Summary in Graph

Exam Summary (GO Classes Test Series 2024 | Linear Algebra | Test 1)

Qs. Attempted:	**20** 5 + 15	**Correct Marks:**	**17** 3 + 14
Correct Attempts:	**10** 3 + 7	**Penalty Marks:**	**1.67** 0.33 + 1.33
Incorrect Attempts:	**10** 2 + 8	**Resultant Marks:**	**15.33** 2.66 + 12.66

Total Questions:	**20** 5 + 15
Total Marks:	**35** 5 + 30
Exam Duration:	**60 Minutes**
Time Taken:	**60 Minutes**

EXAM RESPONSE　　EXAM STATS　　FEEDBACK

Technical

Q #1　Multiple Select Type　Award: 1　Penalty: 0　Linear Algebra

Which of the following statements are true? Note that more than one statement may be true. You should indicate all the true statements.

A. Eigen values can never be zero.
B. Eigen vectors can never be the zero vector.
C. If $\vec{x}$ is an eigen vector of a matrix A, then $2\vec{x}$ is also an eigen vector of A.
D. The maximum possible rank of 5×7 matrix is 5.

Your Answer: B;C;D　Correct Answer: B;C;D　Correct　Discuss

Q #2　Numerical Type　Award: 1　Penalty: 0　Linear Algebra

If $\det \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = 3$, what is $\det \begin{bmatrix} a_{31} & a_{32} & a_{33} \\ 2a_{21} & 2a_{22} & 2a_{23} \\ a_{11} & a_{12} & a_{13} \end{bmatrix}$?

| Your Answer: -3 | Correct Answer: -6 | Incorrect | Discuss |

Q #3 | Multiple Choice Type | Award: 1 | Penalty: 0.33 | Linear Algebra

Let $\mathbf{u} = \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} -4 \\ 17 \\ -13 \end{bmatrix}$.

It can shown that $3\mathbf{u} - 4\mathbf{v} + \mathbf{w} = \mathbf{0}$. Use this fact to find a solution to the system $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{bmatrix} 2 & 4 \\ 5 & 1 \\ -1 & 3 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad \text{and } \mathbf{b} = \begin{bmatrix} -4 \\ 17 \\ -13 \end{bmatrix}$$

A. $\mathbf{x} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$

B. $\mathbf{x} = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$

C. $\mathbf{x} = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$

D. $\mathbf{x} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$

| Your Answer: C | Correct Answer: C | Correct | Discuss |

Q #4 | Multiple Select Type | Award: 1 | Penalty: 0 | Linear Algebra

A, B and C are arbitrary, non-singular matrices. $\det\left(A^T B\right)$ must equal which of the following?

A. $\det(A)\det(B)$

B. $\dfrac{\det(B)}{\det(A)}$

C. $\det\left(A^{-1}\right)\det(B)$

D. $\dfrac{\det(B)}{\det(A^{-1})}$

| Your Answer: A;D | Correct Answer: A;D | Correct | Discuss |

Q #5 | Multiple Choice Type | Award: 1 | Penalty: 0.33 | Linear Algebra

Which of the following vectors are eigenvectors of the matrix: $\begin{bmatrix} 1 & 3 & 6 \\ 2 & 1 & 4 \\ 1 & 0 & 3 \end{bmatrix}$

A. $\begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$

B. $\begin{bmatrix} -2 \\ -2 \\ 1 \end{bmatrix}$

C. $\begin{bmatrix} 0 \\ 1 \\ -5 \end{bmatrix}$

D. None of these

Your Answer: D   **Correct Answer: B**   Incorrect   Discuss

Q #6   Multiple Choice Type   Award: 2   Penalty: 0.67   Linear Algebra

Let a, b be in $\mathbb{R}$. Consider the three vectors

$$\boldsymbol{v}_1 = \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix}, \quad \boldsymbol{v}_2 = \begin{bmatrix} 0 \\ b \\ 1 \end{bmatrix}, \quad \boldsymbol{v}_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$$

For which values of a and b are $\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3$ independent?

   A. $a = 0$ and $b = 1$
   B. $a \neq 0$ and $b \neq 1$
   C. $a = 0$ and $b \neq 1$
   D. $a \neq 0$ and $b = 1$

Your Answer: B   **Correct Answer: B**   Correct   Discuss

Q #7   Multiple Choice Type   Award: 2   Penalty: 0.67   Linear Algebra

- $\mathrm{S}1$: A set of two vectors in $\mathbb{R}^n$ is Linealy dependent if at least one vector is a multiple of the other.
- $\mathrm{S}2$: A set of n vectors in $\mathbb{R}^n$ is Linealy independent if and only if none of the vectors are a multiple of any other vector.

