COMP 2402 Class Notes

Java Collections Framework (JCF)

The Java Collections Framework (JCF) is a unified architecture for representing and manipulating collections.

A collection — sometimes called a container — is simply an object that groups multiple elements into a single unit. Collections are used to store, retrieve, manipulate, and communicate aggregate data. Typically, they represent data items that form a natural group, such as a poker hand (a collection of cards), a mail folder (a collection of letters), or a telephone directory (a mapping of names to phone numbers). If you have used the Java programming language — or just about any other programming language — you are already familiar with collections.

In order to use the JCF you can import it like this.

```java
import java.util.*
```

Sorting

This is how to sort strings based on length by using anonymous object [**Comparator**].

```java
Collections.sort(list, new Comparator<String>() {
    public int compare(String x, String y) {
        return x.length() - y.length();
    }
});

// or you can use lambda function
list.sort( (String o1, String o2) -> o1.compareTo(o2)
```

```java
);

// if you want to sort by length and also
alphabetically
Collections.sort(list,new Comparator<String>() {
    public int compare(String x, String y) {
        // if not same length, use length
        if(x.length() != y.length()) {
            return x.length() - y.length();
        }
        // else compare as strings
        return x.compareTo(y);
    }
});
```

The **compare(x,y)** method works by moving an element left if the **compare(x,y)** method returns a negative integer, and moves the element right if the **compare(x,y)** returns a positive integer. [difference between x and y]

```
(-) x < y
(0) x = y
(+) x > y
```

Maps [Hashmap]

Also known as dictionaries in Swift or C#...

- Cannot have duplicate entries

```java
Map<String, Integer> map = new HashMap<>();
map.put("Java", 6);
map.put("Swift", 10);
map.put("C#", 7);
map.put("Ruby", 9);
```

```
// this will print out every value in the map [foreach]
for(String str : map.keySet()) {
    System.out.println(str + " : " + map.get(str))
}

map.get(key); // fast operation, returns null if no key
found
```

List

Continuing from previous example...

Map.Entry is just a key-value pair

```
List<Map.Entry<String,Integer>> entryList = new
ArrayList<>();
entryList.addAll(map.entrySet); // set containing all
the elements

for(Map.Entry<String,Integer> entry : entrylist) {
    System.out.println(entry.getKey() + " : " +
entry.getValue() );
}
```

Deque [ArrayDeque]

Fast for reading/writing at *start* or *end* of list. Basically just a flexible stack/queue.

```
Deque<String> dq = new ArrayDeque<>();
dq.addFirst("second");
dq.addFirst("first");
dq.addLast("penultimate");
dq.addLast("last");
```

Priority Queue

Essentially: uses a heap instead of a tree, in order to keep a certain one on top. So first element is 'sorted' and then rest is unsorted.

Not good for sorting, or random access.

```
Queue<String> pq = new PriorityQueue<>();
pq.addAll(list);

System.out.println(pq.remove());    // remove smallest
element
```

If alphabetical, one that starts with 'a' will be removed. After first element, the queue is not sorted. Removing one will promote next smallest to the top

Asymptotic Notation [Big O]

Used to analyze complexity of algorithms, to find faster, or which ones requires more space.

Comparing data structures

- Time
- Space
- Correctivenes

Growth rates proportioanl to n

- If input doubles in size, how much will runtime increase?

Runtime as a count of primative operation

- This is machine independent

- Proportional to exact runtimess

```
for(int i = 0; i < n; i++) {
    arr[i] = i;
}
```

Runtime:

- **1**: assignment [int i = 0]
- **n+1**: comparisons [i < n]
- **n**: increments [i++]
- **n**: array offset calculations [arr[i]]
- **n**: n indirect assignments [arr[i] = i]

Definition of Big O

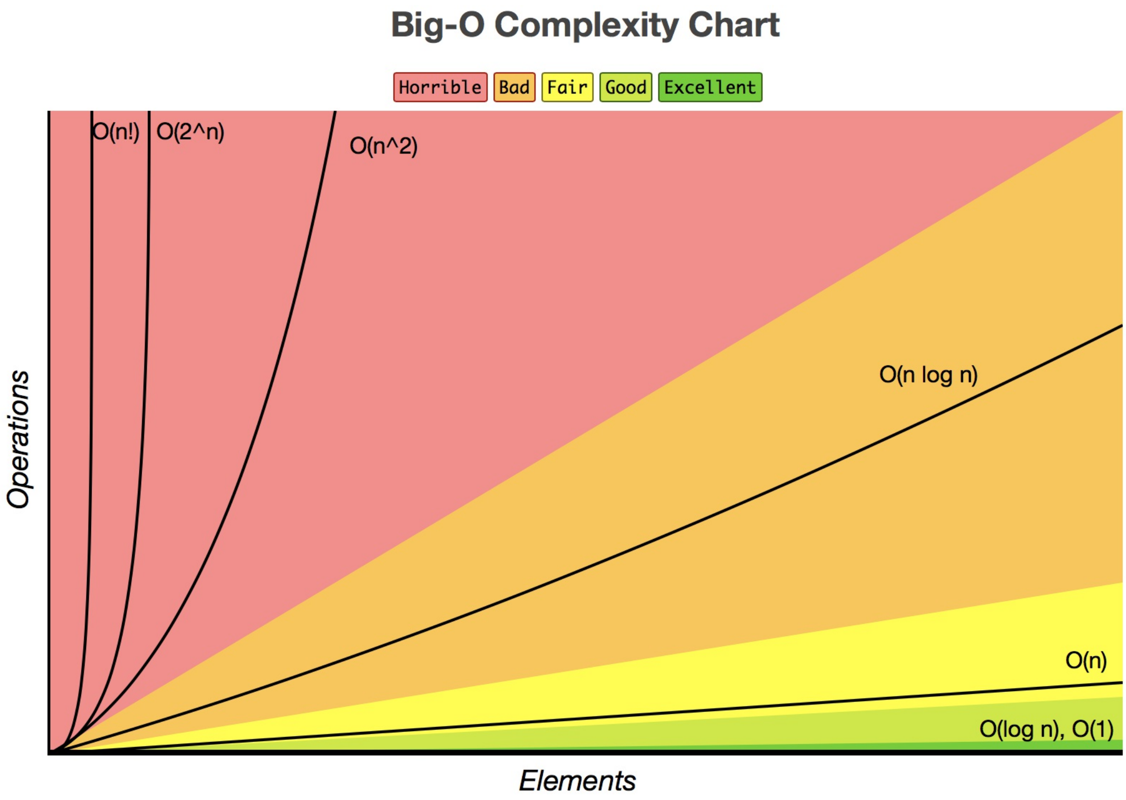
After a certain point, g(x) will grow as fast [or faster] than f(x)

- g(x) is the upper limit to f(x)

`O(g(n)) ∀ (f(n) < c•g(n))`

Orders of growth

Complexity	Name
O(1)	Constant
O(log n)	Logarithmic
O(n)	Linear
O(n log n)	Quasilinear
O(n^2)	Quasilinear
O(2^n)	Exponential
O(n!)	Factorial



