Image Classification, Linear Classifiers and Losses

Deep Learning
26 September 2024
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SAPIENZA
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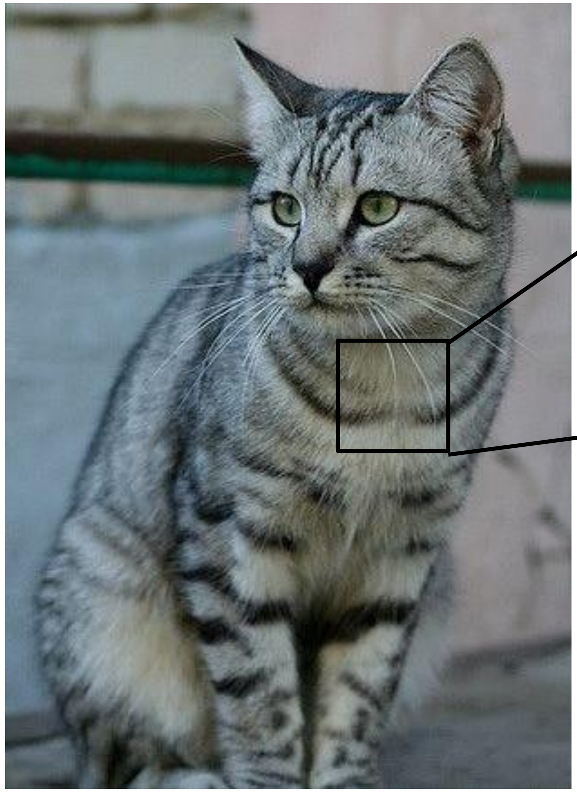
Image Classification: A core task in Computer Vision

(assume given set of discrete labels)
{dog, cat, truck, plane, ...}

$\longrightarrow$ cat

The Problem: Semantic Gap



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```
[[105 112 108 111 104  99 106  99  96 103 112 119 104  97  93  87]
 [ 91  98 102 106 104  79  98 103  99 105 123 136 110 105  94  85]
 [ 76  85  90 105 128 105  87  96  95  99 115 112 106 103  99  85]
 [ 99  81  81  93 120 131 127 100  95  98 102  99  96  93 101  94]
 [106  91  61  64  69  91  88  85 101 107 109  98  75  84  96  95]
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 [122 121 102  80  82  86  94 117 145 148 153 102  58  78  92 107]
 [122 164 148 103  71  56  78  83  93 103 119 139 102  61  69  84]]
```

What the computer sees

An image is just a big grid of numbers between [0, 255]:

e.g. 800 x 600 x 3
(3 channels RGB)

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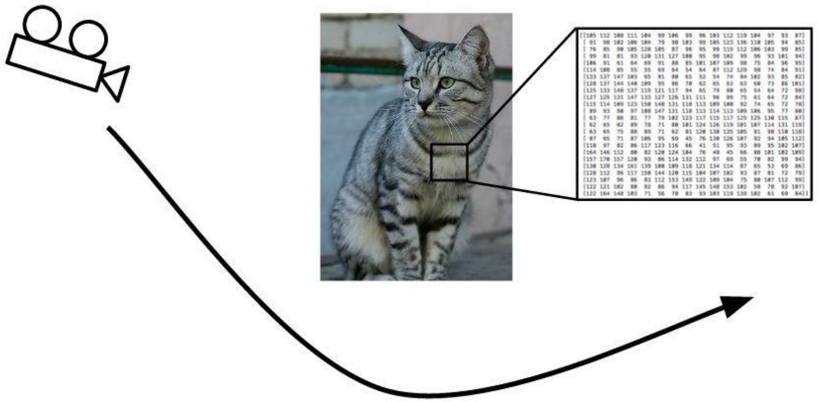
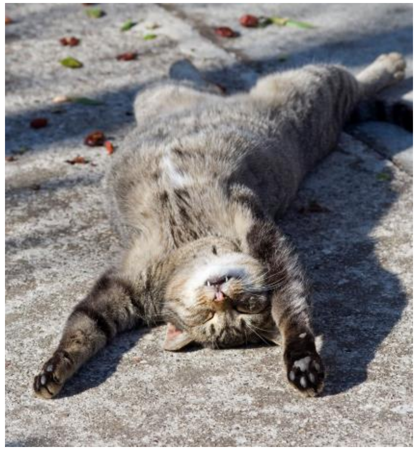
Recall: Challenges of recognition

Viewpoint



Illumination

Deformation



Occlusion

Clutter

Intraclass Variation

An image classifier

```
def classify_image(image):
  # Some magic here?
  return class_label
```

Unlike e.g. sorting a list of numbers,

no obvious way to hard-code the algorithm for recognizing a cat, or other classes.

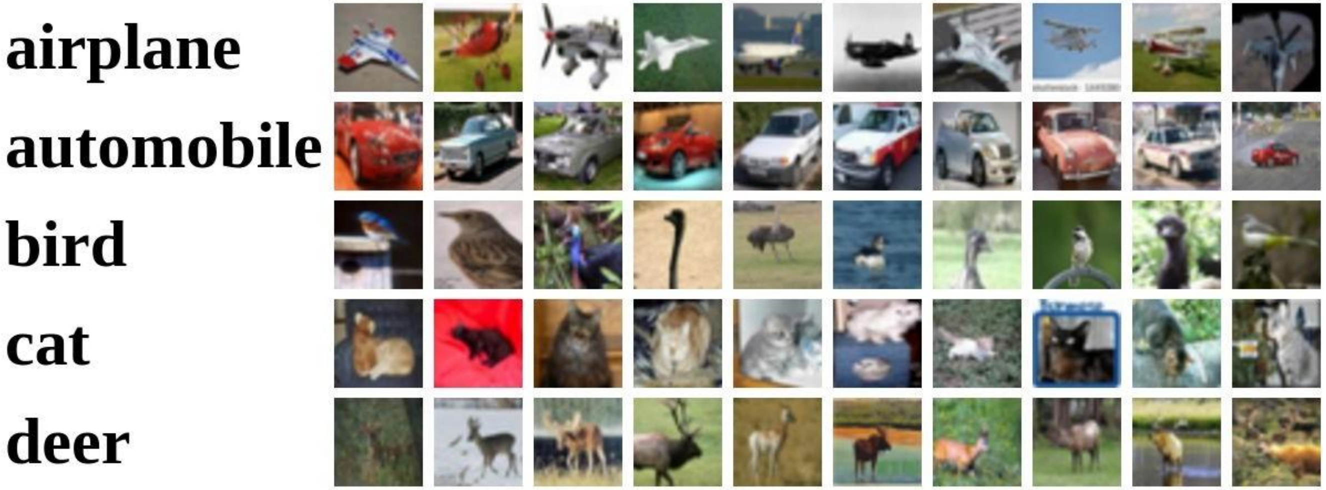
Machine Learning: Data-Driven Approach

1. Collect a dataset of images and labels
2. Use Machine Learning to train a classifier
3. Evaluate the classifier on new images

```python
def train(images, labels):
