Chapter 3: Recursive Functions

WMC CS Club

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§1 Definintion of Recursion

A definition that defines an object in terms of itself is said to be recursive. In computer science, recursion refers to a function or subroutine that calls itself, and it is a fundamental paradigm in programming. A recursive program is used for solving problems that can be broken down into sub-problems of the same type, doing so until the problem is easy enough to solve directly.

There are usually two components in a recursive function: the recursion and the base case.

§2 Common Recursive Functions

Fibonacci Numbers
A common recursive function that you've probably encountered is the Fibonacci numbers: 0, 1, 1, 2, 3, 5, 8, 13, and so on. That is, you get the next Fibonacci number by adding together the previous two.

Factorials
Consider the factorial function, $n! = n \times (n-1) \times (n-2) \times ... \times 3 \times 2 \times 1$, with 0! defined as having a value of 1.

Digit Sum
Try to come up with a recursive function that calculate the digit sum of a number. For example, the digit sum of 2021 is $2 + 0 + 2 + 1 = 5$.

Remark 2.1. Try to write a program using a language you like to solve the above problems.

§3 Recursive Functions in ACSL

Remark 3.1. This ACSL category focuses on mathematical recursive functions rather than programming algorithms. While many mathematical functions can be done iteratively more efficiently than they can be done recursively, many algorithms in computer science must be written recursively.

Example 3.2

Find $f(11)$ if $f(x) = \begin{cases} f(x-3)+1 & \text{if } x > 0 \\ 3x & \text{otherwise} \end{cases}$

Example 3.3

Evaluate $h(13)$ given $h(x) = \begin{cases} h(x-7)+1 & \text{when } x > 5 \\ x & \text{when } 0 \le x \le 5 \\ h(x+3) & \text{when } x < 0 \end{cases}$

Example 3.4

Find the value of $f(12,6)$ given $f(x,y) = \begin{cases} f(x-y, y-1)+2 & \text{when } x > y \\ x+y & \text{otherwise} \end{cases}$

§4 Practice Problems

1. Recursive function $L(x)$ gives the x^{th} (1-indexed) term of the sequence 1,3,5,7,9,...
2. Find $g(12)$ given $g(\zeta) = \begin{cases} g(\zeta-2)-3 & \text{if } \zeta \ge 10 \\ g(2\zeta-10)+4 & \text{if } 3 \le \zeta < 10 \\ \zeta \times \zeta + 5 & \text{if } \zeta < 3 \end{cases}$
3. Find $f(12,7)$ if $f(x,y) = \begin{cases} f(x-1, y+2)+3 & \text{if } x > y \\ 2f(x+1, y-1)-5 & \text{if } x < y \\ x^2+y & \text{if } x = y \end{cases}$
4. Determine $f(15,12)$ given $f(x,y) = \begin{cases} f(y-1, x-2)+4 & \text{if } x > 10 \\ f(x+3, y-3)+2 & \text{if } 5 \le x \le 10 \\ 3x-2y & \text{if } x \le 4 \end{cases}$
5. $f(x)$ takes any positive integers and $f(x) = \begin{cases} 1 & \text{if } x = 1 \\ f(\frac{x}{8}) & \text{if } x \,\%\, 8 = 0 \\ f(x-1)+1 & \text{otherwise} \end{cases}$. Find the maximum value of $f(x)$ for $x \le 100$.
6. A recursive function $E(x)$ takes a positive integer number, if x is divisible by 11, $E(x) = 0$. Otherwise, $E(x) \neq 0$. Try to come up with at least 2 recursive functions that can do this (Hint: direct recursion and alternating digit sum).
7. $k(x)$ computes the super digit sum of a positive integer. A super digit sum repeatedly calculates the digit sum of the number until it becomes a single digit number. For example, $k(1999) = k(1+9+9+9) = k(28) = k(2+8) = k(10) = k(1+0) = k(1) = 1$.
8. Find a recursive function $f(m,n)$ such that $f(m,n)$ always equals to $\frac{(m+n)!}{m!n!}$ for all positive integers m and n.