Summary in Graph

Exam Summary (GO Classes 2024 | Discrete Mathematics | Test 1)

Qs. Attempted:	15 (5 + 10)	Correct Marks:	9 (3 + 6)
Correct Attempts:	6 (3 + 3)	Penalty Marks:	1.67 (0.33 + 1.33)
Incorrect Attempts:	9 (2 + 7)	Resultant Marks:	7.33 (2.66 + 4.66)

Total Questions:	15 (5 + 10)
Total Marks:	25 (5 + 20)
Exam Duration:	45 Minutes
Time Taken:	45 Minutes

EXAM RESPONSE | EXAM STATS | FEEDBACK

Technical

Q #1 | Multiple Choice Type | Award: 1 | Penalty: 0.33 | Mathematical Logic

For a given predicate $\mathrm{P(x)}$, you might believe that the statements $\forall \mathrm{xP(x)}$ or $\exists \mathrm{xP(x)}$ are either true or false.

To prove $\exists \mathrm{x\ P(x)}$ is false:

A. Give an example of an element n in the domain for which $P(n)$ is true.
B. Give an example of an element n in the domain for which $P(n)$ is false.
C. Show for every element n in the domain, that $P(n)$ is true.
D. Show for every element n in the domain, that $P(n)$ is false.

Your Answer: D | **Correct Answer: D** | Correct | Discuss

Q #2 | Multiple Choice Type | Award: 1 | Penalty: 0.33 | Mathematical Logic

Consider the following proposition : $\mathrm{A}_n = \underbrace{(p \to (q \to (p \to (q \to (\ldots)))))}_{\text{number of p's + number of q's = n}}$. Which of the following is false for A_n : A. For every $n > 2, \mathrm{A}_n$ is a tautology. B. For every $n > 2, \mathrm{A}_n$ is a contradiction. C. For every $n = 2, \mathrm{A}_n$ is a contingency. D. For every $n > 2, \mathrm{A}_n$ is Not contingency.

Your Answer: D | Correct Answer: B | Incorrect | Discuss

Q #3 | Multiple Select Type | Award: 1 | Penalty: 0 | Mathematical Logic

Which of the following is the negation of “there is a successful person who is grateful”?

A. There is a successful person who is ungrateful.
B. Every grateful person is unsuccessful.
C. Every unsuccessful person is grateful.
D. Every successful person is ungrateful.

Your Answer: B;D | Correct Answer: B;D | Correct | Discuss

Q #4 | Multiple Select Type | Award: 1 | Penalty: 0 | Mathematical Logic

Consider the following predicates.

- $\text{Rabbit}(x) = x$ is a rabbit.
- $\text{Cute}(x) = x$ is cute.

Consider the following statement E, where the domain of every variable is set of all animals in a jungle J.
$\text{E} = \forall x(\text{Rabbit}(x) \land \text{Cute}(x))$

If statement E is true, then which of the following is true?

A. There is no animal other than rabbits in the jungle J.
B. Every rabbit is cute in jungle J.
C. It is possible that there is some animal in J who is not a rabbit but is cute.
D. There is some rabbit who is cute in jungle J.

Your Answer: A;B | Correct Answer: A;B;D | Incorrect | Discuss

Q #5 | Multiple Select Type | Award: 1 | Penalty: 0 | Mathematical Logic

Consider the statement S : "For all natural numbers n, if n is prime, then n is antisocial."
You do not need to know what antisocial means for this problem, just that it is a property that some numbers have and others do not.
Assume that we know that natural number 10 is not a prime number, $\&$ that natural number 7 is a prime number.

Which of the following statements can be inferred from the statement S?

A. 10 is antisocial.
B. 10 is not antisocial.
C. 7 is antisocial.
D. 7 is not antisocial.

Your Answer: C | Correct Answer: C | Correct | Discuss

Q #6 | Multiple Select Type | Award: 2 | Penalty: 0 | Mathematical Logic

Suppose $P(x, y)$ is some binary predicate defined on a very small domain of discourse: just the integers $1, 2, 3$, and 4. For each of the 16 pairs of these numbers, $P(x, y)$ is either true or false, according to the following table (x values are rows, y values are columns).

	1	2	3	4
1	T	F	F	F
2	F	T	T	F
3	T	T	T	T
4	F	F	F	F

For example, $P(1, 3)$ is false, as indicated by the F in the first row, third column.

Which of the following statements are false?