  # Machine learning!
  return model
```

```python
def predict(model, test_images):
  # Use model to predict labels
  return test_labels
```

Example training set



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Machine Learning: Data-Driven Approach

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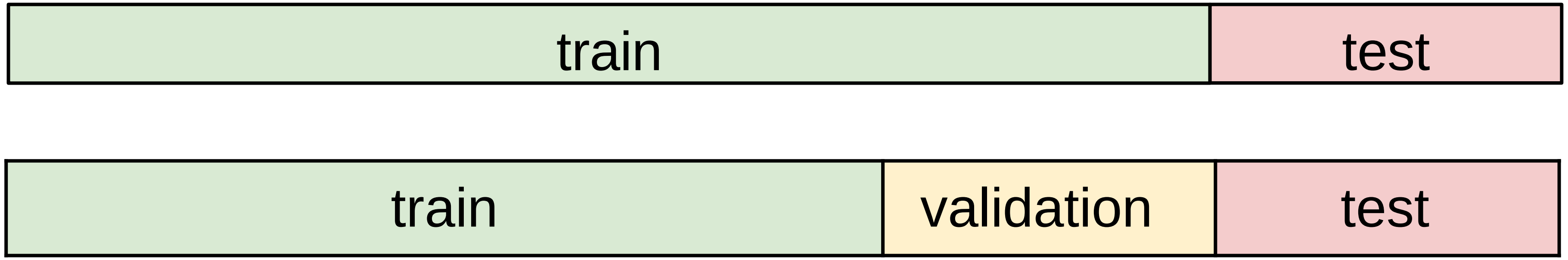

Recall CIFAR10

50,000 training images
each image is **32x32x3**

10,000 test images.

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Parametric Approach: Linear Classifier

$$f(x,W) = Wx$$

Image

Array of **32x32x3** numbers
(3072 numbers total)

$f(\mathbf{x},\mathbf{W})$ → **10** numbers giving class scores

W

parameters
or weights

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Parametric Approach: Linear Classifier

