# Lagrangian Coherent Set Detection with Topological Advection


**Rida Naveed Ilahi**

Mount Holyoke College


## Motivation

Fluid mixing and scalar transport (of e.g. pollutants, temperature, oxygenation, etc.) are central to many problems in oceanography, meteorology, global climate modeling, and engineering. Lagrangian Coherent Structures (LCSs), important geometric features of flow fields, constitute the main organizing principle for these transport and mixing phenomena. However, most LCS detection algorithms require full knowledge of the flow field. For discrete advected 2D particle trajectories, Lagrangian Coherent Sets are good stand-ins for elliptic LCSs.

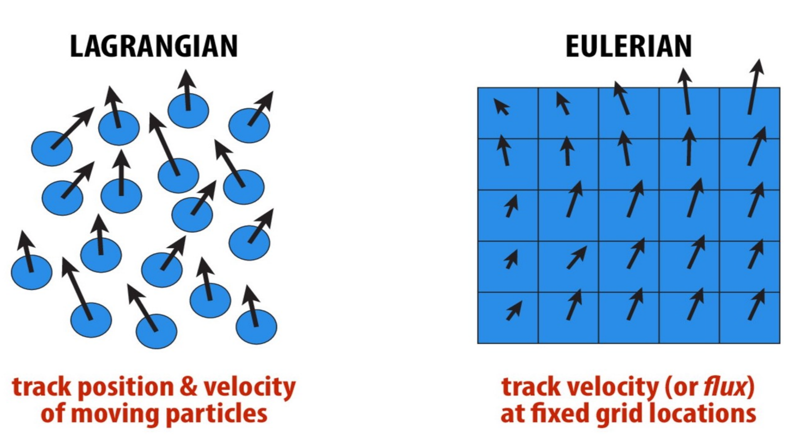

## Abstract

Lagrangian Coherent Sets (LC-Sets) are sets of trajectories which can be enclosed by a material curve which does not appreciably stretch over the time period of interest. Such a curve acts as a barrier to transport. Here we introduce an algorithm which automatically detects coherent sets given Lagrangian particle trajectory data. This algorithm uses ideas from topology and computational geometry to efficiently identify and visualize coherent sets.


## Algorithm Overview

**Input Data:** 2D Lagrangian particle trajectories

- *2D flows* are important (interfacial flows) and are amienable to powerful topological tools.
- *Lagrangian* view aligns with common experimental output and gives objective (coordinate-independent) measures of LC-Sets.




**Topological Advection**

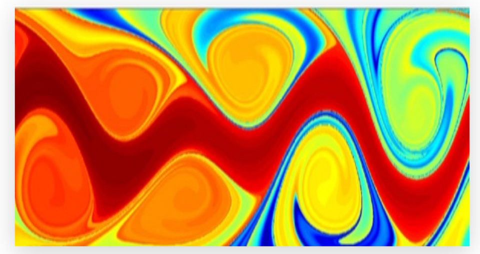
- Topologically equivalent material curves (loops) are encoded in a coordinate system given by their intersection number with the edges of a triangulation (with particles at vertices).
- The triangulation and intersection coordinate are updated (via edge flips) as the particles move.

**Coherent Set Finding**

- Loops surrounding adjacent pairs of points are evolved forward and sorted by final length
- Combine adjacent loops that do not appreciably stretch. Continue "growing" loops. Final loops bound LC-Sets.

**Output:** Sets of trajectories that are coherent

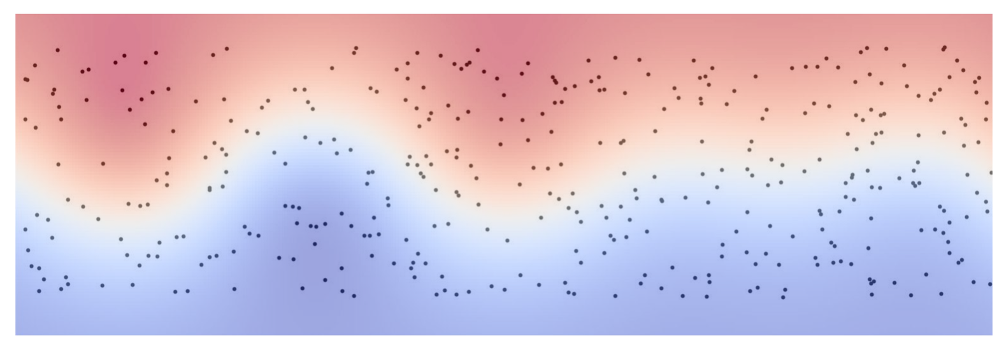
## Bickley Jet - Illustrative Example



Hadjighasem et al. (2017)

