PRL SUMMER INTERNSHIP SIP-2019

REG. NO. : 19STS0310

Optical properties of different glass types and wave aberrations

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July 11, 2019

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Abstract

Optical properties of different glass-types and their dispersion relationship with various Physical parameters like temperature and pressure have been discussed in this work. The work extends to different glass-types belonging to different catalogues which are listed in Zemax optical design software. The standard Mathematical equations for dispersion and thermal analysis has been applied using python coding environment. The results of the dispersion relation and thermal analysis have been rigorously verified with ground-truth. Another model has been created for finding the OPD(Optical Path Difference) for rays traveling through optical systems. Optical Path Difference and associated Aberrations are also discussed in this work.

1 Introduction

In this project, work is divided into three parts because it has three different objectives. The **gitlab** account has been created for uploading codes and backup of project's various models which were created on python. Git bash commands and basics of gitlab features are learned through this project. There are so many optical properties of the different glass-types. First one is dispersion of the different glass-types. Dispersion of the light depends on the dispersion relation. In general, Dispersion relation is dependency of refractive index to wavelength of the light. Therefore, different wavelength would have different refractive index which create the chromatic aberrations in the optical elements. In short, Dispersion relation is expression between refractive index of medium and wavelength. There are so many dispersion formula to find dispersion relation. ZEMAX is software which uses the different catalogues to find the aberrations and dispersion relation. These catalogues are provided by manufacturing companies for different glass types. Catalogues contain the all details of glass-types and its constants values for dispersion relations. Using that, model has been created which extracts the dispersion data for different glass-types to put into dispersion formulas. That gives the relation between refractive index and wavelength.

Now, the refractive index changes with temperature and pressure of the surrounding of glass-type. Temperature and pressure changes the medium or structure of the glass and that gives the variations in the refractive index. Again one compatible model has been created which can take various inputs from catalogues to find the these variations for different glass-types. That model gives the three relations, (1) change of rate of absolute refractive index vs temperature (2)change of rate of relative refractive index vs temperature and (3) refractive index vs temperature. These relations would have different trends at different pressure. This model also verify the calculated data with reference data.

Verification of the model is done by calculating error of the calculated data with reference data for 50 difference glass-types from different catalogues. The error has come out as around $10^{-}10\%$ for all glass-types which is very small. This verification is done for dispersion relation. The verification of thermal data has been done for 5 different glass-types. The error from the model has come out as around 0.01% which is very less. The reference data for thermal analysis were in graphical plots not in the raw data. Because of that, another software "GetData Graph Digitizer" has been used to find the raw data for those plots.

The OPD and aberrations are very concerning topics for today. OPD and aberrations create the blur images for object. Different types of first order aberrations are discusses in the following sections. Various type of aberrations are associated with a given OPD. These Aberrations can be retrieved through the knowledge of OPD. The OPD has been found through the second model which also has been created in python. OPD has been found by simple geometry of the ray called as ray tracing method. The ray tracing method gives the final height of the image at image plane. Results of the spherical and coma aberrations has been shown.

The first section explains the basics of dispersion to find the dispersion relation. The dispersion relation changes with temperature and pressure therefore, the thermal analysis theory included into section of dispersion. Next part of the report is results and verification of the data for dispersion and thermal analysis. The next part is basics of OPD and aberrations. Results of the OPD from model has been included in the next section. The codes for these model has been included in Gitlab public account at foot-notes. Last part of the section is appendix in which basics of the starting project and new learning has been discussed for this particular project.

2 Dispersion of light and thermal analysis in the glasses

Following sections explain the basics of dispersion and thermal analysis theory for different glass-types.

Git Source code of the model :
https://gitlab.com/rid181198/optical-properties-of-different-glass-types-and-aberrations.git

2.1 Objective :1 What is Dispersion relation

Refraction is bending of the light through out the medium and it depends on the material or glass type. Refractive index is calculated by ratio of speed of light in vacuum space and speed of light in medium .

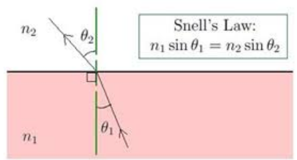

Figure 1: Snell's refraction law

In water, Refractive index is 1.33 means that light speed slows 33 % of original value. Refraction of the light is expressed through snell's law of refraction which is given in the figure 1. In the figure the angle is changing after the refraction but refractive index is assumed to be constant. For constant value of refractive index, the angle would also be constant with different wavelengths. But here it is not case because refractive index of medium depends on the wavelength of light.

Refractive index depends on the wavelength of the light. Different wavelengths have different refractive index and bend with different angles. For chromatic rays, all colors are refracted with different directions because of the phenomena. This called as **dispersion of light** in medium. First approximation for relation is made by cauchy approximation. Cauchy expression is given by following equation,

$$n(\lambda) = B + \frac{C}{\lambda^2} \tag{1}$$

In this (1), B and C are constants and depend on the material type. Constants could be found by experimentally with known refractive index of particular wavelength. Formation of rainbow and other phenomena are due to the dispersion.
Light is made up of electromagnetic oscillations, the different colours have different wavelengths and different frequencies. In vacuum all of them move with the same speed c. Whenever light enters a dielectric medium this separation happens and the phenomenon is known as dispersion. There are more **cauchy expression** for dipersion,

$$n(\lambda) = B + \frac{C}{\lambda^2} + \frac{D}{\lambda^4} \tag{2}$$

These all constants are determined by experimentally and give the proper relation between refractive index and wavelength. The first two terms would suffice to give an accurate value of n. The derivative of (2) is given by,

$$\frac{dn}{d\lambda} = -\frac{B}{\lambda^3} \tag{3}$$

This is **normal dispersion** behaviour for light. But this behaviour is valid only for certain wavelength range. Outside the range there is absorption region where refractive has not valid value . refractive index suddenly decreases very fast and does not obey the Cauchy's law, if we approach the absorption region. Further increasing the wavelength once again refractive index becomes large. Again the behaviour is quite similar to the visible region for the increase in wavelength. This type of behaviour known as **anomalous dispersion**. There are many absorption region and with same repetition of pattern. The refractive index couldn't be found in absorption region because substance absorbs that particular wavelength completely because of the resonance effects in electronic spectra and vibration spectra of material.

There are important facts about the normal dispersion is,
(1) refractive index of substance increases with decreasing of wavelength (2) The rate of increase becomes greater at shorter wavelength.

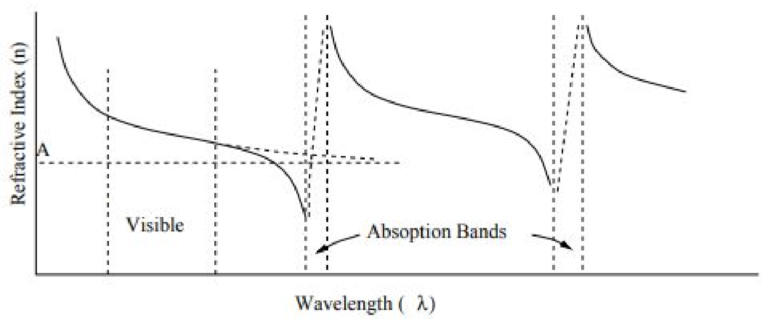

Figure 2: Anomalous dispersion

In the 2, there is band between visible and infrared region with following cachy expression. To derive the expression for anomalous dispersion, sellmeier assumed that all elastically bound particles in the medium oscillate with a natural frequency ω_0 which correspond to a wavelength λ_0 in the vacuum. The final equation looks like,

$$n^2 = 1 + \frac{A\lambda^2}{\lambda^2 - \lambda_0^2} \tag{4}$$

In (4), when $\lambda = \lambda_0$, the trend would diverge and give the same result as in figure 2. To explain many absorption bands, we have to assume different species of electrons with different natural frequencies ω_j corresponding to wavelengths λ_j in the substance and then

$$n^2 = 1 + \Sigma_j \frac{A_j\lambda^2}{\lambda^2 - \lambda_j^2} \tag{5}$$

2.2 Elementary theory of dispersion : sellmeier equation

Effects of electromagnetic field on dielectric medium will be given by electromagnetic theory,

$$D = \varepsilon E \tag{6}$$

. When there is external electric field in the dielectric medium the total displacement is,

$$D = \varepsilon_0 E + P = \epsilon_0(1+\chi)E \tag{7}$$

. The factor χ is called the electric susceptibility of the substance and the factor $(1+\chi) = \epsilon_r$ is nothing but the relative permittivity. And vector P is polarization vector.

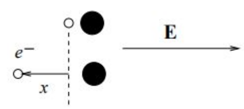

Figure 3: Oscillation of electron from mean position

When an electric field is applied to a substance, electrons of the molecules of that substance are displaced from there mean position as shown in the fig 3. Electron would not leave molecule but oscillate with that frequency and also generate dipole moment inside medium. Polarization per unit volume is given by,

$$P = -Nex \tag{8}$$

In equation (8), N is total number of electrons and x is displacement of electron. Also electric field is in x- direction and have negative sign because of electron negative charge. Here we are ignoring oscillation of positive ions and electron will oscillate with certain frequency ω_0. But light has oscillate electric field then displacement would also oscillate with that time period. Here dampening force comes in equation because of the shielding effects, other atoms effects on electron which try to bound more in atom. For forced oscillation differential equation is given by ,

$$\ddot{x} + r\dot{x} + \omega_0^2 x = -\frac{eE}{m} \tag{9}$$

similarly we can write equation (9) as,

$$\ddot{P} + r\dot{P} + \omega_0^2 P = -\frac{Ne^2E}{m} \tag{10}$$

From maxwell's equations, $\nabla \times E = -\dot{B}$ and $\nabla \times H = \dot{D}$, $\nabla \cdot E = 0$

$$\nabla \times \dot{B} = \mu_0 \ddot{D}$$
$$\nabla \times (\nabla \times E) = -\mu_0\epsilon_0\ddot{E} - \mu_0\ddot{P}$$
$$\nabla^2 \cdot E = \mu_0\epsilon_0\ddot{E} + \mu_0\ddot{P}$$

From all these equations (10) we can get,

$$\ddot{P} = \frac{1}{\mu_0}(\nabla^2 \cdot E - \frac{1}{c^2}\ddot{E}) \tag{11}$$

Differentiating equation (10) w.r.t time,

$$(\frac{\partial^2}{\partial t^2} + r\frac{\partial}{\partial t} + \omega_0^2)\ddot{P} = \frac{Ne^2}{m}\ddot{E} \tag{12}$$

Using equation (11) eliminating P in equation (12),

$$(\frac{\partial^2}{\partial t^2} + r\frac{\partial}{\partial t} + \omega_0^2)(\nabla^2 \cdot E - \frac{1}{c^2}\ddot{E}) = \mu_0\frac{Ne^2}{m}\ddot{E} \tag{13}$$

when a plane electromagnetic wave polarised along the x direction and E will become $E_0 = e^{i(kz-\omega t)}$ and put in equation (13),

$$(\omega^2 - ir\omega + \omega_0^2)(-k^2 + \frac{\omega}{c^2}) = -\frac{\mu_0 Ne^2\omega^2}{m} \tag{14}$$

Further arrangements in equation (14),

$$k^2 = \frac{\omega^2}{c^2}(1 + \frac{\mu_0 c^2 Ne^2}{m(\omega_0^2 - ir\omega - \omega^2)}) \tag{15}$$

In the equation (15) $v = \frac{\omega}{k}$ is phase velocity and $n = \frac{c}{v}$ is refractive index of the medium. So equation will become,

$$n^2 = (1 + \frac{\mu_0 c^2 Ne^2}{m(\omega_0^2 - ir\omega - \omega^2)}) \tag{16}$$

For refractive index being as real,$\omega_0^2 - \omega^2 >> r\omega$ which means that dampening is very slight,

$$n^2 = (1 + \frac{\mu_0 c^2 Ne^2}{m(\omega_0^2 - \omega^2)}) \tag{17}$$

relation between wavelength and angular frequency is $\lambda = \frac{2\pi c}{\omega}$ then

$$n^2 = (1 + \frac{\mu_0 Ne^2\lambda_0^2\lambda^2}{4\pi^2 m(\lambda^2 - \lambda_0^2)}) \tag{18}$$

and finally by assuming constants equation will look like sellmeier formula for dispersion.

2.3 Abbe number : measurement of dispersion

Abbe number is measurement of the strength of dispersion by transparent material. Low abbe number means that dispersion is high vice versa. Abbe number of the material is defined by equation,

$$V_D = \frac{n_D - 1}{n_F - n_C} \tag{19}$$

In equation (19) $n_D, n_F, andn_c$ are refractive index of material at wavelengths of Fraunhofer D,F and C lines(589.3 nm, 486.1 nm, 656.3 nm respectively). Abbe number are useful number for classifying the different glasses. For example, **flint glass** has abbe number V < 50 and **crown glass** has V > 50. Typical values of V range from around 20 for very dense flint glass, around 30 for **polycarbonate plastics**, and up to 65 for very light crown glass, and up to 85 for **fluor-crown glass**.
Abbe numbers are only a useful measure of dispersion for visible light, and for other wavelengths, or for higher precision work, the group velocity dispersion is used. An Abbe diagram is produced by plotting the Abbe number of a material versus its refractive index. Glasses can then be categorised by their composition and position on the diagram.

Next section explains the thermal analysis of the different glass types.

2.4 Objective :2Thermal analysis and related equations

The refractive index of optical glasses change with temperature, the extend of changing RI is depended on the glass type and on the wavelength. It also depends on pressure of surroundings so that would give the two part of refractive index **(1)Absolute refractive index** and **(2) Relative refractive index**. We can find the rate of change of refractive index from temperature range -100 C^o to 140 C^o. We need thermal coefficients for calculation of changes of refractive index with temperature. These coefficients are given in aforementioned catalogs. In the section 6.5, TD line gives the details of thermal coefficients. there are special glasses with negative temperature coefficients in an optical system can help to keep wave front deformations caused by temperature changes on a minimum level. Such glasses are called **athermal glasses**.
From fundamental equation of refractive index relative to vacuum is given by generally,

$$n_{abs}^2(\lambda) = 1 + a \cdot \frac{N}{V} \cdot f \cdot \frac{\lambda^2 \cdot \lambda_0^2}{\lambda^2 - \lambda_0^2} \tag{20}$$

In the equation (20), $n_{abs}^2(\lambda)$ is refractive index in vacuum at certain temperature. Rest of are constants depending on glass type. Differentiating this equation with respect to temperature will give simplify form of the derivative equation is,

$$\frac{dn_{abs}(\lambda, T)}{dT} = \frac{n^2(\lambda, T_0) - 1}{2 \cdot n(\lambda, T_0)} \cdot (D_0 + 2 \cdot D_1 \cdot \Delta T + 3 \cdot D_2 \cdot \Delta T^2 + \frac{E_0 + 2 \cdot E_1 \cdot \Delta T}{\lambda^2 - \lambda_{TK}^2}) \tag{21}$$

In the equation (21), T_0 is reference temperature which is also given in the catalogs. T is variable of temperature in celcius. ΔT is difference between variable and reference temperature. And $D_0, D_1, D_2, E_0, E_1, \lambda_{Tk}$ are constants depend on glass type. The refractive index values given in catalogs are in 101330 Pa pressure and they are called as relative refractive index. So $n(\lambda, T_0)$

is refractive index in catalog at reference temperature T_0 and pressure is 101330 Pa.The change is refractive index with temperature is given by,

$$\Delta n_{abs}(\lambda, T) = \frac{n^2(\lambda, T_0) - 1}{2 \cdot n(\lambda, T_0)} \cdot (D_0 \cdot \Delta T + \cdot D_1 \cdot \Delta T^2 + \cdot D_2 \cdot \Delta T^3 + \frac{E_0 \dot{\Delta} T + \cdot E_1 \cdot \Delta T^2}{\lambda^2 - \lambda_{TK}^2}) \tag{22}$$

$$n_{abs}(\lambda, T) = n_{abs}(\lambda, T_0) + \Delta n_{abs}(\lambda, T) \tag{23}$$

$$n_{rel}(\lambda, T) = \frac{n_{abs}(\lambda, T)}{n_{air}(\lambda, T, p)} \tag{24}$$

From all equations, (22),(23), and (24) we can get relation between the refractive index and temperature with specific pressure. We can get equation for derivative of relative refractive index with temperature,

$$\frac{dn_{rel}(\lambda, T)}{dT} = \frac{\frac{dn_{abs}(\lambda, T)}{dT} - n_{rel}(\lambda, T) \cdot \frac{dn_{air}(\lambda, T, p)}{dT}}{n_{air}(\lambda, T, p)} \tag{25}$$

