PeerJ

Dear Dr. Schuster,

Thank you for your submission to PeerJ.

It is my opinion as the Academic Editor for your article - Integer linear programming outperforms simulated annealing for solving conservation planning problems - that it requires a number of Minor Revisions.

My suggested changes and reviewer comments are shown below and on your article 'Overview' screen.

If you address these changes and resubmit, there's a very good chance your article will be accepted (although this isn't guaranteed).

Resubmission

1. Use the line numbers in your review PDF when reading the comments from your editor and reviewers, and when writing your rebuttal letter:

Download review PDF

2. Download your resubmission checklist:

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3. Edit and resubmit when ready:

Edit and resubmit

Although not a hard deadline, we expect you to submit your revision within the next 40 days.

With kind regards,

Alison Boyer

Academic Editor, PeerJ

Editor comments (Alison Boyer)

MINOR REVISIONS

We have received three in-depth reviews of your paper, indicating that the paper is overall a valuable contribution, but that there are specific issues that need to be addressed in a revision. I look forward to reading a revised version soon.

**Thank you very much for your and the reviewer’s efforts in helping to improve our paper. We have tried our best to accommodate reviewer comments as best as we could and think this now represents a much improved version of our paper.**

Reviewer 1 (Anonymous)

Basic reporting

Well presented manuscript with good placement in the literature

Experimental design

Clear experimental design and description of study.

Validity of the findings

conclusions well supported with data and follow logically from the description of the study design

Comments for the Author

This study explores the performance of a new approach to solving the spatial conservation prioritization problem that currently is commonly currently solved with the simulated annealing approach in Marxan. Recent advances in integer linear programming (ILP) have enabled the scale of problem that Marxan can handle by approximating optimal solution to now be feasible with an ILP approach, which can find an optimal solution. An R package (prioritizR) provides this functionality to conservation planners and researchers. Here, the authors explore the variation in recommended conservation portfolio (suggested protected areas) and the time to solution for a set of conservation scenarios that are run in Marxan, prioritizR with the optimization performed with Gurobi or prioritizR with the optimization performed with Rsymphony (two different options that differ in their availability to practitioners vs academics). They find that on all fronts, the Gurobi based solutions were superior (met the conservation targets at lower cost) and drastically faster than Marxan. The Rsymphony based solutions also were cheaper and faster than Marxan, although slower than Gurobi.

This paper is well written and generates important evidence supporting a new spatial conservation approach that can outperform the current industry leading approach. I have provided several suggestions for improvement and small edits but in general I think this paper is well presented, clearly written, and nicely executed.

**Thank you very much for this positive feedback and your recommendations for improvement.**

Broad comments:

1. I would have liked to see a little bit more information in the introduction on what the advances in ILP were that now permit it to handle nonlinear constraints and penalties. I expect that this is covered in the Haight and Snyder, or Bayer references, but a sentence or two of the technical details would make a reader more convinced that the time is right for a move away from simulated annealing.

**Thank you for this suggestion. We have added the following sentence to the Introduction to provide more information on this (Line XX):  
“Beyer et al. (2016) recently introduced a linearization solution to the nonlinear constraint problem to find efficient solutions in an ILP framework, which greatly improved the utility of ILP for solving conservation planning problems.”**

2. It was not clear whether the broad range of cost savings that could be achieved from the ILP over Marxan approach was due solely to the variation in problem specifications or if there was also an aspect of Marxan potentially providing different solutions to the same problem across different runs (because of the simulated annealing). Please add a bit of detail that comments on the variation due to difference in approach as opposed to the variation due to marxan’s approximate solutions.

**Thank you for raising this point. We have added text in several location to make it clear that we are comparing variation in problem specification, as we always compared EILP to the best/cheapest SA solution for a given problem. Specifically, we added to Methods (Line XX) “To allow for a fair contrast between SA and EILP, that focuses on algorithmic comparisons and not within SA variation, we focused our results and discussion on the best solution achieved with Marxan across 10 repeat runs.”; Results (Line XX) “when comparing EILP results to the best (cheapest) solution for a Marxan scenario.”.**

Editorial comments.

Lines 22-23: It would help the reader if you could specify that this reduction in cost was due to being able to calculate the optimal solution as opposed to an approximation.

