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Optimal Hedging via Filtered Historical Simulation

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Abstract

This paper presents an analysis and state-of-the-art review of the Filtered Historical Simulation (FHS) method proposed by Giovanni Barone-Adesi, Kostas Giannopoulos, and Les Vosper. The FHS method holds considerable significance in the field of finance as it provides a means to estimate the Value-at-Risk (VaR) of financial portfolios by incorporating non-linearities and conditional volatilities observed in financial data.

The primary objective of this study revolves around optimizing portfolio hedging techniques utilizing the FHS method. Specifically, we apply this methodology to a particular portfolio comprising a swaption, employing currency swaps as a hedging mechanism. Moreover, we conduct a comparative analysis with an alternative approach. By subjecting the portfolio to various yield curve shock scenarios, we evaluate the efficacy of currency swaps as a hedging instrument. These analyses offer valuable insights into the relative performance of different hedging strategies. Through a comprehensive comparison and analysis of the FHS method and the yield curve steepener shock, this paper examines the drawbacks and limitations associated with each approach.

Contents

1 Introduction

In today's complex financial markets, managing and mitigating risks associated with investment portfolios is of paramount importance. Financial institutions, investors, and portfolio managers strive to protect their capital by employing effective hedging strategies. Traditional risk measurement techniques, such as historical simulation, have been widely used for estimating the Value-at-Risk (VaR) of portfolios. These models involve generating scenarios based on historical price changes for all variables in a portfolio, providing a more realistic estimation of its risk by relying on the empirical distribution of asset returns. However, these methods often fall short in capturing the intricate dynamics and non-linearities prevalent in financial data, limiting their accuracy and reliability.
To address these limitations, advanced risk measurement methodologies have emerged, one of which is the Filtered Historical Simulation (FHS) method. The FHS method, developed by Giovanni Barone-Adesi, Kostas Giannopoulos, and Les Vosper, incorporates a filtering mechanism to account for time-varying properties and conditional volatilities in financial assets. By capturing the non-linearities and dynamics of financial data, the FHS method offers a more accurate estimation of portfolio risk, enabling investors to make informed decisions and develop optimal hedging strategies.

The FHS method combines elements of historical simulation and time series filtering techniques, such as autoregressive conditional heteroscedasticity (ARCH) models or generalized autoregressive conditional heteroscedasticity (GARCH) models. By incorporating these models, the FHS method captures the volatility clustering and persistence often observed in financial time series data.
The implementation of the FHS method involves several steps. First, the historical returns of the assets comprising the portfolio are simulated. Next, the returns are filtered using a time series model to capture the time-varying properties of the assets' volatilities. The filtered returns are then used to estimate the portfolio's VaR by calculating the desired quantile of the filtered return distribution.
One of the key advantages of the FHS method is its ability to capture both the conditional volatility and higher-order moments of financial returns. This makes it particularly useful for risk measurement in portfolios that contain assets with complex and non-linear dependencies. By accounting for the time-varying nature of volatilities, the FHS method can better capture extreme market events and tail risk, providing a more comprehensive assessment of portfolio risk.

The primary objective of this paper is to explore the concept of optimal hedging using the Filtered Historical Simulation method. By integrating the FHS approach into the hedging framework, we aim to identify the most effective hedging strategies for a specific portfolio. In particular, we focus on the portfolio's composition, which includes a swaption as a derivative instrument.
To achieve our goal, we will adopt a systematic research methodology. We will first provide a comprehensive overview of the FHS method, delving into its theoretical foundations and mathematical framework. Next, we will explore the concept of optimal hedging and discuss the various factors that influence the choice of hedging strategies. We will analyze the benefits and drawbacks of different approaches, including the FHS method, to determine which technique offers the most robust and effective hedging solution for the swaption portfolio.
In order to substantiate our discoveries and evaluate the efficacy of the FHS-derived hedging strategy, we intend to perform empirical analyses employing historical market data. By contrasting the outcomes obtained from the FHS methodology with alternative hedging approaches, such as yield curve steepener shocks, we aim to assess their respective levels of effectiveness in mitigating portfolio risk.
Finally, we will critically analyze the limitations and drawbacks of the FHS method and discuss potential avenues for further research and improvement. By highlighting the strengths and weaknesses of the FHS method in the context of optimal hedging, we aim to provide practical insights and contribute to the ongoing discourse on risk management and portfolio optimization.

