

Sorting Algorithms Summary

In-place: means modifying list to be sorted [as opposed to returning new sorted list]. **Stable:** means the order of elements with equal values is preserved.

Lower bound on Comparison-based Sorting

For a comparison-based algorithm, the expected number of comparisons is $\Omega(n \cdot \log(n))$.

Merge Sort

Sort list by merging sorted sub-lists, reduces total number of comparisons needed.

1. Divide list into equally sized halves until 1 element per sub-list
2. Sort sub-list [recursively]
3. Merge sub-list using comparator

- Comparison-based
- **Not in-place**
- **Stable**
- **Runs in $O(n \cdot \log(n))$ time**

// performs at most $n \cdot \log(n)$ comparisons

Heap Sort

Sort by traversing down a heap tree.

1. Create a heap from list

2. Delete the root node [in list, $a[n-1]$]
 - Now root is in $a[n]$, since $n--$
3. Heapify [make sure heap property is preserved]
4. Repeat steps 2–4 until no more elements to sort

- Comparison-based
- **In-place**
- **Not stable**
- **Runs in $O(n \log(n))$ time**

// performs at most $2n \cdot \log(n) + O(n)$ comparisons

Quick Sort

Sort using a randomly selected value as partition point, sorting sub-lists. Since random selection, might choose worst value [ideally middle value].

1. Randomly select value
2. Add all values less than to left sub-list, otherwise add to right sub-list
3. Repeat until 1 element in sub-list

- Comparison-based
- **In-place**
- **Not stable**
- **Runs in $O(n \log(n))$ [expected] time**

// performs at most $2n \cdot \log(n) + O(n)$

Comparison-based Algorithms

	Comparisons	In-place	Stable
Merge Sort	$n \cdot \log(n)$ [worst-case]	no	yes
Heap Sort	$1.38n \cdot \log(n) + O(n)$ [worst-case]	yes	no

Quick Sort	$2n \bullet \log(n) + O(n)$ [expected]	yes	no
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Merge Sort:

- Fewest comparisons
- Does not rely on randomization [guaranteed runtime]
- Not in-place [expensive memory usage]
- Stable
- Much better at sorting a linked list
 - no additional memory is needed with pointer manipulation

Quick Sort:

- Second fewest comparisons
- Randomized [expected runtime]
- In-place [memory efficient]
- Not stable

Heap Sort:

- Most comparisons
- Not randomized [guaranteed runtime]
- In-place [memory efficient]
- Not stable

Counting Sort

Counting array is used to keep track of duplicates; it is then used to construct the sorted list.

- Not comparison-based
- **Not in-place**
- **Not stable**
- **Runs in $O(n+k)$ time**
 - n integers
 - range of $0 \dots k$

// efficient for integers when the length is roughly equal to maximum value $k-1$

Radix Sort

Sorts w -bit integer with counting sort on d -bits per integer [least to most significant]

- Not comparison-based
- **Not in-place**
- **Not stable**
- **Runs in $O(c \bullet n)$ time**
 - n w -bit integers
 - range of $0 \dots (n^c - 1)$