Politecnico di Milano

Financial Engineering

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Assignment 7

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1 Determine the upfront X% [Pricing]. Solve computing the spot vols.

In order to compute our structured bond following the swap term sheet, we first computed the caplet spot volatilities given the caplet flat volatilities solving system 1 via bootstrap, where ΔC is the difference between two consecutive caps (with maturities corresponding to the ones of the flat volatility surface).

$$\begin{cases} \Delta C = \sum_{i=t_j+1}^{t_{j+1}} caplet(T_i, \sigma_i) \\ \sigma_i = \sigma_{t_j} + \dfrac{T_i - T_{t_j}}{T_{t_{j+1}} - T_{t_j}} (\sigma_{t_{j+1}} - \sigma_{t_j}) \end{cases} \tag{1}$$

Where t_j are the maturities of the flat volatilities given, while each σ_i represents one spot volatility. The caplets flat volatilities under 1Y have been supposed to be all equal to each other $\sigma_1 = \sigma_2 = \sigma_3 = \sigma_{flat_1 Y}$.
By doing so, we obtained the following Spot Volatility Surface:

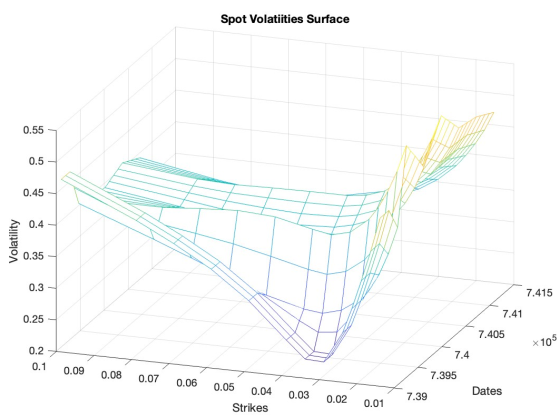
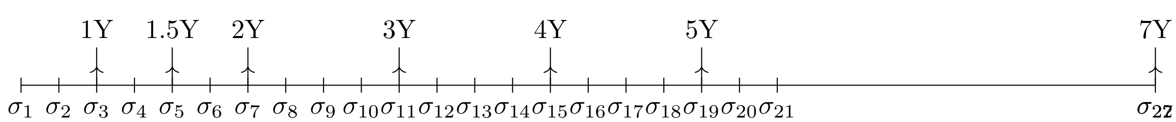


Figure 1: Spot Volatility Surface

Then, using the *interp2()* function - with *cubic spline* method in the strikes - we interpolated in the given caplet strikes to finally get the volatilities needed.

To help you visualize below there is a drawing of our time line for the spot volatilities.



Then we computed the upfront (X) of the contract imposing its NPV equal to zero:

$$X = s_{spread} \cdot BPV - s \cdot BPV_s + 1 - B(t_0, t_{end}) - 1 \cdot B(t_0, t_1) + B(t_0, t_{end}) + \sum_{i=1}^{3} Cap_i - c \cdot \delta_1 \cdot B(t_0, t_1)$$

where:

- s_{spread} = spol of the bank XX = 2%
- s = spol of the I.B. = 1.1%

- BPV = BPV starting from 3 months
- BPV_s = BPV starting from 6 months
- c = initial coupon
- Cap_i = price of the i^{th} Cap, where the three caps are the ones between 0-3 years, 3-5 years and 5-7 years.

We obtained **X = 8.65**%.

2 Delta-bucket sensitivities

We computed the Delta bucket sensitivities by shifting by 1bp the rates of the most liquid financial instruments. We shifted the rates once at a time and we iterated the process for all of them. Then we computed the DVO1 for all the shifts as below:

$$DVO1 = X_{new} - X_{old}$$

where X represent the upfront.
The table below shows the DV01 computed not considering the notional of the portfolio.