3072x1

$$\boxed{f(x,W) = \boxed{W}\boxed{x} + }\boxed{b}$$ 10x1

10x1 10x3072

Image

Array of **32x32x3** numbers
(3072 numbers total)

$f(x,W)$ → **10** numbers giving class scores

W

parameters
or weights

Neural Network

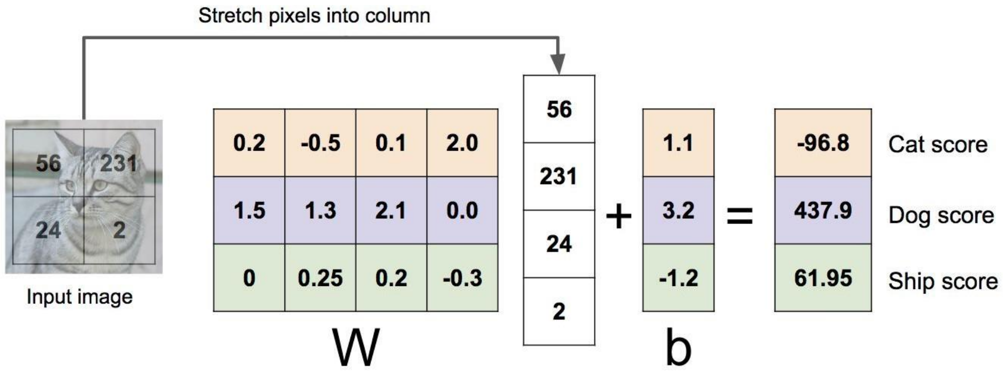
Linear
classifiers

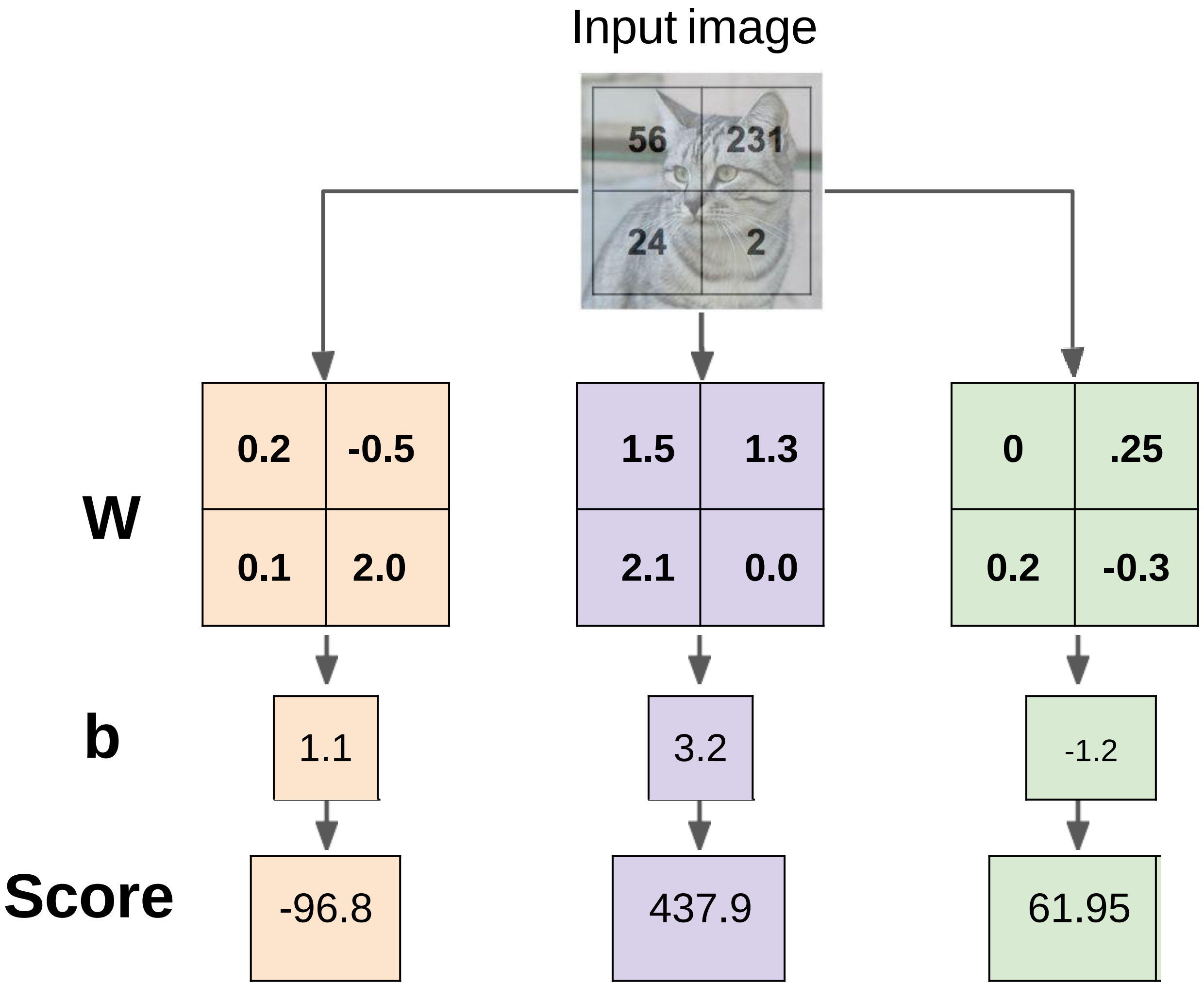
This image is CC0 1.0 public domain

Example with an image with 4 pixels, and 3 classes (cat/dog/ship)

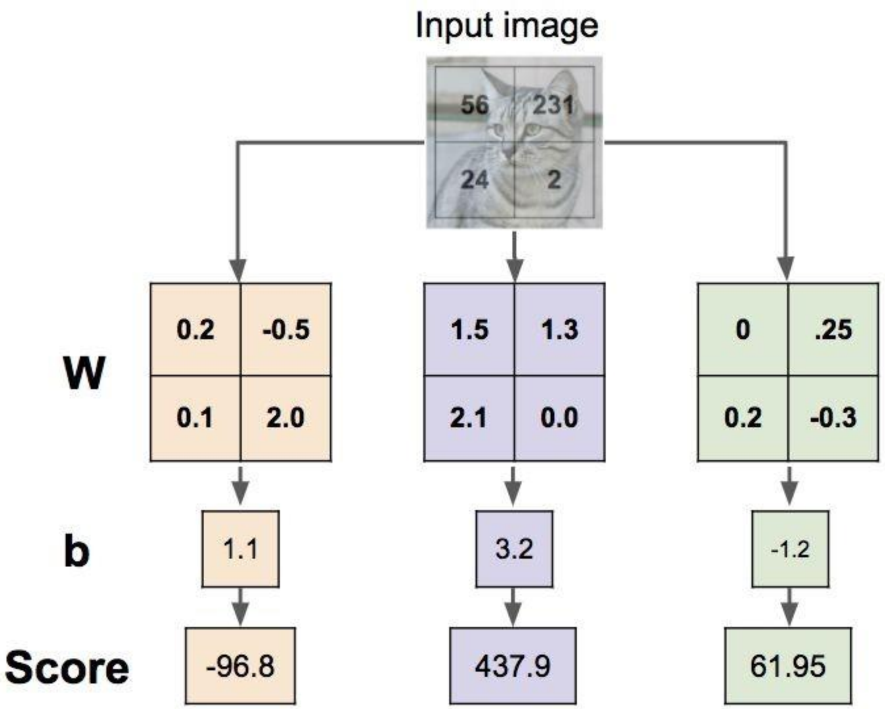
Algebraic Viewpoint

$$f(x,W) = Wx$$

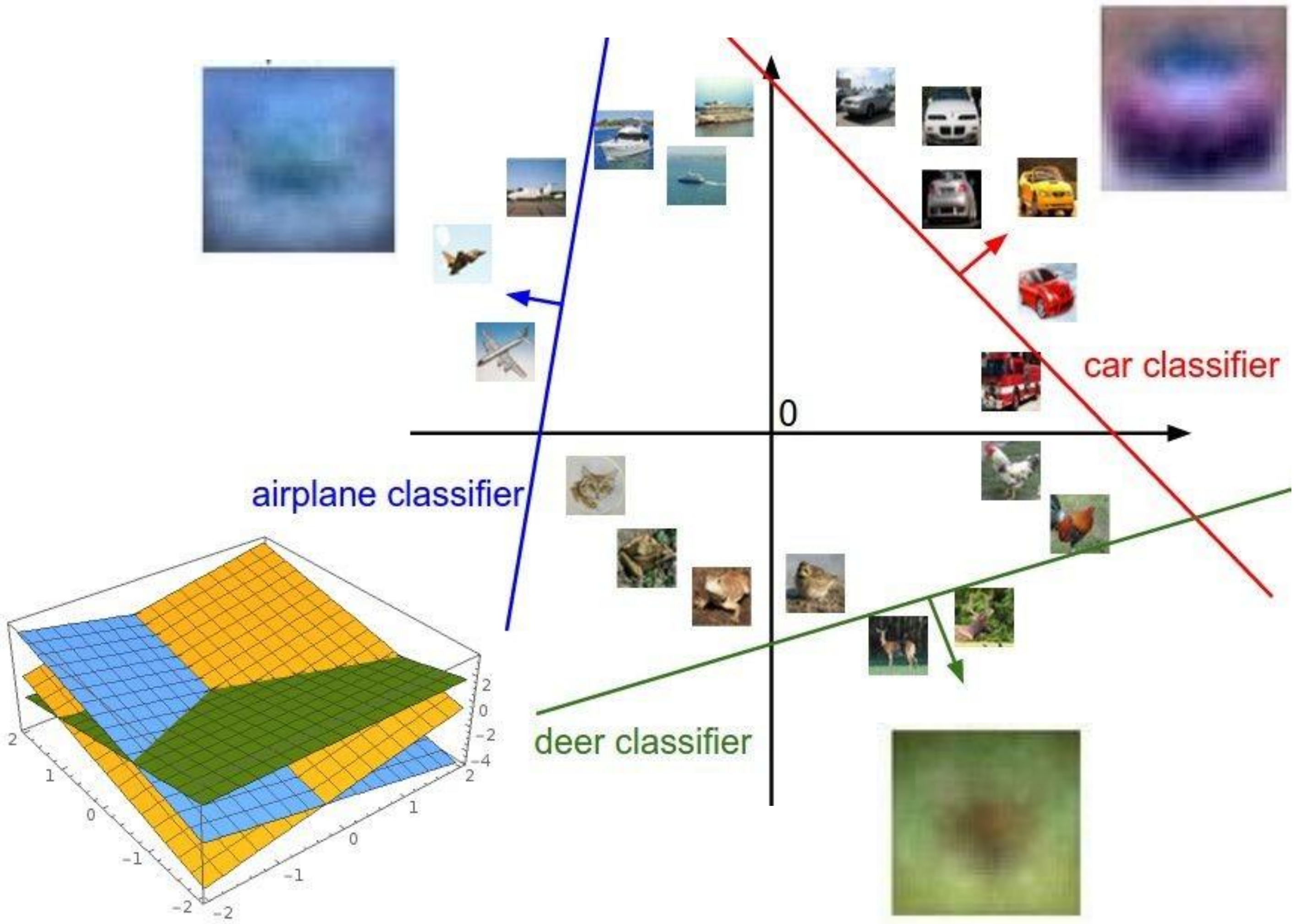




Interpreting a Linear Classifier: Visual Viewpoint



Interpreting a Linear Classifier: Geometric Viewpoint



$$f(x,W) = Wx + b$$

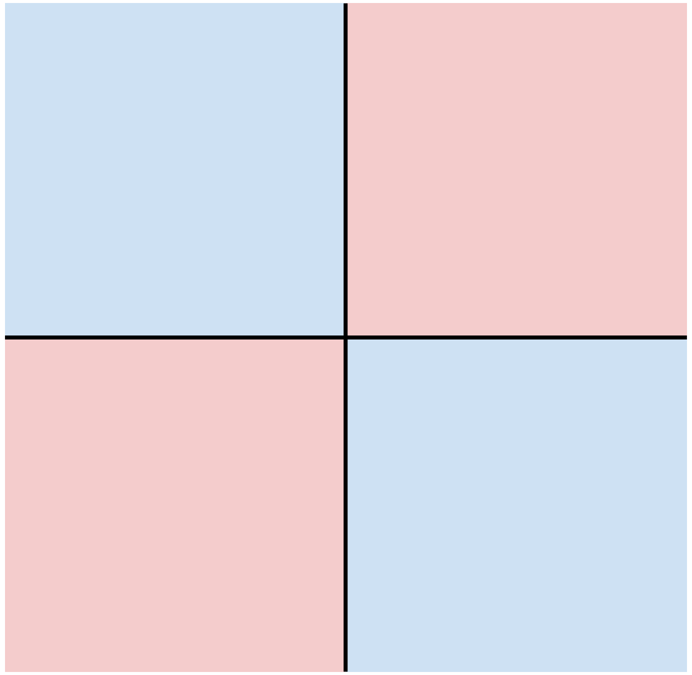
Array of **32x32x3** numbers
(3072 numbers total)