Tips

- Only largest values matter
- Drop all coefficient
- Log bases are all equivalent

Example

```
public class BigO {
    public static void main() {
        String str = "";
        int n = 100;    // O(1)

        for(int i = 0; ,< n; i++) {      // O(n)
            str += "x";     // O(1) but n times
        }

        for(int i = 0; i < n; i+=2) {    // n/2 times -
> O(n)
            str += "y"      // O(1)
```

```
        }

        // this is roughly the same as if n was n/2
with O(n)
        for(int i = 0; i < n; i*=2) {    // O(log n)
            str += "y"       // O(1)
        }
    }
}
```

Array-based Data Structures

ArrayStack [ArrayList]

- Implements **List** interface with an array
- Similar to ArrayList
- Efficient only for stack operations [back]
- superceded by ArrayDeque
- **get(), set() in O(1)**
- **add(), remove() in O(1 + n-i)**
    - good for write at the back

Stacks vs List

Stack	List
push(x)	add(n,x)
pop()	remove(n-1)
size()	size()
peek(x)	get(n-1)

List Interface

- get(i) / set(i,x)
    - Access element i, and return/replace it

- size()
    - number of items in list
- add(i,x)
    - insert new item x at position i
- remove(i)
    - remove the element from position i

dereferencing: getting the address of a data item

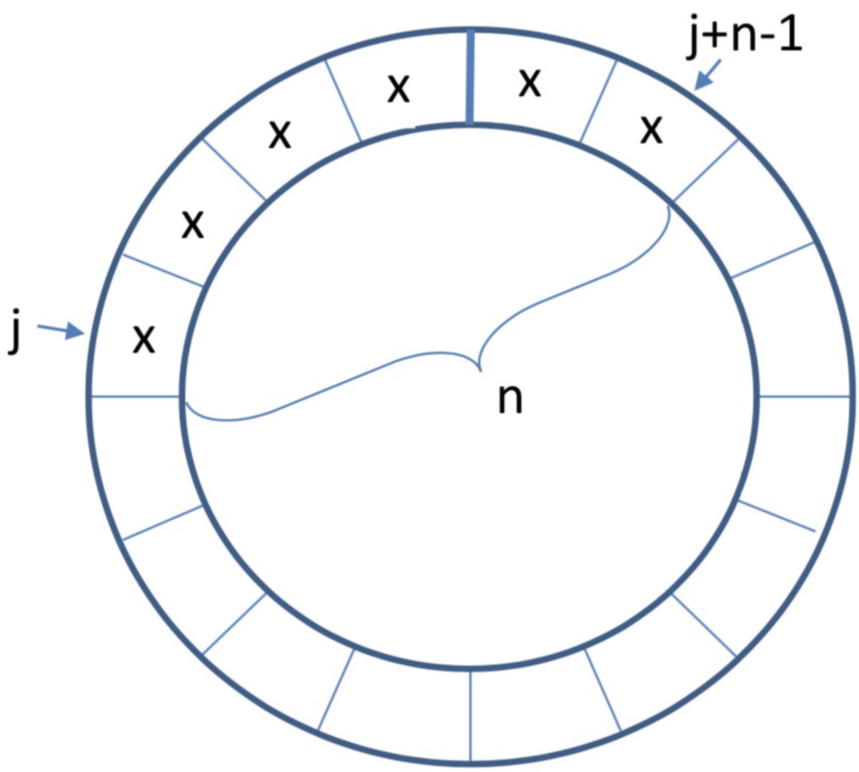
Amortized Cost

When an algorithm has processes that may be much longer but usually is quick, so you take the average. [roughly]

e.g. resizing an an array when adding/removing

ArrayQueue & ArrayDeque

Allow for efficient access at front and backs.



ArrayQueue

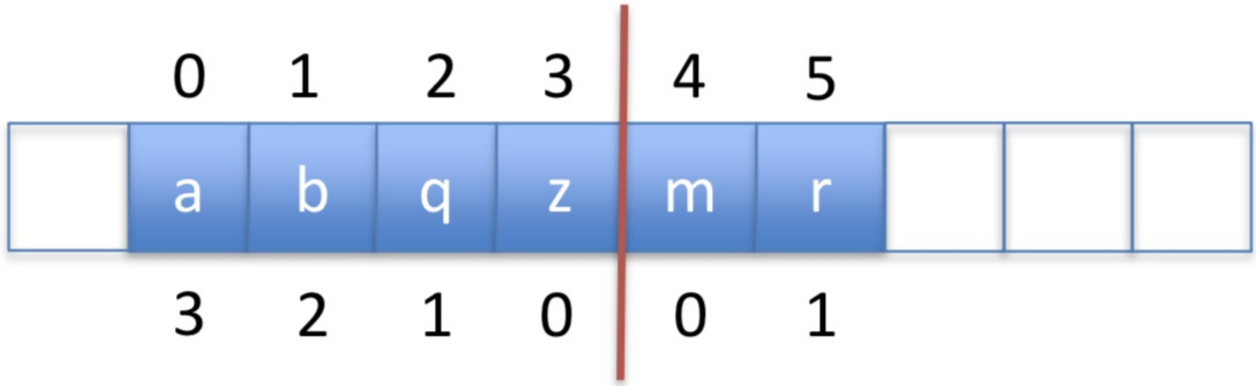
- Implements **Queue** and **List** interfaces with an array
- Cyclic array, (n: number of elements, j: 'index' of last element)
- **get(), set() in O(1)**
- **add(), remove () in O(1 + min(i, n-i))**
    - quick to write at front or back
    - cannot access anywhere else
- **resize is O(n)**

ArrayDeque

- Implements **List** interface with an array
- **get(), set() in O(1)**
- **add(), remove() in O(1 + min(i, n-i))**
    - quick to write at front or back
    - not so quick to access middle
- **resize is O(n)**

DualArrayDeque

- Implements **List** interface
- Uses two **ArrayStacks** front-to-front
- Since arrays are quick to add to the end, this makes front and back operations fast
- May be rebalanced if one array is much larger than the other
- Use Potential Function to decide when to rebalance
- **get(), set() in O(1)**
- **add(), remove() in O(1 + min(i, n-i))**
    - quick to write at front or back, but not middle



Potential Function

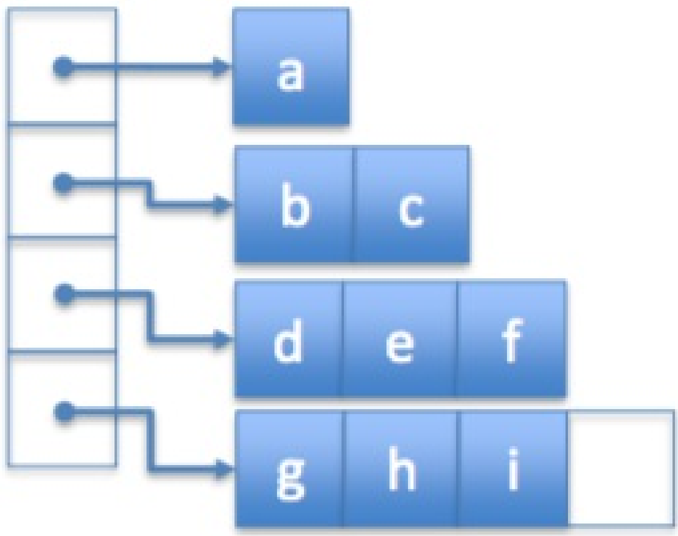
Define a potential function for the data structure to be the absolute difference of the sizes of the two stacks

```
P = | front_array.size - back_array.size |
```