	Date	DV01 (*e-3)
Depos	02-Mar-23	0.001166350318549
Depos	02-May-23	0.010869365456787
Future	15-Mar-23	-0.00306278110927
Future	21-Jun-23	-0.00492087246999
Future	20-Sep-23	-0.00570618496617
Future	20-Dec-23	-0.00492759840860
Future	20-Mar-24	-0.00379805712486
Future	19-Jun-24	-0.00246461618280
Future	18-Sep-24	-0.00088841868192
Swap	03-Feb-25	-0.00239961914344
Swap	02-Feb-26	0.009955188411195
Swap	02-Feb-27	-0.02100983758235
Swap	02-Feb-28	0.010312628766218
Swap	02-Feb-29	-0.02476625741165
Swap	04-Feb-30	0.302044985172639

3 Total Vega

In order to compute the Vega of our contract we computed the Vega of each Cap that compose it. The Vega of a Cap was computed by summing the Vega of each caplet that constitutes the Cap. We calculated it using the following formula:

$$\nu_{caplet} = \delta_i \cdot L_i \cdot \Phi(d_1) \cdot \sqrt{t_i - t_0}$$

where:

$$d_1 = \frac{log(\frac{L_i}{K})}{\sigma_i \sqrt{t_i - t_0}} + \frac{1}{2}\sigma_i \sqrt{t_i - t_0}$$

Without considering the notional of the portfolio we obtained the following results:

Vega cap1	Vega cap2	Vega cap3	Total Vega
0.036531534995205	0.040300059211493	0.051467136069149	0.128298730275846

4 The coarse-grained buckets and hedging with swaps

In our function, *compute_DV01_coarse_grained_swaps.m* we computed the coarse grained DV01 bucket for the 3 periods, 0-2Y, 2Y-5Y and 5Y-7Y. For the first period, we shifted all the rates taking into account the weights computed via *compute_weights* function and considering that all the weights for futures and depos are equal to 1. For the second and third period, we shifted only the swaps with weights computed with the same function as before applied in a new time interval. In addition, in all three cases we have created a new discount curve made with the new shifted ratesSet, and we bootstrapped to find our spot volatilities to finally calculate the new upfront and compute the DV01 coarse grained bucket as the difference between the new upfront and the upfront from the previous question.

DV01_cg_bucket(0-2y)	DV01_cg_bucket(2-5y)	DV01_cg_bucket(5-7y)
-1.36055192996865e-05	-2.15087251017043e-05	0.000284798141627393

Then, we have selected 3 swaps with maturities 2Y-5Y-7Y, and computed their notional such that the delta is equal to zero in the hedged portfolio. For the calculation of DV01 of the 3 swaps, we used our function *DV01_swap* where we shift in a parallel way all the rates in ratesSet by 1bp. The DV01 of the swap is computed as the difference between the NPV shifted and NPV not shifted.

At the end, we computed the swap notionals starting from the one with longest maturity using the following triangular system:

$$N_7 = -\text{Notional} \times \text{DV01\_cg\_bucket(5-7y)}/\text{DV01\_swap\_7}$$
$$N_5 = -\left(\text{Notional} \times \text{DV01\_cg\_bucket(2-5y)} + N_7 \times \text{DV01\_swap\_7}\right)/\text{DV01\_swap\_5}$$
$$N_2 = -\left(\text{Notional} \times \text{DV01\_cg\_bucket(0-2y)} + N_5 \times \text{DV01\_swap\_5} + N_7 \times \text{DV01\_swap\_7}\right)/\text{DV01\_swap\_2}$$

N_2	N_5	N_7
-2.1371e+06	3.4025e+07	-2.3112e+07

5 Hedge the Vega with an ATM 5y Cap

To Hedge the total Vega, we first computed the Vega of our ATM (strike = mid swap rate) 5y cap, then we got the cap Notional using the following formula:

$$N_{\text{cap}} = -\frac{\text{total\_vega} \times \text{Notional}}{\text{vega\_cap\_5y}} \tag{2}$$

N_{cap} = -8.3547e+07

Then to hedge the total portfolio as in d, we considered a new portfolio composed as the old one plus N_{cap} of the 5y ATM cap and we hedged it as we did in question (d). We therefore obtained the new notionals for the IR swaps following these formulas:

$$N_{7,\text{new}} = N_7 - \frac{N_{\text{cap}} \cdot \text{DV01cg,bucket,5y(3)}}{\text{DV01swap,7}}$$
$$N_{5,\text{new}} = N_5 - \frac{N_{\text{cap}} \cdot \text{DV01cg,bucket,5y(2)} + N_7 \cdot \text{DV01swap,7}}{\text{DV01swap,5}}$$
$$N_{2,\text{new}} = N_2 - \frac{N_{\text{cap}} \cdot \text{DV01cg,bucket,5y(1)} + N_5 \cdot \text{DV01swap,5} + N_7 \cdot \text{DV01swap,7}}{\text{DV01swap,2}}$$

$N_{2,\text{new}}$	$N_{5,\text{new}}$	$N_{7,\text{new}}$
1.7777e+08	1.9218e+08	3.4978e+07