2 State of the art

The FHS is based on the combination of GARCH modelling (parametric) and historical portfolio returns (non-parametric) to estimate the last trading day's volatility and then calculate the portfolio's VaR. In this simulation, we rely on the empirical distribution derived from the historical return series without imposing any specific theoretical distribution on the data. In order to ensure that the returns are i.i.d, we take steps to eliminate any serial correlation (by adding MA term) and volatility clusters (captured by GARCH model) present in the dataset.

$$r_t = \mu r_{t-1} + \theta\varepsilon_{t-1} + \varepsilon_t \qquad \varepsilon_t \sim N(0, h_t) \tag{1}$$

$$h_t = \omega + a(\varepsilon_{t-1} + \gamma)^2 + \beta h_{t-1} \tag{2}$$

where in the equation (1) is the ARMA(1,1) modelling of r_t return, with μ and θ are respectively the term of AR(1) and MA(1). And equation (2) is GARCH(1,1) modelling of random residuals ε_t, which defines volatility of the random residuals ε_t as a function of last period volatility h_{t-1}, closest residual ε_{t-1} and constant ω.

For a suitable use of our historical simulation the residuals need to be standardized. We define

$$e_t = \frac{\varepsilon_t}{\sqrt{h_t}} \sim N(0, 1) \tag{3}$$

e_t are daily standardised residuals.

Remember that our aim is to predict the empirical distribution of r_t, that will give us the forecast of asset price, which will be obtained by the following process:

2.1 Simulating a single pathway

For the one day ahead forecast, the variance h_t of period t+1 can be calculated at the end of period t using equation (2) with last estimate residual ε_t.

1. We randomly draw standardised residual returns from the dataset. (equation (3))
2. The first-drawn standardised residual is scaled by the deterministic volatility forecast one day ahead:
$$z_{t+1} = e_1\sqrt{h_{t+1}} \tag{4}$$
3. The one day a head forecast of the asset price is given by:
$$p_{t+1} = p_t + p_t(\mu r_t + \theta z_t + z_{t+1}) \tag{5}$$

For i=2,3,..,10 , we have to simulate the volatility h_t of this corresponding forecast:

$$\sqrt{h_{t+i}} = \sqrt{\omega + \alpha(z_{t+i-1})^2 + \beta h_{t+i-1}} \tag{6}$$

We replicate the above procedure to obtain 5000 simulations for a time horizon of 10 days (i=10).

2.2 Simulating multiple pathways

For the case where multiple assets are involved, the simulation process is applied individually to each asset by using the above approach. To overcome the problem of correlation between assets, the random selection of strips of residuals will correspond to a date in the past common to all assets, which means that the simulated prices and volatilities for each distinct asset are derived from standardized residuals corresponding to the same date.

3 Empirical study

This section describes the application of FHS to a dataset, with daily quotes (MID) of EUR IRS for tenors from 1y to 10y over the period 1999-2008. Interest rates swaps (IRS) receive fixed rate at annual frequency (bond basis) and pay quarterly LIBOR 3m.

3.1 Shocks Simulation via FHS

Given the daily IRS, we first Bootstrap our Zero coupon rates for each day in the dataset, this means that we will have 10 value of Zero coupon rates corresponding to tenors from 1y to 10y. We then define our returns on the new dataset of Zero coupon rate:

$$r_t = \ln \frac{s_t}{s_{t-1}} \tag{7}$$

Where s_t is the Zero coupon rate corresponding to a tenor at date t.

For each historical return series, we fit the ARMA-GARCH(1,1) using equation(1) and (2), to produce i.i.d. standardised residuals. These standardised residuals are then scaled again to reflect current and forecast volatilities.

Table 1: Estimated Parameters for ARMA(1,1)

Return series of ZCR	μ	θ
1y	0.5177	-0.3659
2y	0.3367	-0.2511
3y	0.7970	-0.7722
4y	0.7777	-0.7662
5y	-0.7844	0.7637
6y	-0.8589	0.8394
7y	-0.7083	0.6809
8y	-0.6950	0.6682
9y	-0.7018	0.6737
10y	-0.7883	0.7638

At this point we can proceed with the procedure explained in [**section 2**].