**Added “, due to ILP’s ability to find optimal solutions as opposed to approximations.”**

Line 53: Clarify that it ILP “historically” was not well suited

**Change made.**

Line 59: It would be good to detail here what an optimal solution would be for readers who are not well versed in the terminology (e.g., something like “optimal being the configuration of protected areas that delivers the desired benefits and the lowest cost”).

**Added sentence (Line XX): “. Optimal being the configuration of protected areas that delivers the desired benefits and the lowest cost”**

Line 77-78: As a reader I wanted to know here how the ILP inability to deal with spatially compact solutions had been resolved

**This is hopefully explained in the response to your first broad comment, which we included on Line XX: “Beyer et al. (2016) recently introduced a linearization solution to the nonlinear constraint problem to find efficient solutions in an ILP framework, which greatly improved the utility of ILP for solving conservation planning problems.”**

Line 100: I’m curious how you dealt with the repeated measures aspect of the sampling locations that have been visited multiple times. Is this something that unmarked has a strategy for accounting for?

**We have added the following text to help clarify this question (Line XX): “This form of distribution modelling, also known as occupancy modelling uses the information from repeat visits to a site to infer estimates of detectability of a species as well as estimates of probability of occurrence.”**

Line 111-112: it might be worth mentioning that the use of tax assessment values as an estimate of conservation cost is an underestimate because tax assessment values are often lower than market value

**We have added a sentence about this to Methods (Line XX): “Using tax assessment values as an estimate of conservation cost is an underestimate because tax assessment values are often lower than market value, but estimates of market values over larger areas are rarely available and tax assessments do provide a good general approximation.”**

Line 162: I initially read this as species per features and was confused. I think you actually mean species (e.g. features)

**Thank you. We removed species to avoid confusion.**

Line 162: the numbers run together and make it seem like one really large number.

**Thank you. This is a very good point. We have added “planning units” to each number to break up the numbers. The section now reads: “9,282 planning units, 37,128 planning units, and 148,510 planning units”**

Line 163-167: Please explain here why you wanted to explore additional Marxan configurations. I think it was to try configurations that increased Marxan’s ability to approximate the optimal solution but not sure from the text.

**Thank you for pointing this out. Your thinking was correct and we have added text now that hopefully helps readers understand our rationale for exploring additional configurations with Marxan (Line XX): “Exploring ranges of values for number of iterations and SPF are recommended for calibration of Marxan to increase Marxan’s ability to approximate the optimal solution (Ardron et al. 2010).”**

Lines 174-182: How did you specify the equivalent problem (BLM modifications) in ILP?

**We have added text to the methods section to provide some information on this (Line XX): “Both Marxan and prioritzr allow a user to specify BLM values as presented here. For the details on the mathematical formulation of the spatial compactness constraint in ILP, please see Appendix S2.”**

**In Appendix S2 we’ve added the mathematical formulation of the systematic conservation planning problem used here, specifically the ILP denotation.**

Line 195: Was the reduction because the ILP solution was able to find cheaper parcels or less parcels?

**It was both and we added text to clarify this (Line XX): “, because it selected cheaper and less parcels in the optimal solution”**

Figure 1: Are the results shown for Marxan from a single run? How much variation might we expect around this line if there were multiple Marxan runs? (I don’t think they are necessary, just a comment on the expected variability in Marxan solutions)

**Thank you for this comment. We have added the following text to the figure legend to clarify this: “, across all Marxan runs that met their target for a given scenario (max = 10)”**

Reviewer 2 (Anonymous)

Basic reporting

Well-written, professional throughout. All sufficient in this section.

Experimental design

All fine, although see comments on the framing of the research question in comments below.

Validity of the findings

All fine, although see major comments 1&2 below which will affect the framing/wording of the findings.

Comments for the Author

This manuscript makes an excellent point – Marxan is used as the gold-standard in conservation planning, but it is often not the best choice of algorithm. The manuscript is well-written overall, however, at the moment some components of the manuscript are lacking in this current form.

**Thank you very much for your encouraging comments. We have edited our manuscript according to your feedback and hope that the current version now better reflects what we tried to do.**

First, the authors have stayed away from a mathematical description of the problems being solved here. I think that is a mistake, and explicitly including the mathematical formulations could clear up or shine a light on a few of my major issues below (points 3-5).

