

# COMP 2402 Class Notes

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## Java Collections Framework (JCF)

The Java Collections Framework (JCF) is a unified architecture for representing and manipulating collections.

*A collection — sometimes called a container — is simply an object that groups multiple elements into a single unit. Collections are used to store, retrieve, manipulate, and communicate aggregate data. Typically, they represent data items that form a natural group, such as a poker hand (a collection of cards), a mail folder (a collection of letters), or a telephone directory (a mapping of names to phone numbers). If you have used the Java programming language — or just about any other programming language — you are already familiar with collections.*

In order to use the JCF you can import it like this.

```
import java.util.*
```

## Sorting

This is how to sort strings based on length by using anonymous object **[Comparator]**.

```
Collections.sort(list, new Comparator<String>() {  
    public int compare(String x, String y) {  
        return x.length() - y.length();  
    }  
});  
  
// or you can use lambda function  
list.sort( (String o1, String o2) -> o1.compareTo(o2)
```

```
);

// if you want to sort by length and also
// alphabetically
Collections.sort(list, new Comparator<String>() {
    public int compare(String x, String y) {
        // if not same length, use length
        if(x.length() != y.length()) {
            return x.length() - y.length();
        }
        // else compare as strings
        return x.compareTo(y);
    }
});
```

The **compare(x,y)** method works by moving an element left if the **compare(x,y)** method returns a negative integer, and moves the element right if the **compare(x,y)** returns a positive integer. [difference between x and y]

```
(-) x < y
(0) x = y
(+) x > y
```

## Maps [HashMap]

Also known as dictionaries in Swift or C#...

- Cannot have duplicate entries

```
Map<String, Integer> map = new HashMap<>();
map.put("Java", 6);
map.put("Swift", 10);
map.put("C#", 7);
map.put("Ruby", 9);
```

```
// this will print out every value in the map [foreach]
for(String str : map.keySet()) {
    System.out.println(str + " : " + map.get(str))
}

map.get(key); // fast operation, returns null if no key
found
```

## List

Continuing from previous example...

**Map.Entry** is just a key-value pair

```
List<Map.Entry<String,Integer>> entryList = new
ArrayList<>();
entryList.addAll(map.entrySet()); // set containing all
the elements

for(Map.Entry<String,Integer> entry : entrylist) {
    System.out.println(entry.getKey() + " : " +
entry.getValue() );
}
```

## Deque [ArrayDeque]

Fast for reading/writing at *start* or *end* of list. Basically just a flexible stack/queue.

```
Deque<String> dq = new ArrayDeque<>();
dq.addFirst("second");
dq.addFirst("first");
dq.addLast("penultimate");
dq.addLast("last");
```

# Priority Queue

Essentially: uses a heap instead of a tree, in order to keep a certain one on top. So first element is 'sorted' and then rest is unsorted.

Not good for sorting, or random access.

```
Queue<String> pq = new PriorityQueue<>();  
pq.addAll(list);  
  
System.out.println(pq.remove());    // remove smallest  
element
```

If alphabetical, one that starts with 'a' will be removed. After first element, the queue is not sorted. Removing one will promote next smallest to the top

## Asymptotic Notation [Big O]

Used to analyze complexity of algorithms, to find faster, or which ones requires more space.

### Comparing data structures

- Time
- Space
- Correctiveness

### Growth rates proportionl to n

- If input doubles in size, how much will runtime increase?

### Runtime as a count of primitive operation

- This is machine independent

- Proportional to exact runtime

```
for(int i = 0; i < n; i++) {
    arr[i] = i;
}
```

Runtime:

- **1**: assignment [int i = 0]
- **n+1**: comparisons [i < n]
- **n**: increments [i++]
- **n**: array offset calculations [arr[i]]
- **n**: n indirect assignments [arr[i] = i]

