Hands On

Question - 3

Average runtime complexity of the non-random pivot.

- $T(n)$ is average time complexity for input size of n

- Last element - pivot

Average number of comparision for an array of size n can be given by:

$$C(n) = n-1 + C(0) + C(n-1)$$

$n-1 \rightarrow$ No. of comparision for partitioning

$C(0) \rightarrow$ No. of comparision for empty sub array.

$C(n-1) \rightarrow$ Average number of comparisions for right sub-array

$T(n)$ is sum of $C(k)$ for $k=0$ to $n-1$, divided by n

$$T(n) = \frac{1}{n} \sum_{k=0}^{n-1} C(k)$$

$$T(n) = \frac{1}{n} \sum_{k=0}^{k=n-1} (k + c(0) + c(k-1))$$

$$T(n) = \frac{1}{n} \left(\sum_{k=0}^{k=n-1} k + \sum_{k=0}^{n-1} c(0) + \sum_{k=0}^{n-1} c(k-1) \right)$$

$$T(n) = \frac{1}{n} \left(\frac{n(n-1)}{2} + n\,c(0) + \sum_{k=0}^{n-1} c(k-1) \right)$$

$$T(n) = \frac{1}{n} \left(\frac{n(n-1)}{2} + n\,c(0) + \sum_{k=-1}^{n=-2} c(k) \right)$$

Let $j = k-1$

$$\sum_{k=-1}^{n-2} c(k) = \sum_{j=-2}^{n-3} c(j)$$

Therefore,

$$T(n) = \frac{1}{n} \left(\frac{n(n-1)}{2} + n\,c(0) + \sum_{j=-2}^{n-3} c(j) \right)$$

This will result into recursive nature of the recursive relation

$\therefore$ Average time complexity of non-random pivot version of Quicksort is $\boxed{O(n \log n)}$