- Adding or removing an element can only increase/decrease 1 to this function

RootishArrayStack

- Implements the **List** interface using multiple backing arrays
- Reduces 'wasted space' [unused space]
- At most: *sqrt(n)* unused array locations
- Good for space efficiency
- **get(), set() in O(1)**
- **add(), remove() in O(1 + n-i)**
    - quick to write at the back

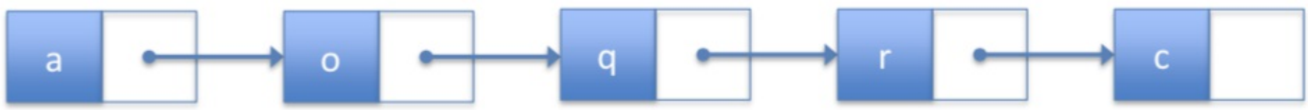


Linked Lists

- Recursive data structure made up of nodes
- Pointers to head and tail, and each node points to the next node
- Efficient add/remove but slow read/write

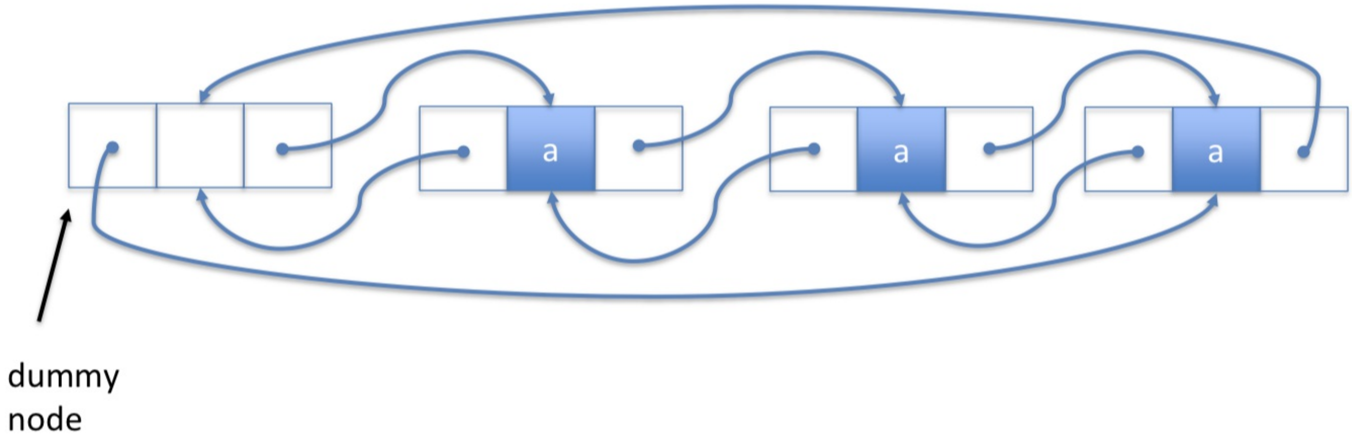
SLList [Singly-Linked List]

- Implements the **Stack** and **Queue** interfaces
- **push(), pop() in O(1)**
- **add(), remove() in O(1)**



DSList [Doubly-Linked List]

- Forward and backwards pointers at each node
- Implements the **List** interfaces
- **get(), set() in O(1 + min(i, n-i))**
- **add(), remove() in O(1 + min(i, n-i))**

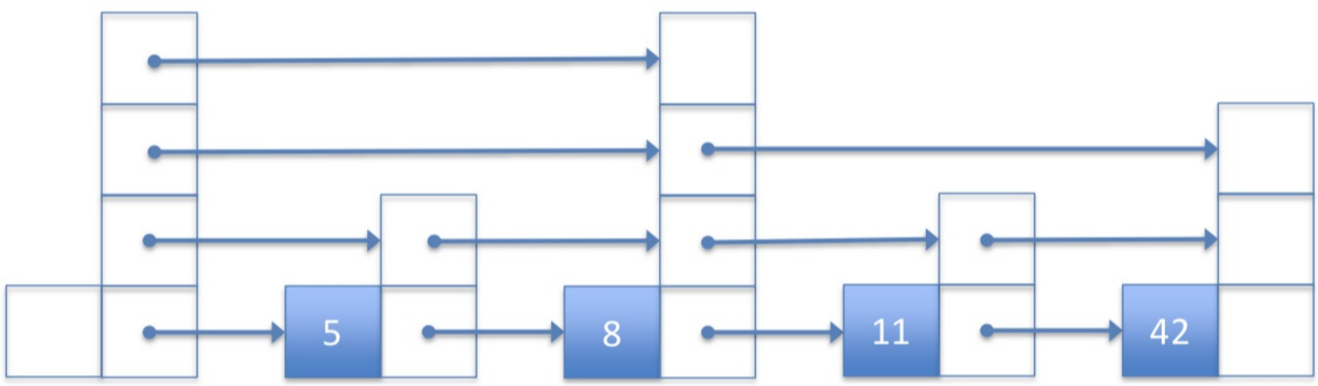


SELList [Space-Efficient Linked List]

- Like a doubly-linked list, but uses block size *b*
- Is a series of **ArrayDeque** with *next* and *prev* pointers
- Implements the **List** interfaces
- **get(), set() in O(1 + min(i, n−i)/*b*)**
- **add(), remove() in O(1 + min(i, n−i)/*b*)**
    - is quicker because you can skip blocks of data

Skiplist

- Like a singly-linked list, with 'skips'
- Randomly generated structure
- Faster searches than linked lists
- Additional nodes with pointers that allow 'skipping'
- Successor search: **find(x) will return smallest value ≥ x**
- **find(), add(), remove() in O(log n)**



List Implementations

	get/set	add/remove
Arrays	*O(1)*	*O(1 + min(i,n−i))*
LinkedList	*O(1 + min(i,n−i))*	*O(1)**
Skiplist	*O(log n)*	*O(log n)*

`*given a pointer to the location, else traversal is necessary`

Definitions

Random variable: a random sample from a group of values

Expected value: average value of a random variable

Indicator variable: random variable with values of 0 or 1

Linearity of Expectation: the expected value of a sum is equal to the sum of expected values

