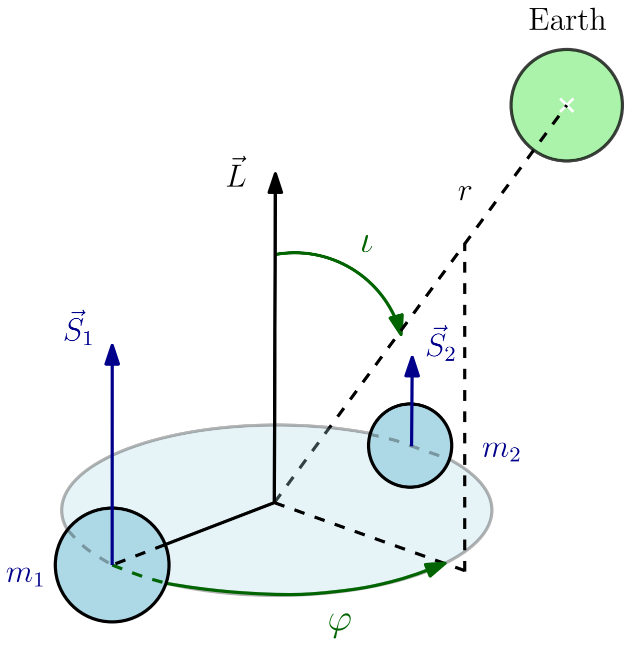


(ι, φ) : line of sight

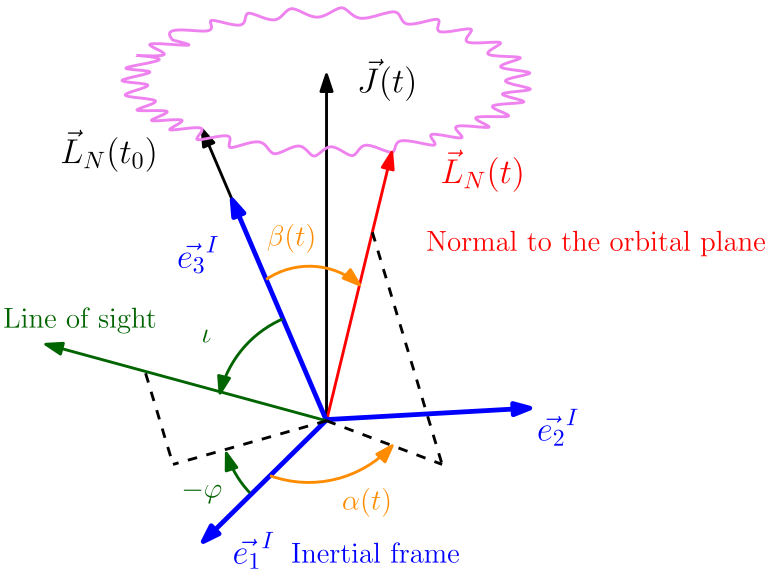


(α, β, γ) : rotation of the frame attached with $\vec{L}_N(t)$

Precession parameters



(ι, φ) : line of sight

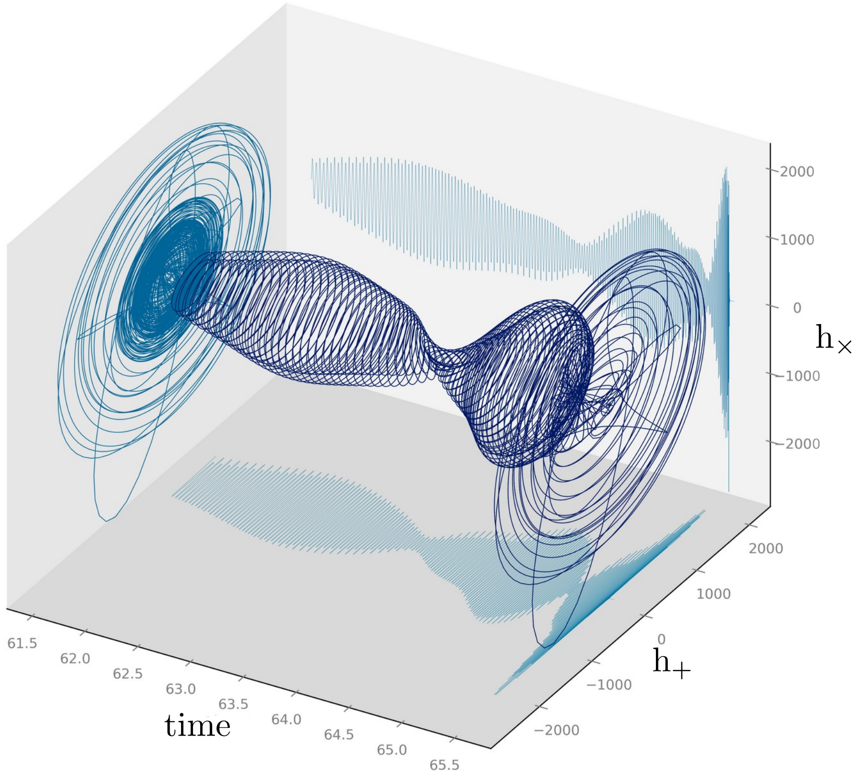


(α, β, γ) : rotation of the frame attached with $\vec{L}_N(t)$

Precessing waveforms

GW signal : $h = h_+ - \mathbf{i}h_\times$

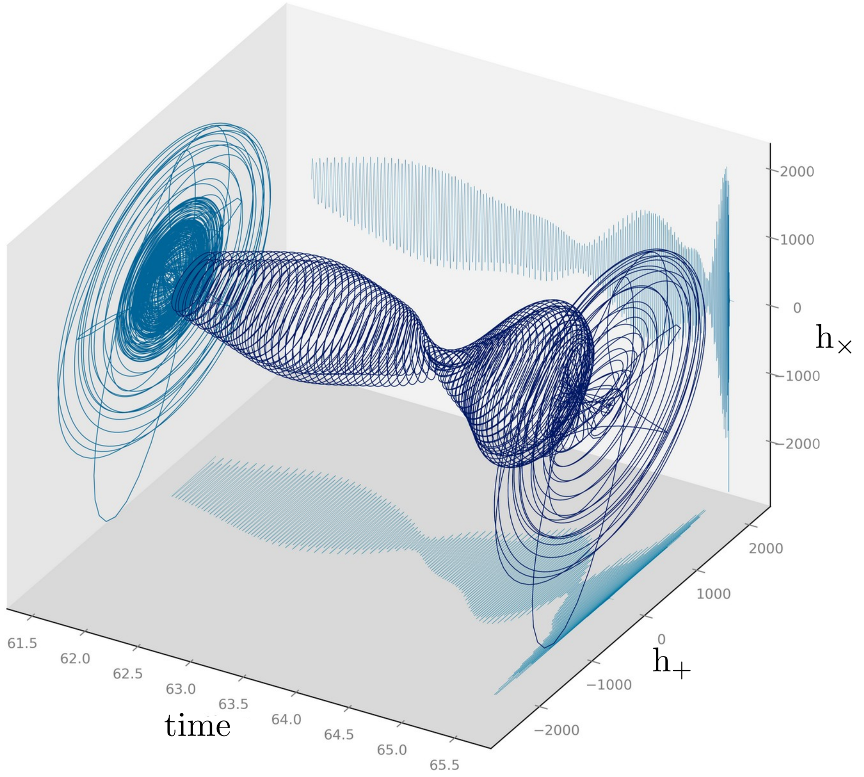
GW are non stationary polarized signals



Precessing waveforms

GW signal : $h = h_+ - \mathbf{i}h_\times$

