

Matrix World : The Picture of All Matrices

I am happy to tell the history of Matrix World—the creation of Kenji Hiranabe in Japan. In April 2020 his friend Satomi Joba asked if I would send him a birthday message as a surprise. He was happy (and very surprised). Kenji combines mathematics with art and with computing : three talents in one. I was the one to be surprised when he sent Matrix World in its first form—without a name, without many of the entries and ideas that you see now, but with the central idea of displaying the wonderful variety of matrices.

Since that first form, Matrix World has steadily grown. It includes every property that would fit and every factorization that would display that property. Interesting that the SVD is in the outer circle and the identity matrix is at the center—it has all the good properties : the matrix I is diagonal, positive definite symmetric, orthogonal, projection, normal, invertible, and square.

Lek-Heng Lim has pointed out the usefulness of matrices M that are **symmetric and orthogonal**—kings and also queens. Their eigenvalues are 1 and -1 . They have the form $M = I - 2P$ (P = symmetric projection matrix). There is a neat match between all those matrices M and all subspaces of \mathbf{R}^n . You may see something interesting (or something missing) in Matrix World. We hope you will ! Thank you to Kenji.

Gilbert Strang

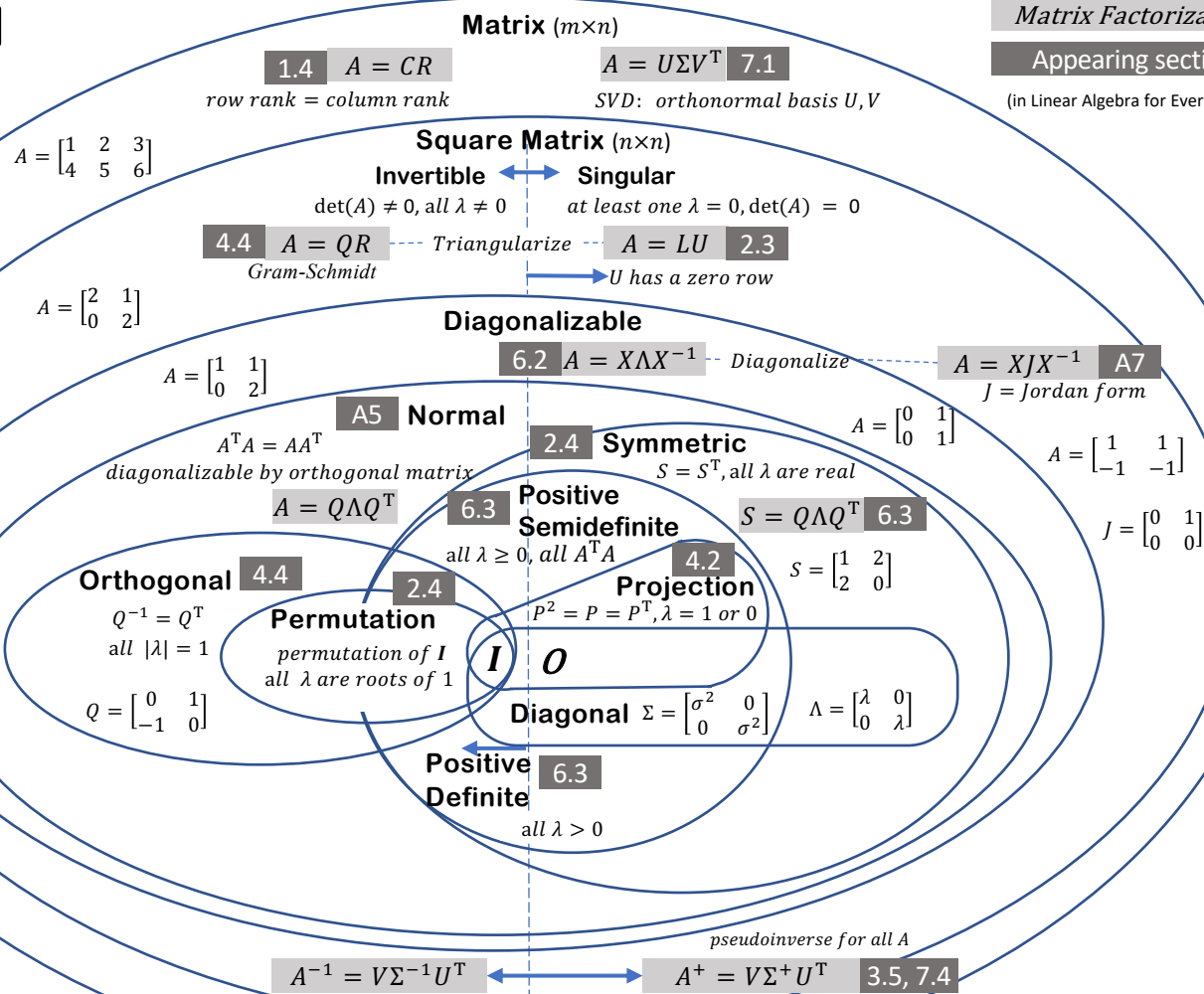
Matrix World

in
Linear Algebra
for Everyone

Matrix Factorization

Appearing section

(in Linear Algebra for Everyone)



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Matrix World

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行列の積分解

解説の節番号

(in Linear Algebra for Everyone)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$1.4 \quad A = CR$$

行 rank = 列 rank

$$A = U\Sigma V^T \quad 7.1$$

SVD: 単位直交基底 U, V

正方行列 ($n \times n$)

可逆 (正則) \longleftrightarrow 非可逆 (特異)

$$\det(A) \neq 0, \forall \lambda \neq 0 \quad \det(A) = 0, \exists \lambda = 0$$

$$4.4 \quad A = QR$$

グラム・シュミット法

$$A = LU \quad 2.3$$

U はゼロ行を持つ

対角化可能

$$6.2 \quad A = X\Lambda X^{-1}$$

対角化

$$A = XJX^{-1} \quad A7$$

J = ジョルダン標準形