**We have now added the mathematical description of the problem as Appendix S2. This includes the actual equations, as well as descriptive text. We decided to refrain from adding this to main text, because we thought most readers might not necessarily explore the maths of the problem formulation (2 of 3 reviewers didn’t).**

Second, some language throughout is misleading, which I detail in points 1 and 2 below. Other major issues follow, but I will point out that I think all of these are fixable with some very careful reframing/rewriting, and some different choices in the results section.

**Please see detailed responses below.**

I do not think any of these issues call into question the fundamental relevance of this work to the conservation planning literature, and I hope the authors can clarify these issues in the manuscript since I hope to see it published.

Major issues

1. The dichotomy between ILP and SA presented here is incorrect, but I think it is just a terminology issue. Integer Linear Programs are a class of problem, with a specific canonical form. Reserve site selection problems largely fit this class. ILP itself is not a solution method/algorithm – in fact, SA is a valid (heuristic) solution method for ILPs. I think the comparison the authors are looking for here is not ‘SA vs ILP’, but ‘Solving ILPs with SA vs [an exact method -- branch and bound, CPLEX, or Gurobi etc]’.

**Thank you for raising this important issue. We have adjusted text throughout to reflect this issue. What we have done was to use the phrase ‘exact integer linear programming solvers’ instead of ‘integer linear programming’, starting with the manuscript title.**

2. Relatedly, the manuscript sometimes uses SA and Marxan interchangeably. Marxan uses a specific problem formulation in order to capture the intended problem (specifically by adding their Boundary Length Modifier), but then uses SA to solve that problem. It’s not clear until fairly late in this manuscript whether the authors are comparing:

• Simulated annealing vs an exact optimisation method to solve the same problem (same objective function, identical constraints); or

• The Marxan formulation of the reserve site selection problem vs an ILP formulation of the reserve site selection problem solved using an exact optimisation method.

The first half of the manuscript feels like it is setting up the first comparison, but the latter is what ends up being done – but it is not clearly stated, the reader is left to figure it out. This manuscript compares two ILP solvers (Gurobi and SYMPHONY) with Marxan. Marxan is a specific implementation of a simulated annealing algorithm, and is built from a very specific problem formulation. I do not think it is fair to claim this is a comparison of ILP solvers and simulated annealing in general for conservation planning problems, it is a comparison between ILP solvers and Marxan.

**Thank you for this comment. We have added the word Marxan in several places in the abstract and reworded part of the Introduction to make it clearer that we compared the Marxan formulation of the min set objective function using SA and EILP. We hope this, as well ass adding more information on the mathematical formulation of the problem will help clarify what we did.**

3. In Materials and Methods, it would be very useful to introduce the problem being solved in either the first or second section. The last parag of the introduction mentions a ‘systematic planning problem’, but it is not yet clear what this actually means in this context.

**Thank you for this comment. To clarify the problem early on, we have expanded the last paragraph of the introduction to provide more details on the systematic conservation planning problem solved. The first part of the paragraph now reads (LINE XX): “Here we compare exact integer linear programming methods with simulated annealing as used in Marxan, for solving minimum set systematic conservation planning problems (Rodrigues et al. 2000) using real-world data from Western North America. The goal of solving the minimum set problem is to find the places that maximize biodiversity, while minimizing reserve cost.”**

4. The Spatial Prioritization section would be easier to follow with a mathematical description of the ILP being tackled (i.e. write out the min set problem, with all constraints), and the Marxan problem. This is related to my issues with the dichotomy elsewhere – this is where that could be cleared up by embracing the math!

**Thank you very much for this suggestion. We have now added Appendix S2, which includes the mathematical description of the ILP problem used here. The Appendix also includes descriptive text, as well as an example to illustrate the use of the equations.**

5. Is there an analogue to the BLM in the ILP formulation used? How does the ILP formulation take into account clumpiness? One of the big arguments for Marxan is the ability to control or explore how clumped the solutions are, so that needs to be addressed here.