Table 2: Historical Asset Residual Returns

Date	1y	2y	3y	...	7y	8y	9y	10y
04-Jan-1999	-2.3488	-1.5157	-0.5867	.	-1.1533	-1.5008	-1.3012	0.1866
05-Jan-1999	0.7062	-0.5726	0.1698	.	-0.7946	-0.7435	-1.0874	0.1309
06-Jan-1999	-1.1095	0.1325	0.5672	.	0.5383	0.7994	0.7292	0.3267
07-Jan-1999	0.3017	0.0279	0.5364	.	-0.4034	-0.4504	-0.4087	0.1669
08-Jan-1999	0.1135	0.0072	0.4206	.	0.2789	0.3053	0.2797	-0.1298
11-Jan-1999	0.0429	0.0019	0.3307	.	-0.1932	-0.2074	-0.1917	0.1010
.	.	.	.	.	.	.	.	.
04-Feb-2007	-0.4432	0.0037	0.0327	.	-0.0144	0.1795	-0.0393	0.0109
.	.	.	.	.	.	.	.	.
31-Dec-2008	-2.9059	-0.9557	0.1211	.	0.1026	-0.1846	0.1493	-0.0492

In particular, in order to simulate asset returns for the following ten days, we start by drawing a random row of historical asset residual returns and re-scale them with the corresponding asset's volatility on

the day of the prediction. The conditional volatility of date of the first prediction, is calculated by substituting the last trading date's residual error and variance into equation (2).
Finally, we simulate the ZC rate by applying equation (5). To produce the ith simulated volatility for the second date ahead we substitute ϵ_{t-1} with the $z^*_{i,t+1}$ in (2). We repeat the above calculations to get the 10-days ahead forecasts of the variances and ZC rates.
We consider a pathway of swap values derived by simulating zero coupon interest rate curves. In the initial scenario, we simulate 10 zero coupon rates for each day of the holding period. This process is replicated 5000 times to generate multiple simulations. For the chosen day (in our example it is day 10), we store the simulation inside a matrix **Simulated shocks** of size 10x5000 where each column presents one simulation (one scenario), and the rows represent the maturities of the swaps (our different assets).

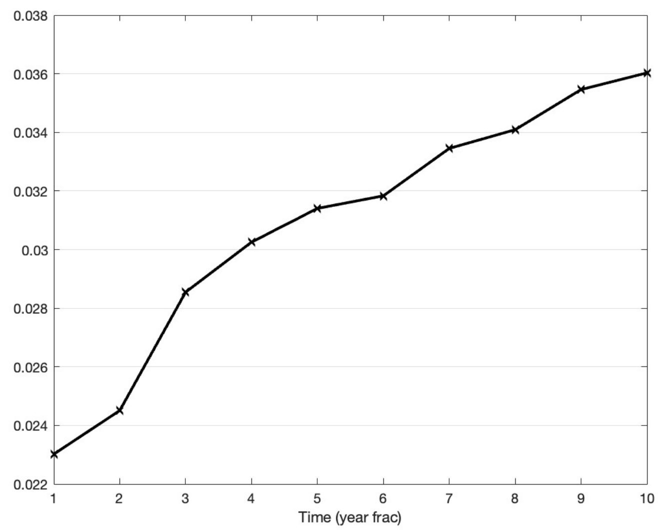

Figure 1: 10 days over of one simulation Zero coupon curve

3.2 Pricing Portfolio and testing different hedging strategy

We consider now an EUR portfolio composed of:

- Long swaption receiver 3y x 5y ATM - Notional EUR 100m
- Vanilla 5 years IRS fixed rate payer - Notional EUR 34m

with hedging ratio originally assigned by the Black delta of the swaption. The Black volatility is 30% and the pricing date of the portfolio is December 31st, 2008.

We first proceeded by deriving the **strike** of the swaption via the computation of the 5y x 3y forward swap rate, using the ZC curve of the 31st December, 2008 obtained from the bootstrap and got a result of: **0.0398**.

To **mark to market** our portfolio at the mid rate, we computed the approximated price of the interest rate swap (by considering a short 5y fixed rate bond, with coupons assigned by the swap rate and face value equal to the IRS notional, and a long cash position with same amount as the IRS notional) and the price of the receiver swaption (by using the Black and Scholes formula with the forward swap rate as underlying). We then multiplied each price by its notional and got a result of: **EUR 3.34m** .

Testing the Hedging strategies:

We first define possible different hedging strategies of the swaption by taking alternative combinations of IRS with 2y, 5y and 10y maturity. For every strategy we first look for the optimal Notional used for hedging.