Physical model : $$\mathrm{h} = \sum_{l=2}^{\infty} \sum_{m=-l}^{l} h_{l,m\,-2}Y_{l,m}(\iota, \varphi)$$



We need to consider several inclinations or other modes than (2,2)

↓

Get non stationary polarized signals

Polarimetric analysis

Bivariate signal : $h = h_+ - \mathbf{i} h_\times$

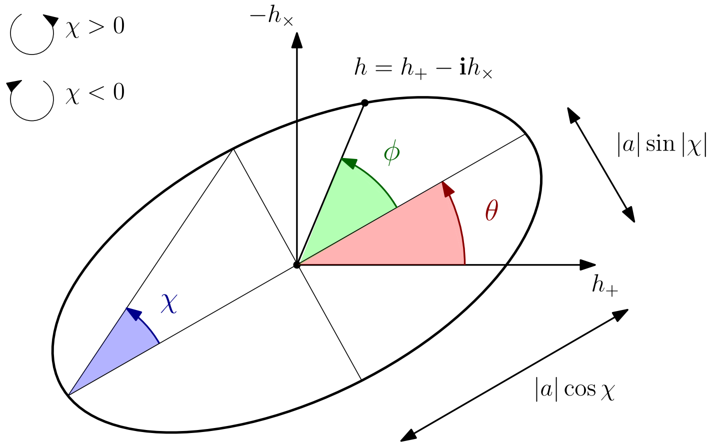
Amplitude, frequency and polarization modulated model :

$$\mathrm{h} = \mathrm{a(t)}\ \mathrm{e}^{\mathbf{i}\theta(t)} \left(\cos\chi(t)\cos\phi(t) + \mathbf{i}\cos\chi(t)\cos\phi(t)\right)$$

$$|\phi'(t)| \gg |\theta'(t)|\,,\ |\chi'(t)|\,,\ \left|\frac{a'(t)}{a(t)}\right|$$