**We have added the following text sections to clarify this:**

**Line XX: “Beyer et al. (2016) recently introduced a linearization solution to the nonlinear constraint problem to find efficient solutions in an EILP framework, which greatly improved the utility of EILP for solving conservation planning problems.”**

**Line XX: “Both Marxan and prioritzr allow a user to specify BLM values as presented here. For the details on the mathematical formulation of the spatial compactness constraint in ILP, please see Appendix S2 and (Beyer et al. 2016).”**

**Appendix S2 includes the mathematical formulation of this, as well as descriptive text explaining the approach used.**

6. Much of the M&M section is written assuming a fairly good knowledge of Marxan. There are some terminology that need to be cleared up:

Species Penalty Functions

Boundary Length Modifiers

Calibration

Iteration

Maybe more that I have missed since I know Marxan, give it a good scour for other jargon.

**Thank you for this suggestion. We have now added Appendix S1 describing this terminology in detail for interested readers to have a look, without the need to read the Marxan Manual or Marxan Good Practice Handbook, which include this information.**

7. In Fig 2, the two objective functions are different for the two problem formulations. Is this deviation in the actual lowest objective function value (which includes a BLM penalty), or the cost of the optimal solution? The latter would be a fairer comparison. For example, the BLM of 100 performs much worse at a 60% target, but it is not clear to me whether that’s just because a larger penalty is being applied to the objective function.

**We had thought about what to show on the y-axis considerably, as we initially thought that cost would make more sense as well. We eventually decided to show the objective function values though, as both SA and EILP are solving the objective function and try to minimize its value (see Appendix S2 for mathematical formulation). This is especially true for case like the implementation of penalties, which are subjective terms that need to be considered in the larger context of the problem formulation. This is why we decided to show the objective function values on the y-axis.**

8. The two results figures use the 10^8 iterations version of the Marxan runs. That seems like a lot – is that standard? It may be a fairer comparison to examine the times at the median number of iterations tested, which would drop the times back a lot. Figure 1 could actually be cool with multiple lines for Marxan, showing multiple interation choices – I think with only a few, it wouldn’t be distracting.

**In keeping in line with the Marxan Manual (“10 million or more is commonly applied on large scale datasets for number of iterations”) we chose 10^8 for Figure 2, given that it has roughly 50,000 planning units and also uses BLM. For Figure 1 we now adjusted the number of planning units down (see next comment) and also adjusted the number of iterations to 10^7, which would be more appropriate for the standard min set problem setup we have used here.**

**We decided against showing multiple lines for Marxan, representing multiple iteration choices, as we think that adding Marxan lines would distract readers from the main comparison of SA and EILP comparisons.**

9. The same point can be made for the number of planning units. The discussion notes that the standard recommendation is not to use Marxan for more than 50,000, yet the results presented here are for more than three times that size. Do the general observations here hold true for a problem where Marxan would actually be considered useful? I’d like to see the results and the figures for a more realistic problem more representative of how Marxan is used in practice. If the message here is really that these other approaches can solve much larger problems than Marxan, then the messaging needs to be changed throughout the manuscript.

**Great point. We have adjusted the scenario we highlight in Figure 1 to represent what Marxan is intended to in practice, which is a problem with 37128 planning units. We’ve also adjusted the number of iterations used to create Figure 1 to 10^7. As a consequence, we have also adjusted text in the Results section to reflect this change. We also recalculated the savings calculations (Line XX) and updated text accordingly.**

10. Although I agree that this is strictly a conservation planning paper (which is fine), it would be good to look outside the field for similar discussions and analyses. The tradeoffs between SA and ILP, or even heuristic vs exact optimisation methods is not a brand new one. The comparison is certainly novel here in a field that so heavily relies upon SA, but a richer context could be drawn from other literatures.

**Thank for this comment. We have now added the following sentence to the Introduction to make it clearer that the SA and ILP comparisons are not new, but that things have changed in favor of ILP in recent years (Line XX): “ The discussion about the pros and cons of linear programming versus heurists, such as SA, in conservation planning spans more than two decades (Cocks and Baird 1989, Underhill 1994, Church et al. 1996, Rodrigues and Gaston 2002, Önal 2004), but the EILP shortcomings mentioned above have largely been overcome in recent years (Beyer et al. 2016).”**

Minor issues

Table 1 is not referred to in the text where it would be very helpful, and it is hard to parse.

**It is referenced in the methods section (Line XX). We have added another line to the table to help make it easier to read.**

Line 51 ‘performing runs’ is a little jargony for this early in an introduction – would be good instead to make it clearer at this point that SA is expensive because it performs many hundreds of simulations to determine the impact of different candidate solutions.

