Assignment 3- Argument

October 10, 2023

Solution to Question 1-a

We aim to show that the premise $\neg(p \vee \neg q) \wedge (\neg p \wedge \neg q) \vee (p \wedge (p \vee \neg q))$ implies p using only the valid argument forms: Modus Ponens, Modus Tollens, Generalization, Conjunction, Construction, Transitivity, and Elimination.

1. **Given Premise**: $\neg(p \vee \neg q) \wedge (\neg p \wedge \neg q) \vee (p \wedge (p \vee \neg q))$
2. **Elimination**:
   - Consider the first part $\neg(p \vee \neg q) \wedge (\neg p \wedge \neg q)$. If this is true, then $\neg p$ is true (via Conjunction and Elimination).
   - Consider the second part $p \wedge (p \vee \neg q)$. If this is true, then p is true (via Conjunction and Elimination).
3. **Conjunction**: Form the conjunction of these implications:
   - If $\neg(p \vee \neg q) \wedge (\neg p \wedge \neg q)$ is true, then $\neg p$ is true.
   - If $p \wedge (p \vee \neg q)$ is true, then p is true.
4. **Construction**: Construct the following statement from step 3.
   - $(\neg(p \vee \neg q) \wedge (\neg p \wedge \neg q)) \Rightarrow \neg p$
   - $(p \wedge (p \vee \neg q)) \Rightarrow p$
5. **Transitivity**: By transitivity of implication, we can form the following:
   - If $\neg(p \vee \neg q) \wedge (\neg p \wedge \neg q) \vee (p \wedge (p \vee \neg q))$ is true, then $\neg p \vee p$ is true.
6. **Generalization and Elimination**: From step 5, generalize to say that either $\neg p$ is true or p is true. Use elimination to say that this disjunction implies p.

Solution to Question 1-b

By these steps and using only the specified argument forms, we can argue that the original premise $\neg(p \vee \neg q) \wedge (\neg p \wedge \neg q) \vee (p \wedge (p \vee \neg q))$ implies p.

We start with the given premise:

$$((p \wedge \neg q) \vee (p \wedge \neg q)) \vee \neg(p \wedge (p \vee q))$$

1. **Elimination (Remove redundancy)**

$$(p \wedge \neg q) \vee \neg(p \wedge (p \vee q))$$

By Elimination, we remove the duplicate $(p \wedge \neg q)$.

2. **Conjunction (Introduce conjunction to eliminate negation of conjunction)**

$$(p \wedge \neg q) \vee (\neg p \vee \neg(p \vee q))$$

We introduce $\neg p$ as a conjunction to the term $\neg(p \wedge (p \vee q))$ to get $\neg p \vee \neg(p \vee q)$.

3. **Generalization (Introduce common terms)**

$$(p \wedge \neg q) \vee \neg p \vee (\neg p \wedge \neg q)$$

We use Generalization to introduce common terms $\neg p$ and $\neg q$ to simplify the expression.

4. **Elimination (Remove redundant terms)**

$$(p \wedge \neg q) \vee \neg p \vee \neg q$$

By Elimination, we remove redundant terms, in this case, $\neg p \wedge \neg q$.

5. **Construction (Combine terms into the final expression)**

$$(p \wedge \neg q) \vee (\neg p \vee \neg q)$$

By Construction, we combine $\neg p$ and $\neg q$ to get $\neg p \vee \neg q$, and combine this with $p \wedge \neg q$ to arrive at $(p \wedge \neg q) \vee (\neg p \vee \neg q)$.

By following these steps using only the argument forms you specified, we've shown that $((p \wedge \neg q) \vee (p \wedge \neg q)) \vee \neg(p \wedge (p \vee q))$ can be simplified to $(p \wedge \neg q) \vee (\neg p \vee \neg q)$.