Politecnico di Milano

Financial Engineering

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Assignment 2

Authors:

Alice Iamoni
Rida Jafar
Chadi Kallas
Filippo Lipari

1 Case Study

1.1 Bootstrap Implementation

For the first part of this assignment we implemented a function that realizes the bootstrap for the discount factors' curve (with a single-curve model).
We first used the mid deposit rates up to 3 months in order to get the discount factors needed to interpolate (on the zero rate) for the one corresponding to the settlement date of the first future. This way, we were able to compute the discount factor corresponding to the expiry date of the first future. By doing the same thing (interpolation/extrapolation for the settlement day's discount and then computing the expiry date's one) for each future, we got the discount factors up to the 18^{th} of December 2024 (the expiry of the 7^{th} future).
We then moved to the swap rates: we first performed an interpolation to obtain the 1 year discount factor. Then, we created a complete set of swap rates and dates - with a modified following convention for the dates. We then proceeded to compute the 2 year discount factor with the 1 year one and so on until the 50^{th}.
In Figure 1 we can observe the plot both for the discount factors and the zero rates for the dates considered.

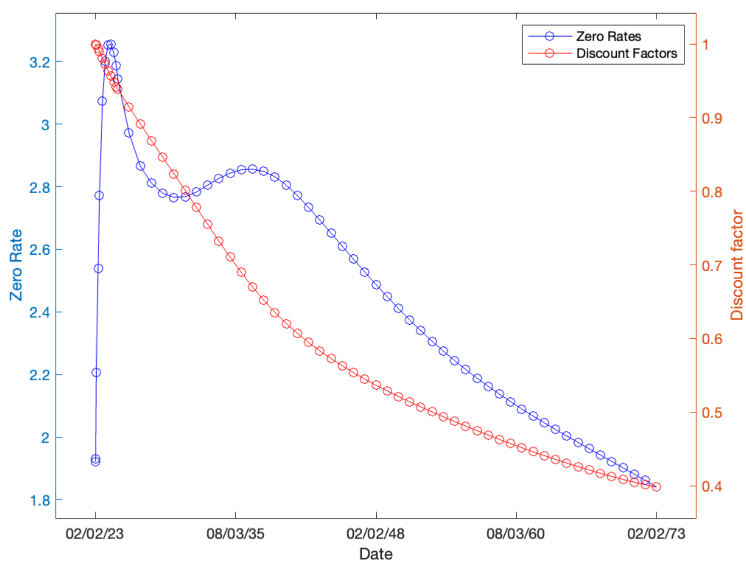

Figure 1: Discount factors and Zero rates

1.2 Q

Bootstrap is a technique used to obtain Discount Factors from quoted rates in the market. These discount factors are essential for pricing various financial instruments, namely bonds, swaps, options, and futures. Indeed, when we need to price instruments, we use the discount factors to discount the future cash flows involved in the instrument at the present time. Thanks to an iterative approach, where the bootstrapping technique computes the discount factors that match the quoted rates of various maturities, we obtain as a result a yield curve. This technique is widely used in finance because it is simple, transparent, standardized, and accurate. In addition to that, it reprices perfectly the set of pre-selected market instruments.
Moreover, we don't need any strong assumption on the form of the term structure. Finally, we have to say that an important condition for this method is the high liquidity of the market, which implies a small bid-ask spread - the difference between the prices on the market for an immediate sell or purchase

of a product. We need highly liquid markets in order to be able to compute many discount factors, for example in our case for long term discount factors we use swap market products because they usually have long-term maturities.

There are also other techniques that can perform the same task, such as interpolation (less accurate), parametric models (more accurate but complexity and computational needs increase), and Monte Carlo simulation (accurate but computationally intensive). The choice of the method depends on the specific needs, but thanks to the previously mentioned results we have great benefits, for instance we can price complex products with non-standard maturities.

2 Exercise

With the discount curve obtained above, for a portfolio composed only by one single swap, a 6y plain vanilla IR swap vs Euribor 3m with a fixed rate 2.8173% and a Notional of 10 Mln € we computed the following interest rates sensitivities:

- DV01-parallel shift;
- DV01Z-parallel shift;
- BPV of the 5y IRS;

The obtained results are the following:

Sensitivity	Unitary	Notional
DV01	5.424154e-04	5424.154957
$DV01^z$	5.60447e-04	5604.475142
BPV	5.42856e-04	5428.561619

We computed DV01 by finding the variation in net present values for our portfolio due to the increase of 1 basis point of the rates used for the bootstrap. For the BPV, instead, we computed the difference on net present values shifting the fixed rate by 1 basis point. Finally, we computed DV01Z as variation of NPVs due to a parallel shift of the zero rates.
We can notice that DV01 and BVP are quite similar for a plain vanilla par swap at trade date, which complies with our theoretical knowledge.
Moreover, we have computed the Macaulay Duration for an IB coupon bond with same fixed rate, expiry and reset dates of the IR swap, and face value equal to IRS notional. We got a value of 5.6055. Based on theory, it has to hold that $DV01^z = MacD * 1bp * Bond_Price$; and this is in compliance with our results, since the price we get for the bond is $9.99216 * 10^6$.

3 Theoretical Exercise

We are now interested in pricing an InterBank Coupon Bond issued on the 31st of Jan '23 with coupon rate equal to the corresponding mid-market 7y swap rate, namely 2.8173%. We assume for the coupons a 30/360 European day count, we impose a face value of 1 and a discount factor up to time i equal to $B(t_0, t_i)$. Therefore the price of a coupon bond can be computed numerically as the sum of the discounted coupons and the discounted face value:

$$P = \sum_{i=1}^{7} (c.\delta(t_{i-1}, t_i) B(t_0, t_i) + 1.B(t_0, t_7) \tag{1}$$

We notice that c is constant, so the price can be written as:

$$P = c \sum_{i=1}^{7} \delta(t_{i-1}, t_i) B(t_0, t_i) + B(t_0, t_7) \tag{2}$$

The resulting value is $P = 1$, so the price of the I.B.coupon bond is equal to its face value and we call the CB at par.

As a matter of fact this resulting value can be deducted without explicit calculations. Since this Interbank Coupon Bond uses the rate of the seventh year swap, we can use the formula below:

$$B(t_0, t_7) = \frac{1 - S_{IR}(t_0, t_7) \sum \delta(t_{n-1}, t_n) B(t_0, t_7)}{(1 + \delta(t_{i-1}, t_i) S_{IR}(t_0, t_7)} \tag{3}$$

It's enough to rearrange the terms to conclude that the present value of all the cash flows is equal to one.