A. $\forall x \exists y P(x, y)$.
B. $\forall y \exists x P(x, y)$.
C. $\exists x \forall y P(x, y)$.
D. $\exists y \forall x P(x, y)$.

Your Answer: B;C | Correct Answer: A;D | Incorrect | Discuss

Q #7 | Multiple Select Type | Award: 2 | Penalty: 0 | Mathematical Logic

Let $P(x)$ and $Q(x)$ be predicates and let D denote the domain of the predicate variable x. Consider the following universal conditional statement,

$$\forall x \in D, P(x) \rightarrow Q(x).$$

Which of the following conditions implies that the above universal conditional statement is true?

A. $P(x) \wedge Q(x)$ is false for all $x \in D$.
B. $P(x) \wedge (\sim Q(x))$ is false for all $x \in D$.
C. $Q(x)$ is true for all $x \in D$.
D. $P(x) \vee Q(x)$ is true for all $x \in D$.

Your Answer: B;C | Correct Answer: B;C | Correct | Discuss

Q #8 | Multiple Select Type | Award: 2 | Penalty: 0 | Mathematical Logic

Let $P(x), Q(x), R(x)$ and $S(x)$ denote the following predicates with domain $\mathbb{Z}$:

$$\begin{aligned}
&P(x): x^2-x-12=0, \\
&Q(x): x \text{ is odd}, \\
&R(x): x<0, \\
&S(x): x^2-9=0.
\end{aligned}$$

Which of the following statements is/are true?

A. $\forall x \in \mathbb{Z}, \quad P(x) \rightarrow Q(x)$
B. $\forall x \in \mathbb{Z}, \quad (P(x) \vee Q(x)) \rightarrow R(x)$
C. $\exists x \in \mathbb{Z}$ such that $P(x) \rightarrow (Q(x) \wedge R(x))$
D. $\forall x \in \mathbb{Z}, \quad S(x) \rightarrow (Q(x) \wedge S(x))$

Your Answer: D | Correct Answer: C;D | Incorrect | Discuss

Q #9 | Multiple Select Type | Award: 2 | Penalty: 0 | Mathematical Logic

Many programming languages support a ternary conditional operator. For example, in C, $\mathrm{C}{+}{+}$, and Java, the expression $x ? y : z$ means "evaluate the boolean expression x. If it's true, the entire expression evaluates to y. If it's false, the entire expression evaluates to z."

In the context of propositional logic, we can introduce a new ternary connective $? :$ such that $p ? q : r$ means "if p is true, the connective evaluates to the truth value of q, and otherwise it evaluates to the truth value of r."

Let p, q, r be three propositional variables. Which of the following is/are correct?

A. Probability of $p ? q : r$, being true is $\frac{1}{2}$.
B. $p ? p : p$ is tautology.
C. $p ? p : (\neg p)$ is tautology.
D. $(\neg p) ? p : (\neg p)$ is tautology.

Your Answer: A;C | Correct Answer: A;C | Correct | Discuss

Q #10 | Multiple Choice Type | Award: 2 | Penalty: 0.67 | Mathematical Logic

Which of the following formulas is a formalization of the sentence :

"There is a barber who shaves all men in the town who do not shave themselves"

Where $\text{shave}(x, y)$ means "x shaves y"

A. $\exists x[\text{Barber}(x) \wedge \exists y[[\text{man}(y) \wedge \neg \text{shaves}(y, y)] \rightarrow \text{shaves}(x, y)]$
B. $\exists x[\text{Barber}(x) \wedge \forall y[[\text{man}(y) \wedge \neg \text{shaves}(y, y)] \wedge \text{shaves}(x, y)]$
C. $\exists x[\text{Barber}(x) \wedge \forall y[[\text{man}(y) \wedge \neg \text{shaves}(y, y)] \rightarrow \text{shaves}(x, y)]$
D. $\exists x[\text{Barber}(x) \wedge \forall y[[\text{man}(y) \rightarrow \neg \text{shaves}(y, y)] \rightarrow \text{shaves}(x, y)]$

Your Answer: A | Correct Answer: C | Incorrect | Discuss

Q #11 | Multiple Select Type | Award: 2 | Penalty: 0 | Mathematical Logic

Translate the following sentences into First-order logic (FOL): " If someone is noisy, everybody is annoyed."

