

# Optical properties of glasses

Ridhesh Goti

June 2019

## 1 Introduction

## 2 Gitlab and Git commands

Gitlab is best platform for development of project when multiple users are involved. Gitlab provides separate space for project. It also provides privacy levels from private to public. There is one SSH key which provides users to use whenever they want to log in into server. SSH key is generated through Git bash command line. RSA,ED25519 are types of SSH keys which are supported by Gitlab. To generate new RSA key,

```
ssh - keygen - o - trsa - b4096 - C"email@example.com"
```

is command. For ED25519,

```
ssh - keygen - ted25519 - C"email@example.com"
```

is command. Once command is entered in Git bash, key is generated automatically. Key should be copied to Gitlab account by command,

```
cat /.ssh/id_ed25519.pub|clip
```

After making account in gitlab, we can make first new project and then add directory and files from computer and edit on the platform. If there are so many users in project then we can make group in gitlab and add them with selected opacity of project. There two types of repository (1) local repository which means that data from any computer client or PC. We can extract data from local repository to gitlab server with commands. This gives the second terminology called as (2) Remote repository which means that data on server or gitlab account. If someone from remotly change the data of gitlab in group project then we can update our local repository easily with one line command in git bash called as **pull**,

```
git pull
```

Before doing this, configuration on gitlab is very important. First to check git has installed or not by command,

`git --version`

In output it'll reply about version of git bash command. Now set the email id and username for account for connection between remote and local repository,

```
git config --global user.name "USER_NAME"
git config --global user.email "email_id@gmail.com"
```

For verification for email and user name

```
git config --global user.name
git config --global user.email
```

First initializing the git through,

`git init`

To start working in locally, we have to clone whole repository in remote to local repository. This is done through, **Clone** is done through two ways,  
(1) via link

```
git clone https://gitlab.com/gitlab-org/gitlab-ce.git
```

(2) via SSH key link

```
git clone git@gitlab.com:gitlab-org/gitlab-ce.git
```

This information is given in the gitlab remote repository data. There are branches in gitlab which is useful feature in the gitlab. With different branches, different users could work on different **branches** and after editing in the files and directory, they can **merge** the files and make unified file. Also merge undo request is also exist in the gitlab. To go in different branches,

```
git checkout -b "branch name"
```

Whenever someone changes the details and path, we can check easily by,

`git status`

We can also edit the files from local to remote server by two lines. First we have to specify filename which has to be modified and gitlab would put that file into one imaginary server space because if we want to go back and undo to update that file then gitlab gives two options. After selecting files, we can **commit** the command by,

```
git add "file name" git commit -m "comments"
```

We can also write comments about changes for every date and time to check progress. Comments are written in commit part. We can add all changes at instant through . sign.

`git add .`

Now we have to just **push** the data from local repository to remote repository (here gitlab) by,

```
git push -u origin master
```

Here **master** is main branch when we had made the account and it is central part of works. We can also change data from different branches by change the path. There is another command which gives the all history of our commits inside local to remote or remote to local.

```
git log
```

There is also undo option for returning to unmodified files which have committed to remote server. We can undo this by,

```
git revert "number"
```

number is code which is given to that commit.

Sometimes there is slightly changes in code or files data which very subtle work to find that. We can find that difference or changes by,

```
git diff
```

But the output of **diff** is not understandable so we can download the editor like **KDiff** in the PC and compare at most three files simultaneously in that editor. So this is basics about the gitlab and git bash command line.

## 3 Aberrations in glass

There are so many aberrations in glass. From seidel aberration theory there are total five aberrations. There is another aberration called as chromatic aberration.

### 3.1 Chromatic aberration

Different wavelengths have different refractive index in glass and different refractive index means it has different focusing power for same glass type. Therefore, it will not give white image of the object. This aberration contains all wavelengths and make image at different focuses. Here in 1 is without chromatic aberration and all wavelengths concentrate on one focus point which means that glass has low dispersion. Therefore, refractive index does not change very much with different wavelengths. And 2 is for chromatic aberration where all wavelengths have different behaviour in glass therefore aberration occur in the glass.

### 3.2 Monochromatic aberration

There are other aberrations with monochromatic light.

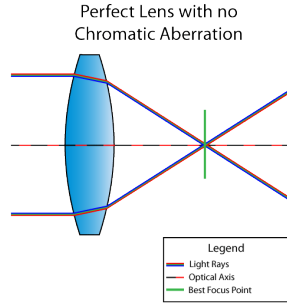


Figure 1: Without chromatic aberration

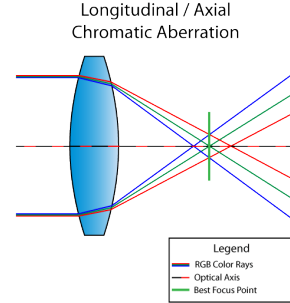


Figure 2: With chromatic aberration

### 3.2.1 Spherical aberration

In this aberration, one monochromatic light is sent to the lens with parallel to optical axis of the lens. The rays near to the optical axis get bend less than rays far from the optical axis. In the 5, to reduce spherical aberration, lens is made with extra optical path which gives the central rays more time to pass through glass and bend more.

### 3.2.2 Coma aberration

An off-axis effect which appears when a bundle of incident rays all make the same angle with respect to the optical axis (source at infinite). Rays are brought to a focus at different points on the focal plane found in lenses with large spherical aberrations an off-axis object produces a comet-shaped image. In the 6 the shape of the image is like comet because of focusing on focal plane at different locations. Comatic aberration is similar to the spherical aberration but this is off axis rather than on optical axis. the peripheral rays produce the smallest image (least magnification) and the coma aberration sign is said to be negative. In contrast, when the peripheral rays are focused further down the axis and produce a much larger image (greater magnification), the aberration is termed positive. To reduce the coma aberration, peripheral rays should be removed.

### 3.2.3 Astigmatism

Astigmatism occurs when there are not spherical curvature. That means curvature in one plane is different than perpendicular plane of it. In the 7, aberration is astigmatism. The black-line in the figure is optical axis. And lens is viewed from some angle. aberration of astigmatism by supposing that the lens has a different focal length in one plane than in the other. Due to the different focal plane in vertical and horizontal, sphere looks like circle or ellipse shape. Although different radii of curvature in different planes is not the usual cause of astigmatism, there is an exception - namely, the human eye. If the radii of

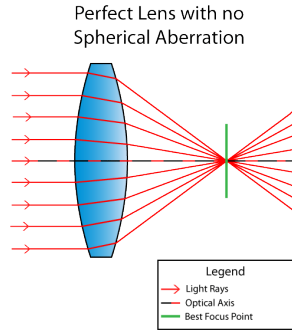


Figure 3: Without chromatic aberration

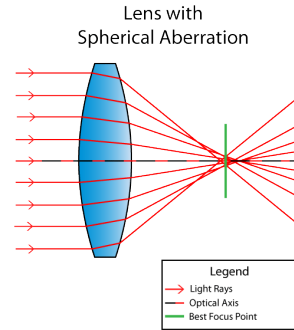


Figure 4: With chromatic aberration

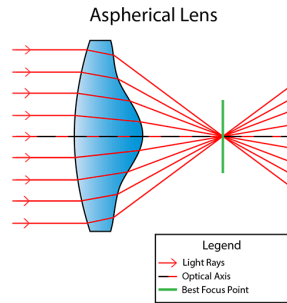


Figure 5: Without chromatic aberration

curvature of the cornea, or of the lens, is different in different planes, then the image on the retina will be astigmatic even on-axis.

### 3.2.4 Curvature of field

Suppose the spherical, coma, astigmatic aberration is resolved but there will be another aberration called as curvature of field. This occurs due to focusing of light at different points on one curvature focal curve. In the 8, all focused points are on one curvature plane instead of plane. Focal surface also called as **petzval** surface. One effective way of dealing with this problem, particularly if detector is a flexible film, is to shape the film-holder so that the film fits along the Petzval surface.