Plot created using [Wolfram Cloud]

Hard cases for a linear classifier

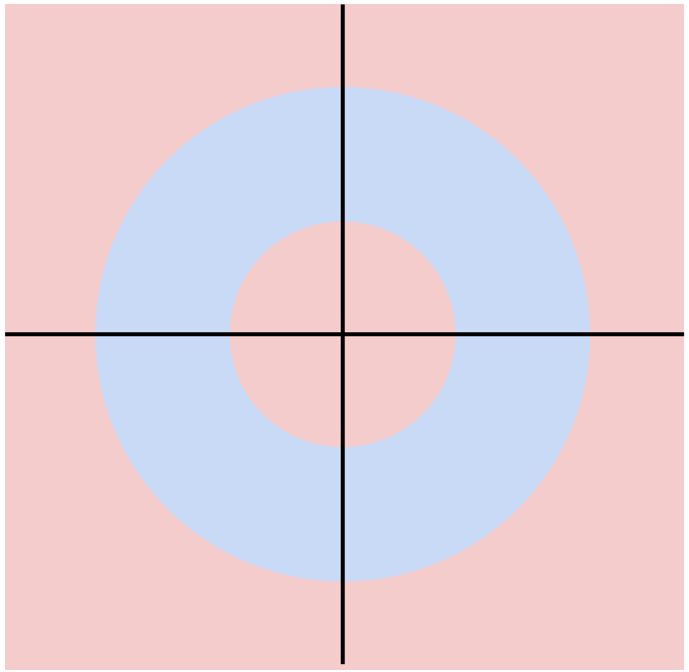
Class 1:
First and third quadrants

Class 2:
Second and fourth quadrants



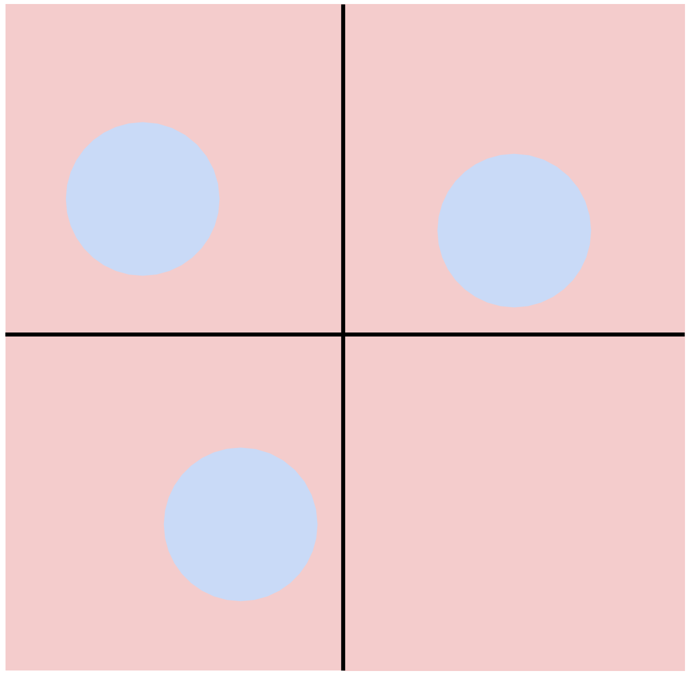
Class 1:
1 <= L2 norm <= 2

Class 2:
Everything else



Class 1:
Three modes

Class 2:
Everything else



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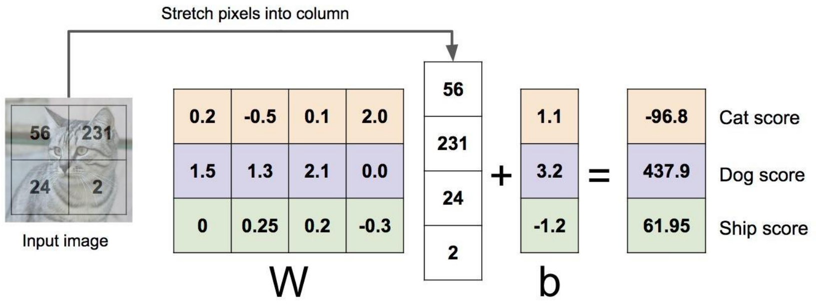
Linear Classifier: Three Viewpoints

Algebraic Viewpoint

$f(x,W) = Wx$

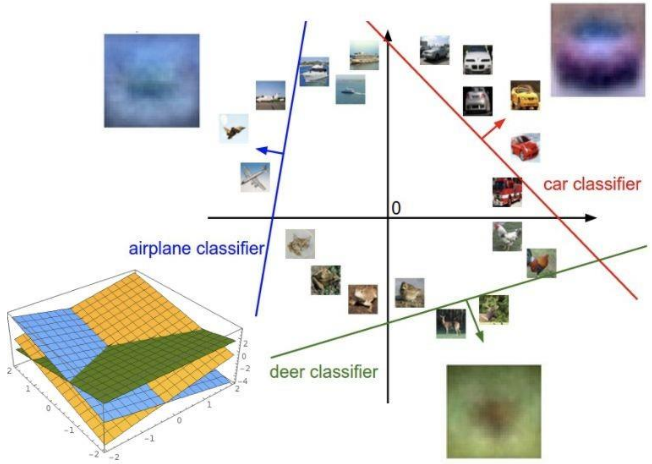


Visual Viewpoint

One template
per class

Geometric Viewpoint

Hyperplanes
cutting up space



So far: Defined a (linear) <u>score function</u> $f(x,W) = Wx + b$

Example class scores for 3 images for some W:

How can we tell whether this W is good or bad?