1. Strategy: hedging with swap 2y
2. Strategy: hedging with swap 5y
3. Strategy: hedging with swap 10y
4. Strategy: hedging with combination of swap 2y, swap 5y, swap 10y
5. Strategy: hedging with combination of swap 2y, swap 5y
6. Strategy: hedging with combination of swap 2y, swap 10y
7. Strategy: hedging with combination of swap 5y, swap 10y

In order to perfectly delta-hedge our portfolio, we want our DV01 to be null. Therefore, for the strategies with only one swap we increased the IRS rates used for the bootstrap of the original curve by 1bps. Then we considered two different portfolios: the first one containing only the EUR 100m swaption, while the second one containing a EUR 1 swap of the kind we chose to hedge with. We calculated the DV01 for both of these portfolios by evaluating the difference in prices before and after the increase in IRS rates. Once found, we could determine the notional of the swap that best hedges the swaption simply by dividing the DV01s of the two portfolios in the following way:

$$N_{swap} = -DV01_{swaption}/DV01_{swap}$$

A similar thing can be done for the hedging strategies with combinations of swaps, but using Bucketed DV01s. We must recall that the sum of the Bucketed DV01s equals the DV01 (except for a small numerical error). Furthermore, if we consider a portfolio with only a 5y-swap and three buckets (2y, 5y, 10y), we will have that the 2y-bucketed DV01 and the 10y-bucketed DV01 will always be null, because the 5y-swap doesn't affect them. The same is true for the similar cases. Therefore for each bucket we can repeat the procedure above, taking a portfolio with only the EUR 100m swaption and another one with a EUR 1 swap with maturity that corresponds to the bucket considered. For instance, if we consider the hedging strategy with all three swaps(**Strategy 4**), we will get the notionals that best delta-hedge the portfolio by calculating:

$$N_{2y-swap} = -(2yBucketDV01_{swaption})/(2yBucketDV01_{2y-swap})$$

$$N_{5y-swap} = -(5yBucketDV01_{swaption})/(5yBucketDV01_{5y-swap})$$

$$N_{10y-swap} = -(10yBucketDV01_{swaption})/(10yBucketDV01_{10y-swap})$$

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The results of these calculations are provided in the table below:

Table 3: Notional for every strategy

	Notional in Million euro (10^6)		
Strategy	Notional 2y swap	Notional 5y swap	Notional 10y swap
1	95.71	0	0
2	0	40.25	0
3	0	0	22.08
4	-39.55	19.23	20.64
5	-39.55	56.76	0
6	-10.89	0	24.58
7	0	2.64	20.64

We test all the 7 strategies against a yield curve steepener shock.

- 2y IRS rate +70bps
- 5y IRS rate +170bps
- 10y IRS rate +190bps

We start by linearizing the steepener shock to obtain shocks for the other maturities. we get:

Maturities	1y	2y	3y	4y	5y	6y	7y	8y	9y	10y
Shocks	35bps	70bps	103bps	136bps	170bps	174bps	178bps	182bps	186bps	190bps

Table 4: Steepener shock of the corresponding IRS

Then we shift our market data composed by the IRS rate of December 31st, 2008. After bootstrapping the newly found data, we can measure the profit and loss of the different hedging strategies that we just found. In order to do this, we simply calculate the difference between the Mark-to-Market of the portfolios before and after the yield curve steepener shock.

Table 5: Profit & Loss

Strategy	1	2	3	4	5	6	7
PL ($*10^6$)	-1.146	0.596	0.794	1.499	1.307	1.013	0.782

First of all, from these results we notice that there are important profits and losses for every strategy, since even if the strategy was delta-hedged it was not gamma-hedged. Furthermore, we notice that **strategy 4** is the one that gets the highest profit after the shock. This means that using all three swaps to hedge the swaption is the strategy that reacts best to this yield curve steepener shock.

3.3 Determine the most effective strategy via VaR calculation by FHS

In the present analysis, we apply the Value at Risk (VaR) methodology to evaluate the performance of the seven strategies mentioned earlier. This analysis involves utilizing the simulated shocks generated by the FHS method. It is important to note that in this case, we work directly with the simulated shocks without applying any shift.

To initiate the VaR analysis, we begin by considering each simulation within our matrix of simulated shocks, denoted as the **Simulated Shocks** matrix with dimensions 10x5000. For each simulation, we compute the new mark-to-market value of our portfolio based on the corresponding strategy. This process enables us to derive the distribution of portfolio values, as illustrated in Figure 2 for strategy 4.

To further analyze the impact on our portfolio, we then subtract the mark-to-market value of the portfolio as of December 31st, 2008, from the computed portfolio values. This subtraction allows us to determine the profit and loss (P&L) distribution associated with our strategies. By subtracting the December 31st, 2008 mark-to-market value, we obtain a measure of the P&L relative to that specific reference point.