### 3.2.5 Distortion

The magnification of an image is image distance divided by object distance, and image distance is different off-axis than on-axis, so the image magnification varies with distance from the axis. If there square object then its distortion look

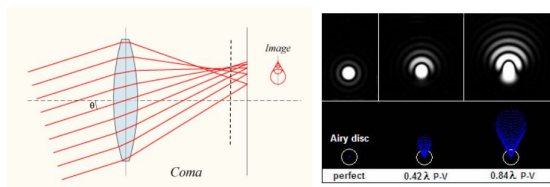


Figure 6: Coma aberration

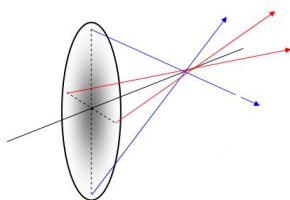


Figure 7: Astigmatism

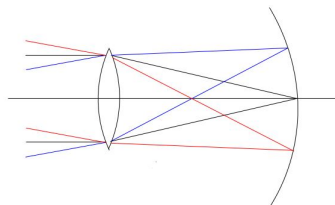


Figure 8: Curvature of field



Figure 9: Pincushion shape



Figure 10: Barrel shape

like in 9 and 10. Here two figures are different depends on certain conditions. Off-axis, the image distance is less than the object distance, so the magnification is less off-axis than on-axis. Barrel distortion results. Off-axis, the image distance is greater than the object distance, so the magnification is greater off-axis than on-axis. Pincushion distortion results.

## 4 Dispersion of light in glasses

Refraction is bend of the light through out the medium and it depends on the material or glass type. Refractive index is calculated by ratio of speed of light in medium and speed of light in vacuum space. In water, Refractive index is 1.33 means that light speed slows 33 % of original value. Sometimes Refractive index depends on the wavelength of the light. Different wavelengths have different refractive index and bend with different angles. For chromatic rays, all colors are refracted with different directions because of the phenomena. This called as **dispersion of light** in medium. First approximation for relation is made by cauchy approximation. Cauchy expression is given by following equation,

$$n(\lambda) = B + \frac{C}{\lambda^2} \quad (1)$$

In this (1), B and C are constants and depend on the material type. Formation of rainbow and other phenomena are due to the dispersion.

Light being electromagnetic oscillations, the different colours have different wavelengths and different frequencies in vacuum for all of them move with the same speed c. Whenever light enters a dielectric medium this separation happens and the phenomenon is known as dispersion. There are more **cauchy expression** for dispersion,

$$n(\lambda) = B + \frac{C}{\lambda^2} + \frac{D}{\lambda^4} \quad (2)$$

These all constants are determined by experimentally and give the proper relation between refractive index and wavelength. The first two terms would suffice to give an accurate value of  $n$ . The derivative of (2) is given by,

$$\frac{dn}{d\lambda} = -\frac{B}{\lambda^3} \quad (3)$$

This is **normal dispersion** behaviour for light. But this behaviour is valid only for certain wavelength range. Outside the range there is absorption region

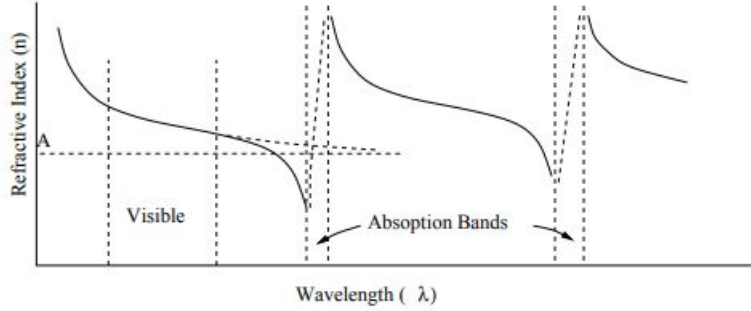


Figure 11: Anomalous dispersion

where refractive has not valid value . refractive index suddenly decreases very fast and does not obey the Cauchy's law, if we approach the absorption region. Further increasing the wavelength once again refractive index becomes large. Again the behaviour is quite similar to the visible region for the increase in wavelength. This type of behaviour known as **anomalous dispersion**. There are many absorption region and with same repetition of pattern. The refractive index couldn't be found in absorption region because substance absorbs that particular wavelength completely because of the resonance effects in electronic spectra and vibration spectra of material. +

There are important facts about the normal dispersion is,

(1) refractive index of substance increases with decreasing of wavelength (2) The rate of increase becomes greater at shorter wavelength. In the 11, there is band between visible and infrared region with following cachy expression. To derive the expression for dispersion, sellmeier assumed that all elastically bound particles in the medium oscillate with a natural frequency  $\omega_0$  which correspond to a wavelength  $\lambda_0$  in the vacuum. The final equation is looks like,

$$n^2 = 1 + \frac{A\lambda^2}{\lambda^2 - \lambda_0^2} \quad (4)$$

In (4), when  $\lambda = \lambda_0$ , the trend would diverge and give the same result as in figure 11. To explain many absorption bands, we have to assume different species of electrons with different natural frequencies  $\omega_j$  corresponding to wavelengths  $\lambda_j$  in the substance and then

$$n^2 = 1 + \sum_j \frac{A_j \lambda^2}{\lambda^2 - \lambda_j^2} \quad (5)$$

## 5 Elementary theory of dispersion

Effect of electromagnetic field on dielectric medium will be given by electromagnetic theory,

$$D = \epsilon E \quad (6)$$



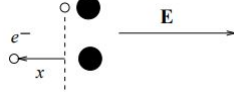


Figure 12: Oscillation of electron from mean position

. When there is external electric field in the dielectric medium the total displacement is,

$$D = \epsilon_0 E + P = \epsilon_0(1 + \chi)E \quad (7)$$

. The factor  $\chi$  is called the electric susceptibility of the substance and the factor  $(1 + \chi) = \epsilon_r$  is nothing but the relative permittivity. And vector  $P$  is polarization vector. When an electric field is applied to a substance electrons of the molecules of that substance are displaced from their mean position as shown in the fig 12. Electron would not leave molecule but oscillate with that frequency and also generate dipole moment inside medium. Polarization per unit volume is given by,

$$P = -Nex \quad (8)$$

In equation (8),  $N$  is total number of electrons and  $x$  is displacement of electron. Also electric field is in  $x$ - direction and have negative sign because of electron negative charge. Here we are ignoring oscillation of positive ions and electron will oscillate with certain frequency  $\omega_0$ . But light has oscillate electric field then displacement would also oscillate with that time period. Here dampening force comes in equation because of the shielding effects, other atoms effects on electron which try to bound more in atom. For forced oscillation differential equation is given by ,

$$\ddot{x} + r\dot{x} + \omega_0^2 x = -\frac{eE}{m} \quad (9)$$

similarly we can write equation (9) as,

$$\ddot{P} + r\dot{P} + \omega_0^2 P = -\frac{Ne^2 E}{m} \quad (10)$$

From maxwell's equations,  $\nabla \times E = -\dot{B}$  and  $\nabla \times H = \dot{D}$  ,  $\nabla \cdot E = 0$

$$\begin{aligned} \nabla \times \dot{B} &= \mu_0 \ddot{D} \\ \nabla \times (\nabla \times E) &= -\mu_0 \epsilon_0 \ddot{E} - \mu_0 \ddot{P} \\ \nabla^2 \cdot E &= \mu_0 \epsilon_0 \ddot{E} + \mu_0 \ddot{P} \end{aligned}$$

From all these equations (10) we can get,

$$\ddot{P} = \frac{1}{\mu_0} (\nabla^2 \cdot E - \frac{1}{c^2} \ddot{E}) \quad (11)$$

Differentiating equation (10) w.r.t time,

$$(\frac{\partial^2}{\partial t^2} + r\frac{\partial}{\partial t} + \omega_0^2) \ddot{P} = \frac{Ne^2}{m} \ddot{E} \quad (12)$$