`Expected height of node [if coin flips were used]:`

```
P(x = 1) = 1/2      // prob. that tails on first flip
P(x = 2) = 1/4      // prob. that tails on second flip
P(x = 3) = 1/8      // prob. that tails on third flip
...
P(x = i) = 1/(2^i)

Thus,
E[x] = i*Sum(1/2^i)       // for all natural numbers
E[x] = Sum(E[I_j])
E[x] = Sum(P(I_j = i))

Indicator variable: 1 if tails, 0 is heads
P(I_1 = 1) = 1
P(I_2 = 1) = 1/2
...
```

```
P(I_j = 1) = 1/(2^(i-1))

let S = Sum(P(x = i)) = 1 + 1/2 + 1/4 + ...

therefore,
S/2 = 1/2 + 1/4 + 1/8 + ...

S - S/2 = 1
=> S = 2

 E[x] = Sum(P(I_j = i)) = S = 1 + 1/2 + 1/4 + ...
 E[x] = 2
```

Expected number of elements in the skiplist:

```
E[n_i] = ?

I_(i,j) = 1 if in list, 0 is not in list
// i is 0...n-1, number of nodes in list

// expected value of sum of indicator(element in list)
E[n_i] = E[ Sum( I_(i,j) ) ]
E[n_i] = Sum( E[ I_(i,j) ] )
E[n_i] = Sum( 1/(2^i) )     // average number values in
each node
E[n_i] = n * ( 1/(2^i) )    // average height(node) *
number of nodes
```

Average height of skiplist:

```
h = # of levels in list

I_i = 1 if level is not empty, 0 if level empty

// expected value of sum of indicator(level not empty)
E[h] = E[ Sum( I_i ) ]       // from 0...infinity [no
```

```
max height]
E[h] = Sum( E[ I_i ] )

I_i ≤ n_i   // if level exists, less likely than number
of nodes

E[I_i] ≤ E[n_i] = n/(2^i)

// use log(n) since we know to prove O(log n)
E[h] = E[ I_i ]{ from [0] to [log(n)] }
       + E[ I_i ]{ from [log(n) + 1] to [infinity] }

E[h] = [ log(n) + 1 ] + [ 1 ]       // because math

E[h]  = log(n) + 2 ≤ log(n) + 3
```

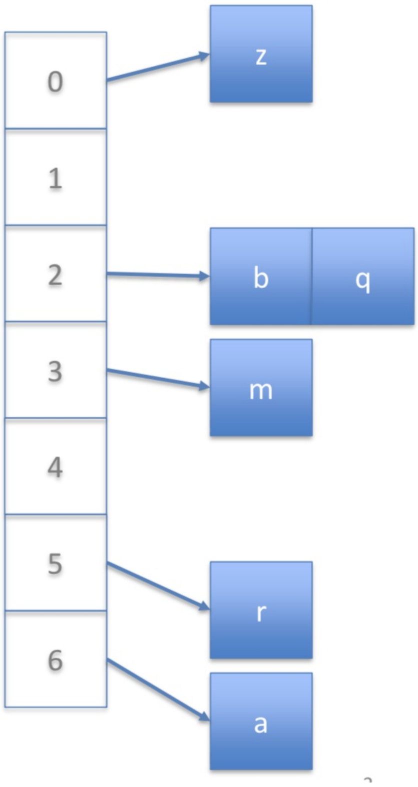
Average length of skiplist:

```
R_i = # of horizontal steps at level ≤ n_i
l = Sum(R_i)

E[R_i] ≤ E[ # node height not promoted ]
E[R_i] ≤ E[ # node height promoted ] - 1
E[R_i] ≤ S - 1      // S = 2 from above
E[R_i] ≤ 1

let sp = total length of search path
E[sp] = E[h] + E[l]
E[sp] = ( log(n) + 3 ) + E[ Sum(R_i) ]
E[sp] = ( log(n) + 3 ) + Sum( E[ R_i ] )
E[sp] ≤ ( log(n) + 3 )
        + Sum(1){ from [0] to [log(n)] }
        + Sum( E[ n_i ] ){ from [log(n) + 1] to n }
E[sp] ≤ ( log(n) + 3 )
        + log(n)
        + Sum( n/(2^i) ){ from [log(n) + 1] to n }
E[sp] ≤ 2*log(n) + 6
```

```
E[sp] = O(log n) + O(1)
```

HashTables

- Unordered sets with fast access
- Associative array
    - Index elements into a range of int
    - for non-integer elements, use hashCode()



Hashing

- Computing an [integer] index into the array

ChainedHashTable

- Implements the **USet** interface
- **find(), add(), remove() in O(n_i)**
    - where *n_i* is based of size of list at index

```
// m ≥ 1 add() / remove() calls, results in O(m) time on
resize()
```

Universal Hashing

```
A hash function, hash(x):int -> {0,...,m-1}, is
universal if, for any elements x, y:

1. if x == y, then hash(x) == hash(y)

2. if x != y, then Prob{ hash(x) == hash(y) } = 1/m
```

- If a hash function gives probablility of 2/m, then it can be called *nearly-universal*

HashCodes

Methods of Java Object:

- .hashCode(), integer representation of object
- .equals(), compare two object references

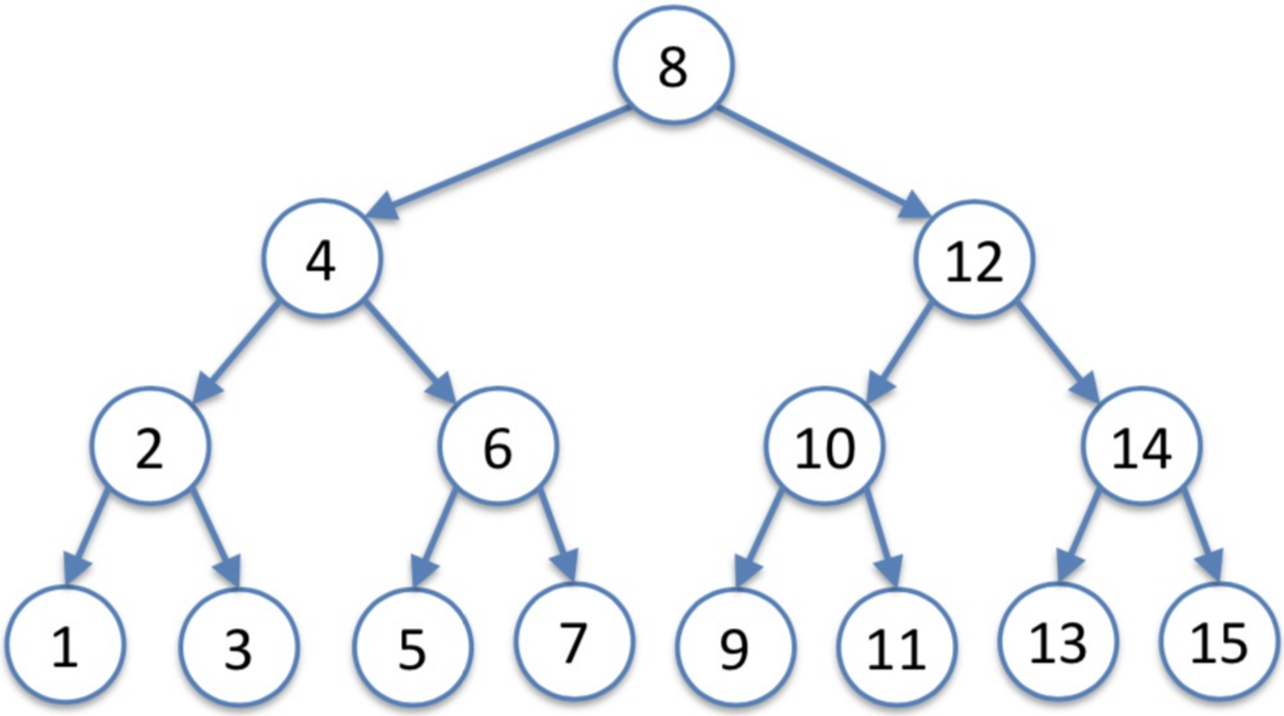
Equal objects must have equal hash codes