Quaternionic embedding :

$$\begin{aligned} h_{\mathbb{H}}(t) &= h(t) + \mathcal{H}\{h\}(t)\mathbf{j} \\ &= a(t)e^{\mathbf{i}\theta(t)}e^{-\mathbf{k}\chi(t)}e^{\mathbf{j}\phi(t)} \end{aligned}$$



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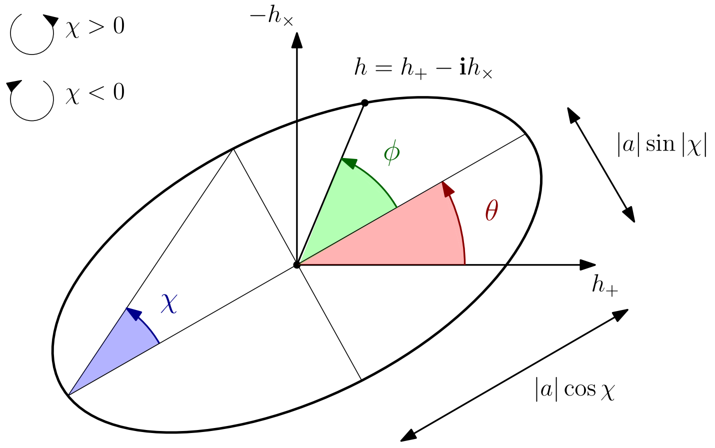
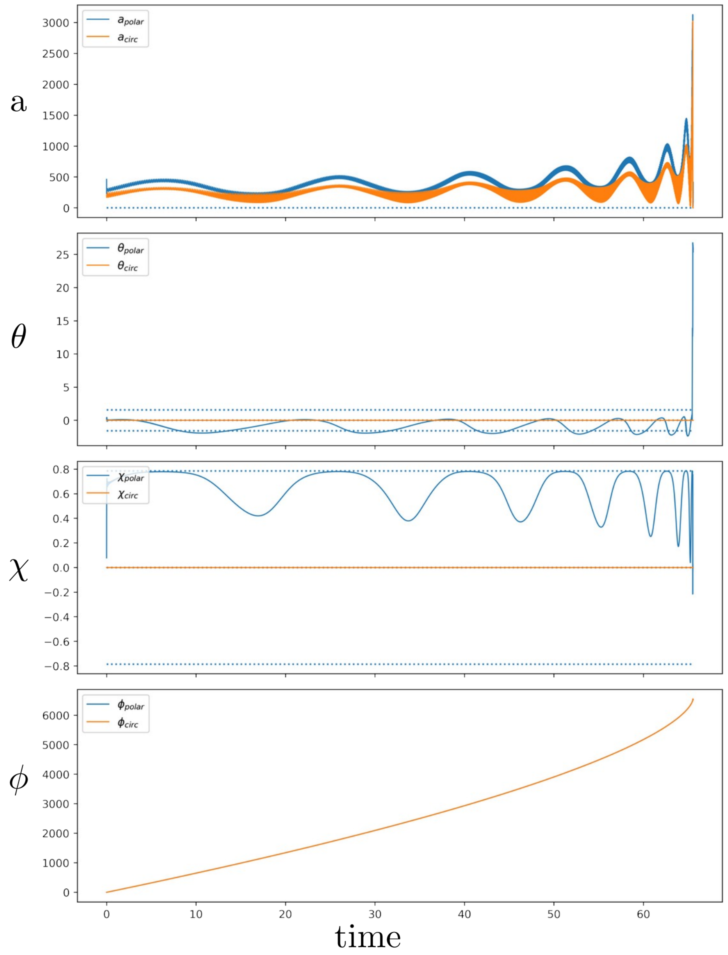
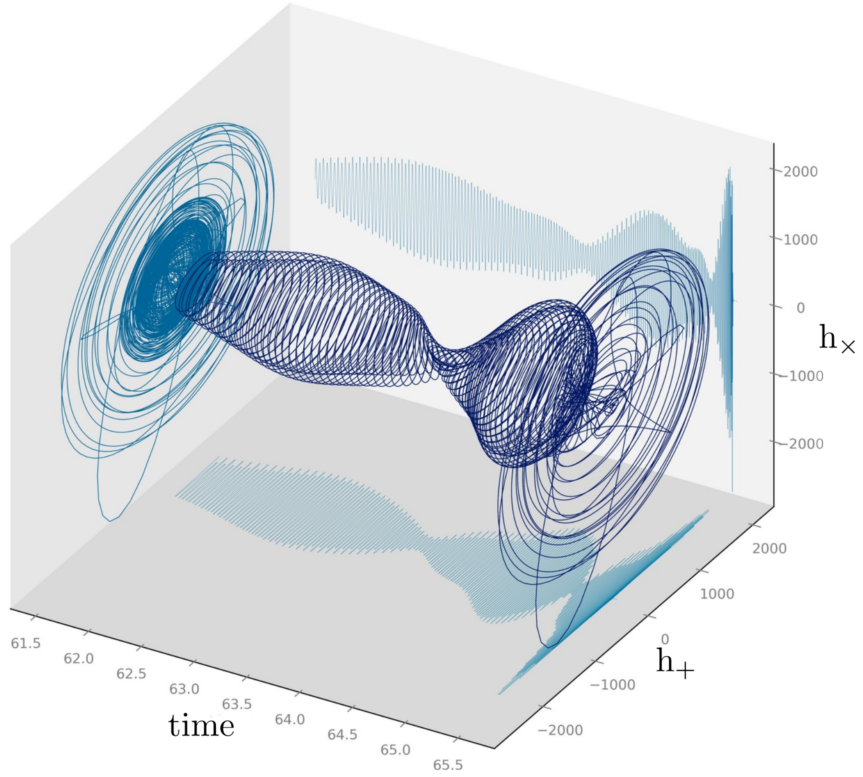


