Politecnico di Milano

Financial Engineering

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Assignment 3

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1 Exercise1

Using the discounts obtained with the case-study on curve bootstrap on the 31st of January 2023 at 10:45 C.E.T., we first interpolated to get the one corresponding to 1 year (2^{nd} of February 2024). We then created the vector discounts_asw containing the necessary discounts for the 1y, 2y and 3y.
We therefore computed the price of the bond (knowing both the discounts and coupons) and then used the following formula to compute the spread in asset swap.

$$S^{asw} = \frac{C(0) - \overline{C(0)}}{BPV(0)} \tag{1}$$

where

1. $BPV(0) = \sum_{i=1}^{3} \delta(t_{i-1}, t_i) B(t_0, t_i)$
2. $C(0) = c * BPV(0) + B(t_0, t_3)$: is the price of the Bond
3. $\overline{C(0)}$ is the dirty price

The result that we obtained for the spread is 0.5206%.

2 Exercise2

Using the discounts obtained with the case-study on curve bootstrap on the 31st of January 2023 at 10:45 C.E.T. we proceeded to consider the obligor ISP with a recovery π equal to 40% and CDS spreads (annual bond): 1y 30 bps, 2y 34 bps, 3y 37 bps, 4y 39 bps, 5y 40 bps, 7y 40 bps.
We started by creating a complete set of CDS spreads and dates with spline interpolation.
Having the CDS spreads, we were then able to perform the bootstrap to compute the survival probabilities at each time step and, by inverting them, also the piecewise-constant λ for each time.
We first computed the intensities and probabilities both neglecting and considering the accrual term.
We then computed them also using the Jarrow & Turnball formula. The results (units of bps for the intensities) can be observed in the tables below.

Probability of survival		
Exact	Approx	JT
0.9950	0.9950	0.9950
0.9887	0.9887	0.9894
0.9816	0.9817	0.9833
0.9742	0.9743	0.9769
0.9670	0.9671	0.9704
0.9603	0.9605	0.9639
0.9542	0.9543	0.9575

Intensities		
Exact	Approx	Difference
50.0001	49.8754	0.1247
63.3922	63.1954	0.1968
72.2357	71.9841	0.2516
75.6983	75.4198	0.2785
73.9212	73.6518	0.2695
69.0517	68.8136	0.2381
64.0243	63.8194	0.2048

Intensities		
JT	Progressive mean	Difference
50.0000	49.8754	0.1246
56.6667	56.5354	0.1313
61.6667	61.6850	0.0183
65.0000	65.1187	0.1187
66.6667	66.8253	0.1586
67.0606	67.1567	0.0961
66.6667	66.6799	0.0133

As expected, the difference between the intensities bootstrapped with the approximated version and the exact one is negligible, being in the order of 10^{-5} at each timestep.
Moreover, comparing the λ obtained with the Jarrow & Turnball formula and the progressive mean of the approximated λ, we notice that, again as expected, they are extremely similar. The order of magnitude of the difference is, infact, also in this case, 10^{-5} at each timestep.
Lastly, we can observe the intensities in the following figure.

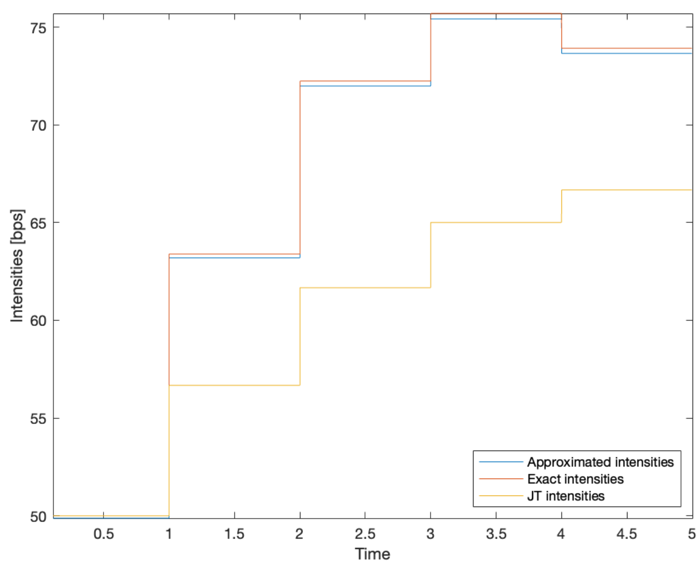

Figure 1: Intensities obtained.

3 Exercise3

We had to simulate the default time, so we simulated the survival probability from a uniform r.v which belongs to the interval [0,1] and then we inverted the relation between the two. In order to do it correctly we had to check whether tau was bigger or lower than theta because this affects the values of the indicator functions in the definition of the survival probability.
Done that, we obtained our set of survival probabilities associated to their corresponding default time and we had to fit the survival probability. This means that our goal was to compute the probability to survive until tau. To do it we needed to estimate the parameters lambda1 and lambda 2.
In order to obtain estimations and confidence intervals for our parameters we exploited different methods. The main one goes through the maximisation of the likelihood of the dataset, but we decided to try also other methods to enrich the validity of our result.
Method 1: by maximizing the likelihood of our dataset we can obtain the parameters that best fits the

dataset and then by exploiting the Fisher Matrix we computed the confidence interval (see Annex3). In addition to that we also provided and other way obtain an estimator for the two parameters by inverting the relation and then taking the mean.
Method 2: we used the liner regression machine to get estimation and the standard error of the parameters. Before doing it, having in mind the real relationship between the probability and time to default, we observed that preprocessing the input could lead us to better results, indeed the survival probability correspond to the cumulative distribution function of an exponential random variable. For this reason, instead of giving as output the vector of probabilities we asked for the log of probabilities. The result was good also without it but still we got an improvement. We should notice that in case of linear regression the results are perfect, this in general can be a bad sign, but in this case is explainable by the fact that we exactly know what is going on between our input and output, and we are replicating it through this linear model which perfectly describes the link.

Estimation of Lambda1		
Method	Estimation	Confidence Interval
Maximum Likelihood	3.9438e-04	[3.3931e-04,4.4945e-04]
Empirical Mean	3.9090e-04	[3.3607e-04,4.4573e-04]
Linear Regression	4.0000e-04	[4.0000e-04,4.0000e-04]

Estimation of Lambda2		
Method	Estimation	Confidence Interval
Maximum Likelihood	9.9921e-04	[9.9302e-04,10.0000e-04]
Empirical Mean	9.9588e-04	[9.8971e-04,10.0000e-04]
Linear Regression	10.0000e-04	[10.0000e-04,10.0000e-04]