Probabilisitc Reasoning Over Time

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March 2023

Introduction

For an intelligent agent to be reason in partially observable environments (such as ones with hidden and observable states), the agent will need to maintain a belief state which represents which states of the world are currently possible; a transition model which the agent will use to predict how the world might evolve in the next time step; and from percepts observed and a sensor model, the agent can update the belief state.

A changing world is modeled using a variable for each aspect of the world state at each point in time. The transition and sensor models may be uncertain: the transition model describes the probability distribution of the variables at time t, given the state of the world at past times, while the observation/emission/sensor model describes the probability of each percept at time t, given the current state of the world.

Generic Temporal Model Setup

Let X_t denote the set of (latent/hidden) state variables at time step t, and E_t denote evidence/observation variables at time step t. Note that we will use the shorthand $X_{1:10}$ to denote $X_1, X_2, ...X_{10}$.

Given a set of hidden and observed variables, the "world" evolves according to a transition model, which is essentially a probability distribution over the latent states. In a first-order Markov model, the transition model is the conditional distribution $P(X_t|X_{t-1})$; the transition model of a second order Markov model is $P(X_t|X_{t-2}, X_{t-1})$. To avoid having an infinite number of distributions, one for each t, we assume that changes are caused by a stationary process – a process of change which are governed by laws that do not themselves change over time.

For the sensor/observation model, we assume that $P(E_t|X_{0:t}, E_{0:t-1}) = P(E_t|X_t)$.

We note that this setup describes a Bayesian network, where the dependency structure flows "forwards" in time for the latent state, and a single emission is produced given each latent state at time t.

Inference in Temporal Models

Filtering: compute belief state – given a sequence of observations, compute what the posterior distribution over the most recent (latent) state is -¿ $P(X_t|e_{1:t})$.

Prediction: compute the posterior distribution of a future state given a sequence of observations -¿ $P(X_{t+k}|e_{1:t})$, for $k > 0$.

Smoothing: compute the posterior distribution of a past state given evidence which extends into the future -¿ $P(X_k|e_{1:t})$ for $0 \leq k < t$.

Most likely path/explanation: compute most likely sequence of latent states given a sequence of observations.

If the transition and emission models are not yet known, they can be learned from observations based on inference. Inference provides an estimate of what transitions actually occurred and of what states generated the sensor readings, and these estimates can be used to update the models. The updated models provide new estimates, and the process iterates to convergence. The overall process is an instance of the expectation-maximization or EM algorithm.

Prediction:

Hidden Markov Models

Kalman Filters