- a.equals(b) => a.hashCode() == b.hashCode()

but, reverse is not true:

- a.hashCode() == b.hashCode() =/> a.equals(b)
  - Same hashcode does not imply, same object
- !a.equals(b) =/> a.hashCode() != b.hashCode()
  - Different object does not mean different hashcode

Binary Trees



Nodes:

- Root: top-most node
- Internal: nodes that have children
- Leaf: nodes with no children
- External: null nodes [children of leaf nodes]

Definitions:

- Depth: distance from root
- level: set of nodes at same depth
- height: largest depth for subtree at node
- size: number of nodes for subtree at node

How to calculate the size of a subtree [recursively]

```
int size(Node u) {
    if(u == null) return 0; // external node
    // add one each time, count this node
    return size(u.left) + size(u.right) + 1;
}
```

How to calculate the height of a subtree [recursively]

```
int height(Node u) {
    if(u == null) return -1; // external node, went too
far
    // add one each time, count this node
    return max( height(u.left) + height(u.right) ) + 1;
}
```

How to calculate the depth of a node [recursively]

```
int depth(Node u) {
    if(u == null) return -1; // root node, went too far
    // add one each time, count this node
    return depth(u.parent) + 1;
}
```

Binary Search Tree [BST]

- Implements the **SSet** interface
- **find(), add(), remove() in O(n)**

Adding Node to Binary Search Tree

```
boolean addNode(x) {
    last = FindLast(x);  // successor search
    if(x == last) return false; // no duplicates
    if(x.value < last.value)
        last.right = x;
    else
        last.left = x;
    return true;
}
```

Removing Node from Binary Search Tree

Case 1: Node is a leaf [no children]

- Just remove node.

Case 2: Node has one child

- Node can be replaced by child node

Case 3: Node has two children

- Replace the node with inorder successor
    - replace with node at lower level, favouring left side

Random Binary Search Trees [RBST]

Balanced trees are statistically more likely.

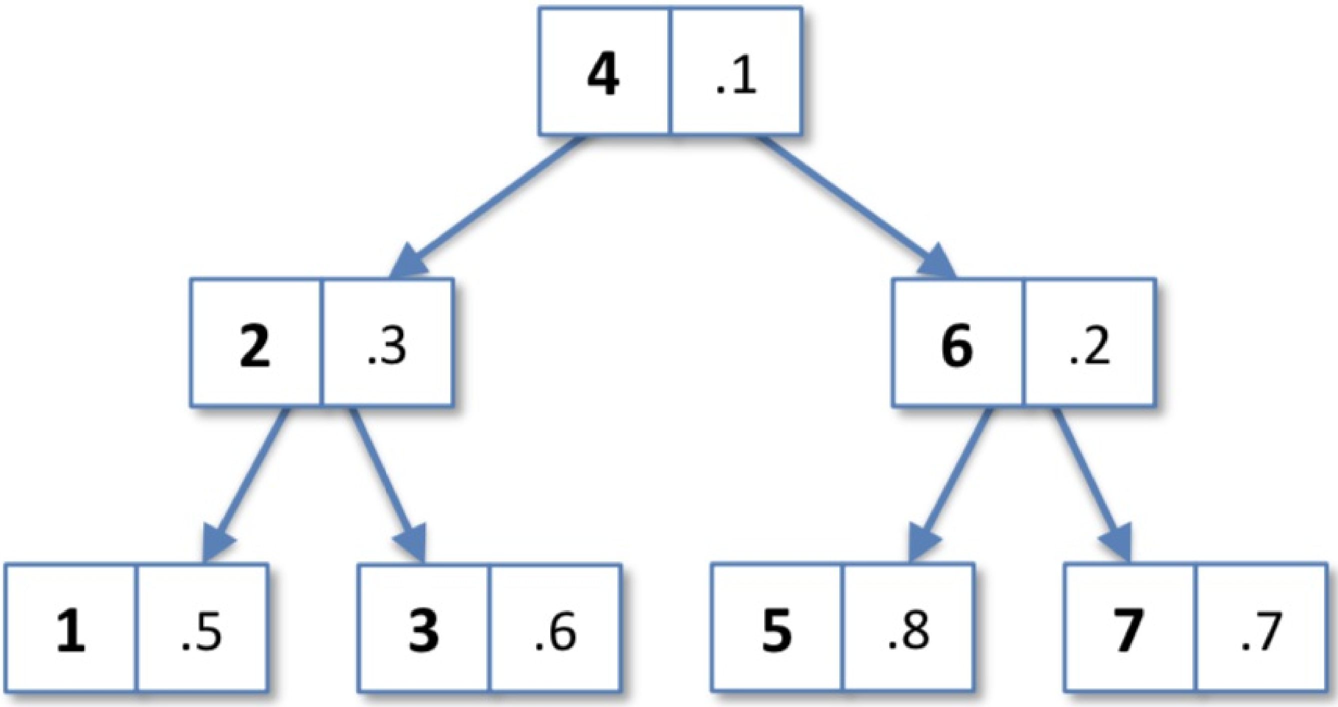
- Implements the **SSet** interface
- **contructed in O(n•log(n))**
- **add(), remove(), find()) in O(log n)**

```
// search path is at most 2•log(n) + O(1)
```

Treaps

Has an extra priority:
Parent priority should be less than child priority.
This has the property of bounding the height of the tree.

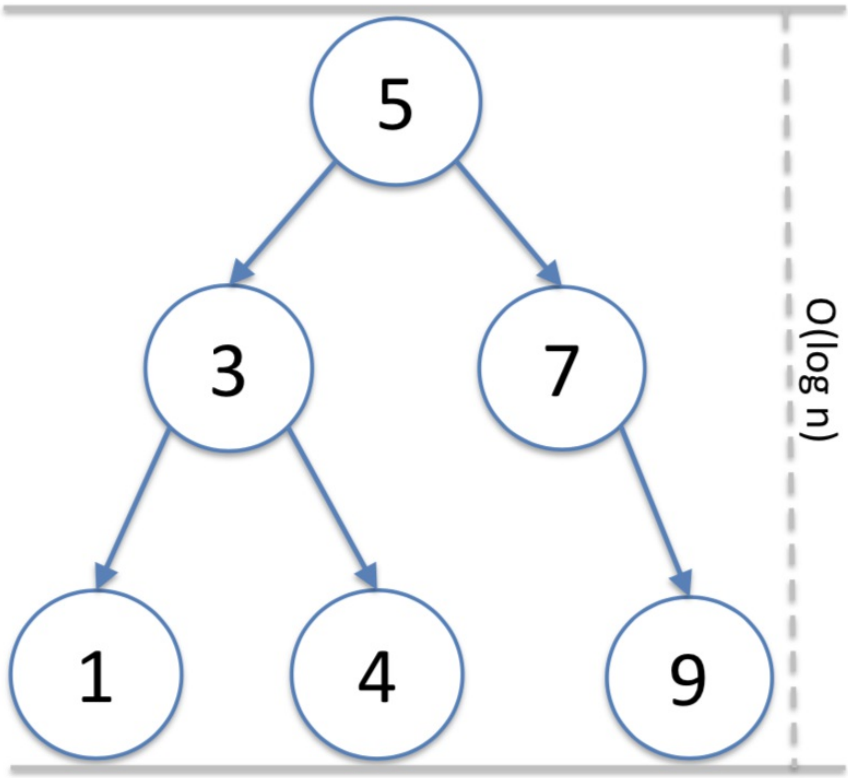
- Implements the **SSet** interface
- Priorities are randomly applied
- **contructed in O(n•log(n))**
- **find(), add(), remove() in O(log n)**



