Politecnico di Milano

Financial Engineering

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Financial Risk Laboratory: 2. Corporate Bond Credit Spread Risk

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1 Q1 to Q3

After bootstrapping the ZC curve we derived two distinct constant values for the hazard rate of Alpha using the two bond prices. We proceeded by computing the default probabilities associated and derived the z spreads using again the bond prices. The results can be observed in the following table:

Bond	1Y	2Y
Hazard rate	56bps	94bps
Default probability	0.56%	1.87%
Z-Spread	56bps	94bps

2 Q4: PIECEWISE CONSTANT HAZARD RATES

We then computed the piecewise constant in time hazard rates:

Hazard rate (0Y-1Y): 56bps

Hazard rate (1Y-2Y): 135bps

and the corresponding default probabilities:

Default probability (1Y): 0.56

Default probability (2Y): 1.89

3 Q5: RATING TRANSITION MATRIX

We lastly proceeded to compute the real-word default probabilities using the rating transition matrix provided using the following formulas:

$$P_{1y} = P_{default\_from\_IG} \tag{1}$$

$$P_{2y} = P_{default\_from\_IG} + P_{HY\_from\_IG} * P_{default\_from\_HY} + P_{staying\_IG} * P_{default\_from\_IG} \tag{2}$$

and obtained the following probabilities:

Default probability (1Y): 1.00

Default probability (2Y): 2.99

4 Q6: DISCUSSION 1

We executed a new bootstrap of the hazard rates and derived a new estimation of the risk-neutral unconditional default probabilities after a drop of the dirty price of the two-years bond to 98.90 $ and obtained the following results:

Hazard rate (0Y-1Y): 56bps

Hazard rate (1Y-2Y): 254bps

Default probability (1Y): 0.56

Default probability (2Y): 3.06

We can observe that since the price of the first bond did not change, then neither the hazard rate relative to the first year nor the probability to default in the first year changed. On the other hand, the hazard rate relative to the $[1y, 2y]$ interval and consequently the probability to default in 2 years increased. This is not surprising since a decrease of the price implies an increase in the risk and therefore in the default probability of the company (in this case in the $[1y, 2y]$ interval, which in turn affects the probability in the whole interval $[t_0, 2y]$).

5 Q7: DISCUSSION 2

We then repeated the whole procedure, changing also the price of the 1 year coupon bond to 98.00 $.

Hazard rate (0Y-1Y): 159bps

Hazard rate (1Y-2Y): 145bps

Default probability (1Y): 1.58

Default probability (2Y): 3.00

In this case, on the other hand, we have a change both in the price both of the 1 year and the 2 years bonds. As we were expecting, both the hazard rates and, consequently, the default probabilities were impacted by this change. Differently from the case above, the increase of the risk is more distributed between the two years and, interestingly, we can notice that the default probability in 2 years is lower than the one we found in the first part of the discussion. We would have expected that lowering both prices we would get a higher probability of default in both the considered times, but we realized that since the hazard rates relative to the first time period rises and the relations between the prices and the hazard rates are highly non-linear, this wasn't neither necessary nor obvious.

6 Q8: DISCUSSION 3

Lastly, we noticed that the risk-neutral unconditional default probabilities derived under the scenario 1 above were not consistent with the real-world unconditional probabilities derived from the transition matrix.

We therefore tried to find new values for Q in order to find consistent probabilities and obtained them by solving the following equations:

First we set the probability to default from investment grade:

$$Q_{1,3} = Defalut\_Probability\_1y \tag{3}$$

Then we used the new value to compute the new probability to stay investment grade:

$$Q_{1,1} = 1 - Q_{1,2} - Q_{1,3} \tag{4}$$

Lastly, we solved the following equation for $Q_{2,3}$:

$$Defalut\_Probability\_2y = Q_{1,3} + Q_{1,2} * Q_{2,3} + Q_{1,1} * Q_{1,3} \tag{5}$$

Using these new values we therefore obtained, as we wanted, the following, coherent values for the probabilities:

Default probability (1Y): 0.56

Default probability (2Y): 3.06