Limitations :

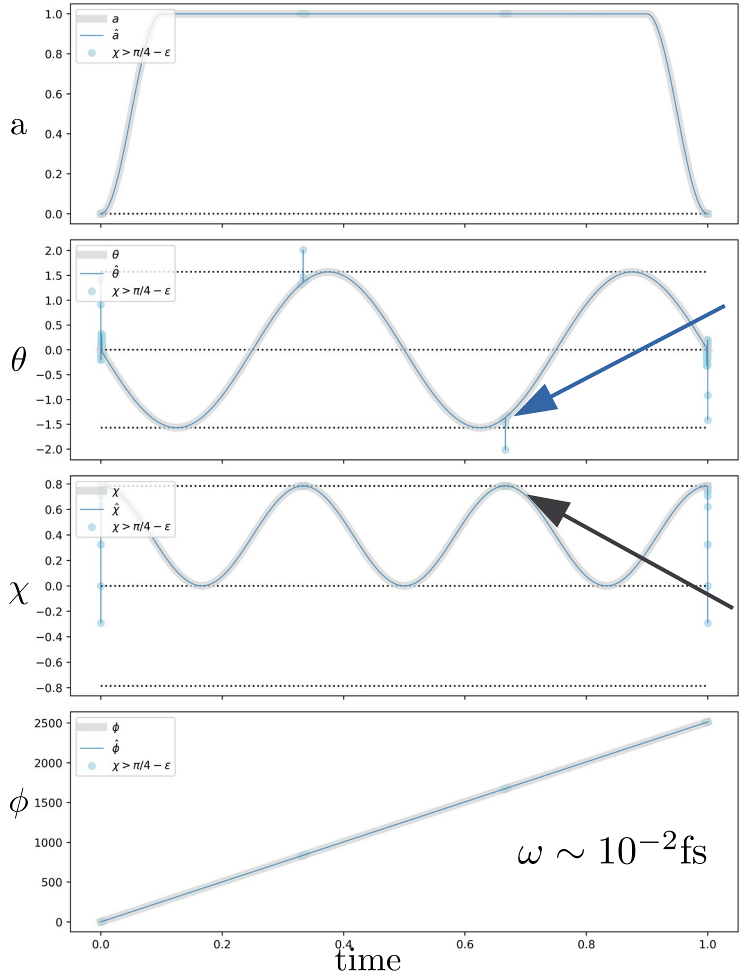
- Gimbal lock
  $\chi(t) = \pm\frac{\pi}{4}\ \forall t \in [t_0 - \delta, t_0 + \delta]$
- Instrumental gimbal lock
  $\left|\chi(t) - \pm\frac{\pi}{4}\right| < \epsilon\,\forall t \in [t_0 - \delta, t_0 + \delta]$
  and $|\phi'(t)| \ll \frac{\mathrm{fs}}{2}$ or $|\phi'(t)| \gg \frac{\mathrm{fs}}{2}$