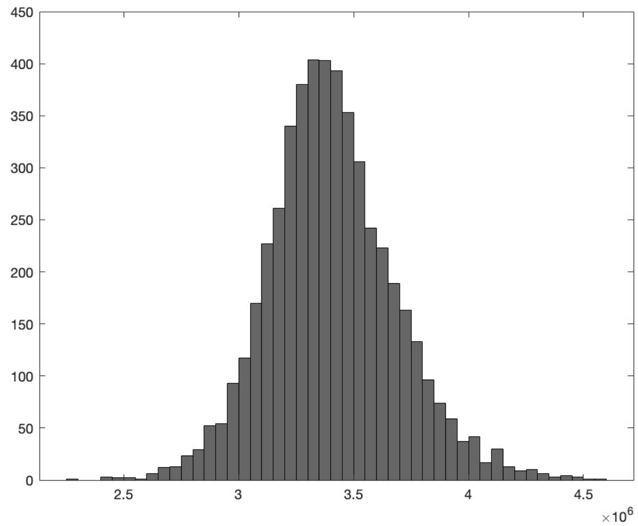

Figure 2: Portfolio value distribution of strategy 4 over 10 day holding period for 5000 simulation runs

After, we sort the simulated P&L in ascending order and we set our confidence level to 99%.
The results of $VaR_{99\%}^{10days}$ obtained for every strategy are presented in the table below:

Table 6: $VaR_{99\%}^{10days}$ $(*10^6)$

Strategy	1	2	3	4	5	6	7
$VaR_{99\%}^{10days}$ $(*10^6)$	1.184	1.023	0.860	0.806	2.345	0.837	0.859

We notice that Strategy 4 has a $VaR_{99\%}^{10days}$ of 0.806 million, which is the lowest when compared to other strategies. This suggests that Strategy 4 may have a lower potential loss at a 99% confidence level compared to the other strategies.

3.4 Discussion

The hedging strategy that appears to be the optimal one found after applying both the steepener shock and the FHS method is **Strategy 4**, which corresponds to hedging the swaption 3y x 5y with a combination of swap 2y, 5y and 10y.
While both methods involve testing different combinations of IRS with different maturities, they approach the problem from different angles. The first method emphasizes hedging effectiveness against a specific shock, while the second method prioritizes risk reduction through VaR calculation.

Therefore using another steepener shock, there is no guarantee that the strategy with the highest profit would be the same.

Let us make a concrete example. We now consider a yield curve steepener shock of this kind:

- 2y IRS rate +70bps
- 5y IRS rate +110bps
- 10y IRS rate +110bps

With the same methodology as before we obtain the new profits and losses for every strategy after this shock, reported in the table below:

Table 7: Profit & Loss for new shock ($*10^6$)

Strategy	1	2	3	4	5	6	7
P&L ($*10^6$)	-0.414	0.282	0.230	0.519	0.563	0.302	0.234

We can clearly see that here the hedging strategy that gets the highest profit is strategy 5, where we only consider the swap 2y and the swap 5y for hedging.

FHS is more reliable than Stress Testing with steepener shock since it is performed not on a specific shock but with a simulation of possible shocks. However, Stress Testing for a particular crisis situations is useful to understand how the chosen hedging strategies might react.

4 Conclusion

Filtered historical simulation enables a rapid assessment of VaR. This efficiency stems from the utilization of a straightforward historical simulation, which is executed daily using a pre-established time-series filter. The computational workload of our computations increases linearly with the number of assets. However, the reliability of our method relies on the effectiveness of the filters employed in our time series analysis; the VaR results can be sensitive to these assumptions, and small changes can significantly impact the risk estimates, better the filter better the VaR estimation.

FHS takes implicitly into account the correlations between the assets by taking a row of random residuals and maintaining the co-movement between the assets. However, this can be an inadequate way of treating the correlation and it can lead to inaccurate risk estimates.

Taking this into account, even if Filtered Historical Simulation can be a useful tool in risk management, it is crucial to understand its limitations and complement it with other risk assessment techniques to obtain a more comprehensive view of the risks involved: Stress testing, for instance, provides insights into tail risks and helps evaluate the impact of severe events that may not be adequately captured by FHS or other historical-based approaches. Other possible risk-assessment approaches can be Parametric VaR, Risk Factor Models or Backtesting.

References

[1] Barone-Adesi, Giovanni, Kostas Giannopoulos, Les Vosper (1999), VaR without correlations for portfolios of derivative securities, Journal of Futures Markets 19.5.