Scapegoat Tree

BST that with height maintained within O(log n), rebuilt if too unbalanced. Limited with integer q, where height ≤ log_{3/2}(q)

- Implements the **SSet** interface
- Rebuild only one search path that triggered rebuild
    - this ensures that not entire tree is rebuilt
- **rebuild() in O(log n) amortized**
- **find(), add(), remove() in O(log n)**




```
// m calls to add() / remove (), results in O( m•log(n) time
spent on rebuild()
```

Binary Search Tree Implementations

	find()	**add()**	**remove()**
BST	*O(n)*	*O(n)*	*O(n)*
RBST / Treaps	*O(log n)* [expected]	*O(log n)* [expected]	*O(log n)* [expected]
Scapegoat Trees	*O(log n)* [amortized]	*O(log n)* [amortized]	*O(log n)* [amortized]
2-4 / RedBlack Trees	*O(log n)* [worst-case]	*O(log n)* [worst-case]	*O(log n)* [worst-case]

Sorted Set Implementations

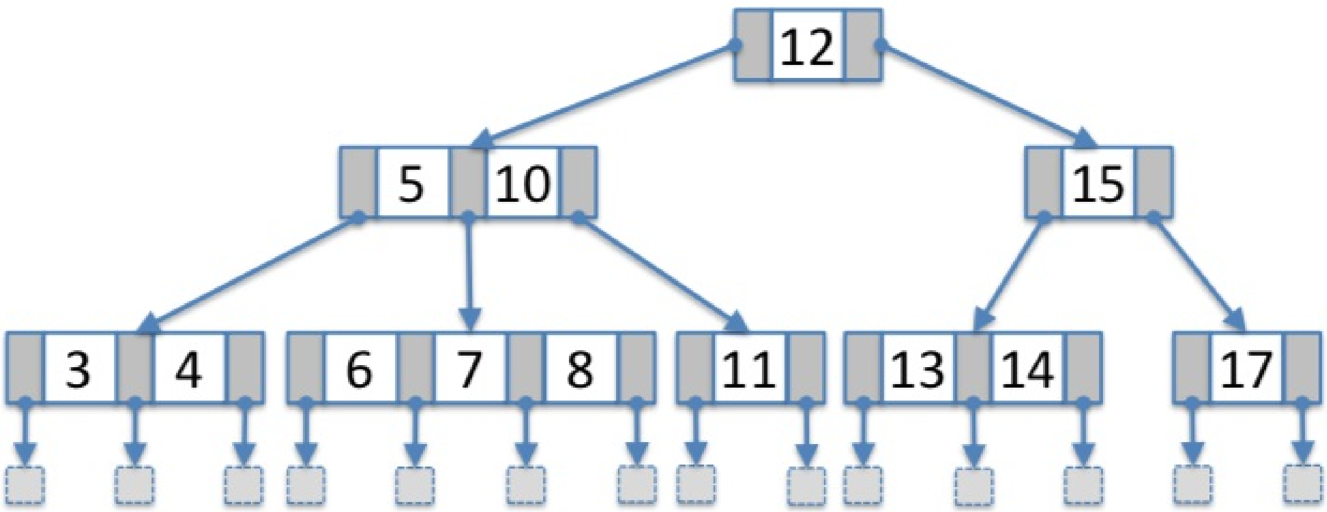
	Runtime
Skiplists	*O(log n)* [expected]

Treaps	*O(log n)* [expected]
Scapegoat Trees	*O(log n)* [amortized]
2-4 / RedBlack Trees	*O(log n)* [worst-case]

2-4 Tree

Tree where every leaf has the same depth.

- Implements the **SSet** interface
- All leaves have equal depth
- All internal nodes have 2-4 children
- **find(), add(), remove() in O(log n) [worst-case]**