In equation (25), we can find $fracdn_{rel}(\lambda, T)dT$ by putting all appropriate values that are given. Where $n_{air}(\lambda, T, p)$ given by,

$$n_{air}(\lambda, T, p) = 1 + \frac{n_{air}(\lambda, 15, p_0) - 1}{1 + 3.4785 \cdot 10^{-3} \cdot (T - 15)} \cdot \frac{p}{p_0} \tag{26}$$

Again In equation (26), $n_{air}(\lambda, 15, p_0)$ is given by,

$$n_{air}(\lambda, 15, p_0) = 1 + 10^{-8} \cdot (6432.8 + \frac{2949810 \cdot \lambda^2}{146 \cdot \lambda^2 - 1} + \frac{25540 \cdot \lambda^2}{(41 \cdot \lambda^2 - 1)}) \tag{27}$$

Here p_0 is 101330 Pa and p is variable. For equation (25), we need

$$\frac{dn_{air}(\lambda, T, p)}{dT} = -0.00367 \cdot \frac{n_{air}(\lambda, T, p) - 1}{1 + 0.00367 \cdot T} \tag{28}$$

With the help of the all coefficients and these equations we can plot between n vs T, $\frac{dn_{rel}(\lambda, T)}{dT}$ vs T and $\frac{dn_{abs}(\lambda, T)}{dT}$ vs T and see how refractive index is changed with temperature and pressure effects.
From the equation (24) we can write n_{rel} as $n_{abs}(\lambda, T) = n_{rel}(\lambda, T) \cdot n_{air}(\lambda, T, P)$, So we can write,

$$n_{rel}(\lambda, T) \cdot n_{air}(\lambda, T, P) = n_{rel}(\lambda, T_0) \cdot n_{air}(\lambda, T_0, P_0) + \Delta n_{abs}(\lambda, T) \tag{29}$$

In equation (29), P_0 is one atmosphere pressure which is given in catalog. Further simplification of equation gives,

$$n_{rel}(\lambda, T) = (n_{rel}(\lambda, T_0) + \frac{\Delta n_{abs}(\lambda, T)}{n_{air}(\lambda, T_0, P_0)}) \cdot \frac{n_{air}(\lambda, T_0, P_0)}{n_{air}(\lambda, T, P)} \tag{30}$$

From equation (30), we can get refractive index of glass type at any temperature and pressure with different wavelengths.

Next section includes the various empirical formulas for dispersion formulas with its constants values and thermal coefficients' values for thermal analysis of the different glass-types.

2.5 ZEMAX - glass catalogues : Dispersion and Thermal data

ZEMAX is software which uses the different catalogues provided by manufacturer of the glass-types and gives the details of abbe number, dispersion and temperature effects on glasses.

2.5.1 Uses of catalogues

In dispersion theory, different wavelengths have different refractive index. There are experimentally verified constants values for all dispersion formula and give the relation between refractive index and wavelengths. There are some catalogues made by manufacturer of glass who provides the details of the glass. From that we can extract the data of glass and its behaviour. For example, schott is manufacturing company for various glass-types who provides the empirical formula for different glass-types and their dispersion and thermal constants for various analysis. Here are some examples of catalogues.

2.5.2 Example of catalogues

These catalogues provide the details of each glass type with its properties like Abbe number, Thermal coefficients, Transmission and Thermal expansion etc. There are more catalogues for different glasses depend on the manufacturing companies of that glass type. For example, **schott** is catalog name and each glasses in that catalogue made by schott company. This catalog contains every details for analysis of glass type. There are many catalogues, **ohara, hoya, birefringent, hikari, sumita, misc, infrared** etc. Each catalogues contain the details of specific glass type. Application of these different glass types require for different fields like in telescope imaging process, glasses in spectacle, windows etc. For that different catalogues are required.

2.5.3 Extension of catalogues

Extension of the catalogue is **AGF** which is basically ANSII file. It could open with notepad with normal texts. ZEMAX software uses this catalogues as AGF and converts into BGF (Binary) file for more efficiency of processing. Here We are using AGF file as texts and extract data from it through **python** platform. We can edit the catalogues as per latest data.

2.5.4 Availability

Catalogues are easily available on online platform. Each manufacturer provide the catalogue of their production glass types. We can download the catalogues by typing **catalogs-name.agf**. Here is link for ohara catalog,

https://www.oharacorp.com/catalog.html

2.5.5 How to read catalogues

In the catalogue, first line has format like this,

$$CC < catalog - name >$$

This tells about the name of catalog. Second line of the AGF file would be,

$$NM < glass - name >< dispersion - formula - number >< MIL >< N_d >< V_d >< Exclude - sub >< Status >< Melt - frequency >$$

In this line MIL represents the reference number for each glass type.N_d is refractive index of glass at wavelength 0.587 micrometers and it is just reference number. V_d is also a reference number and it is abbe number of glass. **Exclude Sub** represents that if it is checked then that glass type is not selected for conversion from model to real glass in application. Exclude sub has only two values, 1 for checked and 0 for unchecked. Status indicates the availability of that glass type in market. There four types of status, **Standard, Preferred, Obsolete, Special**. Standard glass generally available in purchase. Preferred glasses are frequently melted and more likely available on demand. Obsolete glasses are no longer available but may be available on demand. Melt frequency indicates that how frequently glass is melted during manufacturing of that glass. 1 represents the high frequency, 2 represents very often and 5 represents infrequently. Glass name part give the glass type for example N-BK7, YGH52 etc. Dispersion formula number is number given for each dispersion formulas. There are total 13 formulas and each formulas has its corresponding number. The dispersion formula number is 1 for Schott, 2 for Sellmeier 1, 3 for Herzberger, 4 for Sellmeier 2, 5 for Conrady, 6 for Sellmeier 3, 7 for Handbook of Optics 1, 8 for Handbook of Optics 2, 9 for Sellmeier 4, 10 for Extended, 11 for Sellmeier 5, and 12 for Extended 2 and 13 for Extended 3. Third line of the file is

$$GC < individual - glass - comment >$$

This line gives the specifications or any extra comments of that glass type. Fourth line is

$$ED < TCE(-30to70) >< TCE(100to300) >< density >< dPgf >< ignore - thermal - expansion >$$

TCE is thermal coefficient expansion and it is used for modelling of linear expansion of glass with temperature. For different temperature ranges, TCE have different values and therefore there are two columns of TCE one for -30 to 70 and second for 100 to 300. Density is density of glass type. dPgf gives the deviation of relative partial dispersion from normal D line. Ignore thermal expansion is useful for non solids materials. For example, gases and liquids have not thermal expansion but its mounted material would solid. So thermal modelling is different for solids and gases. For gases and liquids, edge effects of mounted material is consider. Fifth line is

$$CD < dispersion - coefficients - 1 - 10 >$$

This gives the all required coefficients of given dispersion formula. Sixth line is

$$TD < D0 >< D1 >< D2 >< E0 >< E1 >< Ltk >< Temp >$$

TD is useful for thermal analysis of glass for example how refractive index of particular wavelength is changed with temperature. Seventh line is

$$OD < relcost >< CR >< FR >< SR >< AR >< PR >$$

Relative cost is relatively to N-BK7 glass type. **CR** stands for climate resistance, **FR** is stain resistance, **SR** is acid resistance, **AR** is alkali resistance, **PR** is phosphate resistance. Eigth line is

$$LD < minlamda >< maxlamda >$$

This gives the validity range of wavelengths which could give proper answer to dispersion formula. Last line is

$$IT < lamda >< transmission >< thickness >$$

This represents the transmissivity of particular wavelength with given thickness of glass type. In catalogue, if there is -1 in data entry that means that data is not available.

2.5.6 Dispersion formulas in glass catalogues

There are total 13 dispersion formulas which are used by different different catalogues for each glass types. With different dispersion formula, refractive index of each wavelength in the light will disperse differently and rise the aberrations. Here is first formula,

(1)schott

$$n^2 = a_0 + a_1\lambda^2 + a_2\lambda^{-2} + a_3\lambda^{-4} + a_4\lambda^{-6} + a_5\lambda^{-8}$$

(2)sellmeier1

$$n^2 - 1 = \frac{K_1\lambda^2}{\lambda^2 - L1} + \frac{K_2\lambda^2}{\lambda^2 - L2} + \frac{K_3\lambda^2}{\lambda^2 - L3}$$

(3)sellmeier2

$$n^2 - 1 = A + \frac{B_1\lambda^2}{\lambda^2 - \lambda_1^2} + \frac{B_2}{\lambda^2 - \lambda_2^2}$$

(4)sellmeier3

$$n^2 - 1 = \frac{K_1\lambda^2}{\lambda^2 - L1} + \frac{K_2\lambda^2}{\lambda^2 - L2} + \frac{K_3\lambda^2}{\lambda^2 - L3} + \frac{K_4\lambda^2}{\lambda^2 - L4}$$

(5)sellmeier4

$$n^2 - 1 = A + \frac{B\lambda^2}{\lambda^2 - C} + \frac{D\lambda^2}{\lambda^2 - E}$$

(6)sellmeier5

$$n^2 - 1 = \frac{K_1\lambda^2}{\lambda^2 - L1} + \frac{K_2\lambda^2}{\lambda^2 - L2} + \frac{K_3\lambda^2}{\lambda^2 - L3} + \frac{K_4\lambda^2}{\lambda^2 - L4} + \frac{K_5\lambda^2}{\lambda^2 - L5}$$

(7)herzberger

$$n = A + BL + CL^2 + D\lambda^2 + E\lambda^4 + F\lambda^6$$

$$L = \frac{1}{\lambda^2 - 0.0028}$$

(8)conrady

$$n = n_0 + \frac{A}{\lambda} + \frac{B}{\lambda^{3.5}}$$

(9)Handbook of optics 1

$$n^2 = A + \frac{B}{\lambda^2 - C} - D\lambda^2$$

(10)Handbook of optics 2

$$n^2 = A + \frac{B\lambda^2}{\lambda^2 - C} - D\lambda^2$$

(11)Extended 1

$$n^2 = a_0 + a_1\lambda^2 + a_2\lambda^{-2} + a_3\lambda^{-4} + a_4\lambda^{-6} + a_5\lambda^{-8} + a_6\lambda^{-10} + a_7\lambda^{-12}$$

(12)Extended 2

$$n^2 = a_0 + a_1\lambda^2 + a_2\lambda^{-2} + a_3\lambda^{-4} + a_4\lambda^{-6} + a_5\lambda^{-8} + a_6\lambda^4 + a_7\lambda^6$$

(13)Extended 3

$$n^2 = a_0 + a_1\lambda^2 + a_2\lambda^4 + a_3\lambda^{-2} + a_4\lambda^{-4} + a_5\lambda^{-6} + a_6\lambda^{-8} + a_7\lambda^{-10} + a_8\lambda^{-12}$$

These are the total 13 formulas for dispersion relation of the glass type. All the corresponding constants are given in the respective catalogs. From these formulas we can find the relation between the refractive index and different wavelengths.

For thermal analysis, The constants are given in the catalogues at ”TD” line which represents the all constants values. For dispersion relation, ”CD” gives the dispersion coefficients. Using this catalogues, we can extract the data from catalogues and find the dispersion relation and thermal analysis. For that the model has been created to find these all data with verification. Next section

2.6 Data extraction through python

Now we have all catalogues with details of every glass type. Once we get the coefficients of glass type for dispersion formula, we can get refractive index with different wavelengths. Plot between refractive index and wavelength gives the proper idea of its trend. Similarly, Thermal analysis is done with python commands with loops and logic process. Once we get all thermal coefficients and reference temperature, we can obtain the change of rate of derivative of absolute refractive index with temperature, the change of rate of derivative of relative refractive index with temperature and trend between refractive index and temperature. These all plots are drawn through python. The code is given in the footnotes. Next section includes the results from model with verification.

3 Results : Dispersion and thermal analysis with verification

There are total four glass types with its information get from different catalogs. Model can take any glass-types from any catalogues but in this report I included only four glass types with its analysis.

3.1 Results from model

Four types of glasses :
(1) N-BK7 from SCHOTT catalog
(2) S-LAH99 from OHARA catalog
(3) K-SFLD14 from SUMITA catalog
(4) J-PSKH1 from HIKARI catalog

3.1.1 N-BK7 from SCHOTT catalog

First of all, we will see how data from catalogs looks like, Here is data from schott catalog for N-BK7,

NM N-BK7 2 517642.251 1.5168 64.17 0 1
GC step 0.5 available
ED 7.100000 8.300000 2.510000 -0.000900 0
CD 1.039612120E+00 6.000698670E-03 2.317923440E-01 2.001791440E-02 1.010469450E+00 1.035606530E+02 0.000000000E+00 0.000000000E+00
TD 1.860000E-06 1.310000E-08 -1.370000E-11 4.340000E-07 6.270000E-10 1.700000E-01 2.000000E+01
OD 1.0000 1.0000 0.0000 1.0000 2.3000 2.3000
LD 3.00000E-01 2.50000E+00
IT 3.00000E-01 5.00000E-02 2.50000E+01
IT 3.10000E-01 2.50000E-01 2.50000E+01
....

To understand what is TD, OD, LD, NM etc, you can refer section 3.5.5. From these data of glass, we can get information about glass through python/model. Here is plot between refractive index vs wavelength by using sellmeier 1 formula.

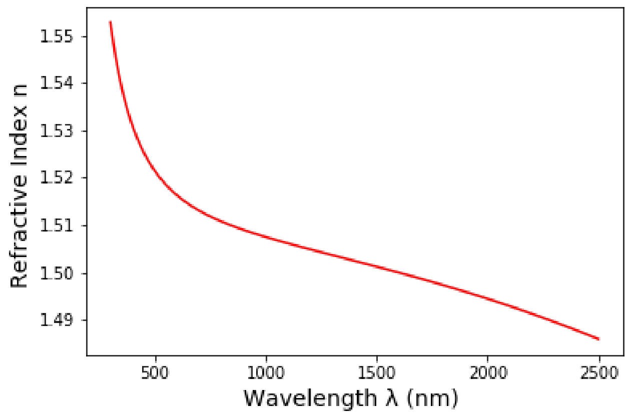

Figure 4: Refractive index vs wavelength for N-BK7 glass

In the figure 4, the curve has negative slope and refractive index is decreasing with increasing of wavelength. Also formula is valid only for certain range of wavelengths. Here range is 300 nm to 2500 nm. Refractive index of N-BK7 at 1060 nm wavelength is 1.5066875568966998 and model can get every value by different inputs. From the equation of abbe number in section 3.3, the value for N-BK7 is 64.13319920721142.

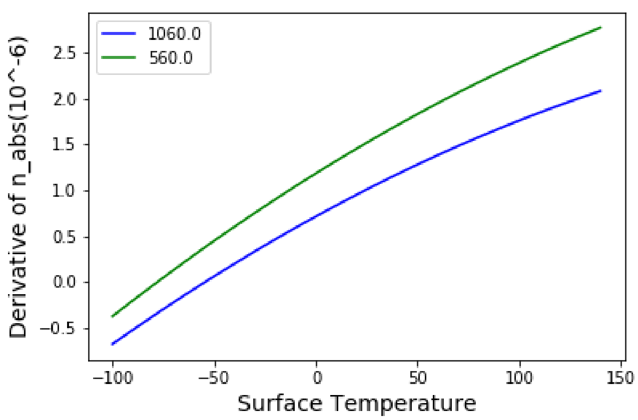

Figure 5: change of rate of derivative of absolute refractive index with temperature at 560 and 1060 nm

For **thermal analysis**, first model find the change of rate of derivative of absolute refractive index with temperature. In the figure 5, there are two curves, one for 560 nm and another for 1060 nm. The rate for 560 nm is greater than

1060 nm wavelength. Here the temperature range is -100 to 140 celcius.

Now refractive index at any given temperature and pressure is calculated from equation (30). In catalog, N-BK7 has reference temperature 20 celcius and model found the refractive index at temperature 40 celcius at pressure 103250 Pa. It gave the answer at wavelength 1060 nm around 1.5067275810747285. And at reference temperature with one atmospheric pressure is 1.5066875568966998. But if we keep temperature constant and decrease the pressure at 90000 Pa of surrounding then value is 1.5067772329259013. As we can see from data for refractive index at constant temperature but different pressure gives the different values. Reason for this trend is that pressure applies on lens surface inward while temperature of glass applies the force outward because of thermal expansion. So if we keep the temperature constant and decrease the pressure then temperature effect will dominate and refractive would be greater than the high pressure condition.