**Thank you for this comment. We have adjusted text to read as follows on Line XX now: “By conducting thousands of simulations to determine the impact of different candidate solutions, Marxan aims to generate solutions that are near-optimal.**

Line 53 – talking about the drawbacks (e.g. the structure of the problem and the time to solve) of ILP in past tense, can that be contextualised that more? Have these problems been solved and are no longer issues, or are you proposing that they are worth the tradeoff?

**We have added the following sentence segment to help clarify this statement (Line XX): “, but the EILP shortcomings have largely been overcome in recent years (Beyer et al. 2016).”**

Line 58 – ‘highly suboptimal’ is confusing wording

**We have rephrased to “no way of knowing how suboptimal their solutions are”**

I do not know what cadastral data means, sorry.

**Thank you for mentioning that. We have replaced cadastral data with property data as one can use both terms interchangeably and property data is likely wider known than cadastral data.**

Line 110 mentions polygons – the context for these has not been set yet. I suggest adding this to the section I suggested at the beginning of M&M describing the problem.

**We have changed polygons to properties to be in line with the previous comment and avoid confusion using multiple terms for the same thing.**

Line 126 – is land cost a socioeconomic cost? I’d be more inclined to call this a financial cost, since there isn’t anything social being captured.

**We have replaced socioeconomic with financial.**

Line 191 – it is disingenuous to present results from uncalibrated Marxan runs, especially when it results in something so extreme as a cost saving of over four thousand percent. That would be a very powerful number if it came from the intended use of Marxan!

**We agree with the reviewer and have removed the sentence in question.**

Line 197 I really like the comparison as illustrated here – how much more you could select in the ILP formulation if you had the same budget as the Marxan optimal solution. If there are other obvious places to make this comparison in the results, I’d love to see this illustrated again. It may not fit neatly anywhere else though, which is fine.

**Thank you. We also thought this was a very useful comparison to show. We couldn’t think of another illustrative example that would fit nicely into this paper, which is why we weren’t able to add in additional examples.**

Reviewer 3 (Jennifer McGowan)

Basic reporting

All good.

Experimental design

All good.

Validity of the findings

All good. I note some of the interpretations go beyond what the paper shows evidence of and have made some suggestions below.

**Thanks very much for your feedback Jen. We really appreciate it.**

Comments for the Author

Thanks for the opportunity to review this paper. Sorry for my delay.

The authors conduct a real- world analysis to support already established findings that ILP is faster and more efficient than Simulated Annealing when it comes to solving the minimum set problem- the most common problem definition in spatial conservation prioritization.

From a technical perspective, the authors present their case clearly. The analysis is robust and the findings do not deviate from previous examples where similar comparisons have been made (e.g. Beyer et al. 2016). To have a real-world example, rather than a simulation-based analysis, is useful for users and a nice contribution to the conversation about tools to underpin planning processes. As a technical document, I think this work is helpful, and having a discussion around open source vs proprietary ILP solvers will be useful for many people looking to use ILP in their analyses.

**Thank you.**

Where I do have some concerns is in the framing towards broader conservation planning guidance and recommendations. While the authors use real-world data, I do not see clear real-world planning evidence offered in the paper to support some of these broader claims.

For example, Line 39: I would disagree that systematic conservation planning is about framing optimization problems. SCP is a ten step process, only one step is about framing and solving the problem for prioritization, the rest is about dialogue, communication, stakeholders, transparency, structure and policies. A SCP process can take 10 years- running a tool takes a week. We should be careful to not over-emphasise the role of tools and algorithms in the much larger effort of SCP. In the end, the algorithm is only a supporting application and the same can be said for the other planning softwares like Marxan and Zonation.

**We agree that framing and solving of the problems we investigate here is a part of systematic conservation planning and have adjusted the sentence in question to better reflect this. The sentence now reads (Line XX): “Systematic conservation planning, on the other hand, is a multi-step process that involves framing conservation planning problems as optimization problems with clearly defined objectives (e.g. minimize acquisition cost) and constraints (Margules and Pressey 2000).”**

Lines 232: the authors encourage “making the switch to ILP as being advisable now that it is computationally feasible” to support SCP processes. I think adding in a clause about “where technical capacity exists” would be prudent. Many decision-makers and technicians in the places where we desperately need to deploy SCP approaches and tools are not even computer or spatially literate, and so I am not sure “who” this message is for and “who” is advising this switch. Some clarification and evidence is needed if a grand statement such as this is going to be made. I think this paper will be of great interest for technically savvy people working on large optimization problems where optimality and speed are desirable- but this is a different target audience than mid-level practitioners in Tanzania or Solomon Islands working to support government decision-making through SCP.