Using equation (11) eliminating  $P$  in equation (12),

$$\left(\frac{\partial^2}{\partial t^2} + r\frac{\partial}{\partial t} + \omega_0^2\right)(\nabla^2 \cdot E - \frac{1}{c^2}\ddot{E}) = \mu_0 \frac{Ne^2}{m} \ddot{E} \quad (13)$$

when a plane electromagnetic wave polarised along the x direction and  $E$  will become  $E_0 = e^{i(kz - \omega t)}$  and put in equation (13),

$$(\omega^2 - ir\omega + \omega_0^2)(-k^2 + \frac{\omega}{c^2}) = -\frac{\mu_0 Ne^2 \omega^2}{m} \quad (14)$$

Further arrangements in equation (14),

$$k^2 = \frac{\omega^2}{c^2} \left(1 + \frac{\mu_0 c^2 Ne^2}{m(\omega_0^2 - ir\omega - \omega^2)}\right) \quad (15)$$

In the equation (15)  $v = \frac{\omega}{k}$  is phase velocity and  $n = \frac{c}{v}$  is refractive index of the medium. So equation will become,

$$n^2 = \left(1 + \frac{\mu_0 c^2 Ne^2}{m(\omega_0^2 - ir\omega - \omega^2)}\right) \quad (16)$$

For refractive index being as real,  $\omega_0^2 - \omega^2 \gg r\omega$  which means that dampening is very slight,

$$n^2 = \left(1 + \frac{\mu_0 c^2 Ne^2}{m(\omega_0^2 - \omega^2)}\right) \quad (17)$$

relation between wavelength and angular frequency is  $\lambda = \frac{2\pi c}{\omega}$  then

$$n^2 = \left(1 + \frac{\mu_0 Ne^2 \lambda_0^2 \lambda^2}{4\pi^2 m(\lambda^2 - \lambda_0^2)}\right) \quad (18)$$

and finally by assuming constants equation will look like sellmeier formula for dispersion.

## 5.1 Abbe number

Abbe number is measurement of the strength of dispersion by transparent material. Low abbe number means that dispersion is high vice versa. Abbe number of the material is defined by equation,

$$V_D = \frac{n_D - 1}{n_F - n_C} \quad (19)$$

In equation (19)  $n_D, n_F$ , and  $n_C$  are refractive index of material at wavelengths of Fraunhofer D, F and C lines (589.3 nm, 486.1 nm, 656.3 nm respectively). Abbe number are useful number for classifying the different glasses. For example, **flint glass** has abbe number  $V < 50$  and **crown glass** has  $V > 50$ . Typical values of  $V$  range from around 20 for very dense flint glass, around 30 for **polycarbonate**

**plastics**, and up to 65 for very light crown glass, and up to 85 for **fluor-crown glass**.

Abbe numbers are only a useful measure of dispersion for visible light, and for other wavelengths, or for higher precision work, the group velocity dispersion is used. An Abbe diagram is produced by plotting the Abbe number of a material versus its refractive index. Glasses can then be categorised by their composition and position on the diagram.

## 6 ZEMAX - glass catalogs

### 6.1 Use of catalogs

In dispersion theory, different wavelengths have different refractive index. There are experimentally verified constants values for all dispersion formula and give the relation between refractive index and wavelengths. There are some catalogs made by manufacturer of glass which provides the details of the glass. From data we can extract the data of glass and its behaviour.

### 6.2 Example of catalogs

These catalogs provides the details of each glass type with its properties like Abbe number, Thermal coefficients, Transmission and Thermal expansion etc. There are more catalogs for different glasses depend on the manufacturing companies of that glass type. For example, **schott** is catalog name and each glasses in that catalog made by schott company. This catalog contains every details for analysis of glass type. There are many catalogs, **ohara**, **hoya**, **birefringent**, **hikari**, **sumita**, **misc**, **infrared** etc. Each catalogs contain the details of specific glass type. Application of this different glass types require for different fields like in telescope, glasses in spectacle, windows etc. For that different catalogs are required.

### 6.3 Extension of catalogs

Extension of the catalog is **AGF** which is basically ANSI file. It could open with notepad with texts. ZEMAX software uses this catalogs as AGF and converts into BGF (Binary) file for more efficiency of processing. Here We are using AGF file as texts and extract data from it through **python** platform. We can edit the catalogs as per latest data.

### 6.4 Availability

Catalogs are easily available on online. Each manufacturer provide the catalog of its glass types. We can download the catalogs by typing **catalogs-name.agf**. Here is link for ohara catalog,

<https://www.oharacorp.com/catalog.html>

## 6.5 How to read catalogs

In the catalog, first line has format like this,

$$CC < catalogname >$$

This tells about the name of catalog. Second line of the AGF file would be,

$$NM < glassname > < dispersionformula - number > < MIL > < N_d > < V_d > < Excludesub > < Status > < Meltfrequency >$$

In this line MIL represents the reference number for each glass type.  $N_d$  is refractive index of glass at wavelength 0.587 micrometers and it is just reference number.  $V_d$  is also just reference number and it is abbe number of glass. **Exclude Sub** represents that if it is checked then that glass type is not selected for conversion from model to real glass in application. Exclude sub has only two values, 1 for checked and 0 for unchecked. Status indicates the availability of that glass type in market. There four types of status, **Standard, Preferred, Obsolete, Special**. Standard glass generally available in purchase. Preferred glasses are frequently melted and more likely available on demand. Obsolete glasses are no longer available but may be available on demand. Melt frequency indicates that how frequently glass is melted during manufacturing of that glass. 1 represents the high frequency, 2 represents very often and 5 represents infrequently. Glass name part give the glass type for example N-BK7, YGH52 etc. Dispersion formula number is number given for each dispersion formulas. There are total 13 formulas and each formulas has its corresponding number. The dispersion formula number is 1 for Schott, 2 for Sellmeier 1, 3 for Herzberger, 4 for Sellmeier 2, 5 for Conrady, 6 for Sellmeier 3, 7 for Handbook of Optics 1, 8 for Handbook of Optics 2, 9 for Sellmeier 4, 10 for Extended, 11 for Sellmeier 5, and 12 for Extended 2 and 13 for Extended 3. Third line of the file is

$$GC < individualglasscomment >$$

This line gives the specifications or any extra comments of that glass type. Fourth line is

$$ED < TCE(-30to70) > < TCE(100to300) > < density > < dPgf > < ignorethermalexpansion >$$

TCE is thermal coefficient expansion and it is used for modelling of linear expansion of glass with temperature. For different temperature ranges, TCE have different values and therefore there are two columns of TCE one for -30 to 70 and second for 100 to 300. Density is density of glass type. dPgf gives the deviation of relative partial dispersion from normal D line. Ignore thermal expansion is useful for non solids materials. For example, gases and liquids have not thermal expansion but its mounted material would solid. So thermal modelling is different for solids and gases. For gases and liquids, edge effects of mounted material is consider. Fifth line is

$$CD < dispersioncoefficients1 - 10 >$$

This gives the all required coefficients of given dispersion formula. Sixth line is

$$TD < D0 > < D1 > < D2 > < E0 > < E1 > < Ltk > < Temp >$$

TD is useful for thermal analysis of glass for example how refractive index of particular wavelength is changed with temperature. Seventh line is

$$OD < relcost > < CR > < FR > < SR > < AR > < PR >$$

Relative cost is relatively to N-BK7 glass type. **CR** stands for climate resistance, **FR** is stain resistance, **SR** is acid resistance, **AR** is alkali resistance, **PR** is phosphate resistance. Eighth line is

$$LD < minlamda > < maxlamda >$$

This gives the validity range of wavelengths which could give proper answer to dispersion formula. Last line is

$$IT < lamda > < transmission > < thickness >$$

This represents the transmissivity of particular wavelength with given thickness of glass type. In catalog, if there is -1 in data that means that data is not available.