## Definition of Big O

After a certain point,  $g(x)$  will grow as fast [or faster] than  $f(x)$

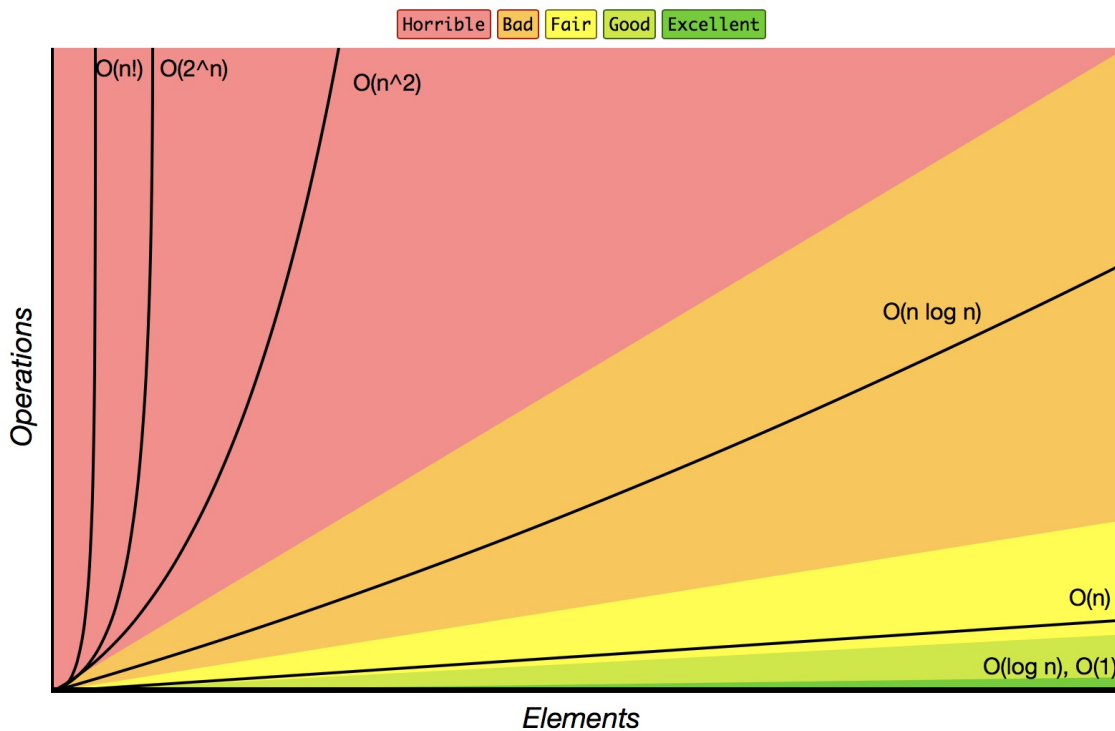
- $g(x)$  is the upper limit to  $f(x)$

$$O(g(n)) \forall (f(n) < c \cdot g(n))$$

## Orders of growth

Complexity	Name
$O(1)$	Constant
$O(\log n)$	Logarithmic
$O(n)$	Linear
$O(n \log n)$	Quasilinear
$O(n^2)$	Quasilinear
$O(2^n)$	Exponential
$O(n!)$	Factorial

## Big-O Complexity Chart



## Tips

- Only largest values matter
- Drop all coefficient
- Log bases are all equivalent

## Example

```
public class BigO {
    public static void main() {
        String str = "";
        int n = 100;    // O(1)

        for(int i = 0; i < n; i++) {    // O(n)
            str += "x";    // O(1) but n times
        }

        for(int i = 0; i < n; i+=2) {    // n/2 times -
            str += "y"    // O(1)
        }
    }
}
```

> O(n)

```

    }

    // this is roughly the same as if n was n/2
with 0(n)
    for(int i = 0; i < n; i*=2) {    // 0(log n)
        str += "y"                // 0(1)
    }
}
}

```

## Array-based Data Structures

### ArrayStack [ArrayList]

- Implements **List** interface with an array
- Similar to ArrayList
- Efficient only for stack operations [back]
- superceded by ArrayDeque
- **get(), set() in  $O(1)$**
- **add(), remove() in  $O(1 + n-i)$** 
  - good for write at the back

### Stacks vs List

Stack	List
push(x)	add(n,x)
pop()	remove(n-1)
size()	size()
peek(x)	get(n-1)

### List Interface

- **get(i) / set(i,x)**
  - Access element i, and return/replace it

- `size()`
  - number of items in list
- `add(i,x)`
  - insert new item `x` at position `i`
- `remove(i)`
  - remove the element from position `i`

