

Data Structure Summary

A data structure is an implementation of an [abstract] interface.

- List
- Queue
- Stack
- Deque [double ended queue]
- Unordered Set [set]
- Sorted Set
- Map [set of key-value pairs]
- Sorted Map [sorted set of key-value pairs (kvp)]

Access and Modification Characteristics

	get/set	add/remove
Arrays	$O(1)$	$O(1 + \min(i, n-i))$
LinkedList	$O(1 + \min(i, n-i))$	$O(1)*$
Skiplist	$O(\log n)$	$O(\log n)$

*given a pointer to the location, else traversal is necessary

Set

Efficient for contains().

SortedSet

Efficient for find().

- Does a successor search [closest value \geq to value]

Maps

Efficient for contains() [kvp]

SortedMap

Efficient for find() [kvp]

Array-based

Efficient for read / write. Expensive insertion / deletion.

ArrayList / ArrayStack

Efficient access anywhere. Efficient insertion / deletion at back [think stack].

- Implements **List** interface with an array
- superseded by ArrayDeque
- **get(), set() in $O(1)$**
- **add(), remove() in $O(1 + n-i)$**
- **resize is $O(n)$ [amortized]**

// for $m \geq 1$ add() / remove() calls, resize() will copy at most $2m$

// the amortized cost of resize() for m calls is $2m/m = O(1)$



ArrayQueue / ArrayDeque

Efficient access anywhere. Efficient insertion / deletion at front and back [think deque].

- Implements **List** interface with an array
- **get(), set() in $O(1)$**
- **add(), remove() in $O(1 + \min(i, n-i))$**

- **resize is $O(n)$** [amortized]

// since ArrayQueue only supports addLast() and removeFirst(), these are $O(1)$



DualArrayDeque

Efficient access anywhere. Efficient insertion / deletion at front and back [think deque].

- Implements **List** interface
- Uses two **ArrayStacks** front-to-front
- May be rebalanced if one array is much larger than the other
- **get(), set() in $O(1)$**
- **add(), remove() in $O(1 + \min(i, n-i))$**



RootishArrayStack

List of Lists, of increasing size. Efficient space [\sqrt{n} wasted space]. Efficient access anywhere. Efficient insertion / deletion at back.

- Implements the **List** interface using multiple backing arrays
- Reduces 'wasted space' [unused space]
- At most: \sqrt{n} unused array locations
- Good for space efficiency
- **get(), set() in $O(1)$**
- **add(), remove() in $O(1 + \sqrt{n-i})$**

// $m \geq 1$ add() / remove() calls, results on $O(m)$ time on resize()



Linked Lists

Efficient insertion / deletion. Expensive access.

Singly Linked List [SLList]

Nodes with pointer to next node. Efficient insertion / deletion. Expensive access.

- Implements the Stack and Queue **interfaces**.
- **get(), set()** in $O(1 + i)$
- **add(), remove()** in $O(1)$



Doubly Linked List [DLList]

Nodes with pointers to previous and next nodes. Efficient insertion / deletion. Expensive access.

- Implements the Stack and Queue **interfaces**.
- **get(), set()** in $O(1 + \min(i, n-i))$
- **add(), remove()** in $O(1 + \min(i, n-i))$



SELList [Space-Efficient Linked List]

Nodes with pointers to previous and next nodes. Values stored as blocks in each node. [you can skip data] Efficient insertion / deletion. Expensive access.

- Implements the **List** interfaces
- wasted space: $\{ n + O(b + n/b) \}$
- **get(), set()** in $O(1 + \min(i, n-i)/b)$
- **add(), remove()** in $O(1 + \min(i, n-i)/b)$

// $m \geq 1$ add() / remove() calls, results in $O(b \cdot m)$ time on
resize()

SkipLists

SLL with additional skipping pointers. Randomly generated structure.
Allows for faster searches.

