(ii) <u>To find the maximum velocity of the kick and angle at which should the kick take place:</u>

<u>Method of Solving the maximum velocity of the kick and angle at which should the kick take place:</u>

In order to solve the second part of the problem, we had to formulate by using the equations form projectile motion. The constraints were basically the width and height of goalpost, which were formulated in terms of the velocity, angle of elevation and angle of deviation. The primary objective was to maximize the power of the striker's kick. But since there is no direct relation between force and our designated variables, we had to modify the objective function as a maximization of horizontal range since we know that range is proportional to power. The higher the power, the higher will be the value of the range. Also it is easier to formulate the objective function as the horizontal range in terms velocity, angle of elevation and angle of deviation.

<u>Problem Details :</u>

The second part of the problem was to calculate the velocity and the necessary angles. We had to be aware of the goal-post's and the football's dimensions. This is because we can only optimize the penalty shot as long as the shot is within the goalpost's boundaries. We ignored the impact of air resistance and assumed that the ball will traverse a projectile motion. This was taken into account when the objective function and the constraint equations were formulated using fundamental physics formulas. This should come out as a nonlinear optimization problem. Hence while solving in MATLAB, the 'fmincon' command was utilized.

Numerical Calculation :

Necessary data regarding the football, goalpost and the penalty area:

(c)Width of the goalpost area, w = 7.3152 m

Height of the goalpost area, h = 2.4384 m

Horizontal distance from the center of the goal area to the penalty spot, l = 10.97m