airplane	-3.45	-0.51	3.42
automobile	-8.87	**6.04**	4.64
bird	0.09	5.31	2.65
cat	**2.9**	-4.22	5.1
deer	4.48	-4.19	2.64
dog	8.02	3.58	5.55
frog	3.78	4.49	**-4.34**
horse	1.06	-4.37	-1.5
ship	-0.36	-2.09	-4.79
truck	-0.72	-2.93	6.14

Loss Functions and Optimization

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**

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Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:



cat	**3.2**	1.3	2.2
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A **loss function** tells how good our current classifier is

Given a dataset of examples

$$\{(x_i, y_i)\}_{i=1}^N$$

Where x_i is image and
y_i is (integer) label

Loss over the dataset is a average of loss over examples:

$$L = \frac{1}{N} \sum_i L_i(f(x_i, W), y_i)$$

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Softmax Classifier (Multinomial Logistic Regression)

Want to interpret raw classifier scores as **probabilities**

cat	**3.2**
car	5.1
frog	-1.7

Softmax Classifier (Multinomial Logistic Regression)

Want to interpret raw classifier scores as **probabilities**

$$s = f(x_i; W)$$

$$P(Y = k | X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

Softmax Function

cat	**3.2**
car	5.1
frog	-1.7

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Softmax Classifier (Multinomial Logistic Regression)

Want to interpret raw classifier scores as **probabilities**

$$s = f(x_i; W)$$

$$P(Y = k | X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

Softmax Function

Probabilities must be >= 0

Probabilities must sum to 1

$$L_i = -\log P(Y = y_i | X = x_i)$$

	Unnormalized log-probabilities / logits		unnormalized probabilities		probabilities
cat	**3.2**	exp →	**24.5**	normalize →	**0.13**
car	5.1		164.0		0.87
frog	-1.7		0.18		0.00

→ $L_i = -\log(0.13)$
$= \mathbf{2.04}$

Maximum Likelihood Estimation
Choose weights to maximize the likelihood of the observed data

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Softmax Classifier (Multinomial Logistic Regression)

Want to interpret raw classifier scores as **probabilities**

$$s = f(x_i; W)$$

$$P(Y = k | X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

Softmax Function

Probabilities must be >= 0

Probabilities must sum to 1

$$L_i = -\log P(Y = y_i | X = x_i)$$

	Unnormalized log-probabilities / logits	$\xrightarrow{\text{exp}}$	unnormalized probabilities	$\xrightarrow{\text{normalize}}$	probabilities		Correct probs
cat	**3.2**		**24.5**		**0.13**	$\rightarrow$ compare $\leftarrow$	**1.00**
car	5.1		164.0		0.87		0.00
frog	-1.7		0.18		0.00		0.00

Cross Entropy

$$H(\underline{P}, \underline{Q}) = H(p) + D_{KL}(P \| Q)$$

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Let's double check...

- Entropy & KL-divergence:

$$H(P^t) = -\sum_y P^t(y) \log P^t(y)$$

$$D_{KL}(P^t||Q) = \sum_y P^t(y) \log \frac{P^t(y)}{Q(y)}$$

- Cross Entropy the sum of both:

$$\begin{aligned} H(P^t, Q) &= H(P^t) + D_{KL}(P^t||Q) \\ &= \sum_y P^t(y) \left(\log \frac{P^t(y)}{Q(y)} - \log P^t(y) \right) \\ &= -\sum_y P^t(y) \log Q(y) \end{aligned}$$

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Let's double check...

- Cross Entropy in our classification case:
  - Target "distribution" / output:

$$P^t(y) = \begin{cases} 1 & y = y_i \\ 0 & y \neq y_i \end{cases}$$

  - Output of the network:

$$Q(y|x_i) = P(Y = k|X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

- Then Cross Entropy Loss for image x_i

$$\begin{aligned} L_i = L(x_i) &= -\sum_y P^t(y) \log Q(y|x_i) \\ &= -1 \cdot \log Q(y_i|x_i) \\ &= -\log P(Y = y_i|X = x_i) \end{aligned}$$

Regularization

λ = regularization strength (hyperparameter)

$$L(W) = \underbrace{\frac{1}{N}\sum_{i=1}^{N} L_i(f(x_i, W), y_i)} + \underbrace{\lambda R(W)}$$

Data loss: Model predictions should match training data

Regularization: Prevent the model from doing *too* well on training data

Simple examples

L2 regularization: $R(W) = \sum_k \sum_l W_{k,l}^2$

L1 regularization: $R(W) = \sum_k \sum_l |W_{k,l}|$

Elastic net (L1 + L2): $R(W) = \sum_k \sum_l \beta W_{k,l}^2 + |W_{k,l}|$

More complex:

Dropout

Batch normalization

Stochastic depth, fractional pooling, etc

Regularization

λ = regularization strength (hyperparameter)

$$L(W) = \underbrace{\frac{1}{N}\sum_{i=1}^{N} L_i(f(x_i, W), y_i)}_{} + \underbrace{\lambda R(W)}_{}$$

Data loss: Model predictions should match training data

Regularization: Prevent the model from doing *too* well on training data

Why regularize?