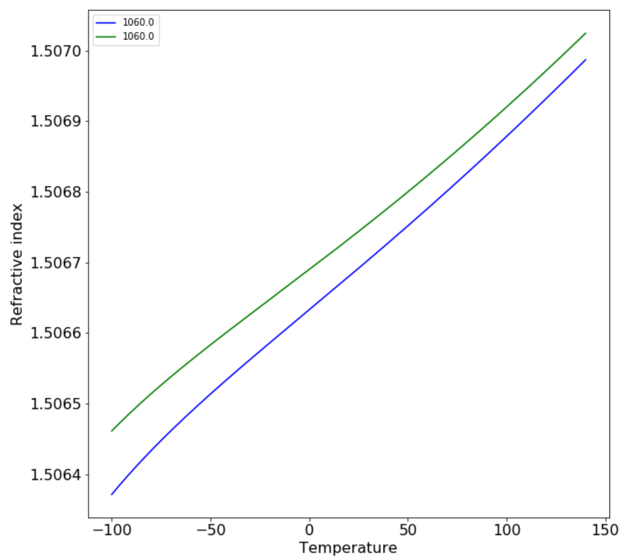

Figure 6: Green color for pressure at P=90000 Pa and Blue color for pressure P = 103250 Pa

Now the plot between refractive index for particular wavelength with temperature is in figure 6. For N-BK7, the refractive is increasing with increasing of temperature.

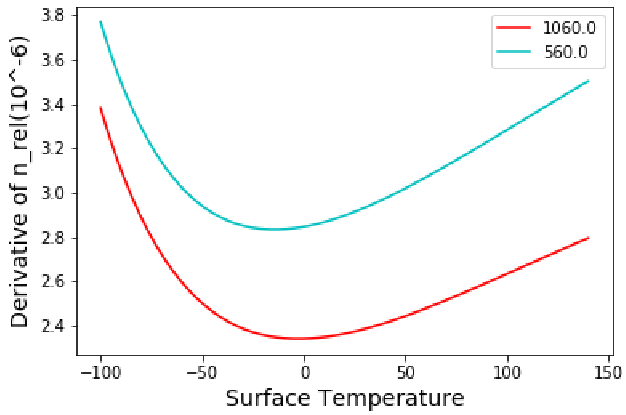

Figure 7: rate is changed drastically at temperature around 0 celcius

Now fourth plot is between change of rate of derivative of relative refractive index with temperature at pressure 103250 Pa. This gives the rate at which relative refractive index changes with temperature. In the figure 7, the rate is decreasing till 0 celcius and increasing after that. So these are all analysis of N-BK7 from given catalog.

3.1.2 S-LAH99 from OHARA catalog

Data from catalog for S-LAH99 is,

NM S-LAH99 2 1 2.001000 29.139473 0 0 -1
GC
ED 7.500000000E+000 0.000000000E+000 5.020000000E+000
5.400000000E-003 0 0
CD 2.391406620E+000 1.314675000E-002 4.392192280E-001 5.532260420E-002
2.383584670E+000 1.612599000E+002 0.000000000E+000 0.000000000E+000
0.000000000E+000 0.000000000E+000
TD -5.060000000E-007 9.590000000E-009 -2.100000000E-011
1.090000000E-006 1.320000000E-009 2.670000000E-001 2.500000000E+001
OD 2.10000 1.00000 -1.00000 2.00000 -1.00000 1.00000
LD 4.04656000E-001 2.32542000E+000
IT 2.80000E-001 0.00000E+000 1.00000E+001
IT 2.90000E-001 0.00000E+000 1.00000E+001
....

From this data of glass, we can get information about glass through python/-model. Here is plot between refractive index vs wavelength by using sellmeier 1 formula.

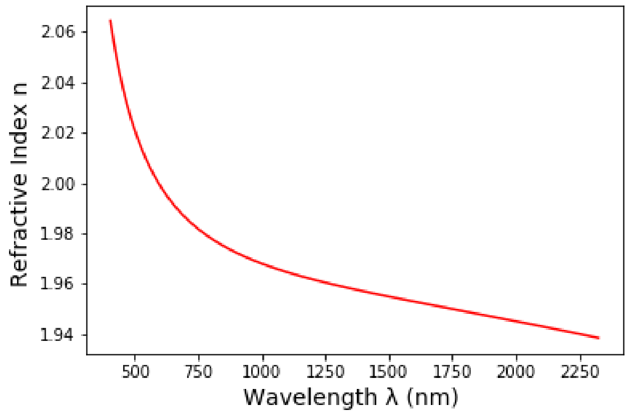

Figure 8: Refractive index vs wavelength for S-LAH99 with sellmeier 1 formula

In the figure 4, the curve has negative slope and refractive index is decreasing with increasing of wavelength. Also formula is valid only for certain range of wavelengths. The range is 404 nm to 2325 nm. Refractive index of S-LAH99 at 1060 nm wavelength is 1.9659499406949175 and model can get every value by different inputs. From the equation of abbe number in section 5.1, the value for S-LAH99 is 29.11920404841508.

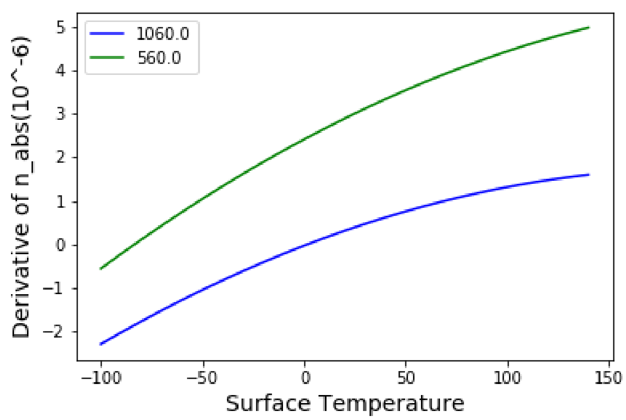

Figure 9: change of rate of derivative of absolute refractive index with temperature at 560 and 1060 nm

For **thermal analysis**, the change of rate of derivative of absolute refractive index with temperature is given in the figure 9. There are two curves, one for 560 nm and another for 1060 nm. The rate for 560 nm is greater than 1060 nm wavelength. Here the temperature range is -100 to 140 celcius. This is same as for N-BK7.

Now refractive index at any given temperature and pressure is calculated from equation (30). In catalog, S-LAH99 has reference temperature 25 celcius and model found the refractive index at temperature 40 celcius at pressure 103250 Pa. It gave the answer at wavelength 1060 nm around 1.965973023911095. And at reference temperature with one atmospheric pressure is 1.9659499406949175. But if we keep temperature constant and decrease the pressure at 90000 Pa of surrounding then value is 1.9660378094776565. As we can see from data for refractive index at constant temperature but different pressure gives the different values. Reason for this trend is same as for N-BK7.

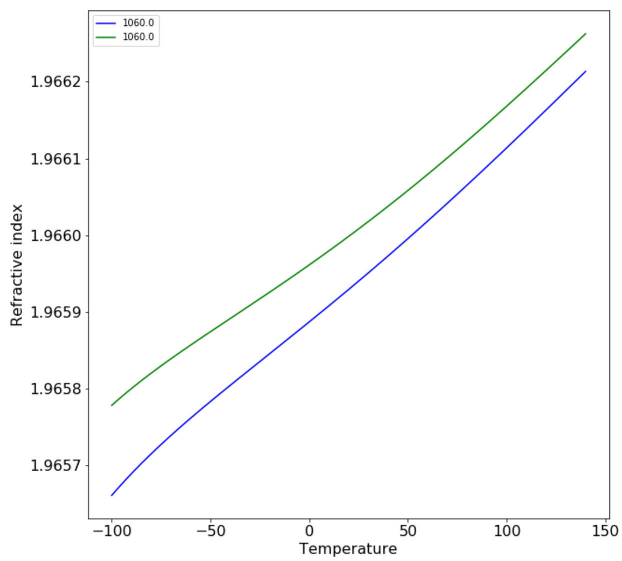

Figure 10: Green color for pressure at P=90000 Pa and Blue color for pressure P = 103250 Pa

Now the plot between refractive index for particular wavelength with temperature is in figure 10. For S-LAH99, the refractive is increasing with increasing of temperature.

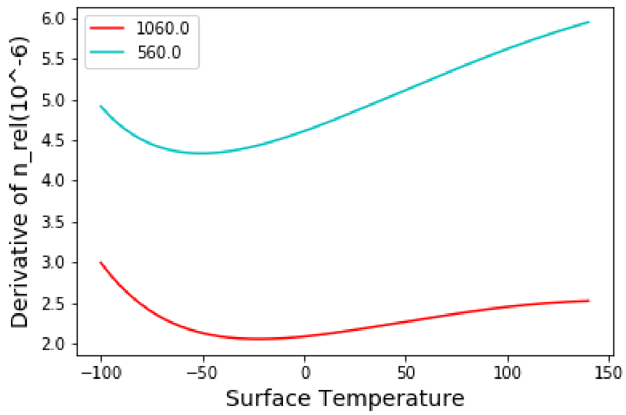

Figure 11: rate is changed drastically at temperature around -50 celcius

Now fourth plot is between change of rate of derivative of relative refractive index with temperature at pressure 103250 Pa. This gives the rate at which relative refractive index changes with temperature. In the figure 11, the rate is decreasing till -50 celcius and increasing after that. So these are all analysis of S-LAH99 from given catalog.

3.1.3 K-SFLD14 from SUMITA catalog

Data from catalog for K-SFLD14 is,

NM K-SFLD14 1 762265 1.76182 26.5 0
ED 8.5 10.2 3.15 0.0144 0
CD 2.9793916E+00 -1.1878549E-02 3.9643280E-02 1.4928665E-03
1.9358213E-05 1.2712921E-05 0.0000000E+00 0.0000000E+00
TD -4.13E-06 1.55E-08 -2.99E-10 9.65E-07 1.53E-09 0.290 2.00E+01
OD -1 1 -1 1 -1 -1
LD 0.4 1.55
IT 0.27 0.000 10
IT 0.28 0.000 10
....

From this data of glass, we can get information about glass through python/-model. Here is plot between refractive index vs wavelength by using schott formula.

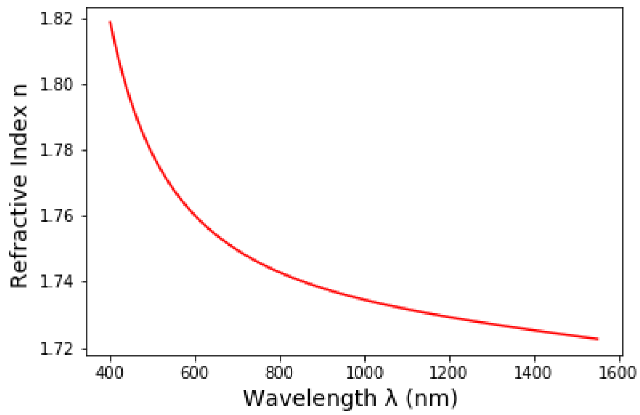

Figure 12: Refractive index vs wavelength for K-SFLD14 with schott formula

In the figure 12, the curve has negative slope and refractive index is decreasing with increasing of wavelength. Also formula is valid only for certain range of wavelengths. The range is 400 nm to 1550 nm. Refractive index of K-SFLD14 at 1060 nm wavelength is 1.7327813923080453 and model can get every value by different inputs. From the equation of abbe number in section 5.1, the value for K-SFLD14 is 26.48690066758293.

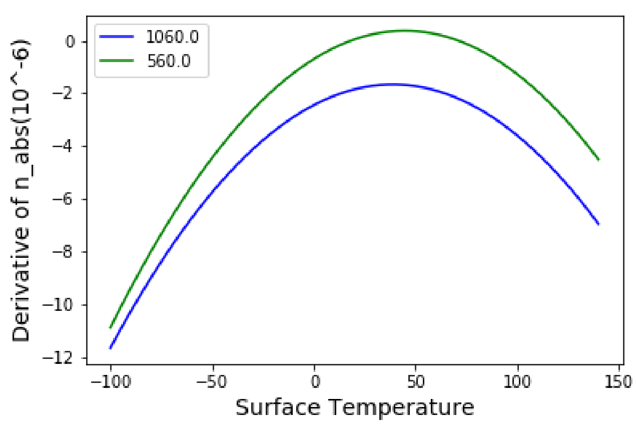

Figure 13: change of rate of derivative of absolute refractive index with temperature at 560 and 1060 nm

For **thermal analysis**, the change of rate of derivative of absolute refractive index with temperature is given in the figure 13. There are two curves, one for 560 nm and another for 1060 nm. The rate for 560 nm is greater than 1060 nm wavelength. Here the temperature range is -100 to 140 celcius. This is same as for N-BK7 and K-LAH99. Here there is slope change from positive to negative in this glass type.

Now refractive index at any given temperature and pressure is calculated from equation (30). In catalog, K-SFLD14 has reference temperature 20 celcius and model found the refractive index at temperature 40 celcius at pressure 103250 Pa. It gave the answer at wavelength 1060 nm around 1.7327685035899263. And at reference temperature with one atmospheric pressure is 1.7327813923080453. But if we keep temperature constant and decrease the pressure at 90000 Pa of surrounding then value is 1.7328256042662116. As we can see from data for refractive index at constant temperature but different pressure gives the different values. Reason for this trend is same as for N-BK7 and K-LAH99.

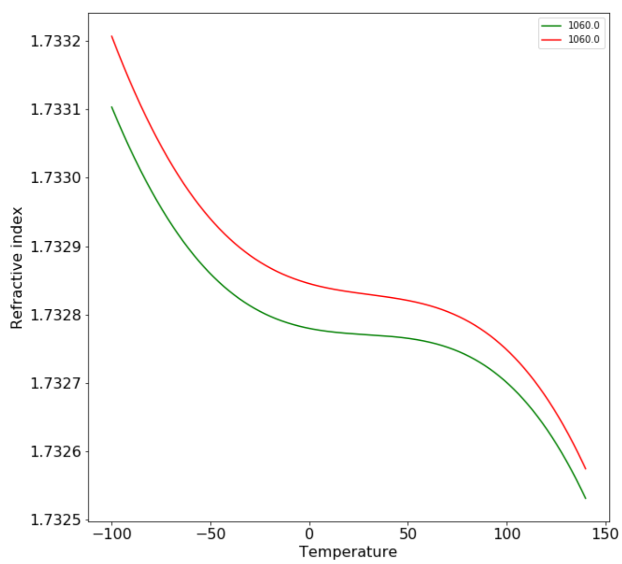

Figure 14: light blue color for pressure at P=90000 Pa and red color for pressure P = 103250 Pa

Now the plot between refractive index for particular wavelength with temperature is in figure 14. For K-SFLD14, the refractive is decreasing with increasing of temperature.

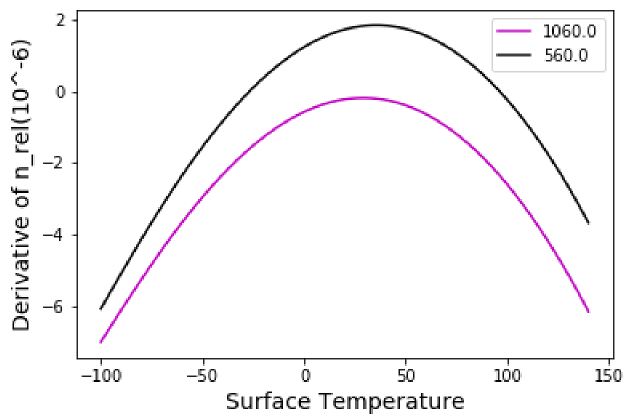

Figure 15: rate is changed drastically at temperature around 30 to 40 celcius

Now fourth plot is between change of rate of derivative of relative refractive index with temperature at pressure 103250 Pa. This gives the rate at which relative refractive index changes with temperature. In the figure 15, the rate is decreasing till 30 to 40 celcius and increasing after that. Here the rate is in negative and positive portion. That means that at some point refractive index does not change very much at that temperature. So these are all analysis of K-SFLD14 from given catalog.