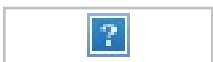
- Implements the **SortedSet** interface
- Successor search: **find(x)** will return smallest value $\geq x$
- **get(), set()** in $O(\log n)$
- **add(), remove()** in $O(\log n)$



After Midterm

HashTable

- Unordered sets with fast access
- Associative array
 - Index elements into a range of int
 - for non-integer elements, use hashCode()



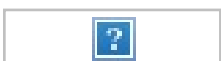
ChainedHashTable

- Implements the **USet** interface
- **find(), add(), remove()** in $O(n_i)$
 - where n_i is based on size of list at index

// $m \geq 1$ add() / remove() calls, results in $O(m)$ time on
resize()

Binary Tree

- Nodes with up to two child nodes



Binary Search Tree [BST]

- Implements the **SSet** interface
- **find(), add(), remove()** in $O(n)$

Random Binary Search Trees [RBST]

Balanced trees are statistically more likely

- Implements the **SSet** interface
- **constructed** in $O(n \log(n))$
- ****** in $O(n)**$
- **find(), add(), remove()** in $O(\log n)$

// search path is at most $2 \cdot \log(n) + O(1)$

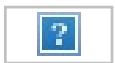
Treaps

Has an extra priority:

Parent priority should be less than child priority.

This has the property of bounding the height of the tree.

- Implements the **SSet** interface
- Priorities are randomly applied
- **constructed** in $O(n \log(n))$
- **find(), add(), remove()** in $O(\log n)$



Scapegoat Tree

BST that with height maintained within $O(\log n)$, rebuilt if too unbalanced

- Implements the **SSet** interface
- Rebuild only one search path that triggered rebuild
 - this ensures that not entire tree is rebuilt
- **rebuild()** in $O(\log n)$ amortized
- **find(), add(), remove()** in $O(\log n)$

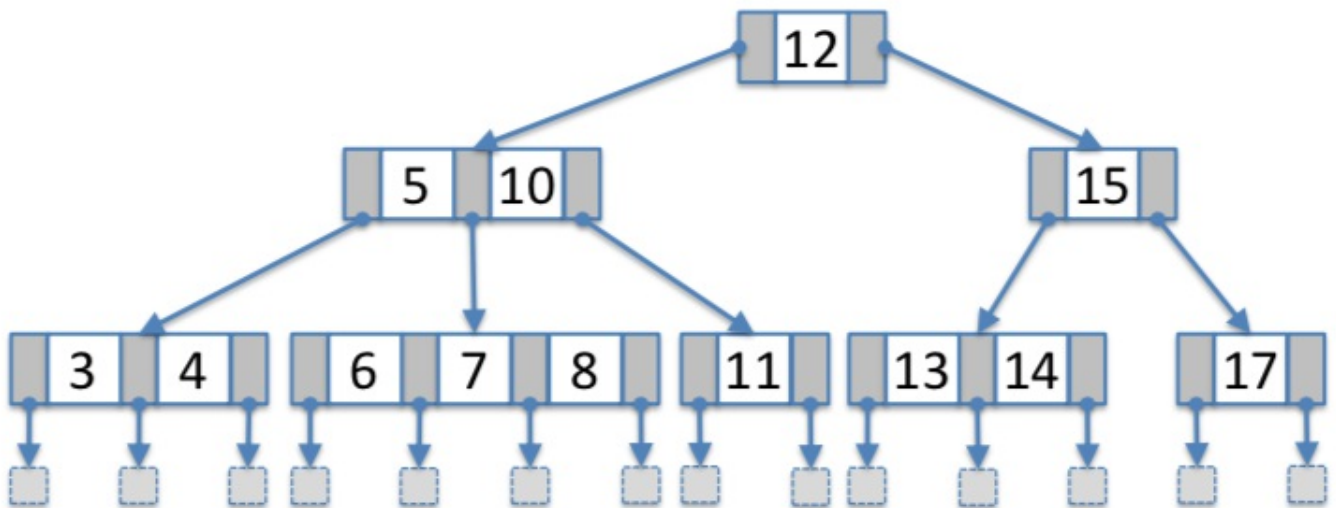


// m calls to add() / remove (), results in $O(m \cdot \log(n))$ time spent on rebuild()

2-4 Tree

Tree where every leaf has the same depth.