## 6.6 Dispersion formulas in glass catalogs

There are total 13 dispersion formulas which are used by different different catalogs for each glass types. With different dispersion formula, refractive index of each wavelength in the light will disperse differently and rise the aberrations. Here is first formula, **(1)schott**

$$n^2 = a_0 + a_1\lambda^2 + a_2\lambda^{-2} + a_3\lambda^{-4} + a_4\lambda^{-6} + a_5\lambda^{-8}$$

**(2)sellmeier1**

$$n^2 - 1 = \frac{K_1\lambda^2}{\lambda^2 - L_1} + \frac{K_2\lambda^2}{\lambda^2 - L_2} + \frac{K_3\lambda^2}{\lambda^2 - L_3}$$

**(3)sellmeier2**

$$n^2 - 1 = A + \frac{B_1\lambda^2}{\lambda^2 - \lambda_1^2} + \frac{B_2}{\lambda^2 - \lambda_2^2}$$

**(4)sellmeier3**

$$n^2 - 1 = \frac{K_1\lambda^2}{\lambda^2 - L_1} + \frac{K_2\lambda^2}{\lambda^2 - L_2} + \frac{K_3\lambda^2}{\lambda^2 - L_3} + \frac{K_4\lambda^2}{\lambda^2 - L_4}$$

**(5)sellmeier4**

$$n^2 - 1 = A + \frac{B\lambda^2}{\lambda^2 - C} + \frac{D\lambda^2}{\lambda^2 - E}$$

**(6)sellmeier5**

$$n^2 - 1 = \frac{K_1\lambda^2}{\lambda^2 - L_1} + \frac{K_2\lambda^2}{\lambda^2 - L_2} + \frac{K_3\lambda^2}{\lambda^2 - L_3} + \frac{K_4\lambda^2}{\lambda^2 - L_4} + \frac{K_5\lambda^2}{\lambda^2 - L_5}$$

(7)herzberger

$$n = A + BL + CL^2 + D\lambda^2 + E\lambda^4 + F\lambda^6$$

$$L = \frac{1}{\lambda^2 - 0.0028}$$

(8)conrady

$$n = n_0 + \frac{A}{\lambda} + \frac{B}{\lambda^{3.5}}$$

(9)Handbook of optics 1

$$n^2 = A + \frac{B}{\lambda^2 - C} - D\lambda^2$$

(10)Handbook of optics 2

$$n^2 = A + \frac{B\lambda^2}{\lambda^2 - C} - D\lambda^2$$

(11)Extended 1

$$n^2 = a_0 + a_1\lambda^2 + a_2\lambda^{-2} + a_3\lambda^{-4} + a_4\lambda^{-6} + a_5\lambda^{-8} + a_6\lambda^{-10} + a_7\lambda^{-12}$$

(12)Extended 2

$$n^2 = a_0 + a_1\lambda^2 + a_2\lambda^{-2} + a_3\lambda^{-4} + a_4\lambda^{-6} + a_5\lambda^{-8} + a_6\lambda^4 + a_7\lambda^6$$

(13)Extended 3

$$n^2 = a_0 + a_1\lambda^2 + a_2\lambda^4 + a_3\lambda^{-2} + a_4\lambda^{-4} + a_5\lambda^{-6} + a_6\lambda^{-8} + a_7\lambda^{-10} + a_8\lambda^{-12}$$

These are the total 13 formulas for dispersion relation of the glass type. All the corresponding constants are given in the respective catalogs. From these formulas we can find the relation between the refractive index and different wavelengths.

## 7 configure optics as new virtual environment and packages

To extract data from catalogs with python, **spyder** is good platform for scientific developments. We have to download anaconda navigator. From anaconda command prompt, environment is good thing to start new project. New environment provides new separate working space which does not affect the other environment or base space. We can download different packages for it. For this project **optical properties of glasses**, I made the optics environment in anaconda and start new programming in spyder editor which is widely useful in scientific tools. To make new environment, we have to just type

**conda create -n optics python=3.7 anaconda**

in anaconda command prompt. Once the environment is created, activation of it is done by,

### activate optics

New packages could be downloaded with command,

**conda install -n optics package-name**

In my optics virtual environment, I have downloaded **numpy**, **matplotlib**, **os**, **scipy** etc in the optics virtual environment with same command. We can import the packages by its requirement and name as short form for further using of packages. This packages contain well defined functions. To import the packages from environment by typing,

**import numpy as np**

to plot graphs, we have to call matplotlib package differently,

**import matplotlib.pyplot as plt**

## 8 Extract data from catalogs by python

Extracting data from catalogs is done by looping process. We have to check each and every line in catalogs to find specific glass type and its details. First we have to get input data from user for glass name with exact code. After using matplotlib package we can find the plot of the refractive corresponding to different wavelengths. Same goes for all analysis in the catalogs. There are thermal coefficients given which are useful in the thermal analysis of the glass type.

## 9 Thermal coefficients formulas and theory

The refractive index of optical glasses change with temperature, the extend of which depending on the glass type and on the wavelength. It also depends on pressure of surroundings so that would give the two part of refractive index **(1) Absolute refractive index** and **(2) Relative refractive index**. We can find the rate of change of refractive index from temperature range  $-100\text{ }^{\circ}\text{C}$  to  $140\text{ }^{\circ}\text{C}$ . We need thermal coefficients for calculation of changes of refractive index with temperature. These coefficients are given in aforementioned catalogs. In the section 6.5, TD line gives the details of thermal coefficients. there are special glasses with negative temperature coefficients in an optical system can help to keep wave front deformations caused by temperature changes on a minimum level. Such glasses are called **athermal glasses**.

From fundamental equation of refractive index relative to vacuum is given by generally,

$$n_{abs}^2(\lambda) = 1 + a \cdot \frac{N}{V} \cdot f \cdot \frac{\lambda^2 \cdot \lambda_0^2}{\lambda^2 - \lambda_0^2} \quad (20)$$

In the equation (20),  $n_{abs}^2(\lambda)$  is refractive index in vacuum at certain temperature. Rest of are constants depending on glass type. Differentiating this

equation with respect to temperature will give simplify form of the derivative equation is,

$$\frac{dn_{abs}(\lambda, T)}{dT} = \frac{n^2(\lambda, T_0) - 1}{2 \cdot n(\lambda, T_0)} \cdot (D_0 + 2 \cdot D_1 \cdot \Delta T + 3 \cdot D_2 \cdot \Delta T^2 + \frac{E_0 + 2 \cdot E_1 \cdot \Delta T}{\lambda^2 - \lambda_{TK}^2}) \quad (21)$$

In the equation (21),  $T_0$  is reference temperature which is also given in the catalogs.  $T$  is variable of temperature in celcius.  $\Delta T$  is difference between variable and reference temperature. And  $D_0, D_1, D_2, E_0, E_1, \lambda_{TK}$  are constants depend on glass type. The refractive index values given in catalogs are in 101330 Pa pressure and they are called as relative refractive index. So  $n(\lambda, T_0)$  is refractive index in catalog at reference temperature  $T_0$  and pressure is 101330 Pa. The change is refractive index with temperature is given by,

$$\Delta n_{abs}(\lambda, T) = \frac{n^2(\lambda, T_0) - 1}{2 \cdot n(\lambda, T_0)} \cdot (D_0 \cdot \Delta T + D_1 \cdot \Delta T^2 + D_2 \cdot \Delta T^3 + \frac{E_0 \Delta T + E_1 \cdot \Delta T^2}{\lambda^2 - \lambda_{TK}^2}) \quad (22)$$

$$n_{abs}(\lambda, T) = n_{abs}(\lambda, T_0) + \Delta n_{abs}(\lambda, T) \quad (23)$$

$$n_{rel}(\lambda, T) = \frac{n_{abs}(\lambda, T)}{n_{air}(\lambda, T, p)} \quad (24)$$