*dereferencing*: getting the address of a data item

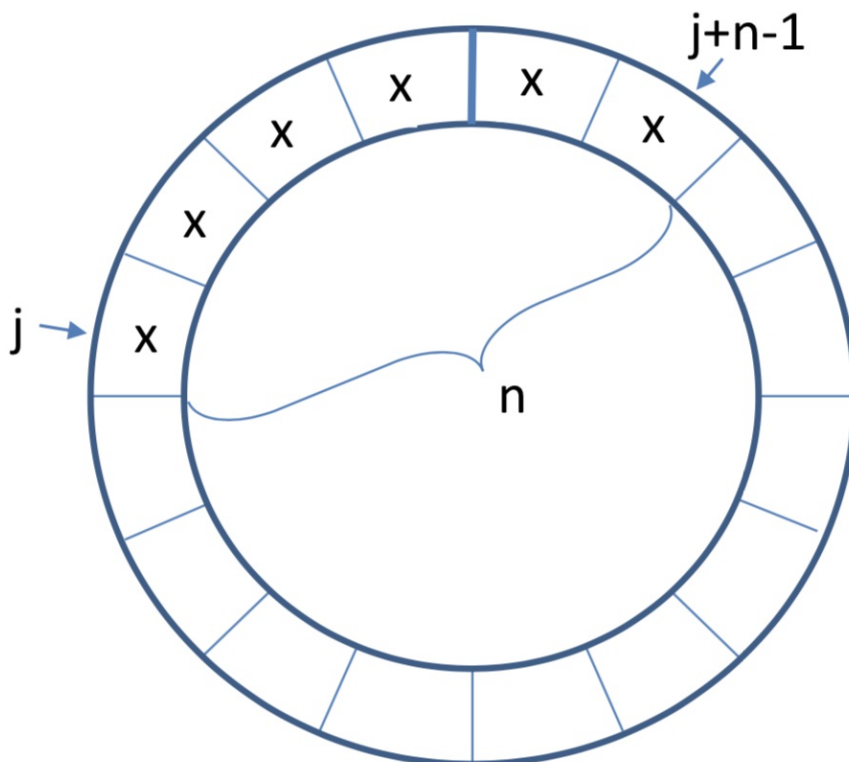
## Amortized Cost

When an algorithm has processes that may be much longer but usually is quick, so you take the average. [roughly]

e.g. resizing an an array when adding/removing

## ArrayQueue & ArrayDeque

Allow for efficient access at front and backs.



## ArrayQueue



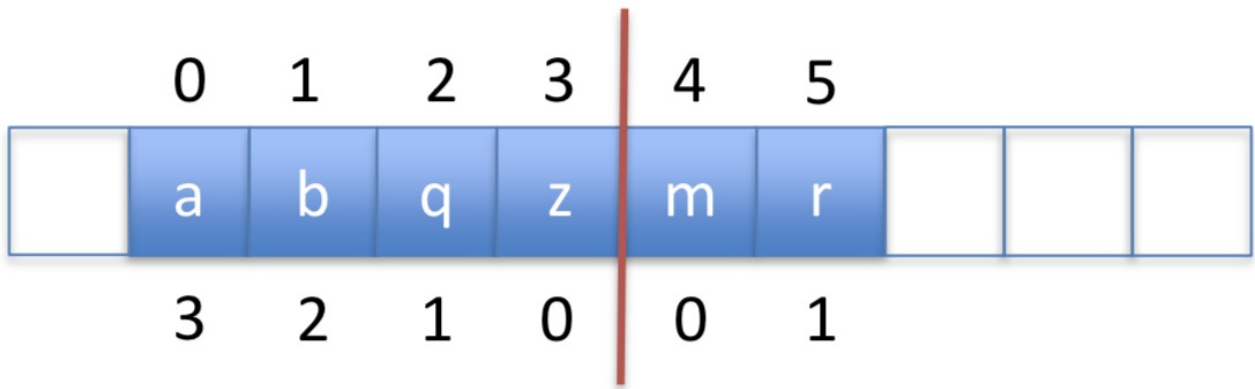
- Implements **Queue** and **List** interfaces with an array
- Cyclic array, (n: number of elements, j: 'index' of last element)
- **get(), set() in  $O(1)$**
- **add(), remove () in  $O(1 + \min(i, n-i))$** 
  - quick to write at front or back
  - cannot access anywhere else
- **resize is  $O(n)$**