- Express preferences over weights
- Make the model *simple* so it works on test data
- Improve optimization by adding curvature

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Regularization: Expressing Preferences

L2 Regularization

$$R(W) = \sum_k \sum_l W_{k,l}^2$$

$$x = [1, 1, 1, 1]$$

$$w_1 = [1, 0, 0, 0]$$

$$w_2 = \boxed{[0.25, 0.25, 0.25, 0.25]}$$

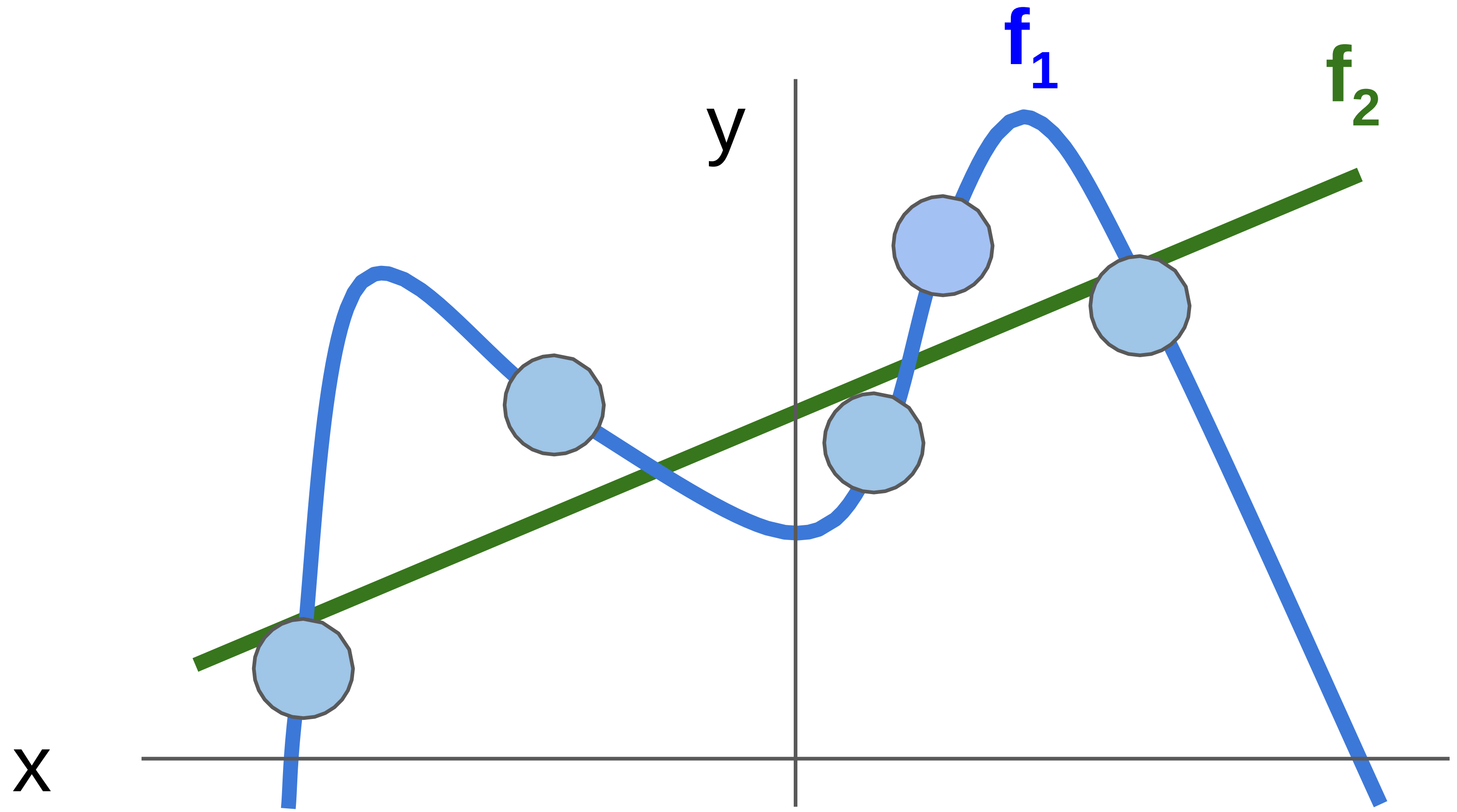
L2 regularization likes to “spread out” the weights

$$w_1^T x = w_2^T x = 1$$

$$R(w_1) = 1^2 + 0^2 + \cdots = 1^2$$

$$R(w_2) = 0.25^2 + 0.25^2 + \cdots = 4 * 0.25^2 = 0.25$$

Regularization: Prefer Simpler Models



Regularization pushes against fitting the data *too* well so we don't fit noise in the data

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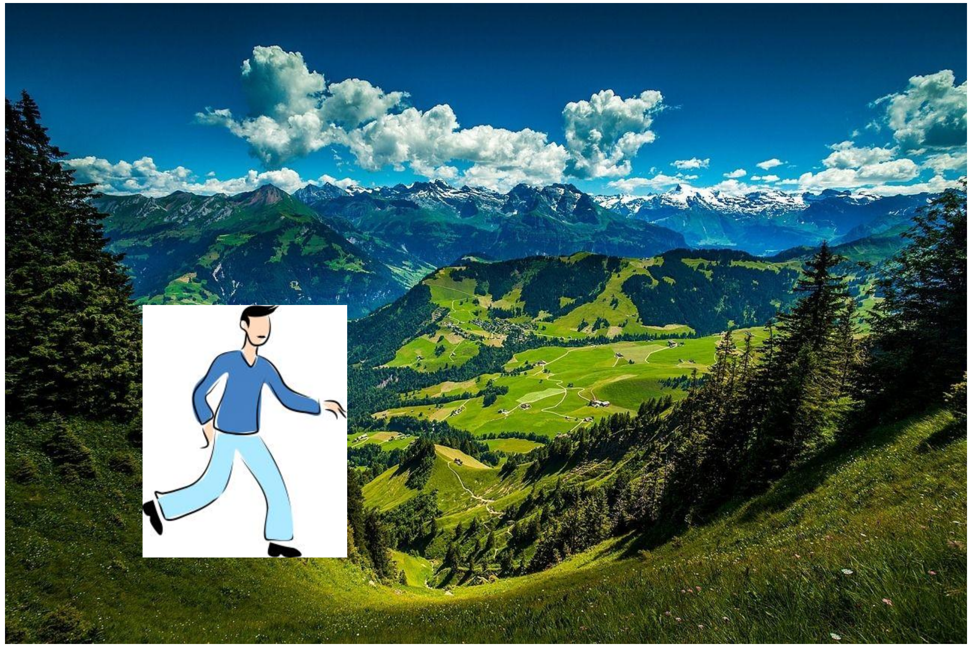
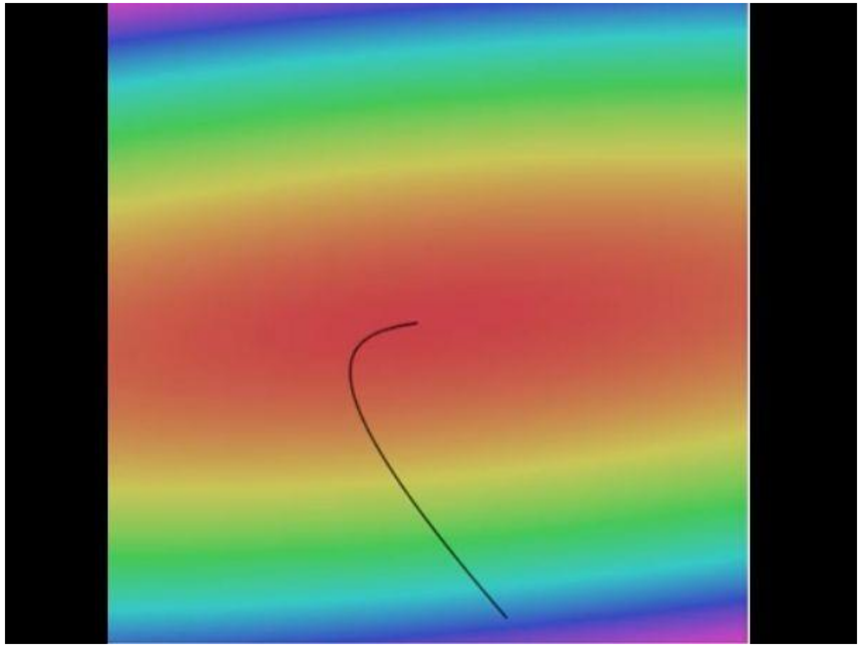
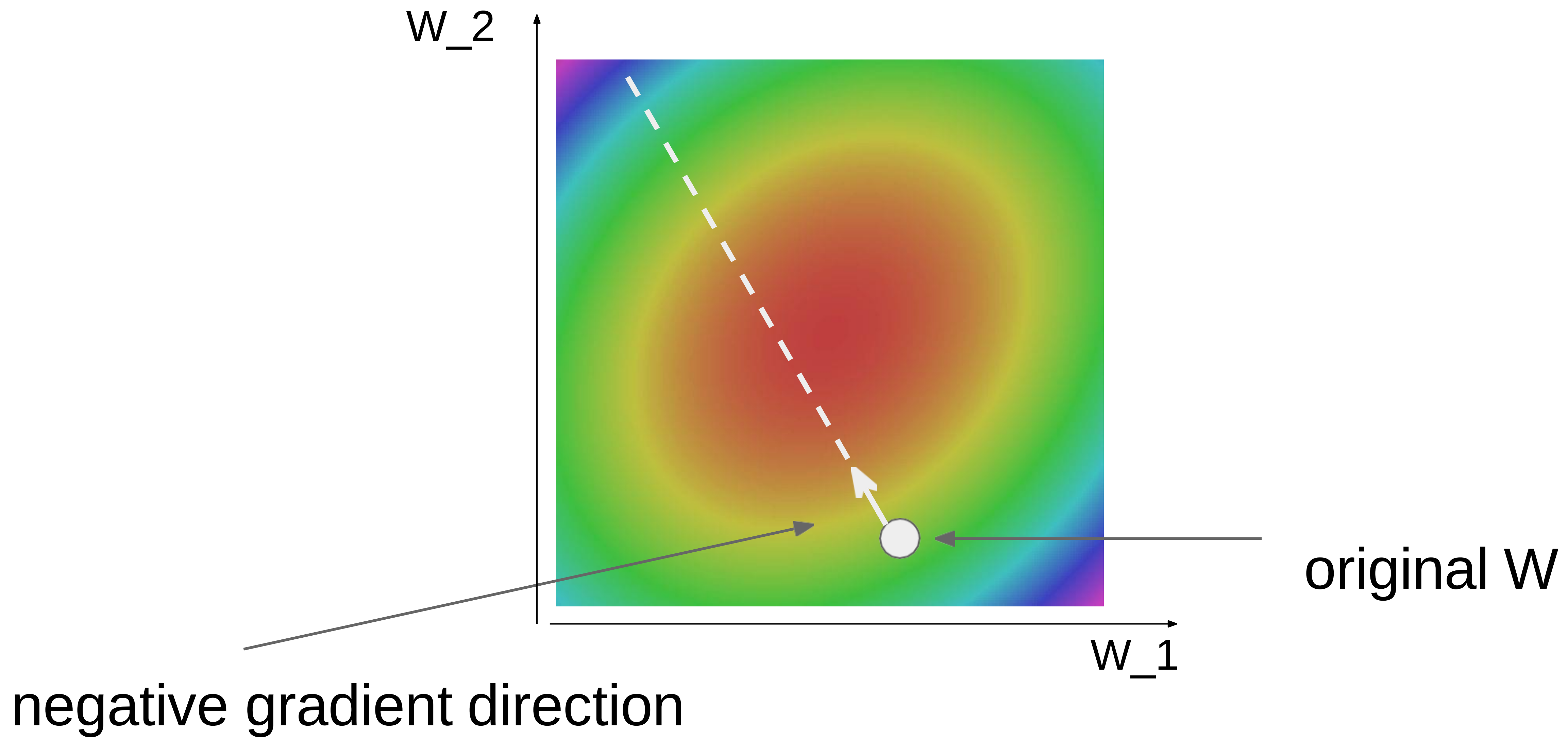
Optimization

Finding the best W: Optimize with Gradient Descent






W_2
W_1
original W
negative gradient direction

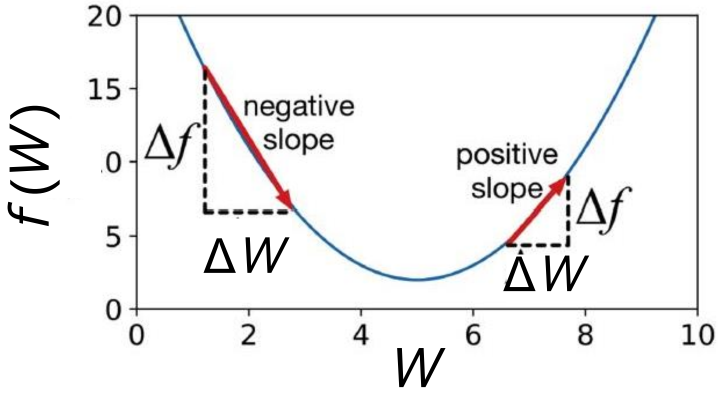
Gradient Descent

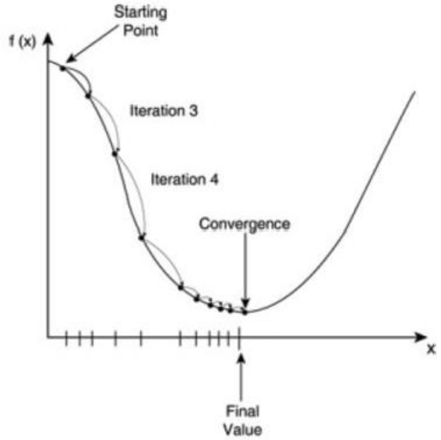
General gradient descent: Start with initial point W_0