3.1.4 J-PSKH1 from HIKARI catalog

Data from catalog for J-PSKH1 is,

NM J-PSKH1 13 1 1.593190 67.900555 0 0 1
GC
ED 1.140000000E+001 0.000000000E+000 4.100000000E+000
1.350000000E-002 0 0
CD 2.502080830E+000 -6.721439070E-003 -5.343137510E-005
1.282644000E-002 1.562053880E-004 1.215935490E-006 9.595508690E-008
0.000000000E+000 0.000000000E+000 0.000000000E+000
TD 0.000000000E+000 0.000000000E+000 0.000000000E+000
0.000000000E+000 0.000000000E+000 0.000000000E+000 2.300000000E+001
OD -1.00000 -1.00000 -1.00000 -1.00000 -1.00000 3.00000
LD 3.88865000E-001 2.05809000E+000
IT 2.80000E-001 0.00000E+000 1.00000E+001
IT 2.90000E-001 0.00000E+000 1.00000E+001
....

From this data of glass, we can get information about glass through python. Here is plot between refractive index vs wavelength by using extended 3 formula.

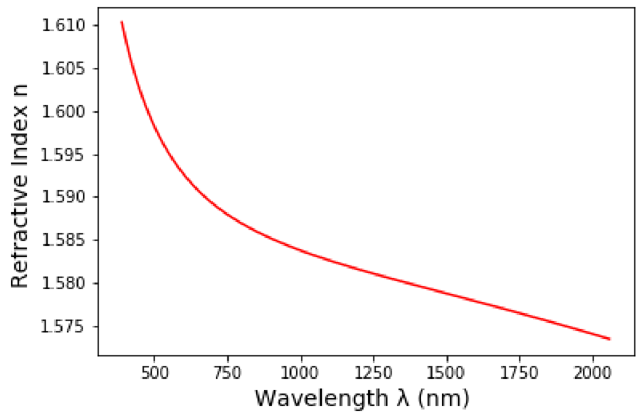

Figure 16: Refractive index vs wavelength for J-PSKH1 with extended 3 formula

In the figure 16, the curve has negative slope and refractive index is decreasing with increasing of wavelength. Also formula is valid only for certain range of wavelengths. The range is 388 nm to 2058 nm. Refractive index of J-PSKH1 at 1060 nm wavelength is 1.5830354695353313 and model can get every value by different inputs. From the equation of abbe number in section 5.1, the value for J-PSKH1 is 67.86486866978349.

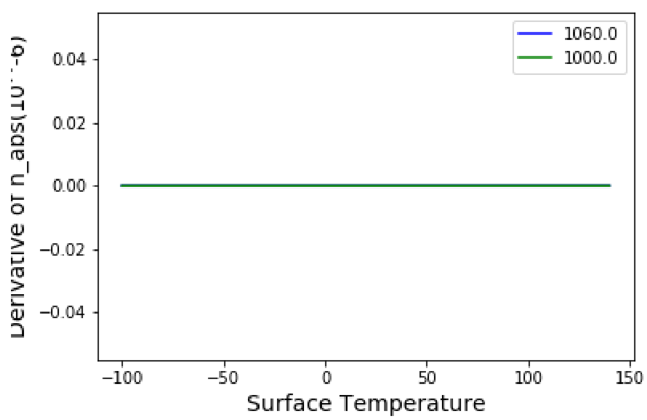

Figure 17: change of rate of derivative of absolute refractive index with temperature at 560 and 1060 nm

For **thermal analysis**, the change of rate of derivative of absolute refractive index with temperature is given in the figure 17. There two curve, one for 560 nm and another for 1060 nm. The rate for 560 nm is same as 1060 nm wavelength. Here the temperature range is -100 to 140 celcius. The plot is constant with value 0. That means this glass type has no change in absolute refractive index

Now refractive index at any given temperature and pressure is calculated from equation (30). In catalog, J-PSKH1 has reference temperature 23 celcius and model found the refractive index at temperature 40 celcius at pressure 103250 Pa. It gave the answer at wavelength 1060 nm around 1.5830508417073328. And at reference temperature with one atmospheric pressure is 1.5830354695353313. But if we keep temperature constant and decrease the pressure at 90000 Pa of surrounding then value is 1.5831030086722035. As we can see from data for refractive index at constant temperature but different pressure gives the different values. Reason for this trend is same as for N-BK7 and K-LAH99.

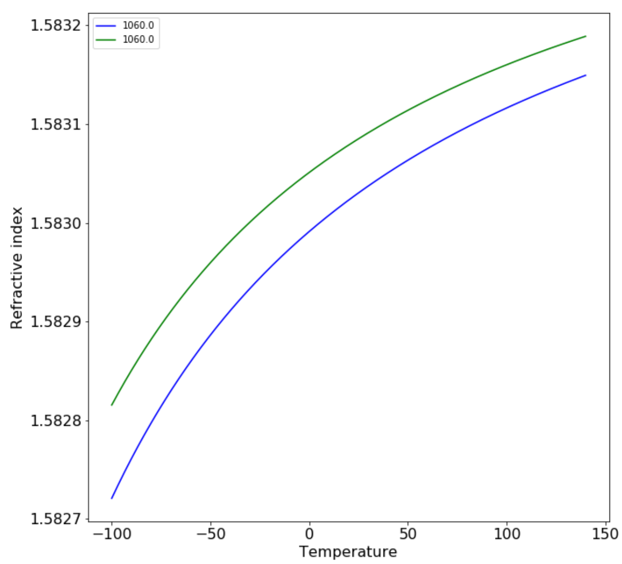

Figure 18: light blue color for pressure at P=90000 Pa and red color for pressure P = 103250 Pa

Now the plot between refractive index for particular wavelength with temperature is in figure 18. For J-PSKH1, the refractive is increasing with increasing of temperature.

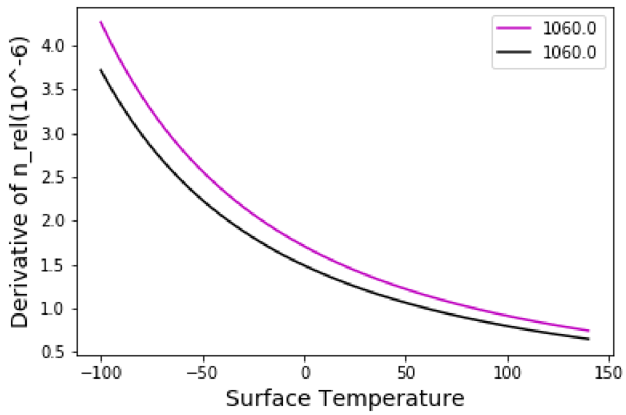

Figure 19: rate is decreasing with temperature

Now fourth plot is between change of rate of derivative of relative refractive index with temperature at pressure 103250 Pa. This gives the rate at which relative refractive index changes with temperature. In the figure 19, the rate is decreasing continuously with temperature. Here the rate is in only positive portion. So these are all analysis of J-PSKH1 from given catalog.

So These four glass type N-BK7, J-PSKH1,K-SFLD14, and S-LAH99 have different behaviour with temperature, their rate of change of refractive index, and change of refractive index with wavelengths. These all data got through python programming. Now we need to verify our data with one reliable resources.

3.1.5 Temperature, Pressure effects on RI of air

The refractive index of the vacuum at any temperature is 1 because there is no medium which can affect the light. Whenever air is present in the space, the refractive index of space can't be exactly 1 because of temperature effects. For different pressure of the air, the temperature affects the RI differently. Here is table for pressure 101325 Pa with different temperature at particular wavelength 1060 nm. These data are come from the model which has been created. From the

Table 1: Refractive index of air with temperature at P =101325, wavelength = 1060 nm

No.	Temperature (C)	Refractive index
1	25	1.0002647665390627
2	30	1.000260389946681
3	35	1.0002561556916556
4	40	1.000252056941361
5	45	1.0002480872936004

table 1, the refractive index of air is decreasing with temperature at wavelength 1060 nm because rising the temperature makes the air less dense.

Another table for pressure P =90000 Pa,

Table 2: Refractive index of air with temperature at P =90000, wavelength = 1060 nm

No.	Temperature (C)	Refractive index
1	25	1.000235173831884
2	30	1.0002312864071188
3	35	1.000227525410797
4	40	1.0002238847739697
5	45	1.0002203588100078

In table 2, the refractive index decreases with decreasing of pressure because the low pressure air would be less dense.

Next section includes the verification of these all data.

3.2 Verification of dispersion relation and thermal analysis data

From aforementioned data, model had plotted refractive index vs wavelength. After model got the reference data from one site which is mentioned in the references, model compared with calculated data with same glass type. The data verification are only for particular wavelength. For verification, model used the python programming to extract web/reference data directly. model includes the code in the footnote. The wavelength is varying with glass types. Here is table which gives the verification of our data,

Table 3: Verification of refractive index data

No.	Glass Name	Glass catalog	Wavelength ($\mu_{ab}m$	Refractive index in web	Refractive index calculated	Difference
1	K5G20	SCHOTT-K	1.435	1.508314166185	1.50831416618502	-2.68673971959287E-14
2	N-SK15	SCHOTT-SK	1.417	1.6055263722658	1.60552637226584	-4.64073224293315E-14
3	N-SSK5	SCHOTT-SSK	1.425	1.6385485784244	1.63854857842444	-4.66293670342565E-14
4	N-BK7	SCHOTT-BK	1.4	1.5024964846769	1.50249648467694	-4.52970994047063E-14
5	N-BAK4	SCHOTT-BaK	1.417	1.5524334945124	1.55243349451242	-2.33146835171282E-14
6	N-FK58	SCHOTT-FK	1.38	1.447776337143	1.44777633714302	-2.8421709430404E-14
7	N-LAK22	SCHOTT-LaK	1.405	1.632632492	1.632632492	3.46E-14
8	N-PK51	SCHOTT-PK	1.4	1.517637108	1.517637108	-3.80E-14
9	N-PSK53	SCHOTT-PSK	1.405	1.604256216	1.604256216	2.66E-15
10	F2HT	SCHOTT-F	1.41	1.596653591	1.596653591	-3.31E-14
11	J-K3	HIKARI-K	1.212	1.505919914	1.505919914	3.64E-14
12	J-SK2	HIKARI-SK	1.212	1.592865567	1.592865567	-1.82E-14
13	J-SSK1	HIKARI-SSK	1.212	1.601477046	1.601477046	4.88E-15
14	J-SF03	HIKARI-SF	1.223	1.806844021	1.806844021	4.71E-14
15	J-LAF7	HIKARI-LaF	1.223	1.723520287	1.723520287	1.44E-14
16	J-KF6	HIKARI-KF	1.212	1.503609009	1.503609009	-3.80E-14
17	J-LLF1	HIKARI-LLF	1.212	1.532465474	1.532465474	1.07E-14
18	J-F2	HIKARI-F	1.212	1.598971601	1.598971601	5.55E-15
19	J-PKH1	HIKARI-PK	1.212	1.507442564	1.507442564	2.75E-14
20	J-BAF8	HIKARI-BaF	1.212	1.606388671	1.606388671	-2.18E-14
21	E-C3	HOYA-C	0.6895	1.514523909	1.514523909	-1.82E-14
22	BSC7	HOYA-BSC	0.6895	1.513343373	1.513343373	-2.13E-14
23	FC5	HOYA-FC	0.6895	1.484504444	1.484504444	-2.38E-14
24	LBC3N	HOYA-LBC	0.6895	1.602243525	1.602243525	2.64E-14
25	E-FD8	HOYA-FD	0.6895	1.680091414	1.680091414	-1.62E-14
26	E-FDS1	HOYA-FDS	0.6895	1.905762088	1.905762088	2.95E-14
27	E-ADF50	HOYA-ADF	0.6895	1.647318779	1.647318779	3.02E-14
28	E-FEL2	HOYA-FEL	0.6895	1.535990459	1.535990459	-3.80E-14
29	PCD51	HOYA-PCD	0.6895	1.589715154	1.589715154	-1.91E-14
30	LAC13	HOYA-LaC	0.6895	1.688020147	1.688020147	2.22E-16
31	S-APL	OHARA-APL	0.6325	1.515705754	1.515705754	4.60E-14
32	BAH27	OHARA-BAH	0.6325	1.698063259	1.698063259	-7.55E-15
33	BAL35	OHARA-BAL	0.6325	1.587108397	1.587108397	-7.99E-15
34	BAM25	OHARA-BAM	0.6325	1.600315139	1.600315139	-1.11E-14
35	L-BBH2	OHARA-BBH	1.35	2.035367854	2.035367854	-4.26E-14
36	BPH35	OHARA-BPH	1.35	1.621582861	1.621582861	2.71E-14
37	BPM51	OHARA-BPM	1.35	1.592398805	1.592398805	-3.35E-14
38	S-FSL5	OHARA-FSL	1.34	1.475402158	1.475402158	-3.31E-14
39	S-LAL56	OHARA-LAL	1.365	1.658079449	1.658079449	-1.55E-15
40	S-NSL3	OHARA-NSL	1.36	1.504340537	1.504340537	-1.24E-14
41	K-SK14	SUMITA-SK	0.955	1.592759399	1.592759399	-3.93E-14
42	K-LaFn5	SUMITA-LaF	0.955	1.727775747	1.727775747	4.55E-14
43	K-SSK4	SUMITA-SSK	0.955	1.606249565	1.606249565	-2.73E-14
44	K-PSK100	SUMITA-PSK	0.955	1.581720436	1.581720436	-1.78E-15
45	K-GIR79	SUMITA-GIR	0.955	1.832273778	1.832273778	-3.73E-14
46	K-FK5	SUMITA-FK	0.955	1.479893339	1.479893339	4.91E-14
47	K-FIR97UV	SUMITA-FIR	0.955	1.421243807	1.421243807	4.82E-14
48	K-BK7	SUMITA-BK	0.955	1.507694575	1.507694575	-4.66E-14
49	K-BOC30	SUMITA-BOC	0.975	1.971646186	1.971646186	2.89E-14
50	K-SFLD8	SUMITA-SFLD	0.975	1.667971361	1.667971361	4.69E-14

From table 3, it is shown that the error is very small. That means data is valid.

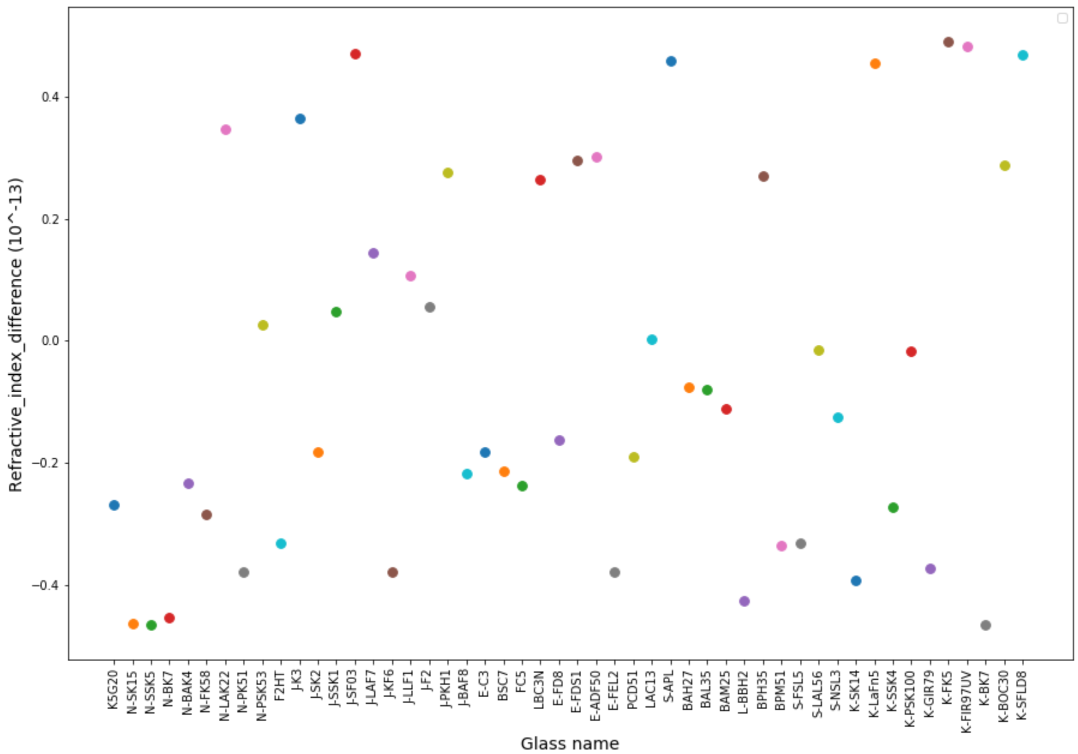

Figure 20: Verification of refractive index

In the figure 20, the plot is between difference of data with reference and glass name. This is verification plot which has very small error.
Now come to data **verification of thermal variations**. From reference data, the verification is done through plotting of difference of change of rate of derivative of absolute refractive index vs temperature, difference of rate of change of derivative of relative refractive index vs temperature, and refractive index vs temperature at particular wavelength. I included only first two parts because there are not any resources for refractive index vs temperature plot. But relation between refractive index and temperature comes from the these rate of n_{abs} and n_{rel} and verification of these rates are valid which are given below,

Model includes total five glasses for verification. (1) N-BK7, (2) F2 (3) N-LAF2 (4) N-PK51 (5) SF57. The raw data from reference data got through software "GetData Graph Digitizer". This software converts graphical data into raw data by tracing the curve.