From all equations, (22), (23), and (24) we can get relation between the refractive index and temperature with specific pressure. We can get equation for derivative of relative refractive index with temperature,

$$\frac{dn_{rel}(\lambda, T)}{dT} = \frac{\frac{dn_{abs}(\lambda, T)}{dT} - n_{rel}(\lambda, T) \cdot \frac{dn_{air}(\lambda, T, p)}{dT}}{n_{air}(\lambda, T, p)} \quad (25)$$

In equation (25), we can find  $\frac{dn_{rel}(\lambda, T)}{dT}$  by putting all appropriate values that are given. Where  $n_{air}(\lambda, T, p)$  given by,

$$n_{air}(\lambda, T, p) = 1 + \frac{n_{air}(\lambda, 15, p_0) - 1}{1 + 3.4785 \cdot 10^{-3} \cdot (T - 15)} \cdot \frac{p}{p_0} \quad (26)$$

Again In equation (26),  $n_{air}(\lambda, 15, p_0)$  is given by,

$$n_{air}(\lambda, 15, p_0) = 1 + 10^{-8} \cdot (6432.8 + \frac{2949810 \cdot \lambda^2}{146 \cdot \lambda^2 - 1} + \frac{25540 \cdot \lambda^2}{41 \cdot \lambda^2 - 1}) \quad (27)$$

Here  $p_0$  is 101330 Pa and  $p$  is variable. For equation (25), we need

$$\frac{dn_{air}(\lambda, T, p)}{dT} = -0.00367 \cdot \frac{n_{air}(\lambda, T, p) - 1}{1 + 0.00367 \cdot T} \quad (28)$$

With the help of the all coefficients and these equations we can plot between  $n$  vs  $T$ ,  $\frac{dn_{rel}(\lambda, T)}{dT}$  vs  $T$  and  $\frac{dn_{abs}(\lambda, T)}{dT}$  vs  $T$  and see how refractive index is changed with temperature and pressure effects.



From the equation (24) we can write  $n_{rel}$  as  $n_{abs}(\lambda, T) = n_{rel}(\lambda, T) \cdot n_{air}(\lambda, T, P)$ ,  
So we can write,

$$n_{rel}(\lambda, T) \cdot n_{air}(\lambda, T, P) = n_{rel}(\lambda, T_0) \cdot n_{air}(\lambda, T_0, P_0) + \Delta n_{abs}(\lambda, T) \quad (29)$$

In equation (29),  $P_0$  is one atmosphere pressure which is given in catalog. Further simplification of equation gives,

$$n_{rel}(\lambda, T) = (n_{rel}(\lambda, T_0) + \frac{\Delta n_{abs}(\lambda, T)}{n_{air}(\lambda, T_0, P_0)}) \cdot \frac{n_{air}(\lambda, T_0, P_0)}{n_{air}(\lambda, T, P)} \quad (30)$$

From equation (30), we can get refractive index of glass type at any temperature and pressure with different wavelengths.

## 10 Data extraction through python

Now we have all catalogs with details of every glass type. Once we get the coefficients of glass type for dispersion formula, we can get refractive index with different wavelengths. Plot between refractive index and wavelength gives the proper idea of its trend. Similarly, Thermal analysis is done with python commands with loops and logic process. Once we get all thermal coefficients and reference temperature, we can obtain the change of rate of derivative of absolute refractive index with temperature, the change of rate of derivative of relative refractive index with temperature and trend between refractive index and temperature. These all plots are drawn through python. In the next section there are some examples of glass type with its details of thermal and dispersion trend.

### 10.1 Glass type with details from python

Here are total four glass types with its information get from different catalogs.

- (1) N-BK7 from SCHOTT catalog
- (2) S-LAH99 from OHARA catalog
- (3) K-SFLD14 from SUMITA catalog
- (4) J-PSKH1 from HIKARI catalog

#### 10.1.1 N-BK7 from SCHOTT catalog

First of all, we will see how data from catalogs looks like, Here is data from schott catalog for N-BK7,

```
NM N-BK7 2 517642.251 1.5168 64.17 0 1
GC step 0.5 available
ED 7.100000 8.300000 2.510000 -0.000900 0
CD 1.039612120E+00 6.000698670E-03 2.317923440E-01 2.001791440E-02
1.010469450E+00 1.035606530E+02 0.000000000E+00 0.000000000E+00
```

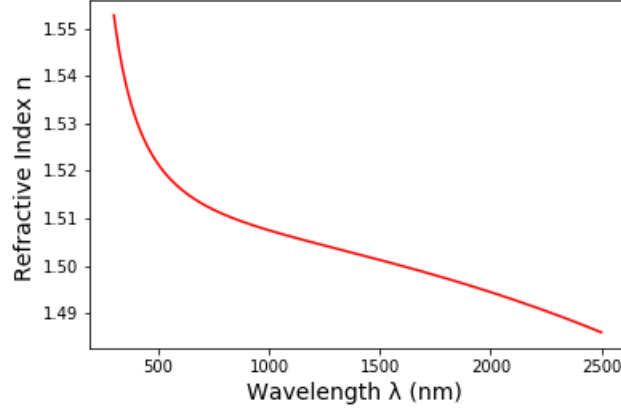


Figure 13: Refractive index vs wavelength for N-BK7 glass

```

TD 1.860000E-06 1.310000E-08 -1.370000E-11 4.340000E-07 6.270000E-10
1.700000E-01 2.000000E+01
OD 1.0000 1.0000 0.0000 1.0000 2.3000 2.3000
LD 3.00000E-01 2.50000E+00
IT 3.00000E-01 5.00000E-02 2.50000E+01
IT 3.10000E-01 2.50000E-01 2.50000E+01
.....

```

To understand what is TD, OD, LD, NM etc, you can refer section 6.5. From this data of glass, we can get information about glass through python. Here is plot between refractive index vs wavelength by using sellmeier 1 formula.

In the figure 13, the curve has negative slope and refractive index is decreasing with increasing of wavelength. Also formula is valid only for certain range of wavelengths. Here range is 300 nm to 2500 nm. Refractive index of N-BK7 at 1060 nm wavelength is 1.5066875568966998 and we can get every value by different inputs. From the equation of abbe number in section 5.1, the value for N-BK7 is 64.13319920721142. For thermal analysis, first I had found the change of rate of derivative of absolute refractive index with temperature. In the figure 14, there two curve, one for 560 nm and another for 1060 nm. The rate for 560 nm is greater than 1060 nm wavelength. Here the temperature range is -100 to 140 celcius.

Now refractive index at any given temperature and pressure is calculated from equation (30). In catalog, N-BK7 has reference temperature 20 celcius and I found the refractive index at temperature 40 celcius at pressure 103250 Pa. It gave the answer at wavelength 1060 nm around 1.5067275810747285. And at reference temperature with one atmospheric pressure is 1.5066875568966998.

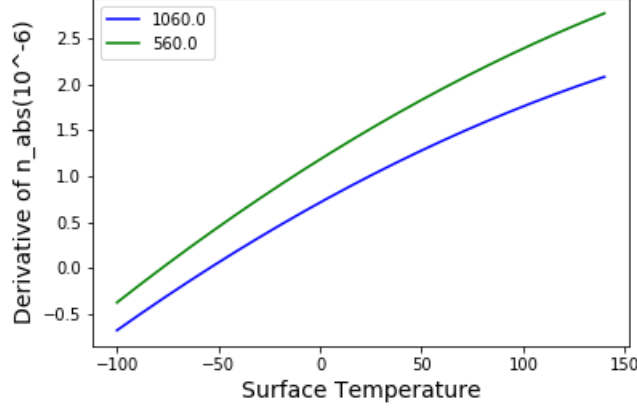


Figure 14: change of rate of derivative of absolute refractive index with temperature at 560 and 1060 nm

But if we keep temperature constant and decrease the pressure at 90000 Pa of surrounding then value is 1.5067772329259013. As we can see from data for refractive index at constant temperature but different pressure gives the different values. Reason for this trend is that pressure is applied the force on lens surface inward while temperature of glass applies the force outward because of thermal expansion. So if we keep the temperature constant and decrease the pressure then temperature effect will dominate and refractive would be greater than the high pressure condition. Now the plot between refractive index for particular wavelength with temperature is in figure 15. For N-BK7, the refractive is increasing with increasing of temperature. Now fourth plot is between change of rate of derivative of relative refractive index with temperature at pressure 103250 Pa. This gives the rate at which relative refractive index changes with temperature. In the figure 16, the rate is decreasing till 0 celcius and increasing after that. So these are all analysis of N-BK7 from given catalog.