Sequence: $W_{t+1} = W_t + \alpha_t d_t$

Steepest Descent:

$d_t = -\nabla f(W_t)$ (we move in the opposite direction of the gradient).






```
# Vanilla Gradient Descent

while True:
  weights_grad = evaluate_gradient(loss_fun, data, weights)
  weights += - step_size * weights_grad # perform parameter update
```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Gradient Descent Variants

- Assume Loss to be: $\mathrm{L}(W) = \frac{1}{n}\sum_i^n L_i(W)$
  - with n the number of training samples
  - L_i the loss for training sample x_i
- Stochastic Gradient Descent:
  - randomly choose one training sample x_i
  - update weights based on loss $L_i(W)$
- Mini-batch training:
  - process a subset of training samples $M \subset \{1, \dots, n\}$
  - update weights based on $L_M(W) = \frac{1}{|M|}\sum_{i \in M} L_i(W)$
- Batch training:
  - process all training samples
  - update weights based on $\mathrm{L}(W) = \frac{1}{n}\sum_i^n L_i(W)$

Stochastic Gradient Descent (SGD)

$$L(W) = \frac{1}{N} \sum_{i=1}^{N} L_i(x_i, y_i, W) + \lambda R(W)$$

$$\nabla_W L(W) = \frac{1}{N} \sum_{i=1}^{N} \nabla_W L_i(x_i, y_i, W) + \lambda \nabla_W R(W)$$

Full sum expensive when N is large!