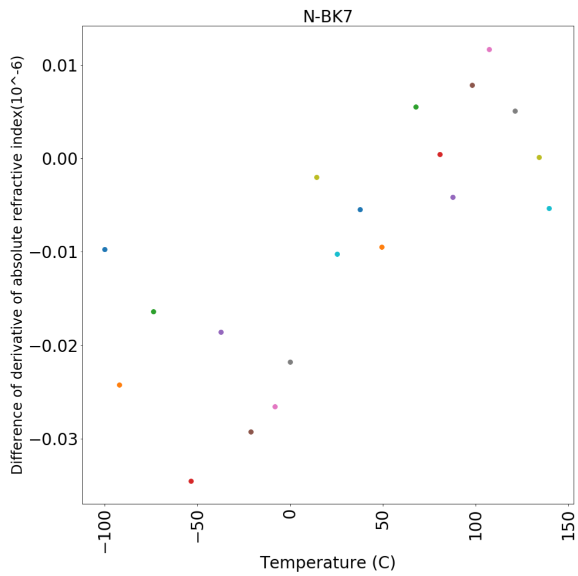

Figure 21: Verification of thermal data for N-BK7 (absolute)

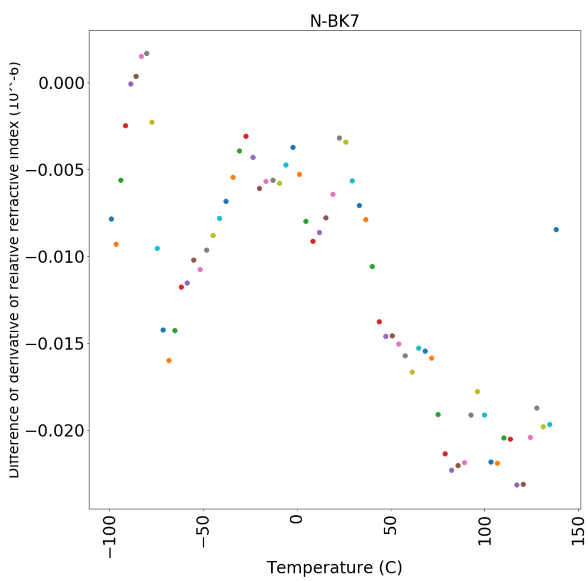

Figure 22: Verification of thermal data for N-BK7 (Relative)

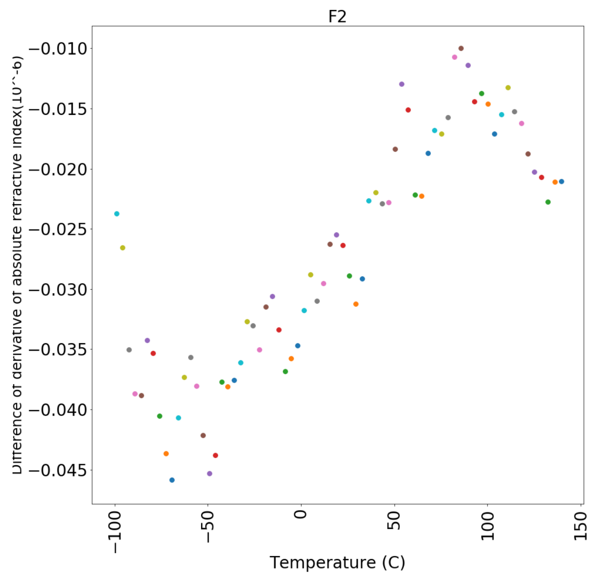

Figure 23: Verification of thermal data for F2(absolute)

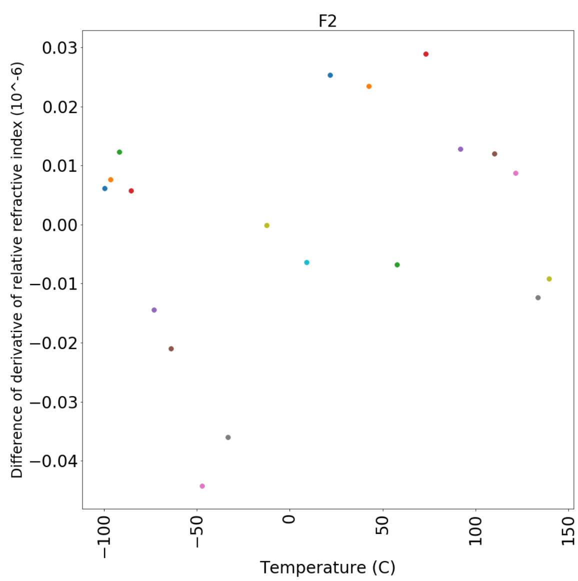

Figure 24: Verification of thermal data for F2(Relative)

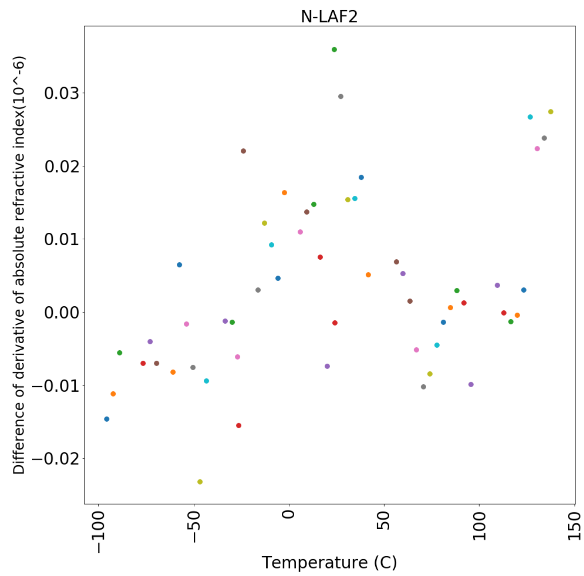

Figure 25: Verification of thermal data for N-LAF2(absolute)

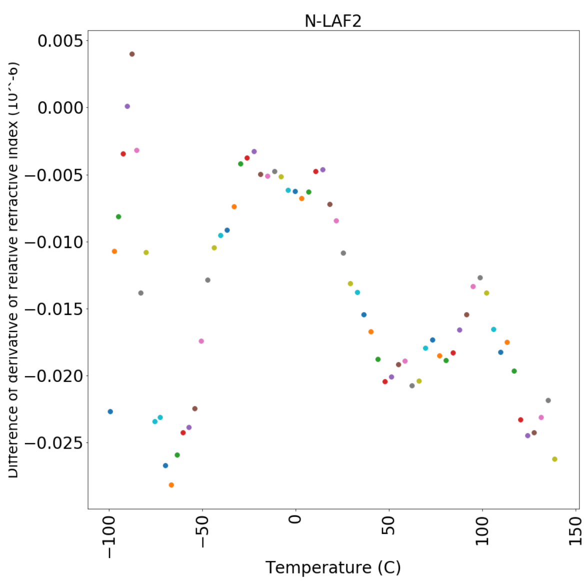

Figure 26: Verification of thermal data for N-LAF2(Relative)

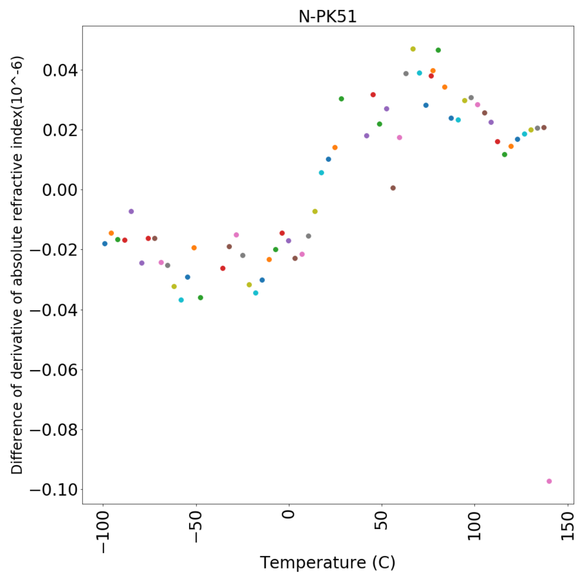

Figure 27: Verification of thermal data for N-PK51(absolute)

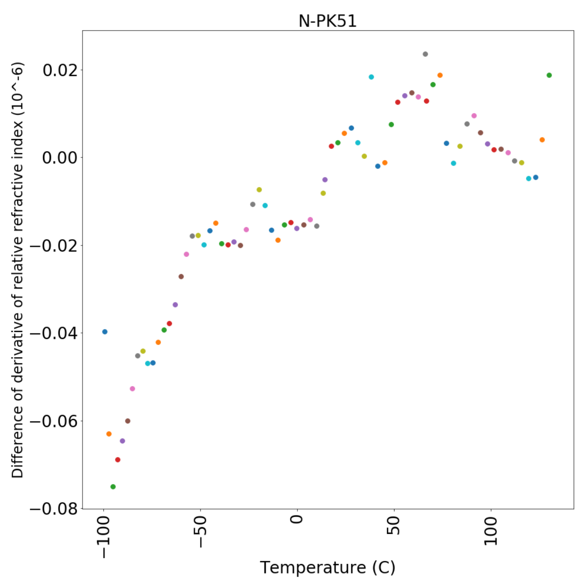

Figure 28: Verification of thermal data for N-PK51(Relative)

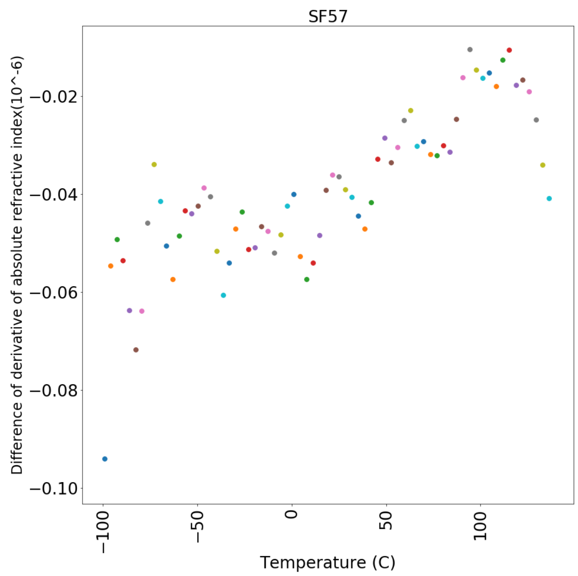

Figure 29: Verification of thermal data for SF57(absolute)

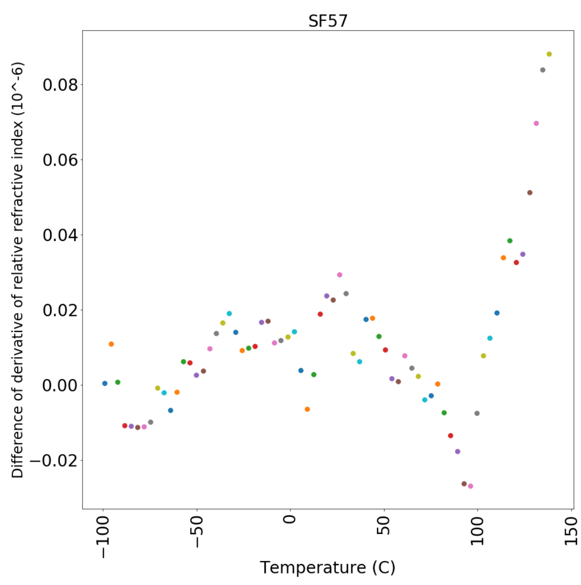

Figure 30: Verification of thermal data for SF57(Relative)

From the figures 21, 22, 23, 24, 25, 26, 27, 28, 29, and 30 the error or difference between given and calculated data is very small around 0.001 which could be negligible. And these verification is done for 1060 nm wavelength. So rates of absolute and relative are taken for 1060 nm wavelength. These are all optical analysis of the different glass types in the given catalog. Table for raw data for verification is given below,

Table 4: Verification of absolute change for N-BK7

Given change in reference	Calculated by formula	Difference(10^-6)
-6.91E-01	-0.68148	-0.00973
-5.84E-01	-0.55973	-0.02421
-2.98E-01	-0.28162	-0.01636
-2.38E-02	0.010703	-0.03453
2.15E-01	0.233072	-0.01857
4.17E-01	0.446435	-0.02924
5.84E-01	0.610641	-0.02653
6.91E-01	0.713158	-0.02175
8.82E-01	0.884131	-0.00199
1.00E+00	1.011646	-0.0102
1.14E+00	1.150037	-0.00545
1.26E+00	1.273399	-0.00948
1.47E+00	1.46122	0.00551
1.59E+00	1.585696	0.000437
1.65E+00	1.650008	-0.00415
1.75E+00	1.745407	0.007863
1.84E+00	1.825202	0.011659
1.94E+00	1.93938	0.005054
2.04E+00	2.039976	0.000108
2.08E+00	2.081324	-0.00534

I included table data for N-BK7 and F2 in table 4,5,6 and 7,8,9 respectively.

These are the results from the model which has been created using python programming. Next section includes the second model to find OPD of the optical system.

4 Aberrations in optical system

There are so many aberrations in glass. From seidel aberration theory there are total five aberrations. There is another aberration called as chromatic aberration.