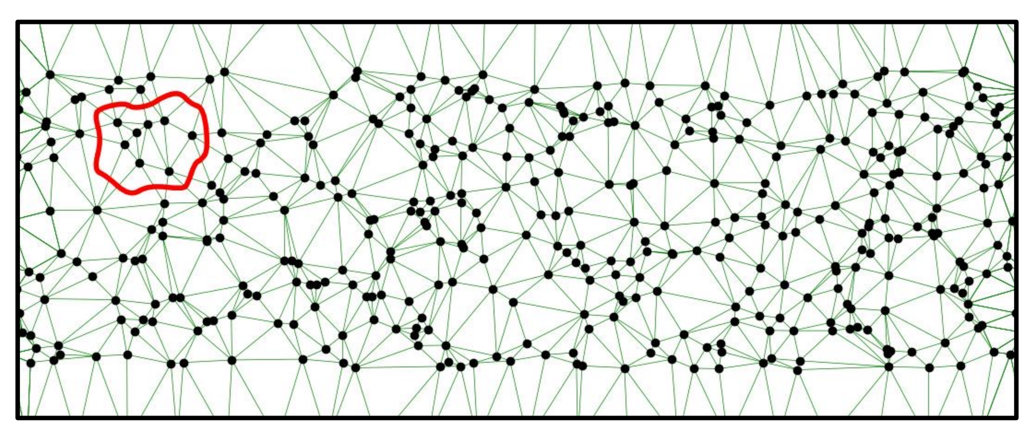
Simplified model for polar zonal jets (e.g. - Arctic polar vortex)

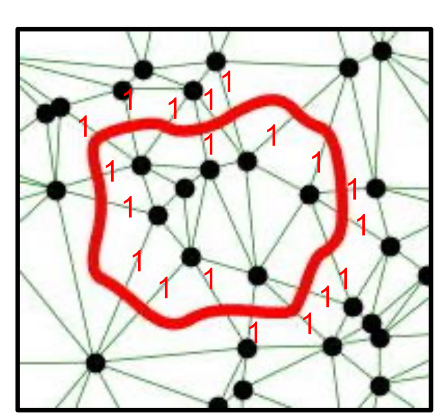
- Periodic boundary conditions in x-direction
- One jet and six vortices (target LC-Sets)
- Time aperiodic forcing (incommensurate Rossby wave numbers)
- Common model for LCS studies



**Bickly Jet stream function with initial locations of advected particles**

## Topological Advection Algorithm





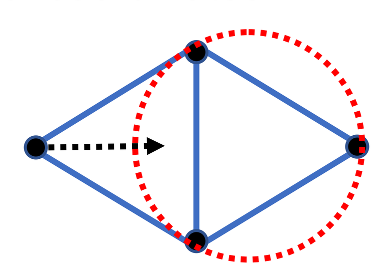
**Loop Coordinates**

- Triangulation data structure (Delaunay) with particles as vertices.
- Loops (closed material curves) are represented by a vector of edge intersection numbers (left).

- All *topologically* equivalent material curves have the same intersection coordinates
- "Length" of loop given by sum of coordinates

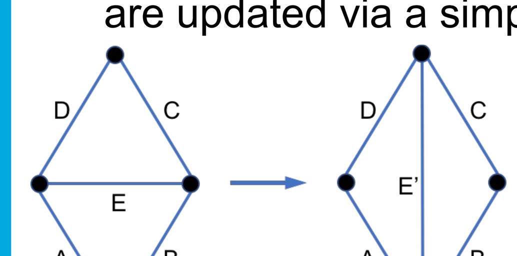
**Triangulation Evolution**

- As the particles move, a Delaunay triangulation is maintained using edge flips: (see QR code for video)
- The intersection coordinates are updated via a simple rule:



Delaunay Event

Flip center edge when point enters circumcircle of other triangle




Edge Flip

Intersection Coordinate Update Rule

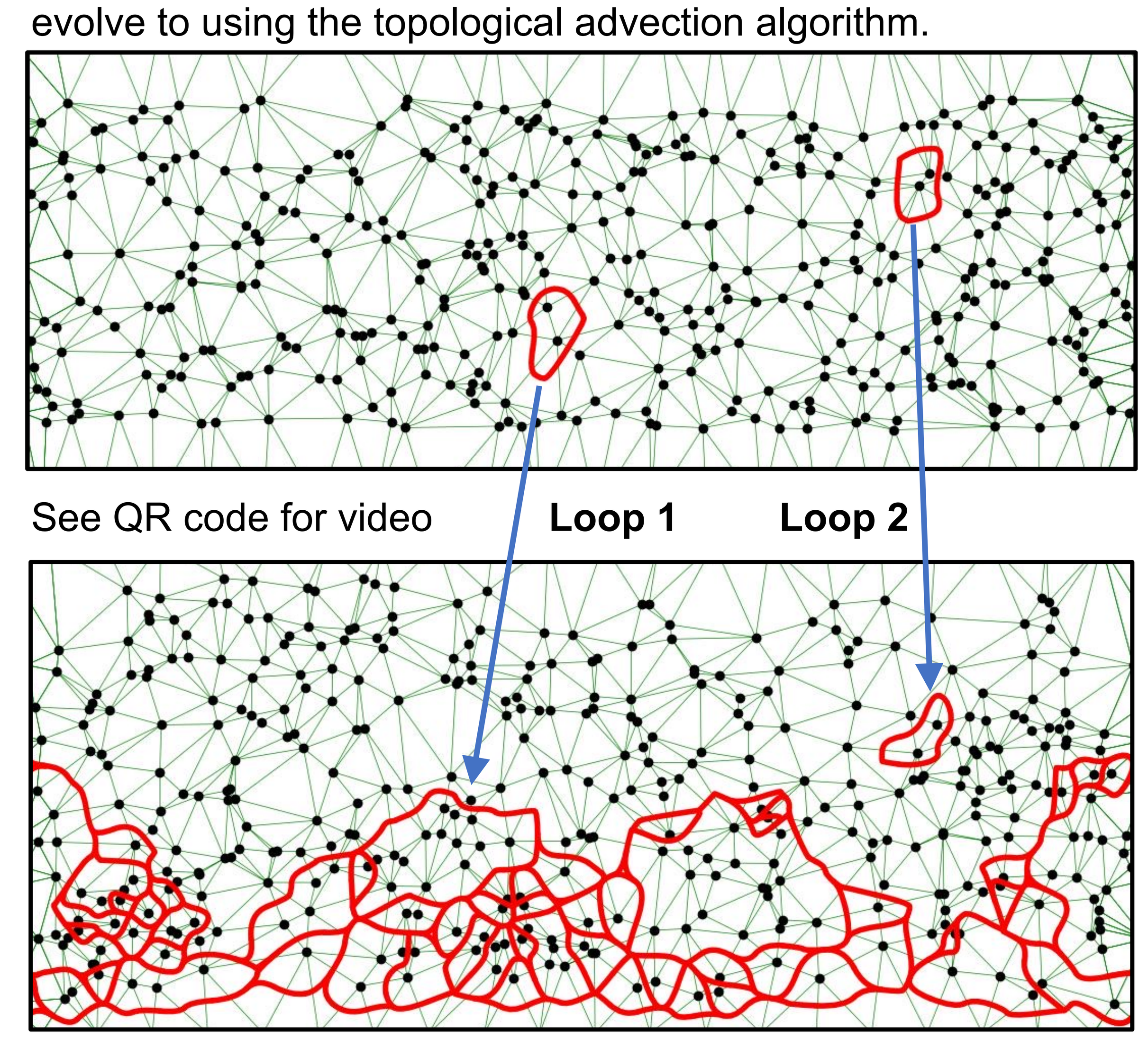
$$E' = \max(A + C, B + D) - E$$

**Can quickly evolve material curves forward in time**

## Coherent Set Detection

**Key Idea**: The boundary of a coherent set experiences minimal stretching ➤ Can use topological advection algorithm to quickly verify a set is coherent.
**Problem:** Number of possible sets is combinatorially large ➤ Start with simple candidate sets and combine to "grow" coherent sets.