- Implements the **SSet** interface
- All leaves have equal depth
- All internal nodes have 2-4 children
- **find(), add(), remove()** in $O(\log n)$ [worst-case]



RedBlack Tree

A self-balancing binary search tree, built off a 2-4 Tree, where each node has a 'colour'.

- Implements the **SSet** interface
- Uses colour to remain balanced when adding / removing
 - There is the same number of black nodes on every root to leaf path
 - i.e. equal sum of colours on any root to leaf path
- No red nodes can be adjacent
 - red nodes must have black parent

- left-leaning: if left node is black, then right node must be black
- **Maximum height of $2 \bullet \log(n)$**
- **find(), add(), remove() in $O(\log n)$ [worst-case]**

Binary Search Tree Implementations

	find()	add()	remove()
BST	$O(n)$	$O(n)$	$O(n)$
RBST / Treaps	$O(\log n)$ [expected]	$O(\log n)$ [expected]	$O(\log n)$ [expected]
Scapegoat Trees	$O(\log n)$ [amortized]	$O(\log n)$ [amortized]	$O(\log n)$ [amortized]
2-4 / RedBlack Trees	$O(\log n)$ [worst-case]	$O(\log n)$ [worst-case]	$O(\log n)$ [worst-case]

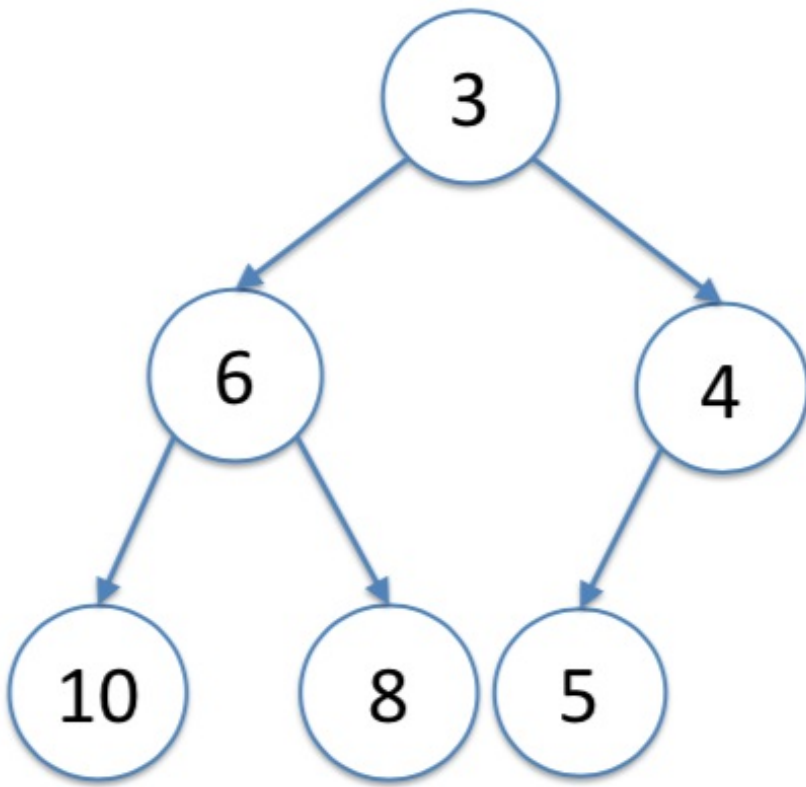
Sorted Set Implementations

	Runtime
Skiplists	$O(\log n)$ [expected]
Treaps	$O(\log n)$ [expected]
Scapegoat Trees	$O(\log n)$ [amortized]
2-4 / RedBlack Trees	$O(\log n)$ [worst-case]

Binary Heaps

A complete Binary Tree that also maintains the heap property.

- Implements the [priority] **Queue Interface**
- Allows to find / remove most extreme node with peek() / remove()
- **add(), remove() in $O(\log n)$**
- **peek() in $O(1)$**



// $m \geq 1$ add() / remove() calls, results in $O(m)$ time on
resize()