Table 5: Verification of relative change for N-BK7

Given change in reference	Calculated by formula	Difference(10^-6)
3.27E+00	3.274056	-0.00802
3.20E+00	3.205571	-0.00948
3.13E+00	3.135894	-0.00578
3.07E+00	3.069411	-0.00264
3.00E+00	3.004968	-0.00022
2.94E+00	2.943821	0.000228
2.89E+00	2.88593	0.001375
2.83E+00	2.830337	0.001545
2.78E+00	2.778867	-0.00241
2.72E+00	2.732037	-0.00968
2.67E+00	2.686575	-0.01437
2.63E+00	2.64344	-0.0161
2.59E+00	2.602141	-0.01439
2.55E+00	2.564013	-0.01189
2.52E+00	2.529449	-0.01163
2.49E+00	2.497772	-0.01031
2.46E+00	2.46929	-0.01086
2.43E+00	2.443086	-0.00973
2.41E+00	2.419828	-0.0089
2.39E+00	2.39904	-0.00791
2.37E+00	2.380885	-0.00691
2.36E+00	2.364979	-0.00552
2.35E+00	2.351595	-0.00401
2.34E+00	2.340197	-0.00317
2.33E+00	2.330842	-0.00437
2.32E+00	2.32339	-0.00616
2.31E+00	2.317713	-0.00576
2.31E+00	2.313689	-0.00569
2.31E+00	2.311208	-0.00585
2.31E+00	2.310164	-0.00481
2.31E+00	2.310459	-0.00378
2.31E+00	2.312001	-0.00532
2.31E+00	2.314706	-0.00803
2.31E+00	2.318492	-0.00918
2.31E+00	2.323283	-0.00869
2.32E+00	2.32901	-0.00782
2.33E+00	2.335606	-0.0065
2.34E+00	2.3429	-0.00323
2.35E+00	2.35104	-0.00346
2.35E+00	2.359874	-0.00569

Table 6: Verification of relative change for N-BK7

Given change in reference	Calculated by formula	Difference(10^-6)
2.36E+00	2.369213	-0.00711
2.37E+00	2.379271	-0.00793
2.38E+00	2.389876	-0.01062
2.39E+00	2.400983	-0.01381
2.40E+00	2.412387	-0.01466
2.41E+00	2.424201	-0.01459
2.42E+00	2.436564	-0.01508
2.43E+00	2.449099	-0.01574
2.45E+00	2.461943	-0.01671
2.46E+00	2.475064	-0.01531
2.47E+00	2.488436	-0.01549
2.49E+00	2.502031	-0.01589
2.50E+00	2.515823	-0.01912
2.51E+00	2.529985	-0.02141
2.52E+00	2.544099	-0.02233
2.54E+00	2.55834	-0.02205
2.55E+00	2.572685	-0.02188
2.57E+00	2.587114	-0.01915
2.58E+00	2.601609	-0.01781
2.60E+00	2.616153	-0.01916
2.61E+00	2.630726	-0.02186
2.62E+00	2.645312	-0.02193
2.64E+00	2.65969	-0.02047
2.65E+00	2.674256	-0.02052
2.67E+00	2.68879	-0.02318
2.68E+00	2.703278	-0.02315
2.70E+00	2.717705	-0.02042
2.71E+00	2.731859	-0.01874
2.73E+00	2.746132	-0.01982
2.74E+00	2.760509	-0.01968
2.77E+00	2.774376	-0.00848

Table 7: Verification of absolute change for F2

Given change in reference	Calculated by formula	Difference(10^-6)
2.50E+00	2.523539	-0.02103
2.48E+00	2.499842	-0.02109
2.45E+00	2.475096	-0.02273
2.43E+00	2.449303	-0.02069
2.40E+00	2.422461	-0.02024
2.38E+00	2.39457	-0.01875
2.35E+00	2.365631	-0.0162
2.32E+00	2.335644	-0.01524
2.29E+00	2.304608	-0.01324
2.26E+00	2.272524	-0.01546
2.22E+00	2.239865	-0.01711
2.19E+00	2.205699	-0.01462
2.16E+00	2.170484	-0.01371
2.12E+00	2.134221	-0.0144
2.09E+00	2.096909	-0.0114
2.05E+00	2.058549	-0.00999
2.01E+00	2.019703	-0.01073
1.96E+00	1.979838	-0.01573
1.92E+00	1.938954	-0.01708
1.88E+00	1.896445	-0.01679
1.83E+00	1.853508	-0.01872
1.79E+00	1.809552	-0.02227
1.74E+00	1.764577	-0.02216
1.70E+00	1.717918	-0.01509
1.66E+00	1.67089	-0.01293
1.61E+00	1.623537	-0.01836
1.55E+00	1.575193	-0.0228
1.50E+00	1.525137	-0.02289
1.45E+00	1.474062	-0.02196
1.40E+00	1.421968	-0.02265
1.34E+00	1.370387	-0.02914
1.29E+00	1.317063	-0.03124
1.23E+00	1.261956	-0.02891
1.18E+00	1.206638	-0.02638
1.12E+00	1.15033	-0.02549
1.07E+00	1.093031	-0.02626
1.01E+00	1.035595	-0.02952

Table 8: Verification of absolute change for F2

Given change in reference	Calculated by formula	Difference(10^-6)
9.45E-01	0.976331	-0.03096
8.87E-01	0.916076	-0.02877
8.24E-01	0.855726	-0.03176
7.61E-01	0.795323	-0.0347
6.97E-01	0.733064	-0.03578
6.34E-01	0.67078	-0.03684
5.73E-01	0.606612	-0.03337
5.10E-01	0.540517	-0.03062
4.44E-01	0.47539	-0.03147
3.75E-01	0.410322	-0.03503
3.09E-01	0.342335	-0.03302
2.41E-01	0.273387	-0.03269
1.69E-01	0.205546	-0.03611
9.82E-02	0.135752	-0.03757
2.69E-02	0.065023	-0.0381
-4.43E-02	-0.00664	-0.0377
-1.21E-01	-0.07705	-0.04382
-1.95E-01	-0.14945	-0.04532
-2.66E-01	-0.22387	-0.04216
-3.37E-01	-0.29923	-0.03806
-4.11E-01	-0.37552	-0.03567
-4.88E-01	-0.45042	-0.03731
-5.67E-01	-0.5262	-0.0407
-6.49E-01	-0.60286	-0.04586
-7.25E-01	-0.6816	-0.04366
-8.02E-01	-0.76125	-0.04055
-8.78E-01	-0.84302	-0.03531
-9.58E-01	-0.92325	-0.03425
-1.04E+00	-1.00311	-0.03886
-1.12E+00	-1.08508	-0.0387
-1.20E+00	-1.16793	-0.03502
-1.28E+00	-1.25296	-0.02653
-1.36E+00	-1.33758	-0.02373

Table 9: Verification of relative change for F2

Given change in reference	Calculated by formula	Difference(10^-6)
2.87E+00	2.865462	0.005967
2.81E+00	2.79969	0.007453
2.72E+00	2.709301	0.012127
2.61E+00	2.608725	0.005561
2.46E+00	2.478892	-0.01461
2.40E+00	2.421079	-0.02108
2.34E+00	2.380092	-0.04438
2.36E+00	2.393282	-0.03614
2.46E+00	2.464525	-0.00024
2.57E+00	2.577903	-0.00647
2.68E+00	2.653308	0.025264
2.81E+00	2.783769	0.023374
2.87E+00	2.878251	-0.00682
3.00E+00	2.971181	0.028819
3.09E+00	3.072942	0.012773
3.17E+00	3.159488	0.01194
3.21E+00	3.205558	0.008727
3.24E+00	3.248061	-0.01235
3.26E+00	3.266383	-0.00924

4.1 Chromatic aberration

Different wavelengths have different refractive index in glass and different refractive index means it has different focusing power for same glass type. Therefore, it will not give white image of the object. This aberration contains all wavelengths and make image at different focuses.

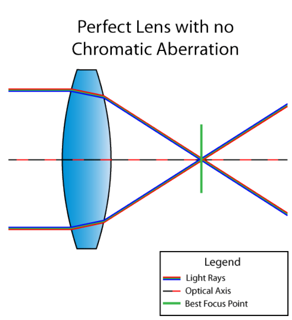

Figure 31: Without chromatic aberration

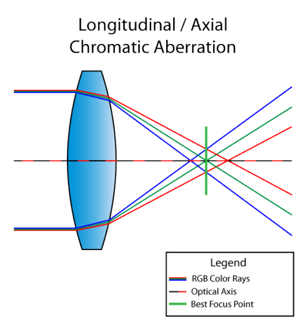

Figure 32: With chromatic aberration

Here in 31 is without chromatic aberration and all wavelengths concentrate on one focus point which means that glass has low dispersion. Therefore, refractive index does not change very much with different wavelengths. And 32 is for chromatic aberration where all wavelengths have different behaviour in glass therefore aberration occur in the glass.

4.2 Monochromatic aberration

There are other aberrations with monochromatic light.

4.2.1 Spherical aberration

In this aberration, one monochromatic light is sent to the lens with parallel to optical axis of the lens. The rays near to the optical axis get bend less than rays far from the optical axis.

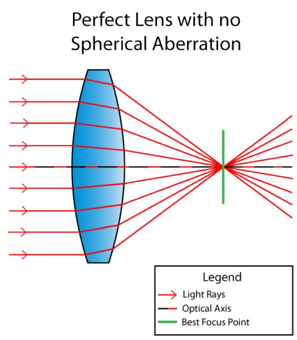

Figure 33: Without chromatic aberration

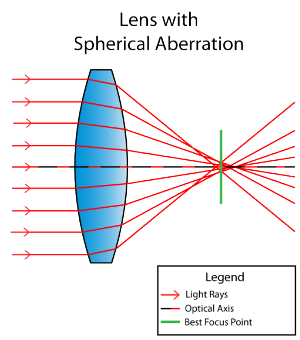

Figure 34: With chromatic aberration

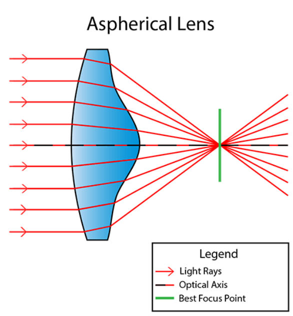

Figure 35: Without chromatic aberration

In the 35, to reduce spherical aberration, lens is made with extra optical path which gives the central rays more time to pass through glass and concentrate at one point.

4.2.2 Coma aberration

An off-axis effect which appears when a bundle of incident rays all make the same angle with respect to the optical axis (source at infinite). Rays are brought to a focus at different points on the focal plane found in lenses with large spherical aberrations an off-axis object produces a comet-shaped image.

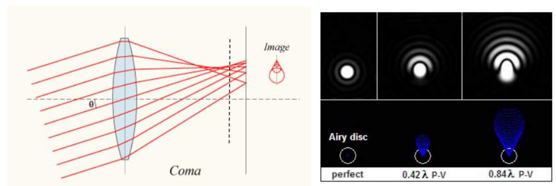

Figure 36: Coma aberration

In the 36 the shape of the image is like comet because of focusing on focal plane at different locations. Comatic aberration is similar to the spherical aberration but this is off axis rather than on optical axis.the peripheral rays produce the smallest image (least magnification) and the coma aberration sign is said to be negative. In contrast, when the peripheral rays are focused further down the axis and produce a much larger image (greater magnification), the aberration is termed positive. To reduce the coma aberration, peripheral rays should be removed.

4.2.3 Astigmatism

Astigmatism occurs when there are not spherical curvature. That means curvature in one plane is different than perpendicular plane of it. In the 37, aberration is astigmatism. The black-line in the figure is optical axis. And lens is viewed from some angle. aberration of astigmatism by supposing that the lens has a different focal length in one plane than in the other. Due to the different focal plane in vertical and horizontal, sphere looks like circle or ellipse shape.

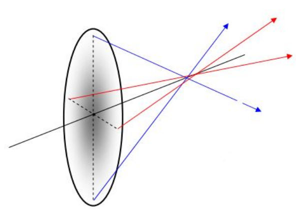

Figure 37: Astigmatism

Although different radii of curvature in different planes is not the usual cause of astigmatism, there is an exception - namely, the human eye. If the radii of curvature of the cornea, or of the lens, is different in different planes, then the image on the retina will be astigmatic even on-axis.

4.2.4 Curvature of field

Suppose the spherical, coma, astigmatic aberration is resolved but there will be another aberration called as curvature of field. This occur due to focusing of light at different point on one curvature focal curve. In the 38, all focused points are on one curvature plane instead of plane. Focal surface also called as **petzval** surface. One effective way of dealing with this problem, particularly if detector is a flexible film, is to shape the film-holder so that the film fits along the Petzval surface.

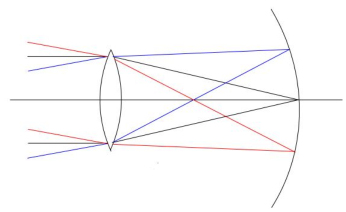

Figure 38: Curvature of field

4.2.5 Distortion

The magnification of an image is image distance divided by object distance, and image distance is different off-axis than on-axis, so the image magnification varies with distance from the axis.

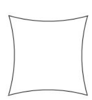

Figure 39: Pincushion shape

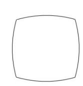

Figure 40: Barrel shape

If there sqaure object then its distortion look like in 39 and 40. Here two figures are different depends on certain conditions. Off-axis, the image distance is less than the object distance,so the magnification is less off-axis than on-axis. Barrel distortion results. Off-axis, the image distance is greater than the object distance, so the magnification is greater off-axis than on-axis. Pincushion distortion results.

Next section explains how the aberrations could be reduced using aspheric surfaces

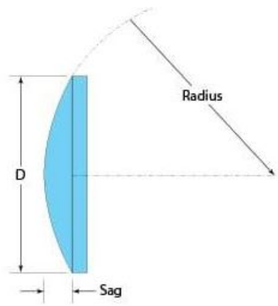

Figure 41: Sag for convex lens

4.3 Reduction in aberrations : Aspheric surface

It is observed that hyperbolic surface reduces the spherical aberration of the glass rather than spherical surfaces. Therefore, Aspherical surface has wide application for correction in spherical aberration, coma aberration etc.

4.3.1 Aspheric surface

To define the aspheric surfaces there is sag factor which varies with height from optical axis of the lens. The sag eqaution is given by,

$$Z(s) = \frac{Cs^2}{1 + (1 - (1 + k)C^2 s^2))^{1/2}} + A_4 s^4 + A_6 s^6 + ... \tag{31}$$

In equation (31), There are only even terms because of the symmetric of the lenses. If we want to design asymmetric lens then we have to add odd terms. Sag for any lens is measured from its line passing through vertex perpendicular to optical axis which is given in figure 41. Basically, sag defines the curvature of surface at particular point on lens. Here C is curvature of radius at vertex of the lens, k is depended on the reference shape of the lens like spherical, parabolic, hyperbolic etc. For example, $k = 0$ it is sphere. Also s is height of the source from optical axis. And last part is constants which are $A_4, A_6, A_8, ...$ etc.
Disadvantage of the aspheircal surfaces are that they are very costly and hard to manufacture particular type of glass shape with good precision.

Example (1) For equation given below,

$$z = \frac{c_x x^2 + c_y y^2}{1 + ((1 - (1 + k_x)c_x^2 x^2 - (1 + k_y)c_y^2 y^2)^{1/2}} \tag{32}$$

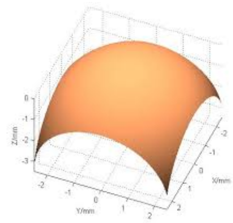

Figure 42: Biconic surface

In the equation (32), the surface is biconic which is graphically shown in the figure 42.

4.3.2 Radius of curvature at each point

In aspherical surfaces, the radius of curvature at each point would be different and it is founded by taking tangent curve at each point and some geometry. The final equation of the radius of curvature at each point is given by,

$$R = \frac{(1 + (z^{'}(s))^2)^{3/2}}{z^{''}(s)} \tag{33}$$

In the equation (33), the radius of curvature is founded by taking derivatives of sag factor. OPD is better way to find out the aberrations. To find OPD for aspheric surfaces, we need to define its surface and after using geometry, we can find the OPD and final height of the image.

Next section includes basics of finding the final height of the image at image plane using matrices.

5 Objective : 3Ray tracing method to find OPD and aberrations

5.1 ABCD matrix for paraxial rays

This matrix is useful tool for describing the effects of optical elements in the system for monochromatic light. The matrix form for any lens with paraxial rays is given by equation,

$$\begin{bmatrix} r_1 \\ \theta_0 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} r_0 \\ \theta_0 \end{bmatrix}$$

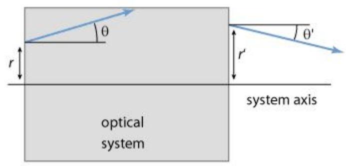

Figure 43: Rays tracing method

Where r_0,θ_0 are initial position of the object and image would have coordinates r_1, θ_0 after passing through the optical elements at image plane. Where A,B,C and D are dependent on the lens types and material. In the figure 43, you can see the coordinate system for rays. Here are some standard examples, (1) Thin lens with focal length f,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{-1}{f} & 1 \end{bmatrix}$$

(2)Translation matrix for the medium

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$$

Where d id distance travelled by the ray. (3) Curved mirror with radius R,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{-2}{R} & 1 \end{bmatrix}$$

(4)Refraction

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{n-n^{'}}{n^{'}R} & \frac{n^{'}}{n} \end{bmatrix}$$

Where $n^{'}$ is refractive index of the medium and n is refractive index of outside the medium. If we have one system with convex lens then we can use this ABCD matrices for each and every element. For example, for convex lens, first ray will encounter with refraction at the first surface. After translation with some distance in lens medium it will again refract to the outside the lens. So we need to multiply each matrices to get final matrix for whole system. Order for multiplication should be maintained. For this example, The first matrix is refraction on coordinates, the second is translation on first and third is refraction on second matrix. But this matrices only use for the paraxial rays approximation.

To get precise image distance and its height, we need to consider oblique rays. So we can't use the ABCD matrix for the aspherical surfaces because it needs to be modified. ABCD matrix is linear relation but when system becomes more complicated the matrix could not be represented as linear or matrix form. ABCD matrix is very useful for simple systems. To find precise answer we need to find optical path difference and height of image which also gives the idea of aberrations of system.

5.2 Optical path difference and ray tracing method to find aberration

When paraxial rays(parallel to optical axis) or oblique rays or peripheral rays(at edge of the lens) are incident on the convex lens, not all rays travel the same distance to the image plane and it create the path difference between rays. In telescope, generally rays are incident on the lens with very small and equal angle. As rays travel different distance other than reference distance called as optical path difference between those two rays. Because of the optical path difference there will be phase difference between rays in the wave-front. There are so many aberrations due to lens material, surface shape and surrounding conditions. There are first, second, third and higher order aberration in the lens. As mentioned earlier, the aberrations could be reduced through shape of the surface of lens.