(d)Diameter of the ball, d = 8.92 inches = 0.2266m

Weight of the ball, W = 0.94 lbs = 4.183 N

g = 9.81 m/s^2

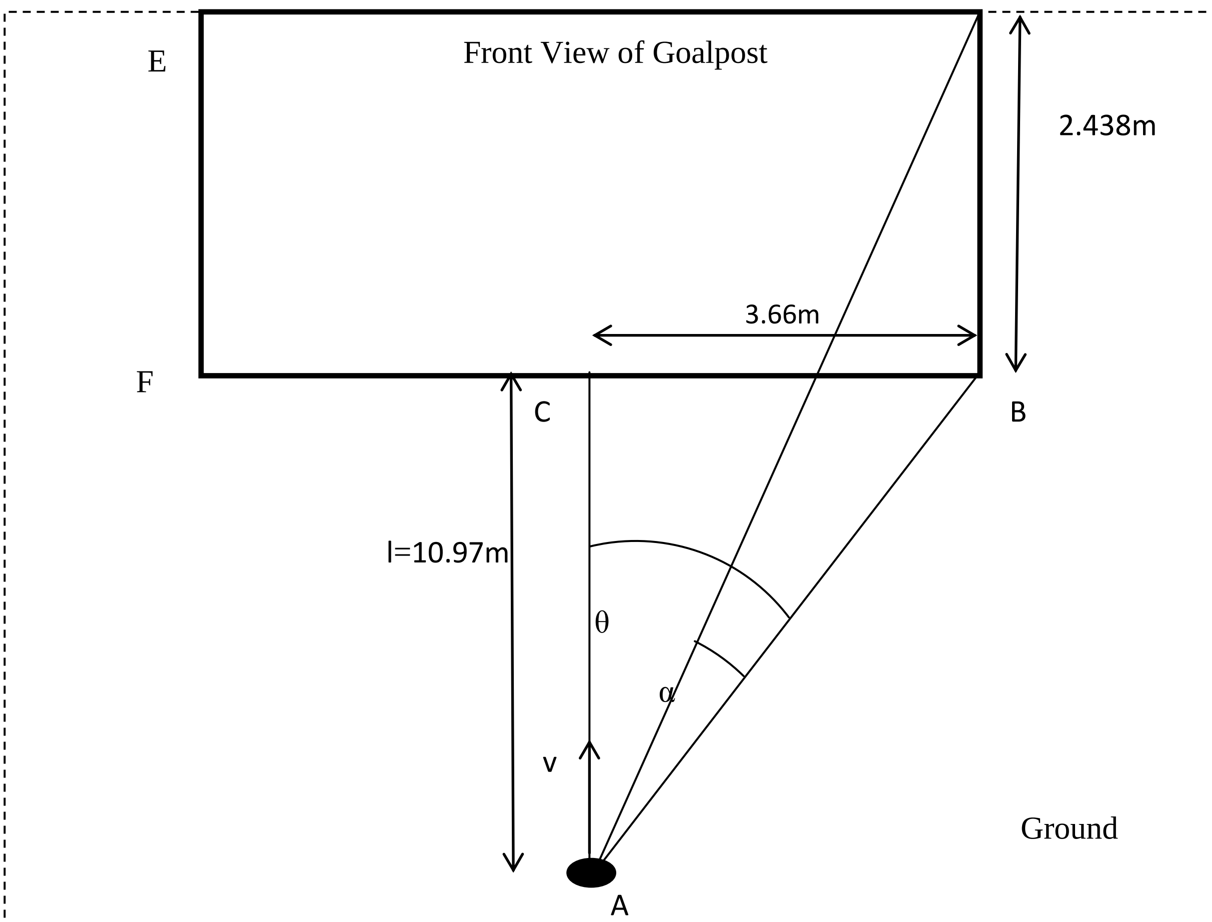

Figure 1: **Front view of the penalty area**

Mathematical modeling for the velocity and angle of the kick:

Decision variables:

θ = angle of deviation from the center to left/right

α = angle of elevation

v= velocity of the ball

Since velocity is a vector quantity, it can resolved along AB as well as AD.

Again, ΔABC and ΔABD are right angled triangles with angle ABC and angle ABD equal to 90 degrees.

$v_{AB} = v.\sec\theta$

$v_{AD} = v.\sec\theta.\sec\alpha$

Horizontal component of $v_{AD}= v.\sec\theta.$

Vertical component of $v_{AD}= v.\sec\theta.\tan\alpha$

Formulation of the Objective Function :

In order to formulate the objective function, we have to come up with a model that signifies maximum power. The direct equation for power is P= F.v

But, applying this equation is hazardous, since it will be quite difficult to model the components of the force quantitatively. It also makes calculation complex and difficult. Therefore, we have come up with the following proposition.

We know that after the ball is kicked, in all fairness it can be assumed that it will follow a projectile motion.

We can assume that the higher the power with which the ball is kicked, the higher will be the range traversed by the ball. This will work as long as the ball enters the net before it reaches a vertical distance greater than the height of the net as shown by the dotted line.

Therefore, the objective function is to maximize the power which is the same thing as maximizing the range.

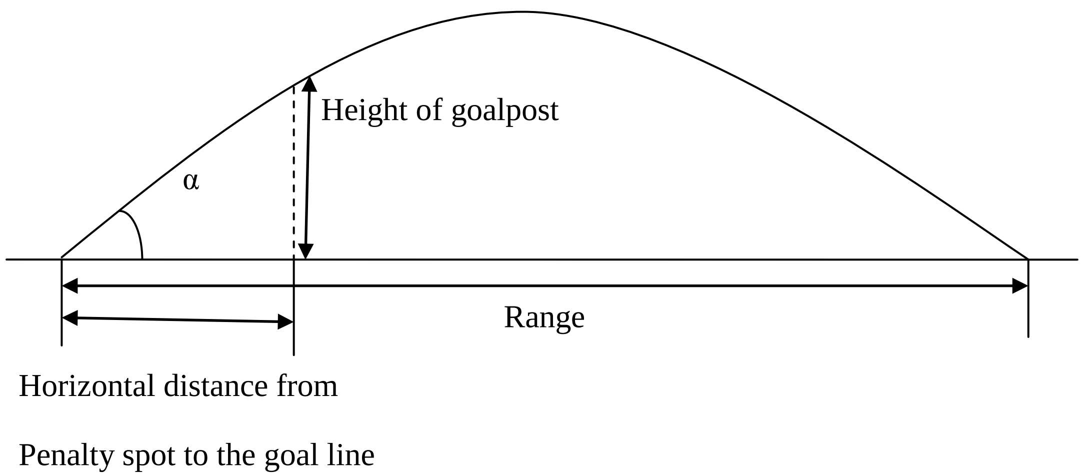

Figure 2: **<u>Side view of the goal area tracking the entire path of the ball beyond the goal</u>**

The range travelled by the ball:

First we need to find the time taken by the ball to reach its highest point during the trajectory.

$v = u + at_{max}$

or, $0 = v\sec\theta\tan\alpha - gt_{max}$

or, $t_{max} = v\sec\theta\tan\alpha/g$

So, total time of flight, $T=2t_{max}= 2v\sec\theta\tan\alpha/g$

Range traversed by the ball $=v\sec\theta * T$

$= v\sec\theta * 2v\sec\theta\tan\alpha/g$

$= 2v^2 \sec^2 \theta \tan \alpha / 9.81$

Formulation of Constraints:

The radius of the ball is 0.1133m. Therefore in order for it to be a successful penalty the maximum length of BC can be 3.66m – 0.1133m = 3.55m

Similarly, if we consider the vertical distance, the maximum length of BD can be 2.438m – 0.1133m = 2.325m

If BD or BC is greater than this value, then the penalty kick will not be successful. These are two of the constraints.

