Project Report:

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Problem statement:

Price a digital option of duration 3 months that pay $1 when the underlying 20-year Japanese government bond rate is increased 25 bps from today's level.

Methodology

Assumptions:

To conduct a thorough analysis of the option, certain assumptions have been made:

1. The Japanese Government rate follows a Geometric Brownian motion.
2. The volatility of the market is constant and known. (Assumed as 17%)
3. The risk-free rate is constant and known. (Assumed as 1%)
4. There are no transaction costs for the purchase of this option.
5. The option is a European - will be exercised only at the expiration.

Thus, we can see that these assumptions resemble those of the Black-Scholes analytic formula for option pricing.

Pricing methodologies:

There are multiple methods that can be used for the calculation of the price of this option:

1. Black – Scholes Analytic formula
2. The binomial asset pricing model
3. Monte Carlo Simulation

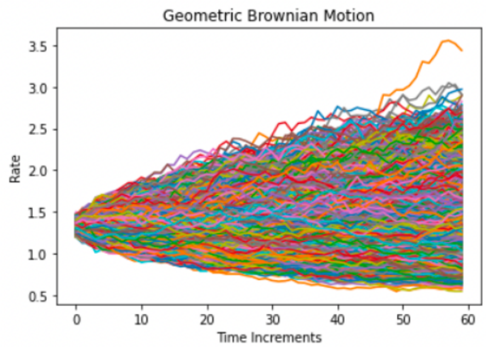
- In this project, we have implemented Black-Scholes analytic model and verified the result using a Monte Carlo simulation for the same.
- The derivation for the Black-Scholes analytic is attached as a separate file titled – 'Black-Scholes derivation for Digital option'.

To assist with this calculation, the current 20-year Japanese government rate as of January 18th (1.33%) was utilized as a key input in the model.
And the rest of the assumptions (Volatility, Risk-free rate, etc.) are as follows:

```
r0 = 1.33          # Today's rate
K = r0 + 0.25      # Strike rate
r = 0.01           # Risk Free rate
sigma = 0.17       # Volatility
T = 0.25           # Time to maturity - 3 months
```

t = 0 # inital time
paths = 1000000 # number of simulations
corr = -0.5 #correlation

In this implementation, we have simulated 1,000,000 values of the Geometric Brownian motion for the day 3 months from today.
If graphed against time, the possible paths of rate would look like the below figure:



Calculating the payoff:

- Thus, here we see that if the rate has reached a value of 0.25% greater than the original value (Today's value), the payoff is considered ($1). Eventually, the average of the total payoff is considered.
- As we increase the number of simulations (paths) the value will become more and more accurate.

We see that, as we compare the results we get from both methods, the value is quite close to each other.

```
Reloaded modules: Option
option value Black-Scholes Analytic 0.020647860736527288
option value monte carlo simulation 0.019111162322012938
```

Scenario Analysis:

What is the fair value of the option one month forward given a range of 20-year Japanese rate change?

Using the code derived above we get the value of the option as - $0.02
This is the value of the option that is maturing 3 months from today. To calculate the value of the option one month forward we can use the forward rate formula -> $F=S_oe^{rT}$ where S_o = spot value (Today's value) r = risk-free rate T = time in years. (1-month in our case = 1/12)
Therefore, F = 0.02 * exp (0.01 * 1/12) ≈ 0.021

Addressing the range of rate change of the 20-year Japanese rates:

- The fair value of the option basically depends on the possibility that the Japanese rates will increase by 25 bps. Thus, the range of the past range of rate changes can help us understand whether the rate will increase by 25 bps.
- If the range of rate change is greater the probability that the rate will change only 25 bps is low thus the fair value of the option will be low. On the other hand, if the range of

rate change is lesser the probability that the rate will change 25 bps is higher, hence the fair value of the option will be higher.

- In other words, **the fair value of the option will be dependent on the volatility of the 20-year Japanese rates.**

Dual Conditional option:

What would be the fair value of the option if a condition that – Nikkei should also drop 5%, and only if both the conditions are met would the option payoff be received?

- It is fair to assume that the Nikkei and the government rates would have some sort of correlation.
- If we consider the macroeconomic factors and their effect on the 2 conditions for this option, we can conclude that often the stock market and government rates would be negatively correlated.
- To get the exact value of this correlation we can run a regression analysis on the historical values of these two attributes and then use this value to simulate the rates for the Monte-Carlo simulation. For demonstration purposes, I have considered the correlation to be -0.5 in this project.

Methodology:

- A way to calculate this dual condition option's fair value would be using Monte Carlo Simulation.
- We simulate the values of the Nikkei and the Government rates, calculate the payoff using the dual condition for the option and then discount the same at the risk-free rate for the time period of the option.

Thus, we can see in main.py we have simulated negatively correlated values and using these pairs of values simulated the Nikkei and the interest rates of the Japanese government.
The payoff is calculated as follows:

```
#calculating payoff for dual condition option
def getDualConOption(self, opt, GBM, Nikkei):
    for i in range(opt.N):
        if (GBM.rtPrices[i] > opt.K and Nikkei.n0/Nikkei.nikkei[i] > 1.05) :
            self.payOff += 1
    return (self.payOff/opt.N)*np.exp(-opt.r*opt.T)
```

Thus, we get the value of this dual-conditional option (all the other conditions same) as 0.00597 ≈ 0.006

Open Question:

What macroeconomic factors make it reasonable or unreasonable to invest in a dual conditional option such as the one described in this project?

The 2 conditions that decide the payoff of the option are affected by the following macroeconomic factors:

1. Inflation

2. Gold prices
3. The Dollar-Yen exchange rate
4. Gross Domestic Product

<u>Inflation</u>:
Effect of inflation on Nikkei – An **increase in inflation** would drive up the prices of goods and services which would mean a reduction in corporate profitability thus the **market would have a downward trend.**
Effect of inflation on Government rates – **Rising inflation** would also lead to an **increase in the interest rates** to reduce spending.

Thus, we see that both these conditions are favorable for the dual conditional option offered in this project as it increases the probability that the government rate would increase by 0.25% and Nikkei would decrease by 5%.

<u>Gold prices:</u>
A decrease in gold prices would drive investors away from the stock market thus increasing the probability that Nikkei would drop by 5%. Most of the time, it is noticed that Gold and interest rates have a negative correlation, thus a decrease in gold prices would increase the interest rates.
Both these conditions make it reasonable to invest in this dual-condition option.

<u>$/¥ ratio:</u>
While the exchange rate of the currencies does not affect the interest rates, the converse is true. An increase in the rate would mean increased foreign investment in the country thus there would also be an increase in the Nikkei.
Thus, this would reduce the probability of both conditions in this dual option being met, making it unfavorable to invest.

<u>Gross – Domestic – Product:</u>
A decrease in the GDP would mean a decrease in the Nikkei too. However, a decrease in GDP would lead to a decrease in interest rates.
This makes this an unfavorable condition to invest in a dual conditional option such as this one.

References:

1. Prabowo, Danang & Asandimitra, Nadia. (2019). Effects of Macroeconomic Variables, World Gold Prices, World Oil Prices, and Dow Jones Index on Japanese Stock Index Nikkei 225. International Journal of Academic Research in Economics and Management Sciences. 8. 2226-3624. 10.6007/IJAREMS/v8-i3/6654.