Computer Science Study Materials

ACSL Study Materials

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Chapter 1: Computer Number Systems

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§1 Introduction

All digital computers, from supercomputers to your smartphone, are electronic devices and ultimately can do one thing: detect whether an electrical signal is on or off. That basic information, called a bit (binary digit), has two values: a 1 (or true) when the signal is on, and a 0 (of false) when the signal is off.

Larger values can be stored by a group of bits. For example, there are 4 different values stored by 2 bits (00, 01, 10, and 11), 8 values for 3 bits, and so on. However, large numbers, using 0s and 1s only, are quite unwieldy for humans. For example, a computer would need 19 bits to store the numbers up to 500,000! We use of different number systems to work with large binary strings that represent numbers within computers. In ACSL, we need to know four types of number systems:

- Binary (Base 2, $\overline{abcd}_2$): 0, 1.
- Octal (Base 8, $\overline{abcd}_8$): 0, 1, 2, 3, 4, 5, 6, 7.
- Decimal (Base 10, $\overline{abcd}_{10}$ or $\overline{abcd}$): 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.
- Hexadecimal (Base 16, $\overline{abcd}_{16}$): 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F.

Consider how we usually represent numbers with decimal number system. If we have a base-10 number $\overline{abcd}$, this number is equal to $10^3a + 10^2b + 10^1c + 10^0d$. This idea works for any bases and any numbers of digits.

§2 Converting with Decimal

The decimal value of a number in an arbitrary base is the sum of each digit multiplied by its place value. Here are some examples of converting from one base into base 10:

Example
Convert 1101_2, 172_8, and $3AD_{16}$ to decimal.

Conversely, the algorithm to convert a number from an arbitrary base requires finding how many times successive powers of the base go into the number, starting with the largest power of the base less than the starting number.

Example
Convert 128, 205 to base 2, 8, and 16, respectively.

§3 Converting between Binary, Octal, and Hex

If you want to convert between number systems other than decimal, you can first convert a number into decimal, then convert that decimal number into the target base. However, there are some shortcuts/ rules. They are fairly straightforward:

1. Converting from octal to binary is simple: replace each octal digit by its corresponding 3 binary bits.
2. Converting from hex to binary is also simple: replace each hex digit by its corresponding 4 binary bits.
3. Converting from binary to either octal or hex is pretty simple as well: group the bits by 3s or 4s (starting at the right), and convert each group.
4. Converting between base 8 and 16 is easy by expressing the number in base 2 (easy to do!) and then converting that number from base 2 (another easy operation)!

Example
Convert 101011_2 to octal and hexadecimal; 54_8 to binary and hexadecimal; $E3_{16}$ to binary and octal.

§4 Practice Problems

1. Convert 156 to binary, octal, and hexadecimal, respectively.
2. Convert $25C_{16}$ to binary, octal, and decimal, respectively.
3. Write 1010111011110101_2 in octal and hexadecimal.
4. Solve for x if $x_{16} = 3676_8$.
5. How many 1s are in the binary representation of 4327_8?
6. Find x if x is a hex that $x = F5AD_{16} - 69EB_{16}$.
7. Evaluate and express the answer in hex: $32_8 + 1011_2 + 352_{10} + AF_{16}$.
8. When convert $AAAAAAAAA_{16}$ to decimal, find the remainder of this number when divided by 5.
9. How many numbers from 100 to 200 in base 10 consist of distinct ascending digits and also have distinct ascending hex digits when converted to base 16?
10. If substring $CADBEF$ is repeated 1000 times in a hex number, find the number of 1s and 0s in its binary representation.