### 10.1.2 S-LAH99 from OHARA catalog

Data from catalog for S-LAH99 is,

```

NM S-LAH99 2 1 2.001000 29.139473 0 0 -1
GC
ED 7.500000000E+000 0.000000000E+000 5.020000000E+000
5.400000000E-003 0 0
CD 2.391406620E+000 1.314675000E-002 4.392192280E-001 5.532260420E-002
2.383584670E+000 1.612599000E+002 0.000000000E+000 0.000000000E+000
0.000000000E+000 0.000000000E+000
TD -5.060000000E-007 9.590000000E-009 -2.100000000E-011
1.090000000E-006 1.320000000E-009 2.670000000E-001 2.500000000E+001

```

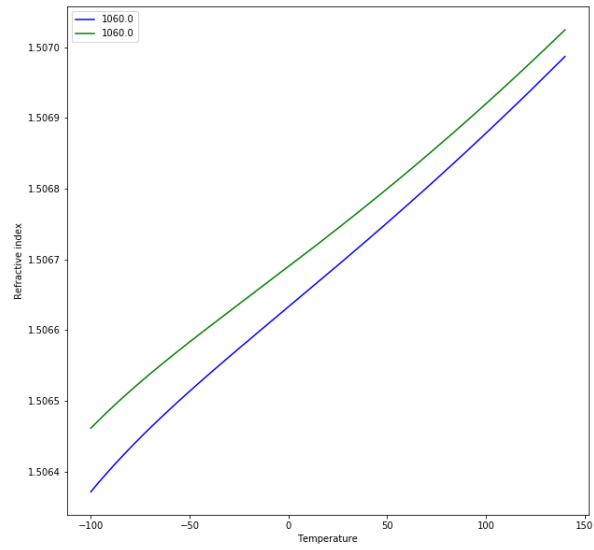


Figure 15: Green color for pressure at  $P=90000$  Pa and Blue color for pressure  $P = 103250$  Pa

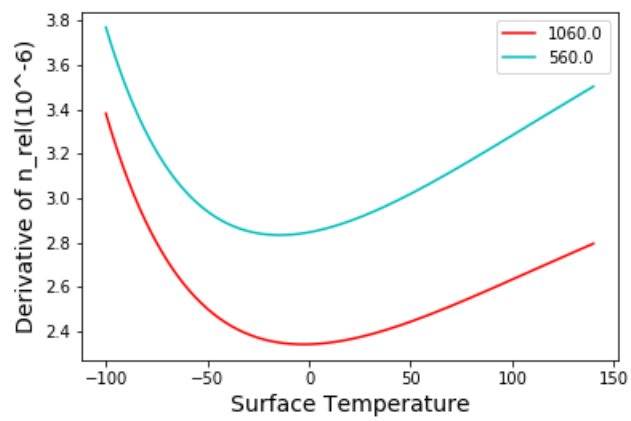


Figure 16: rate is changed drastically at temperature around 0 celcius

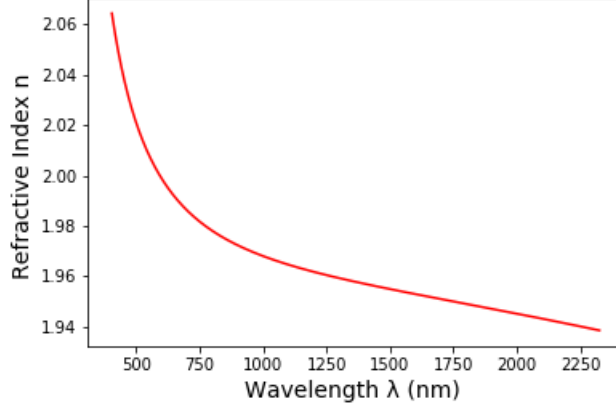


Figure 17: Refractive index vs wavelength for S-LAH99 with sellmeier 1 formula

```
OD 2.10000 1.00000 -1.00000 2.00000 -1.00000 1.00000
LD 4.04656000E-001 2.32542000E+000
IT 2.80000E-001 0.00000E+000 1.00000E+001
IT 2.90000E-001 0.00000E+000 1.00000E+001
.....
```

From this data of glass, we can get information about glass through python. Here is plot between refractive index vs wavelength by using sellmeier 1 formula.

In the figure 13, the curve has negative slope and refractive index is decreasing with increasing of wavelength. Also formula is valid only for certain range of wavelengths. The range is 404 nm to 2325 nm. Refractive index of S-LAH99 at 1060 nm wavelength is 1.9659499406949175 and we can get every value by different inputs. From the equation of abbe number in section 5.1, the value for S-LAH99 is 29.11920404841508. For thermal analysis, the change of rate of derivative of absolute refractive index with temperature is given in the figure 18. There two curve, one for 560 nm and another for 1060 nm. The rate for 560 nm is greater than 1060 nm wavelength. Here the temperature range is -100 to 140 celcius. This is same as for N-BK7.

Now refractive index at any given temperature and pressure is calculated from equation (30). In catalog, S-LAH99 has reference temperature 25 celcius and I found the refractive index at temperature 40 celcius at pressure 103250 Pa. It gave the answer at wavelength 1060 nm around 1.965973023911095. And at reference temperature with one atmospheric pressure is 1.9659499406949175. But if we keep temperature constant and decrease the pressure at 90000 Pa of surrounding then value is 1.9660378094776565. As we can see from data

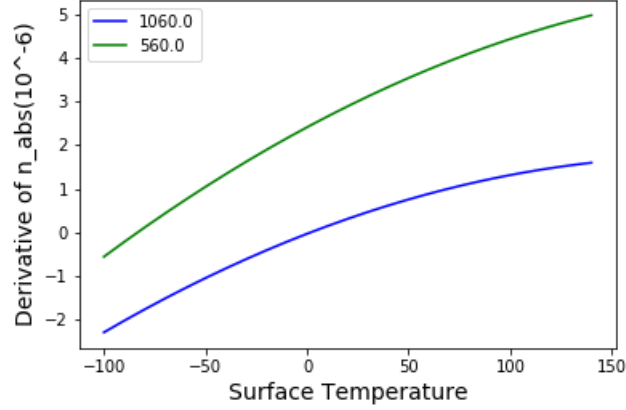


Figure 18: change of rate of derivative of absolute refractive index with temperature at 560 and 1060 nm

for refractive index at constant temperature but different pressure gives the different values. Reason for this trend is same as for N-BK7. Now the plot between refractive index for particular wavelength with temperature is in figure 19. For S-LAH99, the refractive is increasing with increasing of temperature. Now fourth plot is between change of rate of derivative of relative refractive index with temperature at pressure 103250 Pa. This gives the rate at which relative refractive index changes with temperature. In the figure 20, the rate is decreasing till -50 celcius and increasing after that. So these are all analysis of S-LAH99 from given catalog.