   A. S1 and S2 both are correct
   B. S1 is correct and S2 is incorrect
   C. S2 is correct and S1 is incorrect
   D. S1 and S2 both are incorrect

Your Answer: B   **Correct Answer: B**   Correct   Discuss

Q #8   Multiple Select Type   Award: 2   Penalty: 0   Linear Algebra

Suppose A is 3 by 4, and $Ax = 0$ has all solutions in the following form -

$$\mathrm{x} = \mathrm{s} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} + \mathrm{t} \begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

We do some unknown elementary row transformations on A and get the following matrix R -

$$R = \begin{bmatrix} 1 & 0 & a & b \\ 0 & 1 & c & d \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

What will be the values of $a, b, c,$ and d?

   A. $a = -1$
   B. $b = 2$
   C. $c = -1$
   D. $d = 1$

Your Answer: A;B;C;D   Correct Answer: A;B;C;D   Correct   Discuss

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Q #9   Multiple Choice Type   Award: 2   Penalty: 0.67   Linear Algebra

Consider the vectors $\mathbf{u} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 1 \\ c^2 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 1 \\ c \end{bmatrix}$, where c is a constant.

Consider two statements.

- S1 : $c = 1$ is the ONLY possibility where b is a linear combination of u and v.
- S2 : $c = -1$ is the ONLY possibility where b is NOT a linear combination of u and v.

Which of the following is/are true?

A. S1 is correct but S2 is incorrect
B. S1 is incorrect but S2 is correct
C. Both are correct
D. Both are incorrect

Your Answer: B   Correct Answer: B   Correct   Discuss

Q #10   Multiple Select Type   Award: 2   Penalty: 0   Linear Algebra

Which of the following is/are TRUE ?

A. Suppose an $m \times n$ matrix A has n pivot columns. Then for each $\mathbf{b} \in \mathbb{R}^m$, the equation $A\mathbf{x} = \mathrm{b}$ has at most one solution.
B. Suppose $Ax = b$ has the solution, the solution unique solution if and only if $Ax = 0$ has unique solution.
C. If A is an $m \times n$ matrix and $Ax = 0$ has a unique solution, then $Ax = b$ is consistent for every b in $\mathbf{R}^m$.
D. If for some matrix A, and some vectors x and b, we have $Ax = b$, then b is linear combination of columns of A.

Your Answer: A;B;C;D   Correct Answer: A;B;D   Incorrect   Discuss

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Q #11   Multiple Choice Type   Award: 2   Penalty: 0.67   Linear Algebra

Consider the matrices A and R given below.

$$A = \begin{bmatrix} 1 & 2 & 1 & b \\ 2 & a & 1 & 8 \\ \star & \star & \star & \star \end{bmatrix}, \quad R = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Suppose we get R after performing some unknown elementary row operations on A. What will be the value of a and b ?

A. $a = 2, b = 5$
B. $a = 0, b = 3$
C. $a = 4, b = 5$
D. None of these

Your Answer: C   Correct Answer: C   Correct   Discuss

Q #12 | Multiple Select Type | | Award: 2 | | Penalty: 0 | | Linear Algebra |

If the characteristic polynomial of A is $= \lambda^2(\lambda - 3)^4$, then the $\mathrm{rank}(A)$ could be

A. 3
B. 4
C. 5
D. 6

Your Answer: A;B   **Correct Answer: B;C** | Incorrect | | Discuss |

Q #13 | Multiple Select Type | | Award: 2 | | Penalty: 0 | | Linear Algebra |

Which of the following is/are FALSE ?

A. Let A be an $m \times n$ matrix, and assume that the matrix A has a pivot in every column. Additionally, assume $A\vec{v} = A\vec{w}$ for some vectors $\vec{v}, \vec{w} \in \mathbb{R}^n$.
Then $\vec{v} = \vec{w}$.

B. If A and B are both invertible $n \times n$ matrices, then AB is invertible.

C. Let A be an $m \times n$ matrix. Then, the homogeneous equation $A\vec{x} = \vec{0}$ is consistent
if and only if the augmented matrix $[A \mid \vec{0}]$ has a pivot in every row.

