Chapter 4: Prefix, Infix, Postfix Notations

WMC CS Club

Dec 16, 2021

§1 The Infix Notation

The way we usually compute math expressions, which is called infix notation, seems easy to us, but it's not the way how computers compute. Instead of infix notation, computers use prefix and postfix notations. Before introducing them, let's review the infix notation.

The expression $5 + \frac{8}{(3-1)}$ clearly has a value of 9. It is written in infix notation as $5 + 8/(3 - 1)$. The value of an infix version is well-defined because there is a well-established order of precedence in mathematics: We first evaluate the parentheses $(3 - 1 = 2)$; then, because division has higher precedence that subtraction, we next do $8/2 = 4$. And finally, $5 + 4 = 9$.

> **Remark 1.1.** The order of precedence is often given the mnemonic of Please excuse my dear Aunt Sue, or **PEMDAS: parentheses, exponentiation, multiplication/division, and addition/subtraction**. Multiplication and division have the same level of precedence; addition and subtraction also have the same level of precedence. Terms with equals precedence are evaluated from left-to-right.

§2 Think Pre/Post-fixedly

What are pre/post-fix notations? Simply put, in prefix notation, each operator is placed before its operands; in postfix notation, each operator is placed after its operand. The prefix and postfix notations of $5 + \frac{8}{3-1}$ are "$+\ 5\ /\ 8\ -\ 3\ 1$" and "$5\ 8\ 3\ 1\ -\ /\ +$", respectively.

§3 Prefix to Infix to Postfix

Here are the algorithms for converting from prefix(postfix) to infix and from infix to prefix(postfix).

From Prefix or Postfix to Infix

One way to convert from prefix (postfix) to infix is to make repeated scans through the expression. Each scan, find an operator with two adjacent operators and replace it with a parenthesized infix expression. This is not the most efficient algorithm, but works well for a human.

From Infix to Prefix or Postfix

1. Fully parenthesize the infix expression. It should now consist solely of "terms": a binary operator sandwiched between two operands.
2. Write down the operands in the same order that they appear in the infix expression.
3. Look at each term in the infix expression in the order that one would evaluate them, i.e., inner-most parenthesis to outer-most and left to right among terms of the same depth.
4. For each term, write down the operand before (after) the operators.

Example 3.1

Convert the prefix expression $\uparrow\ +\ *\ 3\ 4\ /\ 8\ 2 - 7\ 5$ to infix expression and evaluate.

Example 3.2

Convert the postfix expression $3\ 4\ *\ 8\ 2\ /\ +\ 7\ 5\ -\ \uparrow$ to infix expression and evaluate.

Example 3.3

Convert the infix notation expression to prefix and postfix notations: $(X = (((A * B) - (C/D)) \uparrow E))$.

§4 Practice Problems

1. Evaluate prefix expression $*\ -\ 5\ 3\ +\ 4\ 9$.
2. Evaluate postfix expression $7\ 3\ 2\ \uparrow\ 4\ 5\ +\ /\ *$.

3. Convert $6 - \frac{2^3}{5-1}$ to prefix and postfix expression.

4. Convert $A^B - C * D/(E + F * G)$ to prefix and postfix expression.

5. An intuitive assumption might be that the prefix notation is the reversed string of the postfix notation of the same expression. Consider why this assumption might be incorrect.