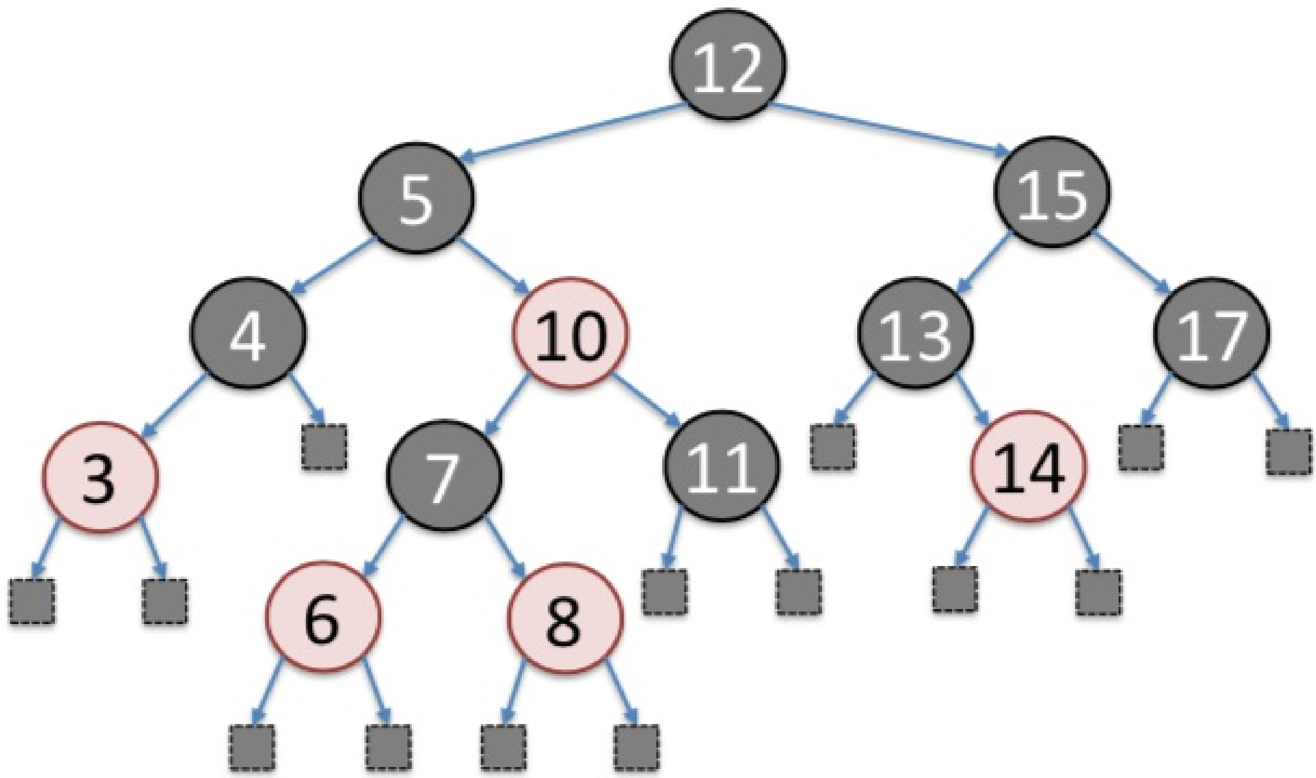


RedBlack Tree

A self-balancing binary search tree, built off a 2-4 Tree, where each node has a 'colour'.

- Implements the **SSet** interface
- Uses colour to remain balanced when adding / removing
    - There is the same number of black nodes on every root to leaf path
    - i.e. equal sum of colours on any root to leaf path
- No red nodes can be adjacent
    - red nodes must have black parent

- left-leaning: if left node is black, then right node must be black
- **Maximum height of 2•log(n)**
- **find(),add(), remove() in O(log n) [worst-case]**



Adding Node to RedBlack Tree

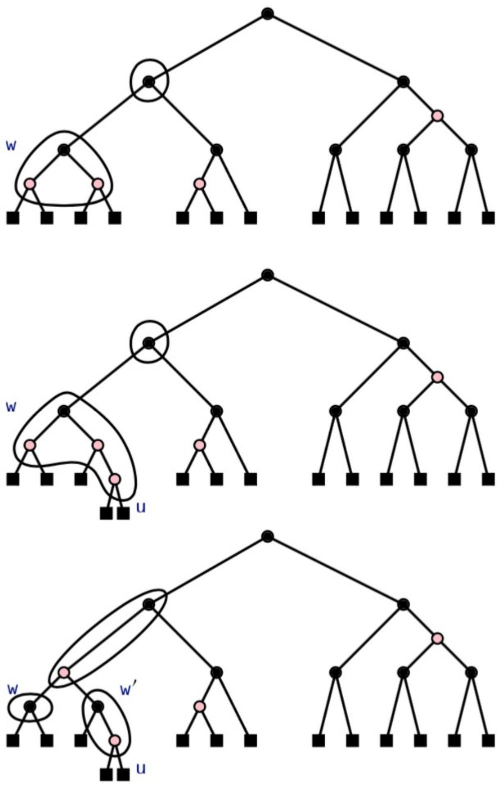
Case 0: black parent...

Case 1: Adding red node with red parent, but black uncle

- Rotate left or right at black grandparent

Case 2: Adding red node with red parent and red uncle

- make grandparent red, and parent and uncle black



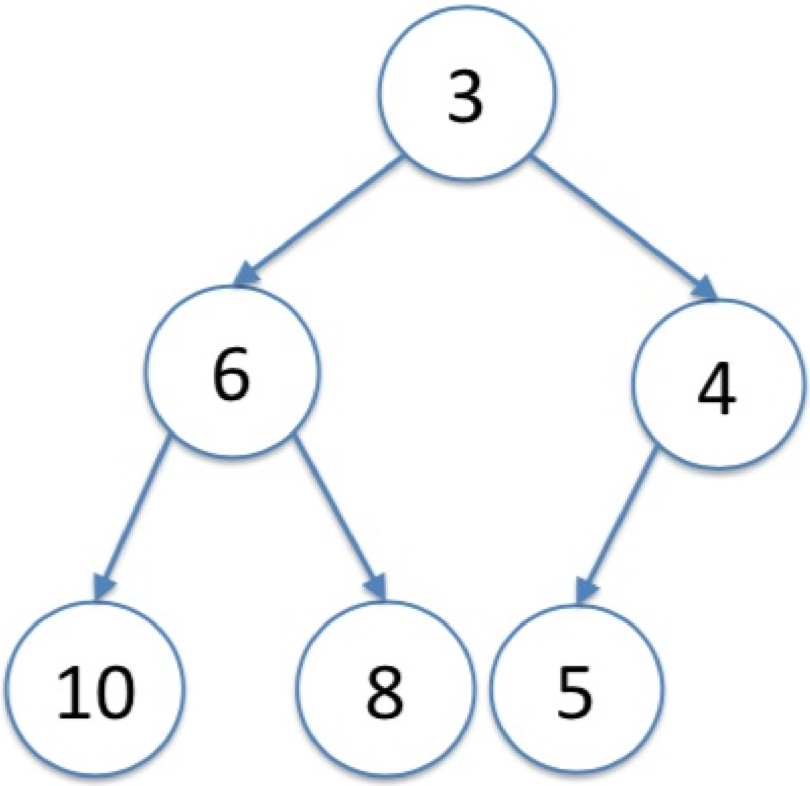
Heaps

Heap Property: Each node is more extreme than [or equal to] its parent.

Binary Heaps

A complete Binary Tree that also maintains the heap property.

- Implements the [priority] **Queue Interface**
- Allows to find / remove most extreme node with peek() / remove()
- **add(), remove() in O(log n)**
- **peek() in O(1)**




```
// m ≥ 1 add() / remove() calls, results in O(m) time on
resize()
```

Eytzinger Method

A method to represent a complete binary tree as an array

Parent: can be found at (i-1)/2 **Left Child:** can be found at 2i+1 **Right Child:** can be found at 2i+2

Meldable Heap

A randomized heap, not bound by an shape or balancing.

- Implements the [priority] **Queue Interface**
- Simpler to implement, and good worst-case time efficiency
- **add(), remove() in O(log n)**

Random Walks

A path through a binary tree [i.e. the expected depth of a node].

- Starting from root node

- Random chance to go to left to right child
- Ends at external nodes

```
// The expected depth of a node is ≤ log(n+1)
```

Sorting Alogrithms

In-place: means modifying list to be sorted [as opposed to returning new sorted list]. **Stable:** means the order of elements with equal values is preserved.

Lower bound on Comparion-based Sorting

For a comparion-based algorithm, the expected number of comparions is $\Omega(n \bullet \log(n))$.