Elliptic case



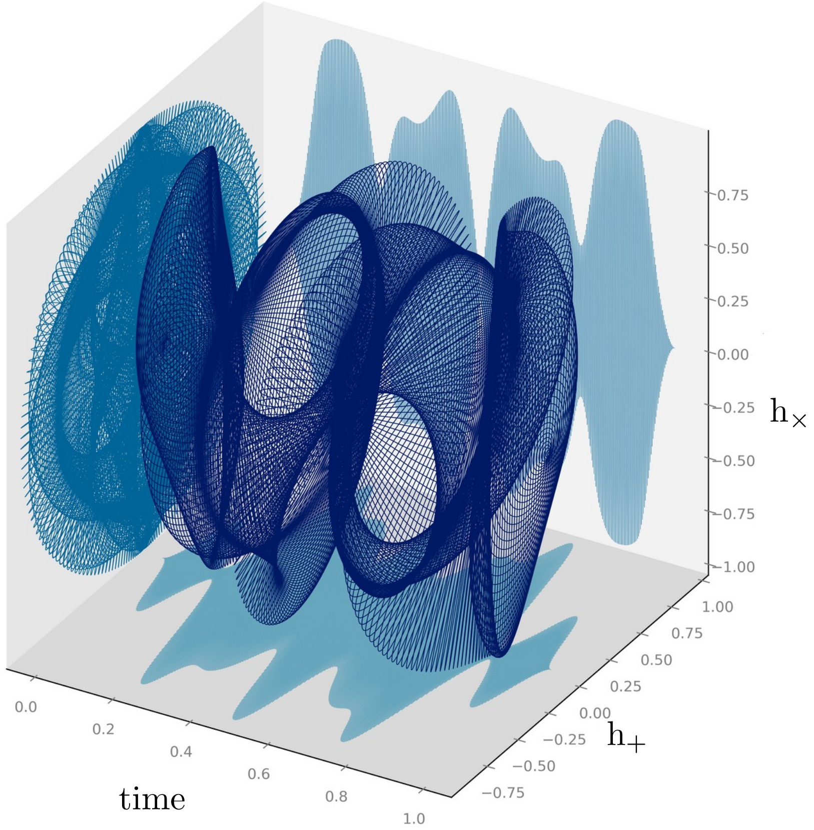


Toy example

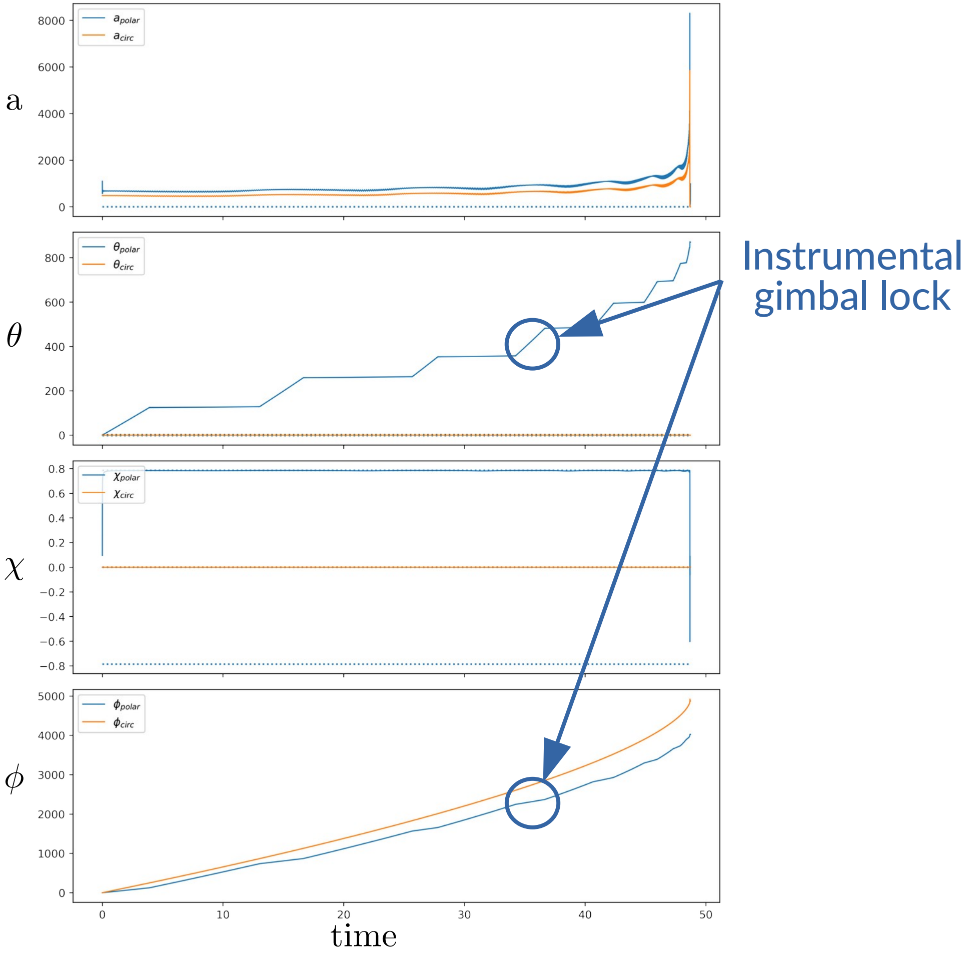


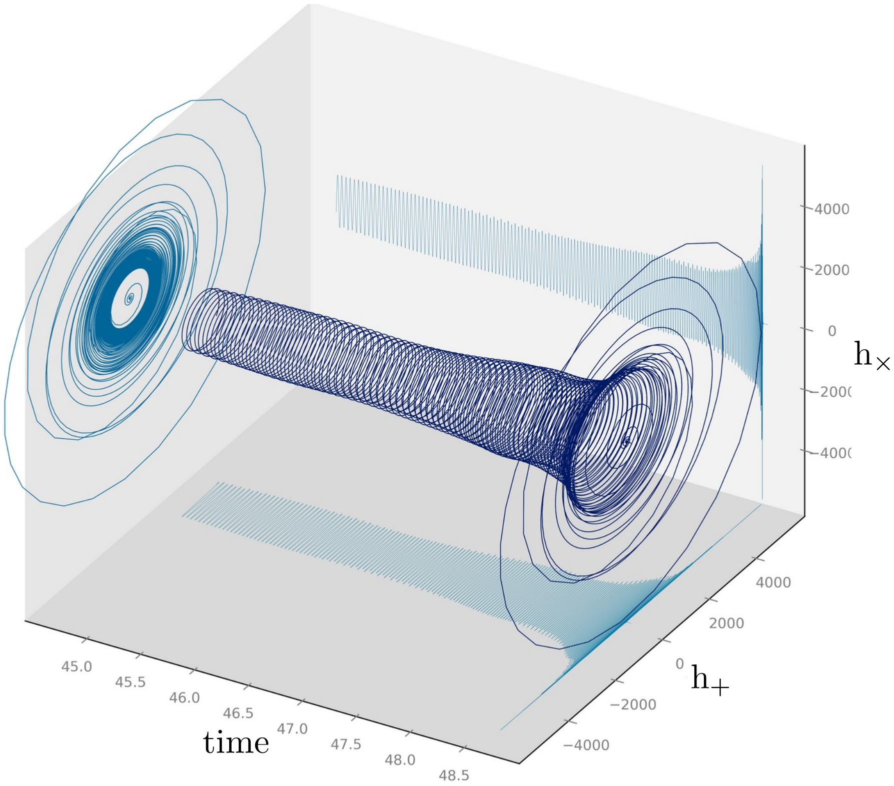
Instrumental gimbal lock

Near circular



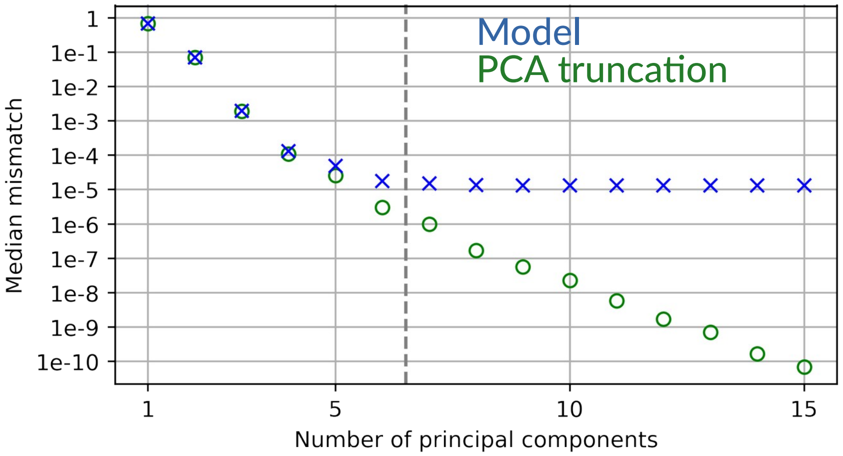
Near circular case



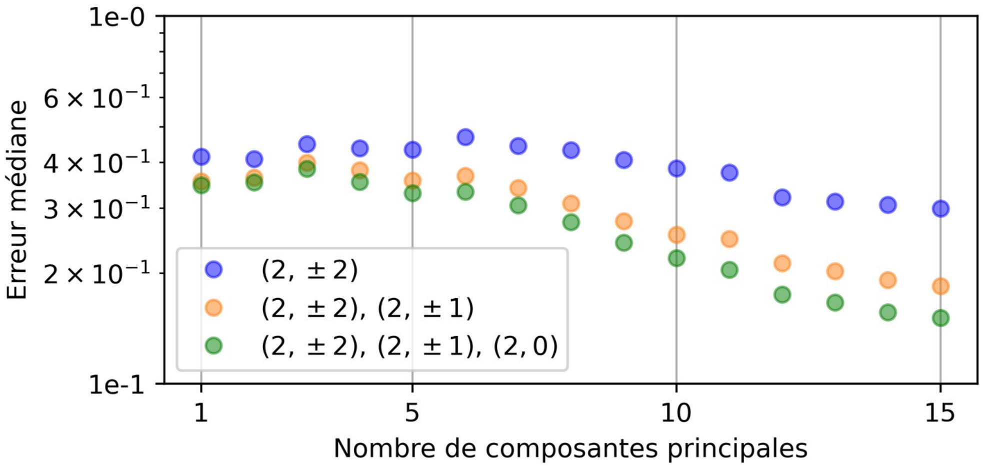