From the aspheric surface, we can define any shape of convex lens with angular symmetric. From basic geometry of the convex lens we can get the OPD and final height of image at the image plane.

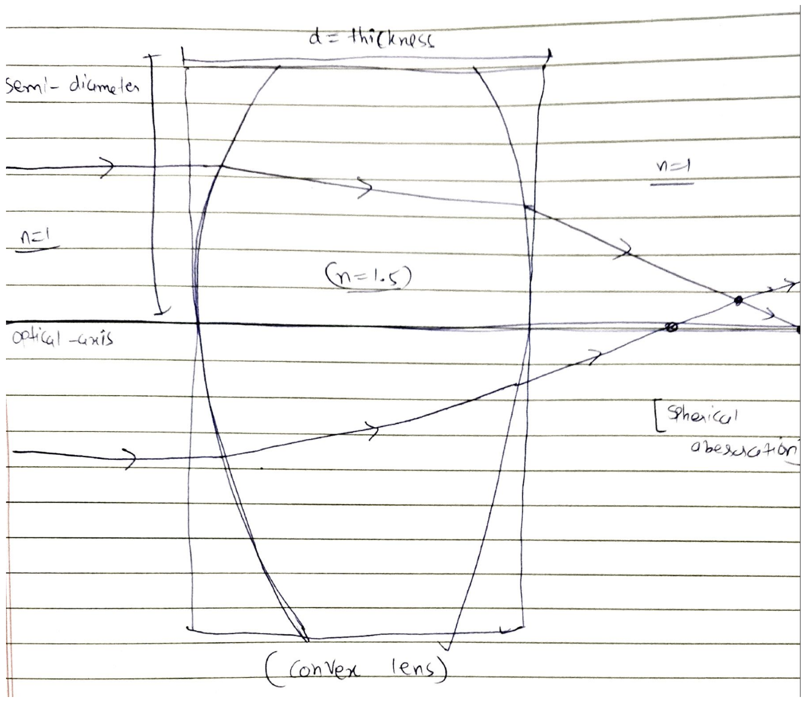

Figure 44: Convex lens with small portion

In the figure 44 the rays travel from initial position to the convex lens with refractive index $n = 1.5$ and transmit through air media. Here thickness is d. semi diameter is distance between vertex to maximum point or edge of the lens. Here two rays concentrate over different regions. This creates the coma and spherical aberration. Coma aberration occurs when the parallel beam incident with small angle. Optical axis is passing through middle of the lens. Aberration occurs because these two rays travel different path and path difference creates the phase difference. Now next part is to find optical path difference with simple geometry in the system and find through model created by python programming.

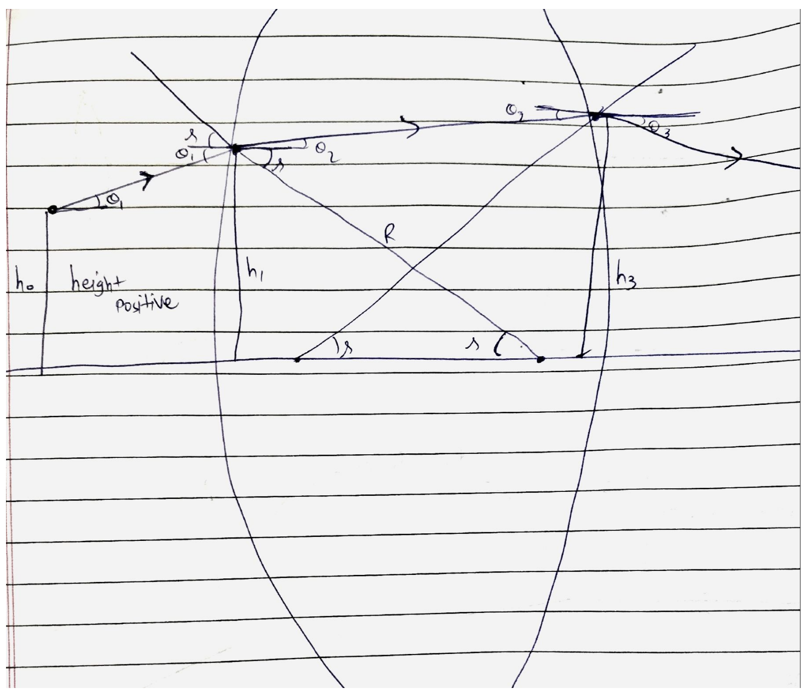

Figure 45: Lens with geometry

Geometry of ray is shown into figure 45. Here initial height is h_0 and incident on the lens with angle θ_1. The angle and height above the optical axis are taken as positive and below optical axis are taken as negative. In this example both part have been taken as positive. Initially, ray emits from height

h_0 with positive angle θ_1. After bending of ray due to medium of lens, the resultant angle becomes θ_2 which could be negative or positive which depends on the snell's law for refraction at the surface of lens. Snell's law states that $n_1 \cdot sin(\theta_1) = n_2 \cdot sin(\theta_2)$. θ_2 has been taken as positive. Second thing is that due to angle θ_1 the height at first surface would be different. Suppose that height is h_1. After travelling through medium, the final resultant angle becomes θ_3 and height becomes h_3. Aforementioned sag factor uses in to defining the surface of the lens, different rays travel different path in the air and glass media.

Also the medium could be any of the glass type which is included in the catalogs. For example if the media is N-BK7 and surrounding temperature is 30 C then refractive index will be 1.506678. So we can use the different glass types data with their thermal analysis and defining the surface structure, we can find the final height of the image and its aberration/optical path difference at different positions.

Geometry of the lens I included simple geometry for convex lens with middle part of it.

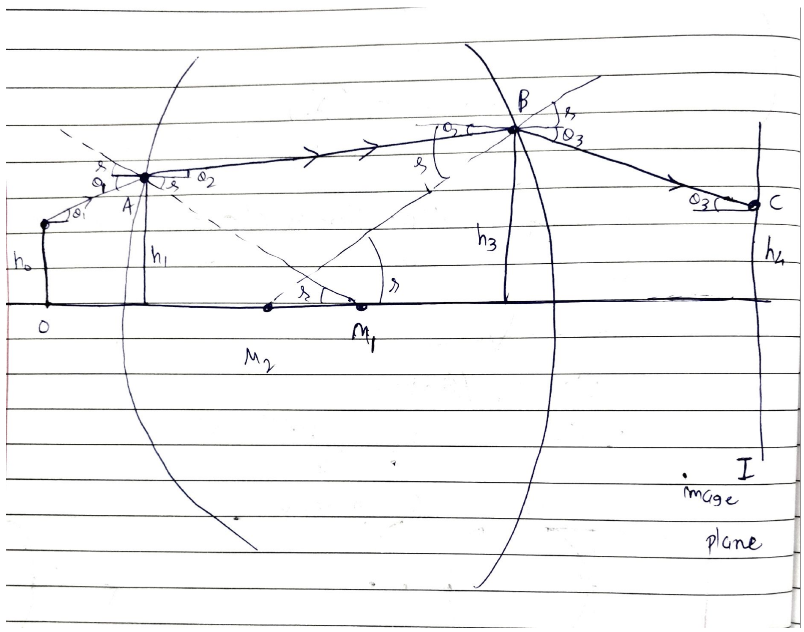

Figure 46: Convex lens

From the figure 46, the ray starts from point O and refracts from first surface of the convex lens. Suppose ray makes the angle θ with horizontal line. From trigonometry of the figure, distance travelled by a ray between O and A would become, $D_1 = L/cos(\theta_1) + Z_1/cos(\theta_1)$. As we can see that, the ray is travelling extra path in the air compared to middle ray due to curvature of the first surface.

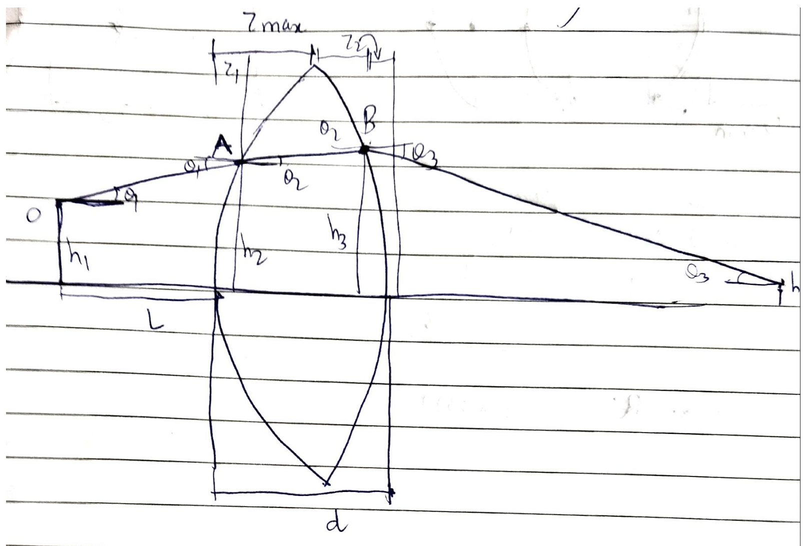

Figure 47: Notations of lens

Where L and Z_1 are distance given in figure 47. Z_1 is sag at that point on surface. Here the surface is spherical so sag of the surface would become, $Z = \frac{h_1^2}{R+(R^2+h_1^2)^{1/2}}$. h is height of the point A on the surface. Both curved surface are assumed to be spherical therefore sag function is same for both surface.

After refraction at first surface, the ray will bend towards horizontal line. In the figure 46, AM_1 is the line passing through center of the curves surface. The angle made by this AM_1 is r and it is given by, $sin(r) = h_1/R$. R is radius of the curved surface. Here the surface is spherical so that R would be constant for each point on the surface. If the surface is aspheric then R is defined by the equation (33). Snell's law applies for only angle made with perpendicular to the surface. Here AM_1 is perpendicular to the surface. So incident angle would become $i = \theta_1 + r$ and angle after refraction become $e = \theta_2 + r$. From snell's law $n_1 \cdot sin(i) = n_2 \cdot sin(e)$, We can find the angle e and subtraction with angle r gives the angle with horizontal line which is basically θ_2. n_1 is refractive index of the air and n_2 is refractive index of the medium which could be found by aforementioned methods to find refractive index for different wavelengths. To find the height at the surface, $h_1 = h_0 + tan(\theta_1)(L + Z_1)$. Here Z_1 is function of h_1 which makes the equation with function of h_1. After solving the equation, it gives the value of h_1.

After getting θ_2 and h_1, ray travels through medium to the point B in the figure 46. On the opposite side of the lens, the surface has negative radius and line passing through center is BM_1 in the figure 46. But here the surface has negative radius and its sag is defined by many ways, but here I defined from the vertex of the surface. Due to that, the function of sag would be same as Z_1. Here new quantity is Z_1max which is at the edge of the surface. Semi-diameter of the surface is distance between optical axis and the edge of the surface. Z_1max is sag at the semi-diameter height.

The distance travelled by ray into medium is given by, $D_2 = (d - 2 \cdot Z_1)/cos(\theta_2)$. d is thickness of the lens. The minimum value of the d is $d_{min} = 2 \cdot Z_1$.

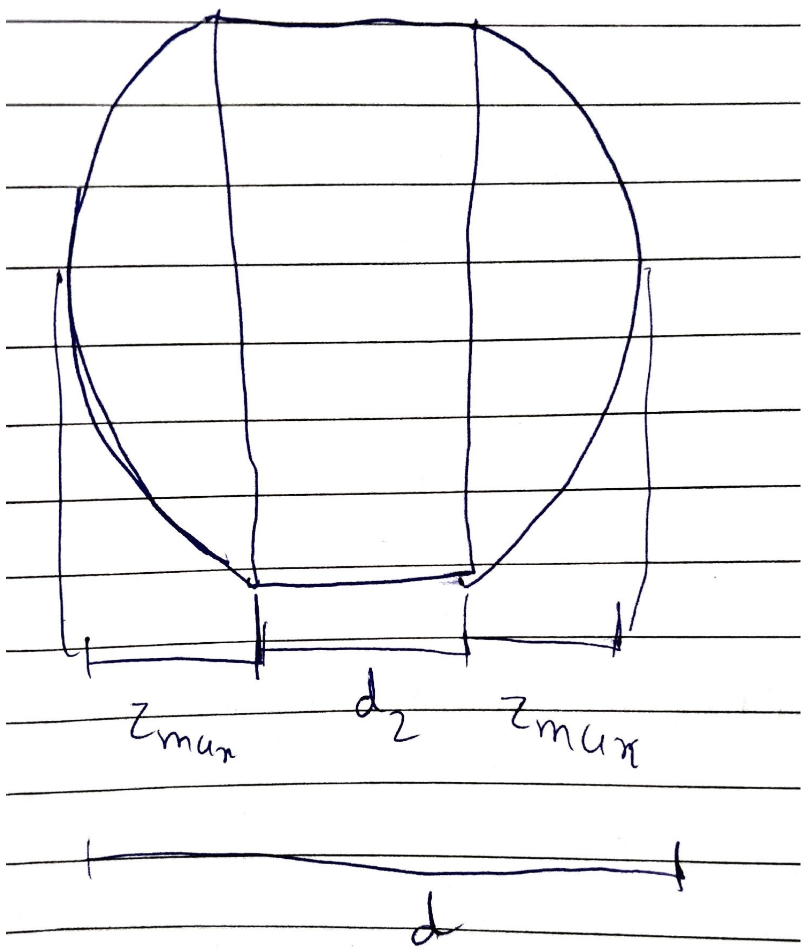

Figure 48: Minimum thickness

If the d is greater than d_{min} then lens would have three parts as shown in the figure 48. Again applying same method for surface two, we will get the θ_3 and h_3 at surface with negative radius.

The distance travelled by ray after refraction at surface two is given by $D_3 = (I + Z_1)/cos(\theta_3)$. I is the image plane distance. Z_1 would be different if the surface two is not spherical. h_3 is the height at point B in figure 46. Final height of the object at image plane is h_4. So this is ray tracing method to find optical path difference and image formation. From optical path difference, we can find the phase difference of the wave.

In the end total distance travelled by ray is $D = D_1 + D_2 + D_3$ where $D_1 = D_1 = L/cos(\theta_1) + Z_1/cos(\theta_1)$, $D_2 = D_2 = (d - 2 \cdot Z_1)/cos(\theta_2)$ and $D_3 = D_3 = (I + Z_1)/cos(\theta_3)$. To find optical distance, the D_1, D_2, D_3 are multiply by the corresponding refractive index of the medium. OPL (Optical path length) = $OPL = n_1 \cdot D_1 + n_2 \cdot D_2 + n_1 \cdot D_3$.
The reference ray is taken as passing through the center of the lens so distance travelled by reference ray is, OPLR (Optical path length reference) = $OPLR = n_1 \cdot L + n_2 \cdot d + n_1 \cdot I$.
OPD (Optical path difference given by $OPD = OPL - OPLR$

From these equation we can find the phase difference from $\delta\phi = \frac{2 \cdot \pi OPD}{\lambda}$. We can use the different separate parts and its effects on the ray with python programming. Using python, model has been defined total ten parts (1) convex surface with air to medium (2) convex surface with medium to air (3) concave surface with air to medium (4) concave surface with medium to air (5) air block for translation of ray (6) medium block for translation in the medium (7) refraction from plane surface (medium to air) (8) refraction from plane surface (air to medium). (9) convex surface medium to medium (10) concave surface medium to medium By combinations of all ten parts, we can make any optical system in python and it gives the OPD and final height, angle with the inputs of initial angle and height.

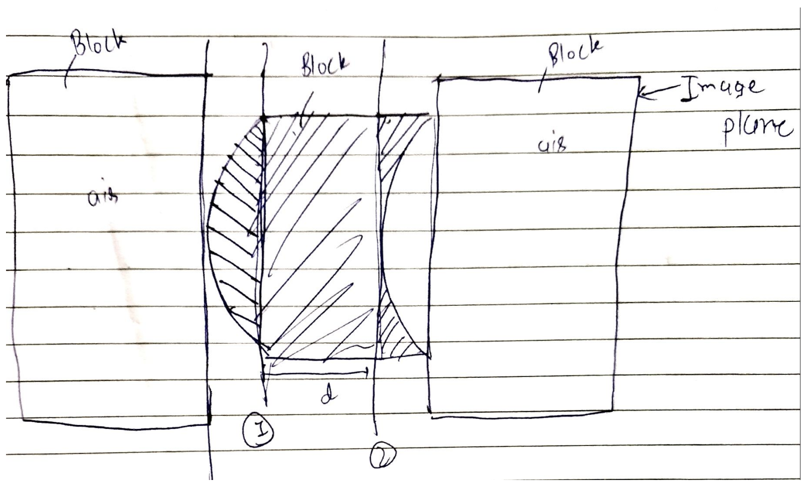

Figure 49: Optical system with parts

For example, we have one optical system given in the figure 49. So model divides the whole system with one block of air medium which defines the initial distance of the object. Second part is convex surface with air medium. Third one is block of medium with given thickness. Fourth part is convex surface with medium to air. Last part is block of air to define the image distance. I applied the necessary equations to find height, angle, and OPD for whole system. The code is given in the appendix.