### 10.1.3 K-SFLD14 from SUMITA catalog

Data from catalog for K-SFLD14 is,

```
NM K-SFLD14 1 762265 1.76182 26.5 0
ED 8.5 10.2 3.15 0.0144 0
CD 2.9793916E+00 -1.1878549E-02 3.9643280E-02 1.4928665E-03
1.9358213E-05 1.2712921E-05 0.0000000E+00 0.0000000E+00
TD -4.13E-06 1.55E-08 -2.99E-10 9.65E-07 1.53E-09 0.290 2.00E+01
OD -1 1 -1 1 -1 -1
LD 0.4 1.55
IT 0.27 0.000 10
IT 0.28 0.000 10
.....
```

From this data of glass, we can get information about glass through python. Here is plot between refractive index vs wavelength by using schott formula.

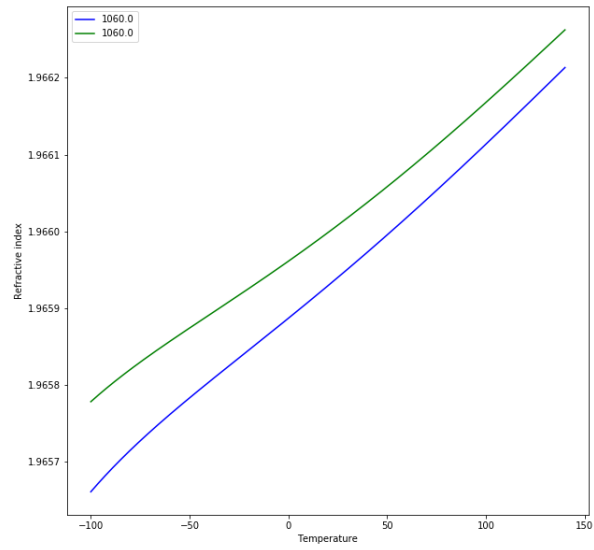


Figure 19: Green color for pressure at  $P=90000$  Pa and Blue color for pressure  $P = 103250$  Pa

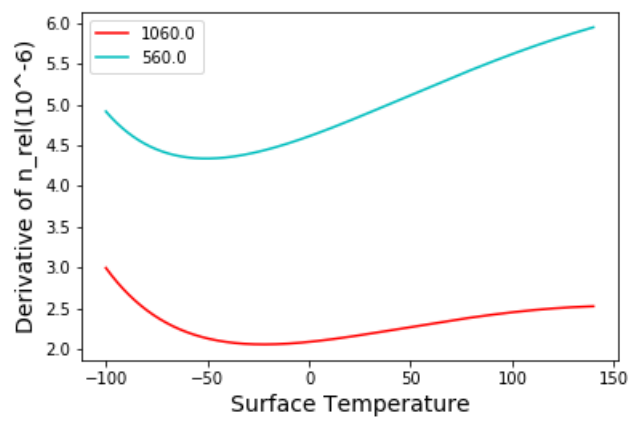


Figure 20: rate is changed drastically at temperature around -50 celcius

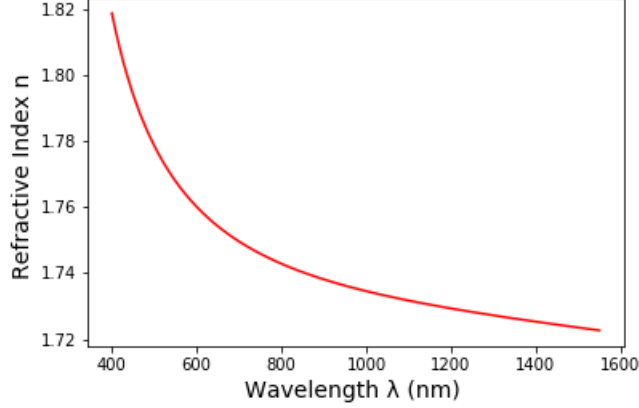


Figure 21: Refractive index vs wavelength for K-SFLD14 with schott formula

In the figure 21, the curve has negative slope and refractive index is decreasing with increasing of wavelength. Also formula is valid only for certain range of wavelengths. The range is 400 nm to 1550 nm. Refractive index of K-SFLD14 at 1060 nm wavelength is 1.7327813923080453 and we can get every value by different inputs. From the equation of abbe number in section 5.1, the value for K-SFLD14 is 26.48690066758293. For thermal analysis, the change of rate of derivative of absolute refractive index with temperature is given in the figure 22. There two curve, one for 560 nm and another for 1060 nm. The rate for 560 nm is greater than 1060 nm wavelength. Here the temperature range is -100 to 140 celcius. This is same as for N-BK7 and K-LAH99. Here there is slope change from positive to negative in this glass type.

Now refractive index at any given temperature and pressure is calculated from equation (30). In catalog, K-SFLD14 has reference temperature 20 celcius and I found the refractive index at temperature 40 celcius at pressure 103250 Pa. It gave the answer at wavelength 1060 nm around 1.7327685035899263. And at reference temperature with one atmospheric pressure is 1.7327813923080453. But if we keep temperature constant and decrease the pressure at 90000 Pa of surrounding then value is 1.7328256042662116. As we can see from data for refractive index at constant temperature but different pressure gives the different values. Reason for this trend is same as for N-BK7 and K-LAH99. Now the plot between refractive index for particular wavelength with temperature is in figure 23. For K-SFLD14, the refractive is decreasing with increasing of temperature. Now fourth plot is between change of rate of derivative of relative refractive index with temperature at pressure 103250 Pa. This gives the rate at which relative refractive index changes with temperature. In the figure 24, the rate is decreasing till 30 to 40 celcius and increasing after that. Here the rate



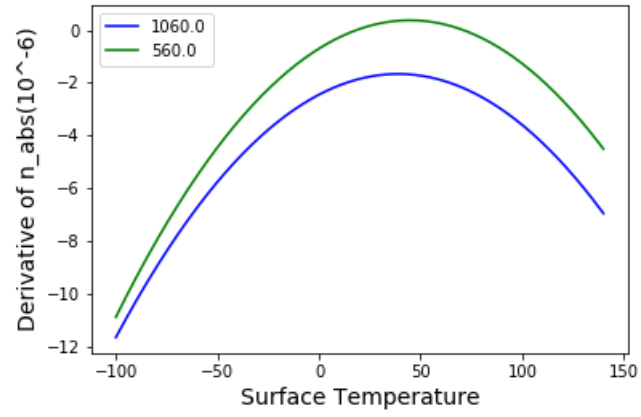


Figure 22: change of rate of derivative of absolute refractive index with temperature at 560 and 1060 nm

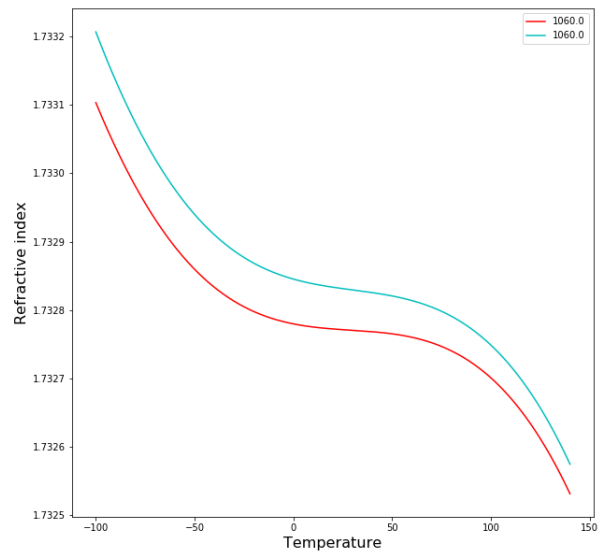


Figure 23: light blue color for pressure at  $P=90000$  Pa and red color for pressure  $P = 103250$  Pa

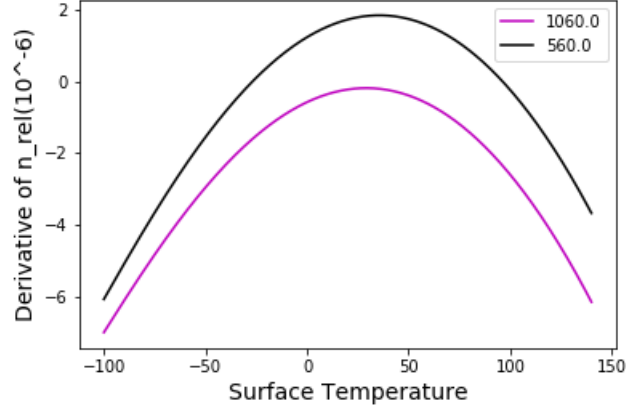


Figure 24: rate is changed drastically at temperature around 30 to 40 celcius

is in negative and positive portion. That means that at some point refractive index does not change very much at that temperature. So these are all analysis of K-SFLD14 from given catalog.