$$A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$$

A5 正規

$$A^T A = A A^T$$

直交行列で対角化可能

$$A = Q\Lambda Q^T$$

対称 2.4

$$S = S^T, \forall \lambda \in \mathbb{R}$$

$$S = Q\Lambda Q^T \quad 6.3$$

$$A = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$$

$$J = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$S = \begin{bmatrix} 1 & 2 \\ 2 & 0 \end{bmatrix}$$

直交 4.4

$$Q^{-1} = Q^T$$

$$\forall |\lambda| = 1$$

$$Q = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

置換 2.4

I の並び替え
 $\forall \lambda$ は 1 の根

射影 4.2

$$P^2 = P = P^T, \lambda = 1 \text{ or } 0$$

I

O

対角

$$\Sigma = \begin{bmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{bmatrix}$$

$$\Lambda = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

正定値 6.3

$$\forall \lambda > 0$$

$$A^{-1} = V\Sigma^{-1}U^T$$

すべての A に対する擬似逆行列

$$A^+ = V\Sigma^+U^T \quad 3.5, 7.4$$



矩阵世界

矩阵分解

对应章节

(在 Linear Algebra for Everyone 中)

矩阵 ($m \times n$)

1.4 $A = CR$

行秩 = 列秩

$A = U\Sigma V^T$ 7.1

SVD: 单位正交基底 U, V

$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

方阵 ($n \times n$)

可逆

$\det(A) \neq 0, \text{all } \lambda \neq 0$

奇异

$\exists \lambda = 0, \det(A) = 0$

4.4 $A = QR$

格拉姆-施密特

三角化

$A = LU$ 2.3

U 有一个零行

$A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$

可对角化

6.2 $A = X\Lambda X^{-1}$

对角化

$A = XJX^{-1}$ A7

J = 约旦型

$A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$

A5 正规

$A^T A = A A^T$

可通过正交矩阵对角化

$A = Q\Lambda Q^T$

2.4 对称

$S = S^T$, 所有 λ 都是实数

$A = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$

$J = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

6.3 半正定

all $\lambda \geq 0$, all $A^T A$

4.2 投影

$P^2 = P = P^T, \lambda = 1 \text{ or } 0$

$S = Q\Lambda Q^T$ 6.3

$S = \begin{bmatrix} 1 & 2 \\ 2 & 0 \end{bmatrix}$

正交

$Q^{-1} = Q^T$

all $|\lambda| = 1$

$Q = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

置换

I 的置换

all λ are roots of 1

I O

对角

$\Sigma = \begin{bmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{bmatrix}$

$\Lambda = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$

正定

6.3

all $\lambda > 0$

所有 A 的伪逆

$A^{-1} = V\Sigma^{-1}U^T$

$A^+ = V\Sigma^+U^T$ 3.5, 7.4