## ArrayDeque

- Implements **List** interface with an array
- **get(), set() in  $O(1)$**
- **add(), remove() in  $O(1 + \min(i, n-i))$** 
  - quick to write at front or back
  - not so quick to access middle
- **resize is  $O(n)$**

## DualArrayDeque

- Implements **List** interface
- Uses two **ArrayStacks** front-to-front
- Since arrays are quick to add to the end, this makes front and back operations fast
- May be rebalanced if one array is much larger than the other
- Use Potential Function to decide when to rebalance
- **get(), set() in  $O(1)$**
- **add(), remove() in  $O(1 + \min(i, n-i))$** 
  - quick to write at front or back, but not middle



## Potential Function

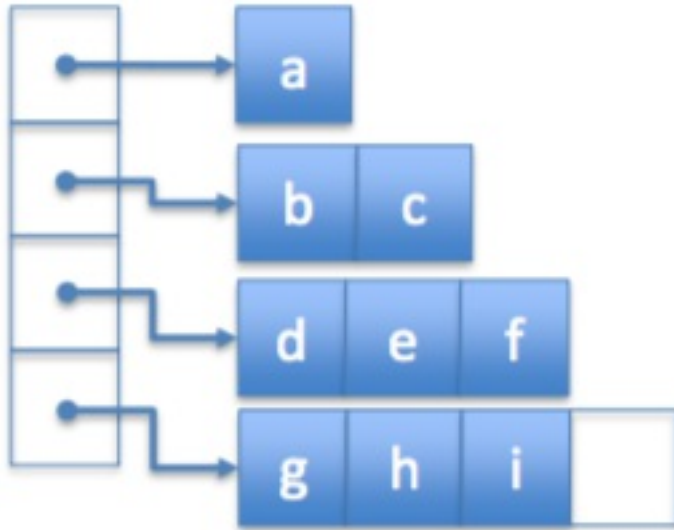
Define a potential function for the data structure to be the absolute difference of the sizes of the two stacks

$$P = | \text{front\_array.size} - \text{back\_array.size} |$$

- Adding or removing an element can only increase/decrease 1 to this function

## RootishArrayStack

- Implements the **List** interface using multiple backing arrays
- Reduces 'wasted space' [unused space]
- At most:  $\sqrt{n}$  unused array locations
- Good for space efficiency
- **get(), set() in  $O(1)$**
- **add(), remove() in  $O(1 + n-i)$** 
  - quick to write at the back

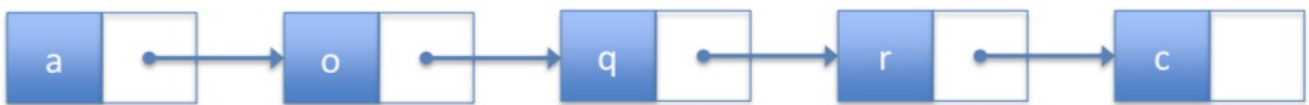


## Linked Lists

- Recursive data structure made up of nodes
- Pointers to head and tail, and each node points to the next node
- Efficient add/remove but slow read/write