Merge Sort

Sort list by merging sorted sub-lists, reduces total number of comparisons needed.

```
1. Divide list into equally sized halves until 1
element per sub-list
2. Sort sub-list [recursively]
3. Merge sub-list using comparator
```

- Comparison-based
- **Not in-place**
- **Stable**
- **Runs in $O(n \bullet \log(n))$ time**

```
// performs at most n•log(n) comparisons
```

Heap Sort

Sort by traversing down a heap tree.

```
1. Create a heap from list
2. Delete the root node [in list, a[n-1]]
    - Now root is in a[n], since n--
3. Heapify [make sure heap property is preserved]
4. Repeat steps 2-4 until no more elements to sort
```

- Comparison-based
- **In-place**
- **Not stable**
- **Runs in O(n•log(n)) time**

`// performs at most 2n•log(n) + O(n) comparisons`

Quick Sort

Sort using a randomly selected value as pivot point, then sorting the sub-lists.
Since random selection, might choose worst value [ideally middle value].

```
1. Randomly select value
2. Add all values less than to left sub-list, otherwise
add to right sub-list
3. Repeat until 1 element in sub-list
```

- Comparison-based
- **In-place**
- **Not stable**
- **Runs in O(n•log(n)) [expected] time**

`// performs at most 2n•log(n) + O(n)`

Comparison-based Algorithms

	Comparisons	In-place	Stable

Merge Sort	*n•log(n)* [worst-case]	no	yes
Heap Sort	*1.38n•log(n) + O(n)* [worst-case]	yes	no
Quick Sort	*2n•log(n) + O(n)* [expected]	yes	no

Merge Sort:

- Fewest comparisons
- Does not rely on randomization [guaranteed runtime]
- Not in-place [expensive memory usage]
- Stable
- Much better at sorting a linked list
    - no additional memory is needed with pointer manipulation

Quick Sort:

- Second fewest comparisons
- Randomized [expected runtime]
- In-place [memory efficient]
- Not stable

Heap Sort:

- Most comparisons
- Not randomized [guaranteed runtime]
- In-place [memory efficient]
- Not stable

Counting Sort

Counting array is used to keep track of duplicates; it is then used to construct the sorted list.

- Not comparison-based
- **Not in-place**
- **Not stable**
- **Runs in O(n+k) time**

    - n integers
    - range of 0...k

```
// efficient for integers when the length is roughly equal
to maximum value k-1
```

Radix Sort

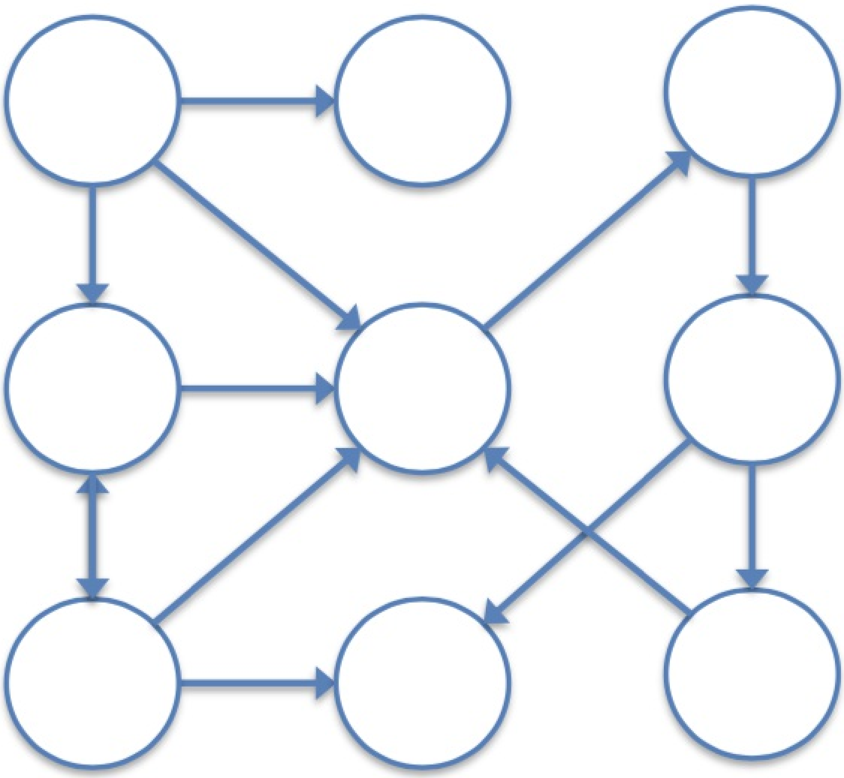
Sorts w-bit integer with counting sort on d-bits per integer [least to most significant]

- Not comparison-based
- **Not in-place**
- **Not stable**
- **Runs in O(c•n) time**
    - n w-bit integers
    - range of 0...(n^c – 1)

Graphs

A graph is a pair of sets: G(V,E)

- **V** is the set of all vertices
- **E** is the set of all edges



Graph Interface

Interface that defines characteristics of a graph

- **addEdge(i,j):** adds an edge between nodes i and j
- **emoveEdge(i,j):** removes edge between nodes i and j
- **hasEdge(i,j):** returns true if edge exists between nodes i and j
- **outEdges(i):** returns set of all outbound edges from node i
- **inEdges(i):** returns set of all inbound edges from node i

	Adjacency Matrix	**Adjacency List**
addEdge	*O(1)*	*O(1)*
removeEdge	*O(1)*	*O(deg(i))*
hasEdge	*O(1)*	*O(deg(i))*
outEdge	*O(n)*	*O(1)*
inEdge	*O(n)*	*O(n+m)*
space used	*O(n^2)*	*O(n+m)*

- `n is the number of nodes`
- `m is the number of edges`

Adjacency Matrix

An *n x n* matrix representing adjacent nodes.

- useful for dense graphs [approx. n^2 edges]
    - memory usage is acceptable
- Matrix algebraic operations to computer property of graph
    - like finding shortest paths between all pairs of vertices

Adjacency List

Stores all outbound edges from a node.

- Is better to use than **Adjacency Matrix** if memory restricted, or for outEdges()

e.g.

Source Node (n)	Adjacent Nodes (m)
0	2,4,5,6
1	2,3
2	5
3	0,6,3
4	1,2,3,5,6
5	0,6
6	4,6

Graph Traversal

We can use Breadth-first or Depth-first search order to visit every node.

Preorder: Start at the root node, go left always, else go right
[visits each node **before its children**]

- Used to copy the tree

Inorder: All nodes from left to right [visually]
[visits each node **after its left** children]

- Useful for getting size of all subtrees

Postorder: Bottom to top, with left priority
[visits node **after its children**]

- Used to delete tree

Breadth-first Search

Go through all adjacent nodes first the.

- Good for finding quickest paths from one node to another [but not unique paths].
    - There could be equally quick paths not found

Process:

- You do this with a queue and list
    - queue stores position we are at
        - add all unseen adjacent nodes to queue and seen list
        - remove value from queue
        - go to removed value, repeat
    - list stores nodes we have seen
        - so that seen values are not added to queue

Depth-first Search

Go through list based of a priority.

- Good for finding node with highest / lowest priority?

Process:

- You do this with a stack and list
  - stack stores position we are at
    - add current node to seen list
    - add smallest unseen adjacent nodes stack [recursively]
  - list stores nodes we have seen
    - so that recursive calls are always to smallest unseen node

Adjacency Matrix vs. Adjacency List

It is better to use **Adjacency List** for **traversals**.

	Adjacency Matrix	Adjacency List
Breadth	*O(n^2)*	*O(n+m)*
Depth	*O(n^2)*	*O(n+m)*