The constraints are that maximum length of BC can be 3.55m and and that of BD can be 2.325m.

These can be written as $BD \leq 2.325$ and $BC \leq 3.55$

Let ,'t' be the time at which the ball crosses the goal line, t= 10.97/v

We know BD = h;

And $t = 10.97/v$;

Applying the equations of projectile motion on v_{AD}:

We know, that analyzing vertically, $v = u + at$

or, $v_{AD}{*} = v\sec\theta \tan\alpha - g * 10.97/v$

or, $v_{AD}{*} = v\sec\theta \tan\alpha - 107.6/v$

We know, $v^2 = u^2 + 2ax$;

or, $v^2\sec^2\theta\tan^2\alpha - 2\sec\theta\tan\alpha * 107.6/v + 11577.8/v^2 = v^2\sec^2\theta\tan^2\alpha - 19.62h$

or, $2\sec\theta\tan\alpha * 107.6/v - 11577.8/v^2 = 19.62h$

or, $\mathbf{h = 10.97\sec\theta\tan\alpha - 590.1/v^2}$

But since BD = h and has to be less than or equal to 2.325m,

$10.97\sec\theta\tan\alpha - 590.1/v^2 \leq 2.325$ -·-·-·-·-·-·-·-·-·-·-·-·-·-·-·-·- constraint 1

Also, $BC \leq 3.55$

Considering ΔABC and applying simple trigonometry we get,

$\mathbf{\tan\theta = BC/10.97}$

or, $10.97 \tan\theta \leq 3.55$ -·-·-·-·-·-·-·-·-·-·-·-·-·-·- constraint 2

Mathematical formulation:

Maximize $Z = 2v^2 \sec^2\theta \tan\alpha / 9.81$

Subject to

$5.48 \sec\theta . \tan\alpha - 590.1/v^2 \leq 2.325$

$10.97 \tan\theta \leq 3.55$

$v, \theta, \alpha \geq 0$

Solve :

>> penalty

			Max	Line search	Directional	First-order	
Iter	**F-count**	**f(x)**	**constraint**	**steplength**	**derivative**	**optimality**	**Procedure**
0	**4**	**-0**	**3.55**				**Infeasible start point**
1	**8**	**-0**	**-0.1293**	**1**	**0**	**Inf**	

Local minimum possible. Constraints satisfied.

fmincon stopped because the size of the current search direction is less than twice the default value of the step size tolerance and constraints are satisfied to within the default value of the constraint tolerance.

<stopping criteria details>

No active inequalities.

ans =

.3236 0 0

<u>Appendix</u> :

i) <u>Code For Objective Function</u>

```
%.m file for the optimization assignment

%This .m file will contain the objective function and any linear

%inequalities (also limits) given in the specified problem

% Initialize function

function [x,fval]= penalty()

%Define options for optimset command (mainly the specific algorithm used)
options=optimset;

%Choose the Active-set algorithm for the problem

options = optimset(options,'Algorithm', 'active-set');

%Display the number of iterations needed

options=optimset(options,'Display','iter');

%Evaluate the optimized variables and minimum value using fmincon function

[x,fval]=fmincon('-2*x(3)^2*(sec(x(1)))^2*tan(x(2))/9.81',[0 0
0],[],[],[],[],[],[],'constraints',options);

%display the optimized variables
```

ii) <u>Code For Constraints</u>

```
% .m file codes for non-linear Constraints

%Initialize function

function [c ceq]=constraints(x)

%Define the constraints in terms of both variables x(1) and x(2)

p=2.325-10.97*cos(x(1))*tan(x(2))-590.1/x(3)^2;

q=3.55-10.97*tan(x(1));
%Write the two non-linear constraints function as a column matrix

c=[p;q];

% Write the equality constraints (if any)
ceq=[];
```

Result :

From the above MATLAB calculation, we observed that the values of v, θ and α are 0.3236, 0 and 0. This signifies that the shot should be along the ground at the center with the calculated velocity. This is also correspondent with our linear solution since there, we also saw that shooting at the center has the highest probability.

Calculation:

This project will play a big part in coming up with a strategy on how to best take a penalty kick. It does not simply end with Cristiano Ronaldo. The same process can be applied to find out any footballer's best penalty kick. Moreover, even goalkeepers can look at this and come up with a strategy of their own against the particular player's best penalty kick. All in all, this will have great significance in the football world.

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