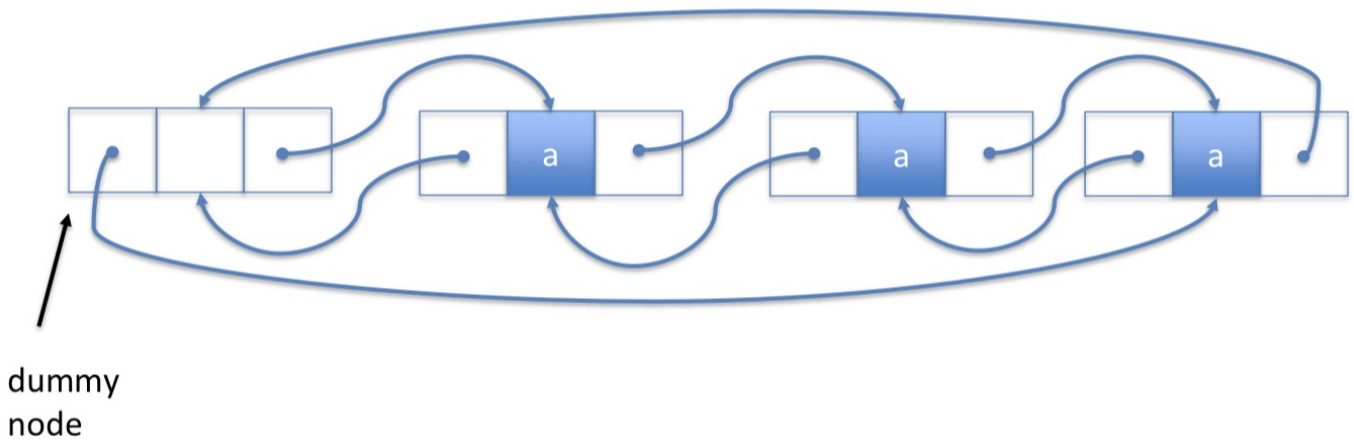
### SLList [Singly-Linked List]

- Implements the **Stack** and **Queue** interfaces
- **push(), pop() in  $O(1)$**
- **add(), remove() in  $O(1)$**



### DSList [Doubly-Linked List]

- Forward and backwards pointers at each node
- Implements the **List** interfaces
- **get(), set() in  $O(1 + \min(i, n-i))$**
- **add(), remove() in  $O(1 + \min(i, n-i))$**



## SELList [Space-Efficient Linked List]

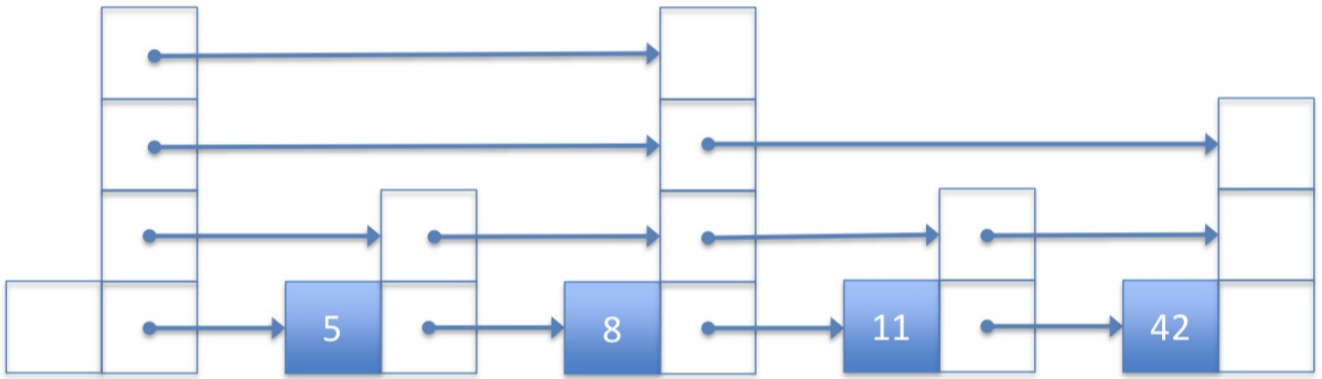
- Like a doubly-linked list, but uses block size  $b$
- Is a series of **ArrayDeque** with *next* and *prev* pointers
- Implements the **List** interfaces
- **get(), set()** in  $O(1 + \min(i, n-i)/b)$
- **add(), remove()** in  $O(1 + \min(i, n-i)/b)$ 
  - is quicker because you can skip blocks of data

## Skiplist

- Like a singly-linked list, with 'skips'
- Randomly generated structure
- Faster searches than linked lists

## SSet Interface

- Additional nodes with pointers that allow 'skipping'
- Successor search: **find(x)** will return **smallest value  $\geq x$**
- **find(), add(), remove()** in  $O(\log n)$



## List Implementations

	get/set	add/remove
Arrays	$O(1)$	$O(1 + \min(i, n-i))$
LinkedList	$O(1 + \min(i, n-i))$	$O(1)$
Skiplist	$O(\log n)$	$O(\log n)$