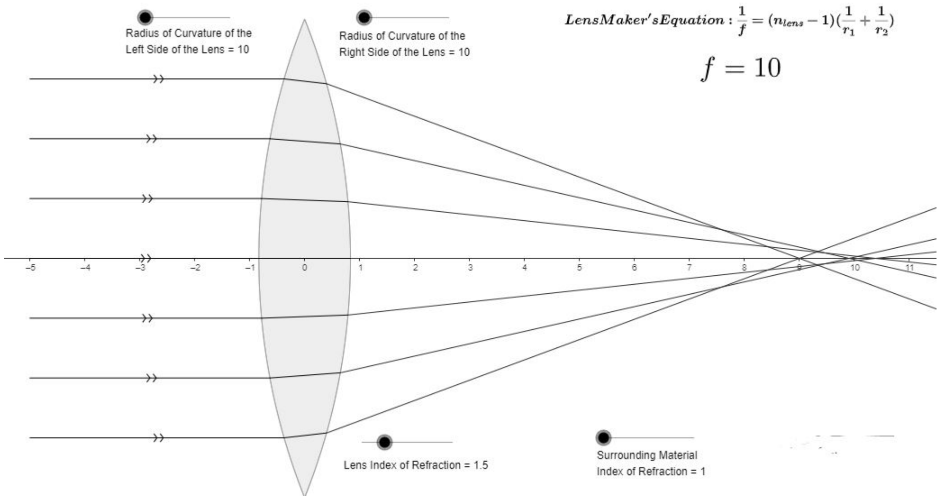

Figure 50: Model-1

This is simple model from reference which is given in the figure 50. In the figure, the most of the rays are meeting at 9 to 10 mm from the lens. That means, not all rays are meeting at the one particular point which is called as spherical aberration. The radius of both curved surface has 10 mm. So image plane would be at 10 mm. The least area of confusion occurring at around 9.4 mm where the image would be clearer than other distance. In the model, the image plane is taken at 9.40 mm where the area of confusion is least than other distance. The refractive index of the medium is 1.5. Model applied these same optical system with same parameters with N-BK7 medium. N-BK7 has refractive index around 1.506687 at 25 celsius and pressure 101330 Pa. From that model gave the plot between final image height vs initial height for angle 0 for all rays from the pupil.

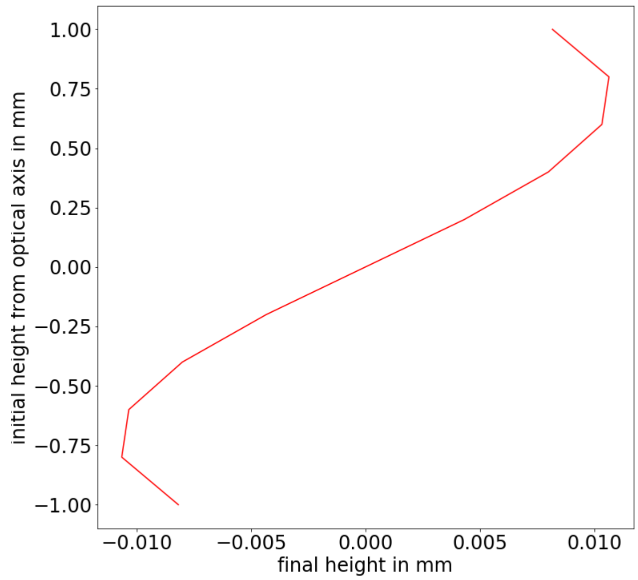

Figure 51: Spherical aberration : Final height of image vs Initial height at angle =0

Because of the refractive index for N-BK7 has slightly greater than 1.5,image plane has been taken at distance 9 mm. The plot is given in figure 51. The height of the image becomes very small compared to the initial. This is also agrees with the reference figure given in 50. Another plot is OPD vs initial height. Here initial height is taken between +1 mm to -1mm with respect to optical axis. So OPD would be in order of 0.01 mm.

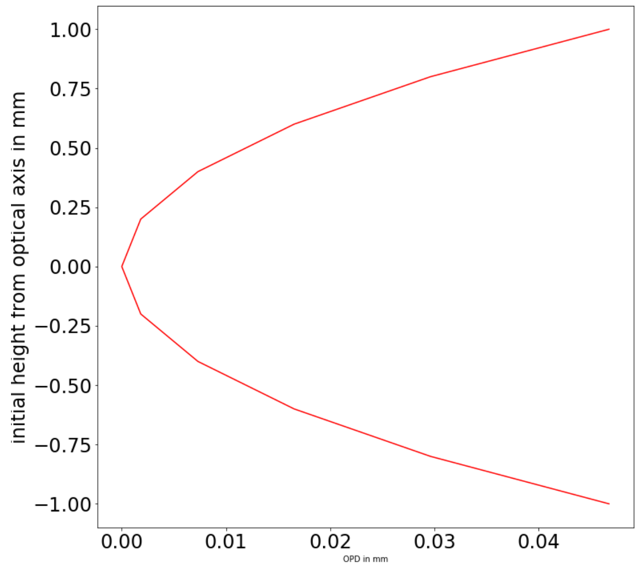

Figure 52: Spherical aberration : OPD vs initial height

In the figure 52, The OPD is in mm and has symmetric values because of the parallel rays. This is for spherical aberration. For coma, the incident angle should be around 0.1 degree.

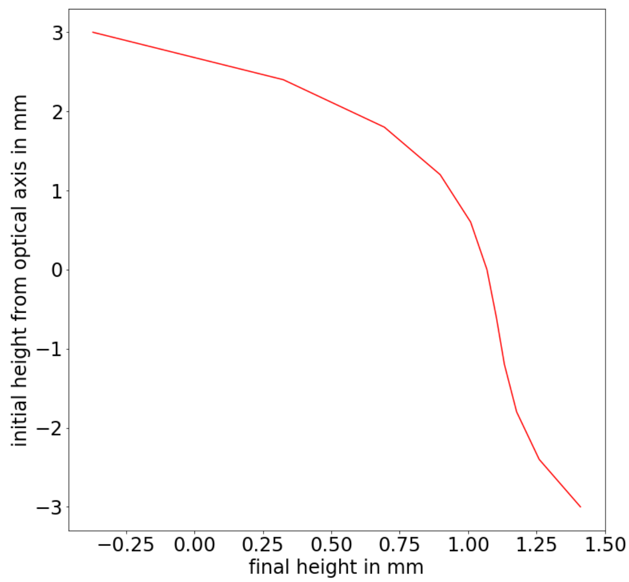

Figure 53: Coma aberration : Final height vs Initial height

Here the incident angle is 4 degree and model gave the plot between Final height vs Initial height in the figure 53.In the figure the final height for all rays between are in positive direction above the optical axis which is also occurred in the coma aberration. Also image plane is taken as 10 mm form middle point of the optical system because where the coma aberration looks clearer than other distance. And the pupil size is 3 mm to -3 mm.

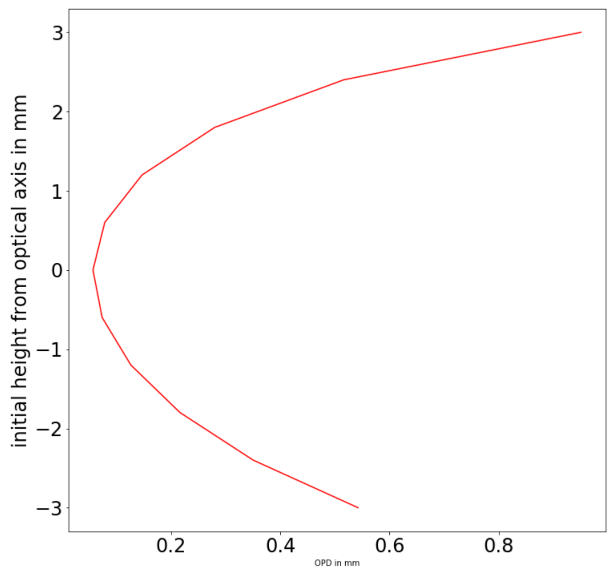

Figure 54: Coma aberration : OPD vs initial height

In the figure 54, the plot is different from spherical aberration. The OPD tends to become linear as the angle of incident increases. Here the reference ray is taken as one which is passing through the center of the lens parallel to the optical axis. So these are all calculated data for given model which is consenting with reference figure. Now we can make arbitrary optical system to find the final image height and OPD from this model. The code is given in footnotes.

The aberration could be found by using OPD and appropriate equations. We can find the phase difference from OPD and its wave-front error which is aberration of the wave. This work is leave it for future work to expanding it further.

6 Applications

The applications of the data is very useful to find dispersion relation for given glass type. As mentioned in the section 4.1.1, the plot between refractive index vs wavelength gives the dispersion relation of N-BK7. There are so many catalogues from manufacturer which are useful to find the dispersion relation for given glass type. We can also find the chromatic aberration from the dispersion relation. Chromatic aberration occurs when different wavelengths of light focus on different points.

Thermal analysis of the glass gives the proper idea about how temperature affects the refractive index of the medium as shown in the section 4.1.1 for N-BK7 and other glass types. In astronomy, telescope uses the different glass types for imaging process. Telescopes are at higher altitude from ground where the temperature and pressure are different from ground conditions. Glass made with reference temperature and pressure which are given in catalogues during manufacturing of it. Also the data and constants in catalogues are at reference temperature. To find the refractive index at given temperature and pressure we need thermal analysis of it. At higher altitudes, telescope's glass has very different conditions than ground. From thermal analysis of glass, we can find the refractive index at given conditions. Telescope needs precise image formation without aberrations. To reduce the aberrations, we need to get precise dispersion relation which is changing with temperature and pressure. SO thermal analysis of glass is very useful in telescopes.

To reduce the aberrations of the telescope or optical systems, aspheric surfaces are very useful tool. Hyperbolic curved surface has less spherical aberration than spherical curved surface. From this we can say that, to reduce higher order aberrations, we need to define the arbitrary aspheric surface and optical system which can reduce the aberrations. Aspherics surfaces and its radius at each point is defined in section 5.3.1 and 5.3.2 .

Ray tracing method is very reliable application for modelling of optical systems like telescope to get precise image with reduction of higher order aberrations. Ray tracing method only use the simple geometry with given aspheric surface. Using geometry, we can find the optical path length, optical path difference, final image height at image plane. These are sufficient outputs for analysing the image formation. From OPD, we can find the phase difference of the wave front. So these are the main applications of the each calculated data from this project. Below section includes the some useful references to get these data. But that work is leave it for future work.

7 Conclusion

From the results and its verification, The data calculated from model is very close to the reference data[3][6]. From that we can find dispersion relation for different glass-types and its thermal analysis like how temperature and pressure affects the refractive index of the medium. The error from the model is around $10^{-}10\%$ for dispersion relation and error for thermal analysis is around 0.001% . With that agreement, The model can find the refractive index of glass at different wavelength with given conditions like temperature and pressure.

Second model is to find OPD and aberrations.This model is nearly agree with reference simulation [12]. Spherical and coma aberration's OPD has been found in the model and its final image at image plane. This work could be done in future to find aberrations in the system.

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A Gitlab and Git commands

Gitlab is best platform for development of project when multiple users are involved. Gitlab provides separate space for project. It also provides privacy levels from private to public. There is one SSH key which provides users to use whenever they want to log in into server. SSH key is generated through Git bash command line. RSA,ED25519 are types of SSH keys which are supported by Gitlab. To generate new RSA key,

ssh-keygen -o -t rsa -b 4096 -C ”email@example.com”

is command. For ED25529,

ssh-keygen -t ed25519 -C ”email@example.com”

is command. Once command is entered in Git bash, key is generated automatically. Key should be copied to Gitlab account by command,

cat /.ssh/id_ed25519.pub — clip

After making account in gitlab, we can make first new project and then add directory and files from computer and edit on the platform. If there are so many users in project then we can make group in gitlab and add them with selected opacity of project. There two types of repository (1) local repository which means that data from any computer client or PC. We can extract data from local repository to gitlab server with commands. This gives the second terminology called as (2) Remote repository which means that data on server or gitlab account. If someone from remotly change the data of gitlab in group project then we can update our local repository easily with one line command in git bash called as **pull**,

git pull

Before doing this, configuration on gitlab is very important. First to check git has installed or not by command,

git –version

In output it'll reply about version of git bash command. Now set the email id and username for account for connection between remote and local repository,

git config −−global user.name ”USER_NAME”
git config −−global user.email ”email_id@gmail.com”

For verification for email and user name

git config −−global user.name
git config −−global user.email

First initializing the git through,

git init

To start working in locally, we have to clone whole repository in remote to local repository. This is done through, **Clone** is done through two ways,
(1) via link

git clone https://gitlab.com/gitlab-org/gitlab-ce.git

(2) via SSH key link

git clone git@gitlab.com:gitlab-org/gitlab-ce.git

This information is given in the gitlab remote repository data. There are branches in gitlab which is useful feature in the gitlab. With different branches, different users could work on different **branches** and after editing in the files and directory, they can **merge** the files and make unified file. Also merge undo request is also exist in the gitlab. To go in different branches,

git checkout -b ”branch name”

Whenever someone changes the details and path, we can check easily by,

git status

We can also edit the files from local to remote server by two lines. First we have to specify filename which has to be modified and gitlab would put that file into one imaginary server space because if we want to go back and undo to update that file then gitlab gives two options. After selecting files, we can **commit** the command by,

git add ”file name” git commit -m ”comments”

We can also write comments about changes for every date and time to check progress. Comments are written in commit part. We can add all changes at instant through . sign.

git add .

Now we have to just **push** the data from local repository to remote repository (here gitlab) by,

git push -u origin master

Here **master** is main branch when we had made the account and it is central part of works. We can also change data from different branches by change the path. There is another command which gives the all history of our commits inside local to remote or remote to local.

git log

There is also undo option for returning to unmodified files which have committed to remote server. We can undo this by,

git revert ”number”

number is code which is given to that commit.

Sometimes there is slightly changes in code or files data which very subtle work to find that. We can find that difference or changes by,

git diff

But the output of **diff** is not understandable so we can download the editor like **KDiff** in the PC and compare at most three files simultaneously in that editor. So this is basics about the gitlab and git bash command line.

B Creating virtual environment for model in anaconda

To extract data from catalogs with python, **spyder** is good platform for scientific developments. We have to download anaconda navigator. From anaconda command prompt, environment is good thing to start new project. New environment provides new separate working space which does not affect the other environment or base space. In this project, ”optics” as new virtual environment has been created to work in it. We can download different packages for it. For this project **optical properties of glasses**, I started new programming in spyder editor which is widely useful in scientific tools. To make new environment, we have to just type

conda create -n optics python=3.7 anaconda

in anaconda command prompt. Once the environment is created, activation of it is done by,

activate optics

New packages could be downloaded with command,

conda install -n optics package-name

In ”optics” virtual environment, I have downloaded **numpy, matplotlib, os, scipy, prettytables etc** in the ”optics” virtual environment with same command. We can import the packages by its requirement and name as short form for further using of packages. This packages contain well defined functions. To import the packages from environment by typing,

import numpy as np

to plot graphs, we have to call matplotlib package differently,

import matplotlib.pyplot as plt

C Extracting data from catalogs by python

Extracting data from catalogs is done by looping process. We have to check each and every line in catalogs to find specific glass type and its details. First we have to get input data from user for glass name with exact name. After using matplotlib package we can find the plot of the refractive corresponding to different wavelengths. Same goes for all analysis in the catalogs. There are thermal coefficients given which are useful in the thermal analysis of the glass type. Thermal coefficients are given in the catalogues. By same process, we can get the plots for thermal analysis.

List of Figures

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