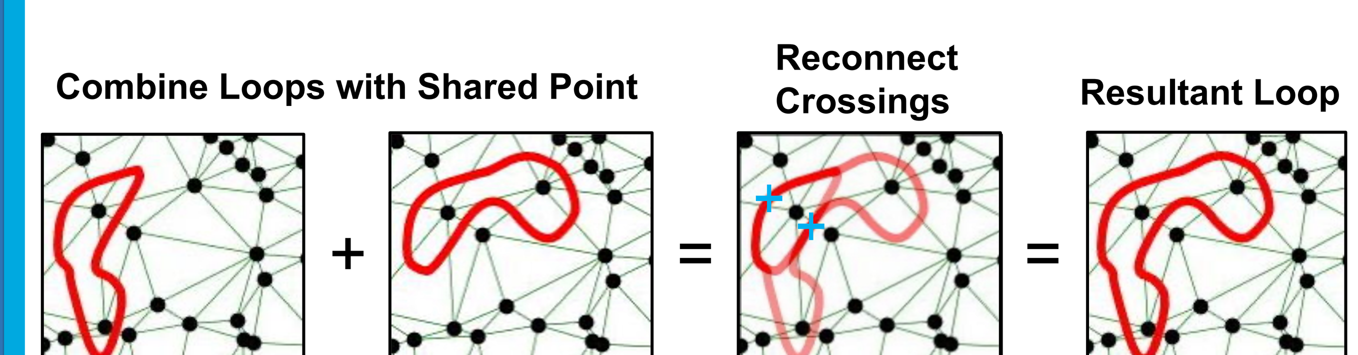
Consider two simple initial loops and the final states they evolve to using the topological advection algorithm.




Loop 1 – appreciable stretching, Loop 2 – minimal stretching

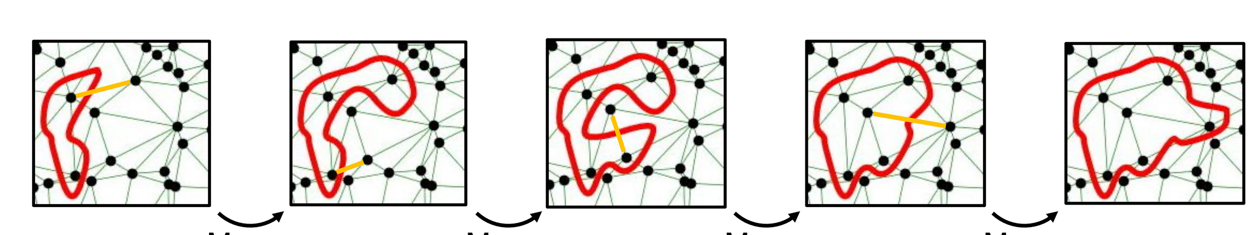
**Simple "Seed" Loops**

- Stretching measured by the ratio of the final to initial length
- Consider the set of simple loops which encircle pairs of points adjacent in the initial triangulation
- Sort by stretching rate and retain the subset with stretching rate smaller than a cutoff value. These are the "seed" loops.