Approximate sum using a **minibatch** of examples
32 / 64 / 128 common

```
# Vanilla Minibatch Gradient Descent

while True:
  data_batch = sample_training_data(data, 256) # sample 256 examples
  weights_grad = evaluate_gradient(loss_fun, data_batch, weights)
  weights += - step_size * weights_grad # perform parameter update
```


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Gradient Descent

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Numerical gradient: slow :(, approximate :(, easy to write :)
Analytic gradient: fast :), exact :), error-prone :(

In practice: Derive analytic gradient, check your implementation with numerical gradient

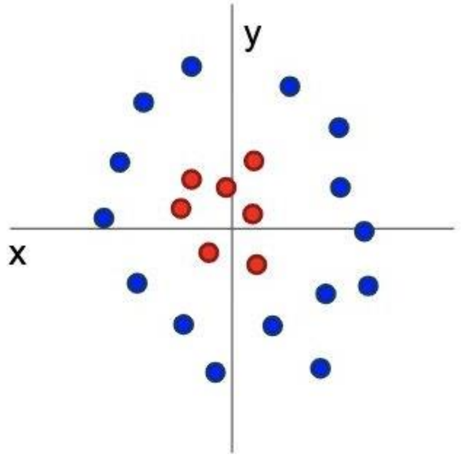
Image Features

Problem: Linear Classifiers are not very powerful

Visual Viewpoint

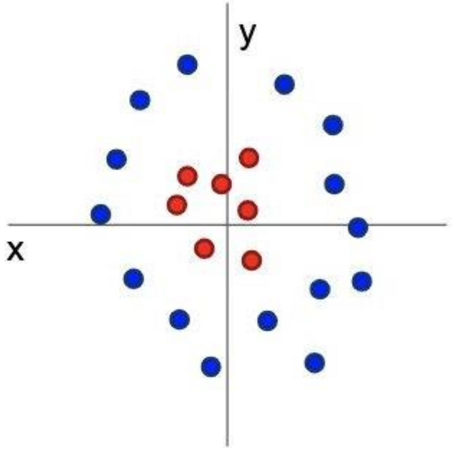
Linear classifiers learn one template per class

Geometric Viewpoint



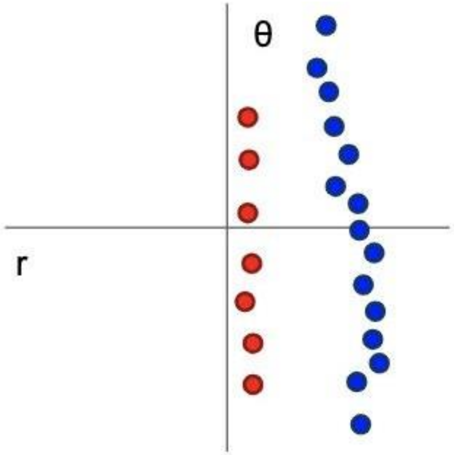
Linear classifiers can only draw linear decision boundaries

One Solution: Feature Transformation

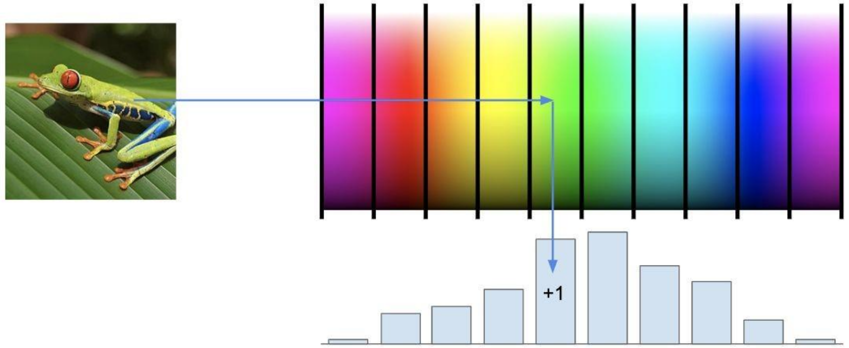


$f(x, y) = (r(x, y), \theta(x, y))$

Transform data with a cleverly chosen **feature transform** f, then apply linear classifier



Color Histogram



Histogram of Oriented Gradients (HoG)

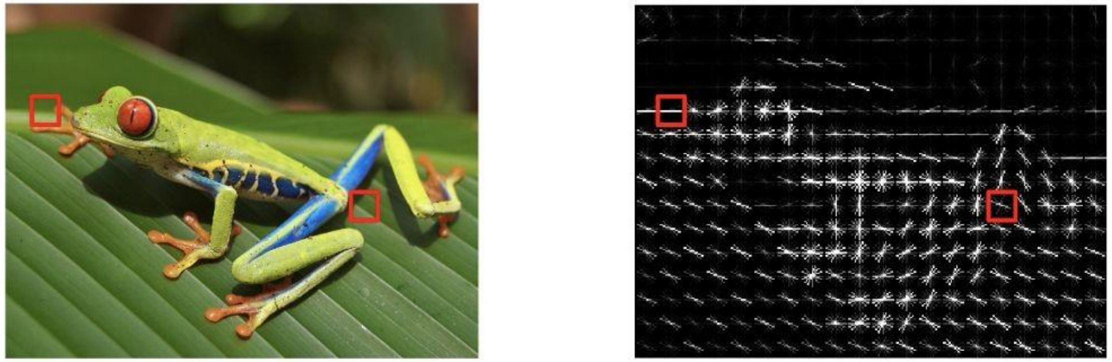


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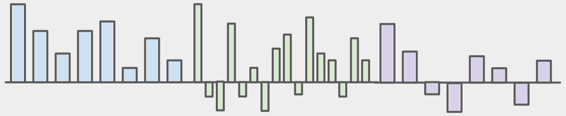
Image features vs ConvNets

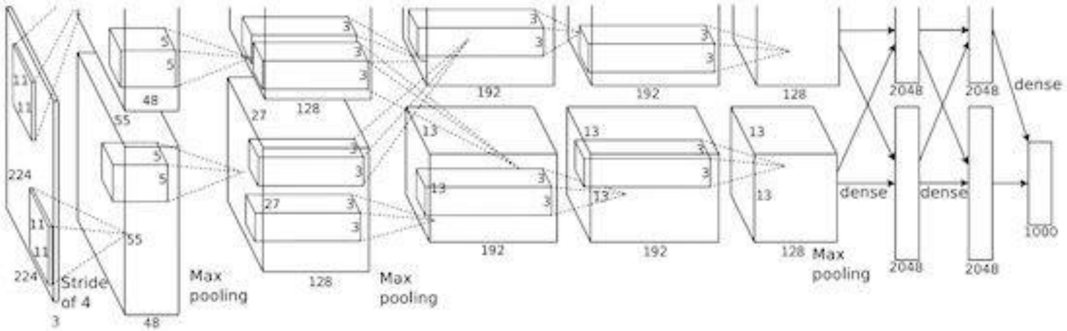
Feature Extraction



f

10 numbers giving scores for classes

training



Krizhevsky, Sutskever, and Hinton, “Imagenet classification with deep convolutional neural networks”, NIPS 2012.

10 numbers giving scores for classes

training

Thank you

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