#### 10.1.4 J-PSKH1 from HIKARI catalog

Data from catalog for J-PSKH1 is,

```
NM J-PSKH1 13 1 1.593190 67.900555 0 0 1
GC
ED 1.140000000E+001 0.000000000E+000 4.100000000E+000
1.350000000E-002 0 0
CD 2.502080830E+000 -6.721439070E-003 -5.343137510E-005
1.282644000E-002 1.562053880E-004 1.215935490E-006 9.595508690E-008
0.000000000E+000 0.000000000E+000 0.000000000E+000
TD 0.000000000E+000 0.000000000E+000 0.000000000E+000
0.000000000E+000 0.000000000E+000 0.000000000E+000 2.300000000E+001
OD -1.00000 -1.00000 -1.00000 -1.00000 -1.00000 3.00000
LD 3.88865000E-001 2.05809000E+000
IT 2.80000E-001 0.00000E+000 1.00000E+001
IT 2.90000E-001 0.00000E+000 1.00000E+001
.....
```

From this data of glass, we can get information about glass through python. Here is plot between refractive index vs wavelength by using extended 3 formula.

In the figure 25, the curve has negative slope and refractive index is decreasing with increasing of wavelength. Also formula is valid only for certain range of

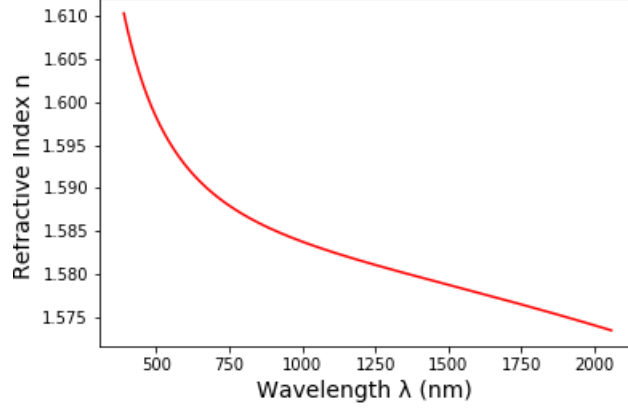


Figure 25: Refractive index vs wavelength for J-PSKH1 with extended 3 formula

wavelengths. The range is 388 nm to 2058 nm. Refractive index of J-PSKH1 at 1060 nm wavelength is 1.5830354695353313 and we can get every value by different inputs. From the equation of abbe number in section 5.1, the value for J-PSKH1 is 67.86486866978349. For thermal analysis, the change of rate of derivative of absolute refractive index with temperature is given in the figure 26. There two curve, one for 560 nm and another for 1060 nm. The rate for 560 nm is same as 1060 nm wavelength. Here the temperature range is -100 to 140 celcius. The plot is constant with value 0. That means this glass type has no change in absolute refractive index

Now refractive index at any given temperature and pressure is calculated from equation (30). In catalog, J-PSKH1 has reference temperature 23 celcius and I found the refractive index at temperature 40 celcius at pressure 103250 Pa. It gave the answer at wavelength 1060 nm around 1.5830508417073328. And at reference temperature with one atmospheric pressure is 1.5830354695353313. But if we keep temperature constant and decrease the pressure at 90000 Pa of surrounding then value is 1.5831030086722035. As we can see from data for refractive index at constant temperature but different pressure gives the different values. Reason for this trend is same as for N-BK7 and K-LAH99. Now the plot between refractive index for particular wavelength with temperature is in figure 27. For J-PSKH1, the refractive is increasing with increasing of temperature. Now fourth plot is between change of rate of derivative of relative refractive index with temperature at pressure 103250 Pa. This gives the rate at which relative refractive index changes with temperature. In the figure 28, the rate is decreasing continuously with temperature. Here the rate is in only positive portion. So these are all analysis of J-PSKH1 from given catalog.

So These four glass type N-BK7, J-PSKH1, K-SFLD14, and S-LAH99 have

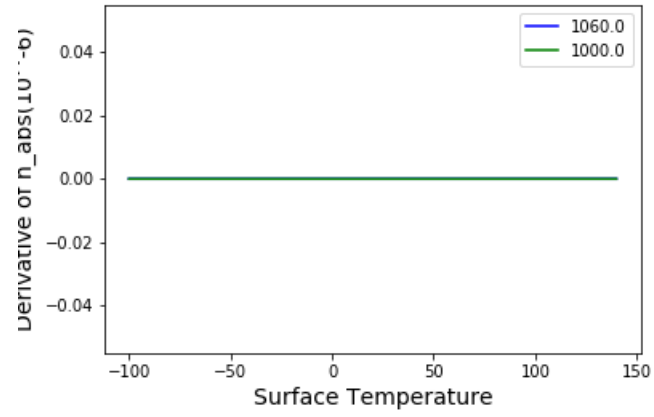


Figure 26: change of rate of derivative of absolute refractive index with temperature at 560 and 1060 nm

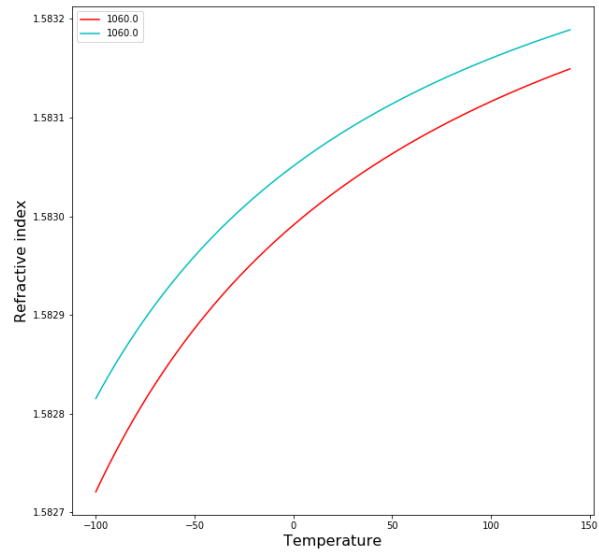


Figure 27: light blue color for pressure at  $P=90000$  Pa and red color for pressure  $P = 103250$  Pa

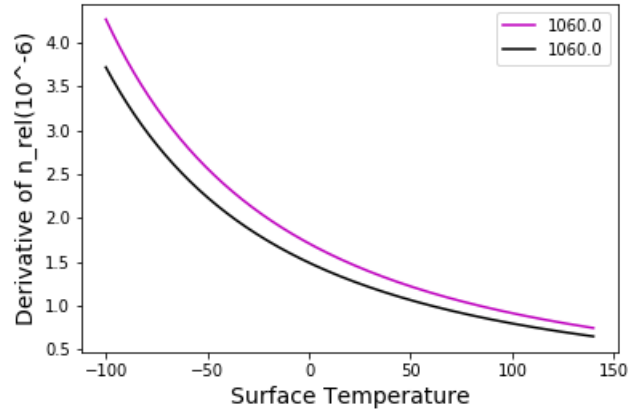


Figure 28: rate is decreasing with temperature

different behaviour with temperature, their rate of change of refractive index, and change of refractive index with wavelengths. These all data got through python programming. Now we need to verify our data with one reliable resources.

## 11 Verification of refractive index and thermal analysis data