**Combining Loops**

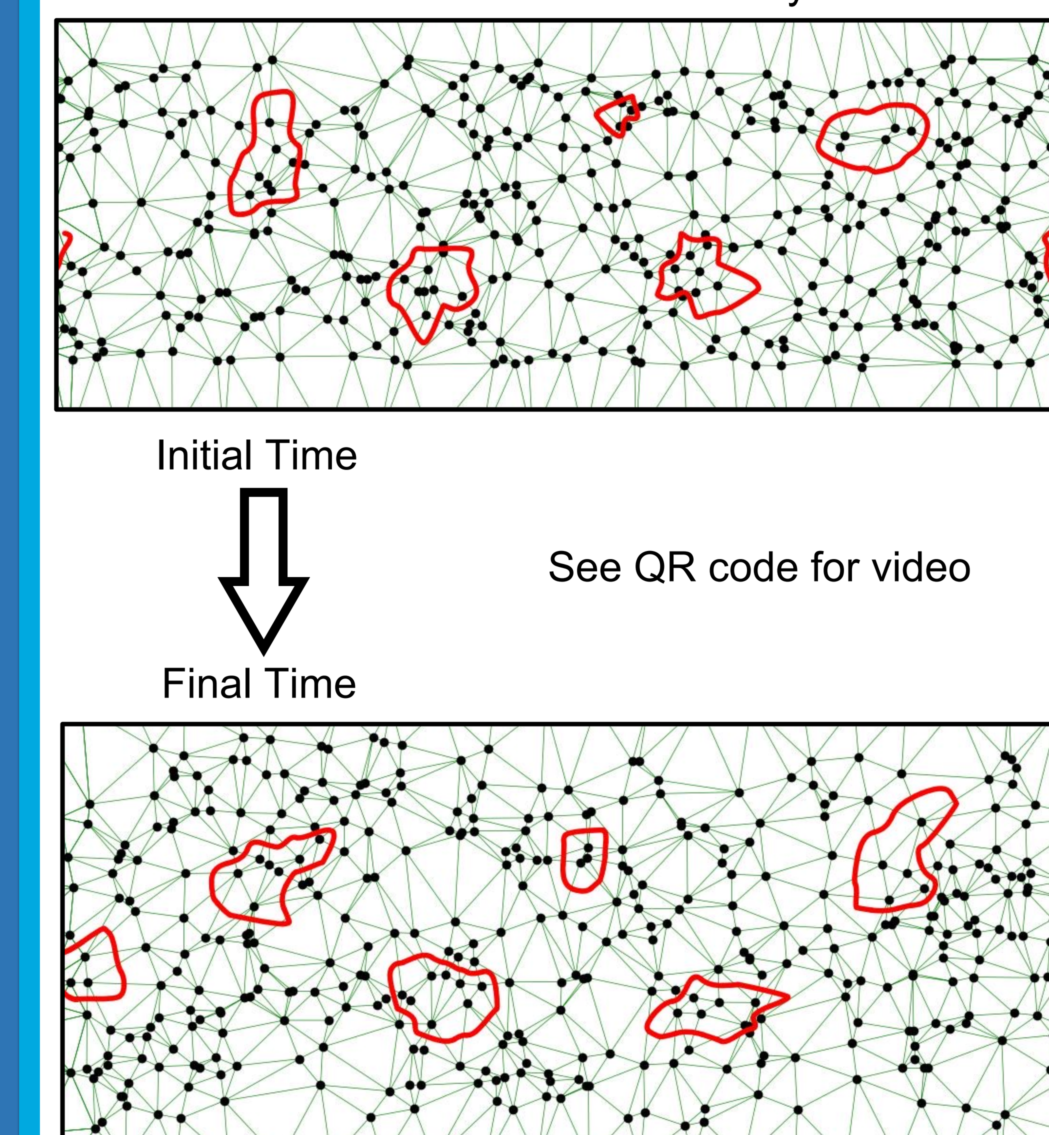
- Seed loops which share a point are combined (see above) and tested for stretching.
- If stretching is below the cutoff, the new loop is retained
- Loops grow through these mergers until any possible additional seed loop would cause stretching above the cutoff.




## Lagrangian Coherent Sets

Once the loop has grown to be as large as possible, we consider the loop to be the boundary of a coherent set and the interior particles comprise the coherent set.

Coherent Sets for the Bickley Jet




## Conclusions/Future Work

The detection of Coherent Structures in 2D Lagrangian particle trajectory data has implications for understanding climate models, oil spills, etc.

Our algorithm uses ideas from topology and computational geometry to quickly find coherent sets within sparse data.

**Future Work:**

Test algorithm on experimental data sets (for example, Jupiter's Great Red Spot)

Perform time complexity analysis, accuracy, and scalability testing of the algorithm

Please scan the QR code to see some example videos!


Summer research was made possible by Mount Holyoke College's LYNK fund