D. Let A be a 3×2 matrix and B a 2×3 matrix. Then the determinant of AB MUST be zero.

Your Answer: A;B;C   **Correct Answer: C** | Incorrect | | Discuss |

Q #14 | Multiple Choice Type | | Award: 2 | | Penalty: 0.67 | | Linear Algebra |

Which one of the following statements is NOT correct?

A. An eigenvalue of A is a scalar λ such that $A - \lambda I$ is not invertible.
B. An eigenvalue of A is a scalar λ such that $(A - \lambda I)v = 0$ has a solution.
C. An eigenvalue of A is a scalar λ such that $Av = \lambda v$ for a nonzero vector v.
D. An eigenvalue of A is a scalar λ such that $\det(A - \lambda I) = 0$.

Your Answer: C   **Correct Answer: B** | Incorrect | | Discuss |

Q #15 | Multiple Select Type | | Award: 2 | | Penalty: 0 | | Linear Algebra |

Let A be $n \times n$ matrix with real coefficients. Which of the following is/are true?

A. A must have an even number of non-real eigenvalues.
B. If $v_1, v_2 \in \mathbb{R}^n$ are eigenvectors of A with different eigenvalues $\lambda_1 \neq \lambda_2$, then v_1 and v_2 are linearly independent.
C. If $v_1, v_2 \in \mathbb{R}^n$ are eigenvectors of A with different eigenvalues $\lambda_1 \neq \lambda_2$, then v_1 and v_2 are orthogonal.
D. The eigenvalues of AB are the product of the eigenvalues of A and B.

Your Answer: B;D   **Correct Answer: A;B** | Incorrect | | Discuss |

Q #16 | Multiple Choice Type | Award: 2 | Penalty: 0.67 | Linear Algebra |

It is known that $A \in \mathbb{R}^{3 \times 3}$ has eigenvalues $\lambda_1 = 0, \lambda_2 = 1, \lambda_3 = 2$.
Which of the following is true?

    A. Linear system $Ax = b$ always has solution for any b
    B. Linear system $Ax = b$ has solution for some $b \neq 0$
    C. Linear system $Ax = b$ can not have solution for any non zero b
    D. Linear system $Ax = b$ has solution iff $b = 0$.

| Your Answer: C | Correct Answer: B | Incorrect | Discuss |

Q #17 | Multiple Select Type | Award: 2 | Penalty: 0 | Linear Algebra |

For any real numbers a, b, c, let $A = \begin{bmatrix} 1 & a & b \\ -a & 1 & c \\ -b & -c & 1 \end{bmatrix}$ and let $B = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$. Which of the following

is/are true ?

    A. A is invertible for all values of a, b, c.
    B. B is not invertible for any values of a, b, c.
    C. B is invertible for all values of a, b, c.
    D. A is not invertible for any values of a, b, c.

| Your Answer: A;B | Correct Answer: A;B | Correct | Discuss |

Q #18 | Multiple Select Type | Award: 2 | Penalty: 0 | Linear Algebra |

Let A be an $m \times n$ matrix and $\mathbf{b}$ be a vector in $\mathbb{R}^m$. Which of the following statements implies that the
matrix equation $A\mathrm{x} = \mathrm{b}$ is consistent?

    A. Every row of A contains a pivot position.
    B. $\operatorname{rank} A = n$.
    C. $m = n$ and A is invertible.
    D. $m < n$.

| Your Answer: A;B;C | Correct Answer: A;C | Incorrect | Discuss |

Q #19 | Multiple Select Type | Award: 2 | Penalty: 0 | Linear Algebra |

Let A be a 12×9 matrix (i.e., A has 12 rows and 9 columns) and b be a vector in $\mathbb{R}^{12}$. We know that the
matrix equation $A\mathbf{x} = \mathrm{b}$ has infinitely many solutions. Which of the following statements are false?

    A. b can be written as a linear combination of the columns of A.
    B. Every row of A has a pivot position.
    C. The columns of A are linearly dependent.
    D. $\operatorname{rank} A \geq 9$.

| Your Answer: A;C | Correct Answer: B;D | Incorrect | Discuss |

Q #20 | Numerical Type | Award: 2 | Penalty: 0 | Linear Algebra |

Let A be 5×4 matrix. If all solutions of $Ax = 0$ are scalar multiple of one nonzero vector then what will be the rank of A?

Your Answer: 3   **Correct Answer: 3**   Correct   Discuss

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