**Thank you. We have added “where technical capacity exists” to the sentence in question (Line XX)**

While reading this paper- the following analogy came to mind:

I find this paper analogous to being told to choose between a Porsche (ILP) and a pick-up truck (SA), and then being told the Porsche is what I should buy because on all counts, it performs much more efficiently than the pick-up truck. It will get me where I need to go faster than anything else. But what if my objective is to build a house? Would I still be told to choose the Porsche? According to the logic of this paper, the answer is yes, because speed and efficiency are the only things driving the choice. In reality, there will be a suite of external factors influencing the choice of car that go far beyond performance efficiency. The “one is superior to the other” message that continuously emerges in the paper, and which is solely driven by the technical efficiency of the algorithm, feels a bit myopic when the authors move beyond the algorithmic comparison to discuss its applicability for integration in broader planning processes.

**We would probably not have chosen this analogy, but we think we understand the point the reviewer is making here. The point of this paper is a technical comparison of SA and ILP, but the points we make do have value beyond the technical aspects of algorithms, given that they have implications beyond the analytical part of systematic conservation planning. If this issue is really about the perceived recommendation against using Marxan for systematic conservation planning, we really recommend an evolution of Marxan (or pimping the pickup truck) and not a replacement with a Porsche. The pick-up truck (Marxan) is using an engine that has a terrible fuel efficiency (SA) and what we recommend is an engine upgrade in the form of a modern engine (ILP). To reflect this in the paper, we have added the following sentence to the Discussion (Line XX): “Given Marxan’s flexibility to use optimization methods other than SA, we hope that a future version of Marxan will include EILP solvers.”**

Prioritzr, Zonation, and Marxan all aim to serve the same role: to support and improve transparency in decision-making for planning. They all have pros and cons and can be deployed in different ways for different users in different political and planning contexts. I would encourage the authors to remove statements suggesting the world transition to ILP because of its superior performance, if their only evidence for this claim is efficiency. To demonstrate otherwise would require a different comparison that is beyond the scope of this paper.

**We respectfully disagree with part of this statement. At a technical level the transition to ILP over SA approaches is worth to recommend a switch from SA approaches to ILP. Beyer et al. 2016, stated so as well, based on a theoretical example: “When solving linear or quadratic conservation planning problems we recommend using ILP over heuristic approaches whenever possible.”**

**In our concluding sentence we give a clear recommendation, which is very much in line with what Beyer et al (2016) recommended: “Given the potential EILP is showing for conservation planning, we recommend users consider adding this modified approach to solving systematic conservation planning problems.”**

**Our ‘real world’ example provides further evidence for this recommendation. Given that the Beyer paper was co-authored by both the maintainer (Matt Watts) and ‘father’ of Marxan (Hugh Possingham), we are not arguing that Marxan be replaced, but really that SA be replaced by ILP to solve Marxan type problems. To make this point clear, we have added the following sentence to the discussion (Line XX): “Given Marxan’s flexibility to use optimization methods other than SA, we hope that a future version of Marxan will include EILP solvers.”**

However, the ILP-based Prioritzr is a promising new addition to the suite of tools we have at our disposal for planning and I think with a minor tweak to some of the framing, this paper will be well-received amongst those individuals looking for more powerful solvers.

**Thank you for this encouraging comment. We have added text throughout based on all reviewer comments that will hopefully include the framing tweaks the reviewer brought up.**

It might also be useful to suggest how ILP and SA can compliment each other in broader planning processes if the authors do want to bring the discussion up to a higher level than technical comparisons.

**We hope that with the problem formulation in Appendix S2, we were able to make it clearer that SA and ILP don’t really complement each other, but are rather two methods to solve the same problem. If the reviewer was referring to Marxan, rather than SA, the we have added the following sentence to discussion to make it clearer what we think could be a useful way forward for Marxan (Line XX): “Given Marxan’s flexibility to use optimization methods other than SA, we hope that a future version of Marxan will include EILP solvers.”**