

Statistics Review

Ricson

July 12, 2017

Contents

1	Probability Theory	4
1.1	Conjugate Priors	4
1.2	Basic manipulations of Multivariate Gaussians	4
1.3	Extremal Value Distributions	4
2	Information Theory	4
2.1	Basics	4
2.2	KL Divergence	4
2.3	Fisher Information Matrix	4
3	Linear Algebra	4
3.1	Identities	4
3.2	Singular Value Decomposition	4
3.3	Principle Component Analysis	4
4	Probabilistic Graphical Models	4
4.1	Viterbi and Forward Backward Algorithm	4
4.2	Message Passing and Loopy Belief Propagation	7
4.3	Efficient Inference in Dense Gaussian CRFS	7
5	Linear and Nonlinear Bayesian Models	7
5.1	Bayesian Linear Regression	7
5.1.1	Evidence Approximation	7
5.2	Bayesian Logistic Regression	7
5.2.1	Laplace Approximation	7

5.3	Spline and Wavelet Bases	7
5.4	Bayesian Neural Networks	7
6	Approximate Inference	7
6.1	Expectation Maximization	7
6.2	Variational Inference	7
7	Sampling Algorithms	7
7.1	Importance Sampling	7
7.2	Monte Carlo Markov Chain	7
7.2.1	Metropolis Hastings	7
7.2.2	Reversible Jump MCMC	7
7.3	Gibbs Sampling	7
8	Optimization	7
8.1	Lagrange Multipliers	7
8.2	Lagrangian Duality	7
8.3	KKT Conditions	7
8.4	SGD Variants	7
8.5	BADMM Algorithm	7
9	Reinforcement Learning	7
9.1	Optimal Control	7
9.1.1	LQR	7
9.1.2	MPC and ILQR	7
9.1.3	Stochastic Optimal Control	7
9.2	Policy Gradients	7
9.3	Natural Policy Gradients	7
9.4	Trust Region Policy Optimization	7
9.5	Guided Policy Search	7
9.6	End to End Deep Visuomotor Policies	7
10	Neural Networks	7
10.1	Variational Autoencoders	7
10.2	Architectures	7
10.2.1	Neural Turing Machine	7
10.2.2	Neural GPU	7
10.2.3	Grid LSTM	7

11 Miscellaneous	7
11.1 Matrix Differentials	7
11.2 Calculus of Variations	7
11.3 Echo State Networks	7
11.4 Gumbel Trick	7
11.5 Support Vector Machines	7
11.6 Gaussian Processes	7
11.7 Independent Component Analysis	7

1 Probability Theory

1.1 Conjugate Priors

1.2 Basic manipulations of Multivariate Gaussians

1.3 Extremal Value Distributions

2 Information Theory

2.1 Basics

2.2 KL Divergence

2.3 Fisher Information Matrix

3 Linear Algebra

3.1 Identities

3.2 Singular Value Decomposition

3.3 Principle Component Analysis

4 Probabilistic Graphical Models

4.1 Viterbi and Forward Backward Algorithm

The key to deriving the forward backward algorithm for the linear-chain CRF is to realize that there is no fundamental difference between HMM and CRF. The likelihood of a linear-chain CRF is defined to be the product of all the potentials. But before we get mixed up in math here: o_i denotes observation and x_i denotes the hidden state.

$$P(\mathbf{x}) = \prod_i \phi(x_i) \prod_{i,i+1} \phi(x_i, x_{i+1})$$

On the other hand, the likelihood of the equivalent HMM model is the following:

$$P(x) = \prod_i P(o_i|x_i)P(x_i|x_{i+1})$$

Note this product is incorrect when $i = n$, but let's ignore that.

Now if you squint really closely, you'll realize that these two equations are actually the same thing. We can just replace $P(o_i|x_i)$ with $\phi(x_i)$ and $P(x_i|x_{i+1})$ with $\phi(x_i, x_{i+1})$. So it suffices to solve inference on the general setting of CRFs. However, we'll start on HMMs simply to avoid a bit of headaches.

We want to solve for $P(x_k|\mathbf{o})$. The key is the following manipulation:

$$\begin{aligned} P(x_k|\mathbf{o}) &= \frac{P(\mathbf{o}|x_k)p(x_k)}{P(\mathbf{o})} \\ &\propto P(\mathbf{o}|x_k)p(x_k) \\ &= P(\mathbf{o}_{1:k}|x_k)P(\mathbf{o}_{k+1:n}|x_k)P(x_k) \end{aligned}$$

Let's take that second term and apply Bayes' Theorem again

$$\begin{aligned} P(\mathbf{o}_{k+1:n}|x_k) &= \frac{P(x_k|\mathbf{o}_{k+1:n})P(\mathbf{o}_{k+1:n})}{P(x_k)} \\ &\propto \frac{P(x_k|\mathbf{o}_{k+1:n})}{P(x_k)} \end{aligned}$$

And substitute it back in...

$$\begin{aligned} P(\mathbf{o}_{1:k}|x_k)P(\mathbf{o}_{k+1:n}|x_k)P(x_k) &= P(\mathbf{o}_{1:k}|x_k) \frac{P(x_k|\mathbf{o}_{k+1:n})}{P(x_k)} P(x_k) \\ &= P(\mathbf{o}_{1:k}|x_k)P(x_k|\mathbf{o}_{k+1:n}) \end{aligned}$$

So now we just need to figure out how to compute $P(\mathbf{o}_{1:k}|x_k)$ and $P(x_k|\mathbf{o}_{k+1:n})$

4.2 Message Passing and Loopy Belief Propagation

4.3 Efficient Inference in Dense Gaussian CRFS

5 Linear and Nonlinear Bayesian Models

5.1 Bayesian Linear Regression

5.1.1 Evidence Approximation

5.2 Bayesian Logistic Regression

5.2.1 Laplace Approximation

5.3 Spline and Wavelet Bases

5.4 Bayesian Neural Networks

6 Approximate Inference

6.1 Expectation Maximization

6.2 Variational Inference

7 Sampling Algorithms

7.1 Importance Sampling

7.2 Monte Carlo Markov Chain

7.2.1 Metropolis Hastings

7.2.2 Reversible Jump MCMC

7.3 Gibbs Sampling

8 Optimization

8.1 Lagrange Multipliers

8.2 Lagrangian Duality

8.3 KKT Conditions

7

8.4 SGD Variants

8.5 BADMM Algorithm

9 Reinforcement Learning