Hyperparameters tuning

Without precession



With precession



Conclusion

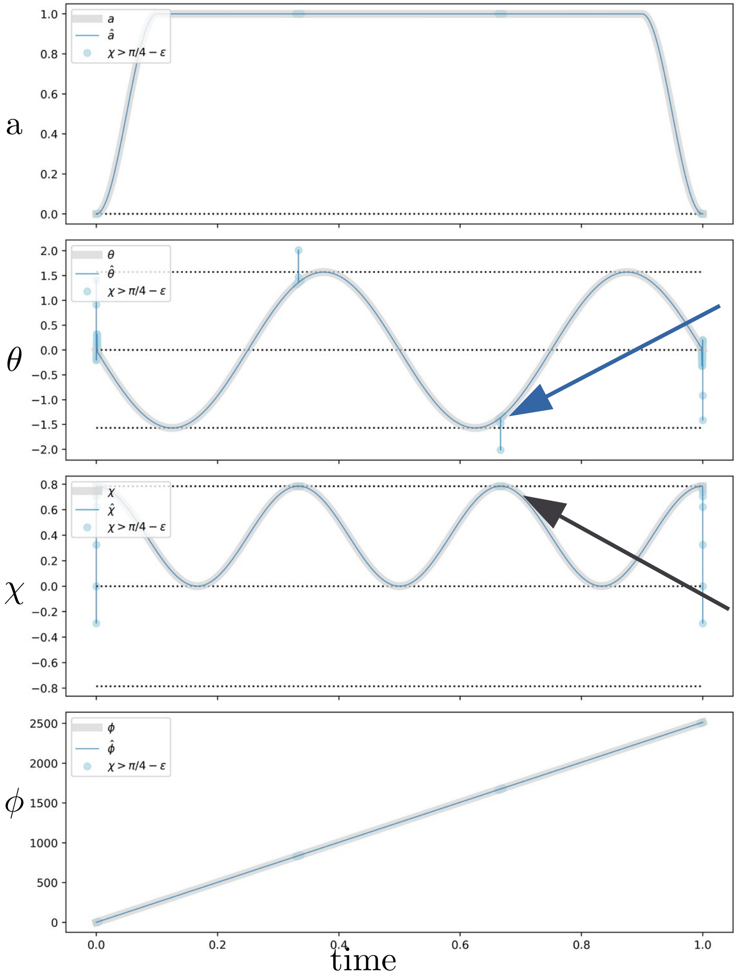
- **Representation problem** due to Euler angle definitions

- We are looking for alternative representations

- **Solutions :**
  - Subsampling
  - Other decomposition than $\mathrm{h}_{\mathbb{H}}(t) = a(t)e^{\mathbf{i}\theta(t)}e^{-\mathbf{k}\chi(t)}e^{\mathbf{j}\phi(t)}$

    ex: $\mathrm{h}_{\mathbb{H}}(t) = a(t)e^{\phi(t)\mu(t)}$
  - (Use instantaneous moments)

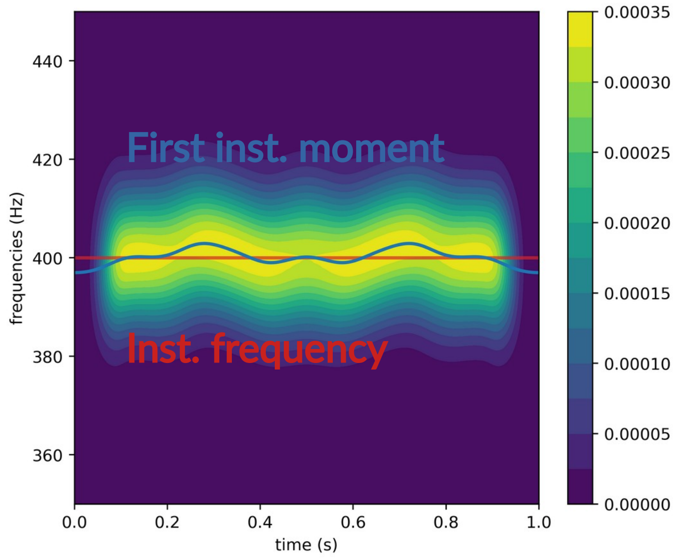
Toy example



Instrumental gimbal lock

Near circular

Idea : use tf-ridge to estimate $\phi(t)$ (fail)