Use the following predicates :

- $\mathrm{N}(x)$: "x is noisy"
- $\mathrm{A}(x)$: "x is annoyed"

Which of the following is correct translation :

A. $\exists x(\mathrm{N}(x) \rightarrow \forall y(\mathrm{A}(y)))$
B. $\exists x(\mathrm{N}(x)) \rightarrow \forall y(\mathrm{A}(y))$
C. $\forall x(\mathrm{N}(x)) \rightarrow \forall y(\mathrm{A}(y))$
D. $\forall x(\mathrm{N}(x) \rightarrow \forall y(\mathrm{A}(y)))$

Your Answer: A;B | Correct Answer: B;D | Incorrect | Discuss

Q #12 | Multiple Choice Type | Award: 2 | Penalty: 0.67 | Mathematical Logic

We define a new quantifier, uniqueness quantifier, the symbol of which is $\exists!$.

For any predicate P and universe U, $\exists! x \mathrm{P}(x)$ means there is exactly one element in the universe for which P is true.

Which of the following statements is(are) Valid ?

I. $\exists! x \mathrm{P}(x) \wedge \exists! x \mathrm{Q}(x) \Rightarrow \exists! x(\mathrm{P}(x) \wedge \mathrm{Q}(x))$
II. $\exists! x(\mathrm{P}(x) \wedge \mathrm{Q}(x)) \Rightarrow \exists! x \mathrm{P}(x) \wedge \exists! x \mathrm{Q}(x)$
III. $\exists! x \mathrm{P}(x) \vee \exists! x \mathrm{Q}(x) \Rightarrow \exists! x(\mathrm{P}(x) \vee \mathrm{Q}(x))$
IV. $\exists! x(\mathrm{P}(x) \vee \mathrm{Q}(x)) \Rightarrow \exists! x \mathrm{P}(x) \vee \exists! x \mathrm{Q}(x)$

A. I, II, IV
B. I, III
C. II, III, IV
D. IV only

Your Answer: C | Correct Answer: D | Incorrect | Discuss

Q #13 | Multiple Select Type | Award: 2 | Penalty: 0 | Mathematical Logic

Consider the following predicates.

- $\mathrm{Rabbit}(x) = x$ is a rabbit.
- $\mathrm{Cute}(x) = x$ is cute.

Consider the following statement E, where the domain of every variable is set of all animals in a jungle J.
$\mathrm{E} = \exists x(\mathrm{Rabbit}(x) \rightarrow \mathrm{Cute}(x))$

If statement E is false, then which of the following is necessarily true?

A. There is no animal other than rabbits in the jungle J.
B. There is no cute animal in the jungle J.
C. There is no cute rabbit in jungle J.
D. There is some rabbit who is not cute in jungle J.

Your Answer: A;B;C | Correct Answer: A;B;C;D | Incorrect | Discuss

Q #14 | Numerical Type | Award: 2 | Penalty: 0 | Mathematical Logic

Let P be a compound proposition over 4 propositional variables : a, b, c, d. We know that for a compound proposition over n propositional variables, we have 2^n rows in the truth table. Every row of the truth table of P is called an "Interpretation" of P. A row in the truth table of P is called "model" iff P is true for that row. Let P be the sentence $(a \wedge b) \vee (b \wedge c)$ How many models are there for P?

Your Answer: 7 | Correct Answer: 6 | Incorrect | Discuss

Q #15 | Multiple Choice Type | Award: 2 | Penalty: 0.67 | Mathematical Logic

Let's make a trip to a new world called "Never Never Land".

Regular, ordinary first-order logic has two quantifiers: $\forall$ and $\exists$.

Now, let's imagine we lived in a world in which these quantifiers didn't exist, and instead we only had one quantifier, N. The quantifier N is the "never" quantifier, and the expression

$$\mathrm{N}x.\ [\text{some formula}]$$

means "[some formula] is never true, regardless of what choice of x we pick." For example, the expression $\mathrm{Nx(P(x))}$ says "There is No element x in the domain, such that $\mathrm{P(x)}$ is true".

For predicates $\mathrm{A(x)}$ and $\mathrm{B(x)}$, Which of the following is the correct expression for "All A's are B's" ?

A. $\neg\mathrm{Nx(A(x) \rightarrow B(x))}$
B. $\neg\mathrm{Nx(A(x) \wedge \neg B(x))}$
C. $\mathrm{Nx(A(x) \wedge \neg B(x))}$
D. $\mathrm{Nx}(\neg A(x) \wedge B(x))$

Your Answer: C | Correct Answer: C | Correct | Discuss