\* given a pointer to a location

## Definitions

**Random variable:** a random sample from a group of values

**Expected value:** average value of a random variable

**Indicator variable:** random variable with values of 0 or 1

**Linearity of Expectation:** the expected value of a sum is equal to the sum of expected values

**Expected height of node [if coin flips were used]:**

```

P(x = 1) = 1/2      // prob. that tails on first flip
P(x = 2) = 1/4      // prob. that tails on second flip
P(x = 3) = 1/8      // prob. that tails on third flip
...
P(x = i) = 1/(2^i)

```

Thus,  
 $E[x] = i \cdot \text{Sum}(1/2^i)$  // for all natural numbers  
 $E[x] = \text{Sum}(E[I_j])$   
 $E[x] = \text{Sum}(P(I_j = i))$

Indicator variable: 1 if tails, 0 is heads

$P(I_1 = 1) = 1$

$P(I_2 = 1) = 1/2$

...

$P(I_j = 1) = 1/(2^{(j-1)})$

let  $S = \text{Sum}(P(x = i)) = 1 + 1/2 + 1/4 + \dots$

therefore,

$S/2 = 1/2 + 1/4 + 1/8 + \dots$

$S - S/2 = 1$

$\Rightarrow S = 2$

$E[x] = \text{Sum}(P(I_j = i)) = S = 1 + 1/2 + 1/4 + \dots$

$E[x] = 2$

### Expected number of elements in the skiplist:

$E[n_i] = ?$

$I_i(j) = 1$  if in list, 0 is not in list

//  $i$  is  $0 \dots n-1$ , number of nodes in list

// expected value of sum of indicator(element in list)

$E[n_i] = E[\text{Sum}(I_i(j))]$

$E[n_i] = \text{Sum}(E[I_i(j)])$

$E[n_i] = \text{Sum}(1/(2^i))$  // average number values in each node

$E[n_i] = n * (1/(2^i))$  // average height(node) \* number of nodes

### Average height of skiplist:

$h = \# \text{ of levels in list}$

$I_i = 1$  if level is not empty,  $0$  if level empty

// expected value of sum of indicator(level not empty)  
 $E[h] = E[ \text{Sum}( I_i ) ]$  // from  $0 \dots \text{infinity}$  [no max height]

$E[h] = \text{Sum}( E[ I_i ] )$

$I_i \leq n_i$  // if level exists, less likely than number of nodes

$E[I_i] \leq E[n_i] = n/(2^i)$

// use  $\log(n)$  since we know to prove  $O(\log n)$

$E[h] = E[ I_i ] \{ \text{from } [0] \text{ to } [\log(n)] \} + E[ I_i ] \{ \text{from } [\log(n) + 1] \text{ to } [\text{infinity}] \}$

$E[h] = [ \log(n) + 1 ] + [ 1 ]$  // because math

$E[h] = \log(n) + 2 \leq \log(n) + 3$

### Average length of skiplist:

$R_i = \# \text{ of horizontal steps at level } \leq n_i$   
 $l = \text{Sum}(R_i)$

$E[R_i] \leq E[ \# \text{ node height not promoted } ]$

$E[R_i] \leq E[ \# \text{ node height promoted } ] - 1$

$E[R_i] \leq S - 1$  //  $S = 2$  from above

$E[R_i] \leq 1$

let  $sp = \text{total length of search path}$

$E[sp] = E[h] + E[l]$

$E[sp] = ( \log(n) + 3 ) + E[ \text{Sum}(R_i) ]$

$$\begin{aligned}
E[sp] &= ( \log(n) + 3 ) + \text{Sum}( E[ R_i ] ) \\
E[sp] &\leq ( \log(n) + 3 ) \\
&\quad + \text{Sum}(1)\{ \text{from } [0] \text{ to } [\log(n)] \} \\
&\quad + \text{Sum}( E[ n_i ] )\{ \text{from } [\log(n) + 1] \text{ to } n \} \\
E[sp] &\leq ( \log(n) + 3 ) \\
&\quad + \log(n) \\
&\quad + \text{Sum}( n/(2^i) )\{ \text{from } [\log(n) + 1] \text{ to } n \} \\
E[sp] &\leq 2*\log(n) + 6 \\
E[sp] &= O(\log n) + O(1)
\end{aligned}$$