$$\int_{\mathbb{R}_+} \omega \, |h_{\mathbb{H}}(\omega)|^2 \, \mathrm{d}\omega = \int_{\mathbb{R}} (\phi'(t) + \theta'(t) \sin 2\chi(t)) \, |h_{\mathbb{H}}(t)|^2 \, \mathrm{dt}$$

R^2 scores

$$R^2(y, \hat{y}) = 1 - \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)}{\sum_{i=1}^{n} (y_i - \bar{y})}$$

PC	1	2	3	4	5	6
a	1.67e-06	0.00231	0.0214	0.00728	1.42	0.177
Φ	1.65e-09	9.44e-07	0.000248	0.00322	0.00401	0.0326

Python library

README

Contact : cyril.cano@gipsa-lab.fr

Overview

This Git repo provides the library mlpgw.py that allows to train a machine-learning model able to regress gravitational-wave waveform from a set of examples as described in this article.

This Git repo includes several notebooks that allow to reproduce the results presented in the paper.

The learning part is mainly based on scikit-learn. This package is included in the required environment.

Take care that in the notebook a gravitational waveform $h(t)$ is denoted There was an error rendering this math block but There was an error rendering this math block in the paper.

Installation

Clone this Git repo and create the environment `gw-generation` by running:

```
conda env create -f environment.yml
```

Activate the environment

```
conda activate gw-generation
```

... and run the following command line from this folder:

```
conda develop .
```

How to generate a waveform using a pre-computed ML model?

This notebook shows how to generate a waveform with a pre-computed ML model.

The pre-computed model is stored in a set of Pickle files (see data/)

Git : https://git.ligo.org/cyril.cano/gw-generation

Python library

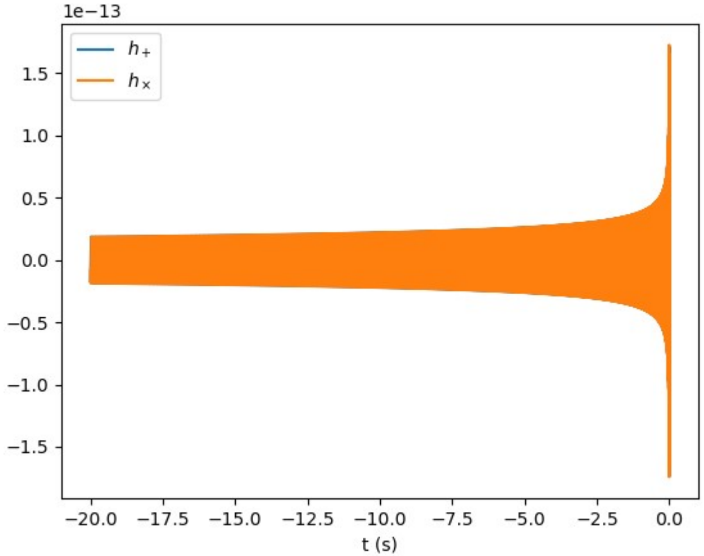
In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
import mlpgw

%matplotlib notebook
```

In [2]:
```
# Dowload the model
model = mlpgw.load_obj('../data/model')
```

In [3]:
```
# Make prediction
h_pred = model.predict(m1=15, m2=5, s1z=0.9, s2z=0.2)
```

In [4]:
```
# Plot it
plt.figure()
plt.plot(h_pred['time'], h_pred['hp'], label=r'$h_+$')
plt.plot(h_pred['time'], h_pred['hc'], label=r'$h_\times$')
plt.xlabel('t (s)')
plt.legend()
plt.show()
```



Git : https://git.ligo.org/cyril.cano/gw-generation

Polarization angle

Each detector : $$F_{+}^{D}(\Omega)h_{+}(t) + F_{\times}^{D}(\Omega)h_{\times}(t) + n(t) = \Re\left\{F^{D}(\iota,\,\varphi,\,\psi)h(t)\right\} + n(t)$$

Line of sight
and
polarization angle

$$F^{D}(\iota,\,\varphi,\,\psi) = e^{-\psi\mathbf{i}}\left[F_{+}^{D}(\iota,\,\varphi) + \mathbf{i}F_{\times}^{D}(\iota,\,\varphi)\right]$$

$$h(t) = h_{+}(t) - \mathbf{i}h_{\times}(t)$$