$\vec{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 3 \end{bmatrix}$ ← vector addition

$\vec{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 3 \end{bmatrix}$

$\vec{v}\vec{w} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ ← linear combination of $\vec{v}$ & $\vec{w}$

- dot of $\vec{v}$ & $\vec{w}$ are not commutative, but cross product is.
- $\vec{v}\vec{w}$ & $\vec{w}\vec{v}$ are not parallel (or are parallel) in not commutative. Dot product is the length of the first vector projected onto the second times the length of the second vector.

$\vec{v}\vec{w} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ ← distributive property

$\vec{v}\vec{w} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ ← scalar multiplication

$\vec{v}\vec{w} = 2 \cdot 1 + 0 \cdot 1 + 0 \cdot 1 + 3 \cdot 1$

- Length, length squared
  - $\|\vec{v}\| = \sqrt{2^2 + 3^2} = \sqrt{13}$ ← length
  - $\|\vec{v}\|^2 = 2^2 + 3^2 = 13$ ← length squared

- Dot product
  - $\vec{v} \cdot \vec{w} = v_1 w_1 + v_2 w_2 + \cdots + v_n w_n$
  - Properties
    - $\vec{v} \cdot \vec{w} = \vec{w} \cdot \vec{v}$ ← $\vec{v} \cdot \vec{w} = \|\vec{v}\| \cdot \|\vec{w}\| \cdot \cos\theta$
    - $\vec{v} \cdot \vec{v} = \|\vec{v}\|^2$ ← $\vec{v} \cdot \vec{v} = \|\vec{v}\| \cdot \|\vec{v}\| \cdot \cos 0$
    - $\vec{v} \cdot \vec{w} = 0$ ← $\vec{v} \cdot \vec{w} = \|\vec{v}\| \cdot \|\vec{w}\| \cdot \cos 90$
  - Triangle inequality
    - $|\vec{v} \cdot \vec{w}| \le \|\vec{v}\| \cdot \|\vec{w}\|$ ← Cauchy-Schwarz inequality
    - $\|\vec{v} + \vec{w}\| \le \|\vec{v}\| + \|\vec{w}\|$ ← triangle inequality
  - Angle between two points
    - $\vec{v} \cdot \vec{w} = v_1 w_1 + v_2 w_2$
    - $\cos\theta = \frac{\vec{v} \cdot \vec{w}}{\|\vec{v}\| \cdot \|\vec{w}\|}$
    - $\cos\theta = \frac{v_1 w_1 + v_2 w_2}{\sqrt{v_1^2 + v_2^2} \cdot \sqrt{w_1^2 + w_2^2}}$
    - $\cos\theta = \frac{v_1 w_1 + v_2 w_2}{\sqrt{v_1^2 + v_2^2 + w_1^2 + w_2^2}}$
    - $\cos\theta = \frac{v_1 w_1 + v_2 w_2}{\sqrt{v_1^2 + v_2^2 + w_1^2 + w_2^2}}$ ← Pythagorean theorem

- Orthogonal: if $\vec{v} \cdot \vec{w} = 0$ ← perpendicular to one another (or) $\theta = 90$ (or) one vector is zero.
  - Projection

  - $\vec{v} \cdot \vec{w} = \text{base} \cdot \text{projection of } \vec{v} \cdot \vec{w} = \vec{v} \cdot \vec{w}$
  - $\vec{v} \cdot \vec{w} = \vec{v} \cdot \vec{w} = \text{Corresponding}$
  - Finding the base vector for $\vec{v}$
    - $\vec{v} \cdot \vec{w} = \vec{v} \cdot \vec{w} = \vec{v} \cdot \vec{w} = \vec{v} \cdot \vec{w} = \vec{v} \cdot \vec{w} = \vec{v} \cdot \vec{w} = \vec{v} \cdot \vec{w} = \vec{v} \cdot \vec{w}$
    - $\vec{v} \cdot \vec{w} = \vec{v} \cdot \vec{w} = \vec{v} \cdot \vec{w} = \vec{v} \cdot \vec{w} = \vec{v} \cdot \vec{w} = \vec{v} \cdot \vec{w} = \vec{v} \cdot \vec{w} = \vec{v} \cdot \vec{w}$ ← the base vector of $\vec{v}$
    - $\vec{v} \cdot \vec{w} = \vec{v} \cdot \vec{w} = \vec{v} \cdot \vec{w} = \vec{v} \cdot \vec{w} = \vec{v} \cdot \vec{w} = \vec{v} \cdot \vec{w} = \vec{v} \cdot \vec{w} = \vec{v} \cdot \vec{w}$ ← the base vector of $\vec{v}$
  - Cross product
    - $\vec{v} \times \vec{w} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \times \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} v_2 w_3 - v_3 w_2 \\ v_3 w_1 - v_1 w_3 \\ v_1 w_2 - v_2 w_1 \end{bmatrix}$ ← cross product
    - Cross product is perpendicular to both vectors
    - Cross product is not commutative
    - Cross product is not associative
    - Cross product is not distributive
    - Cross product is not cancellative
    - Cross product is not transitive
    - Cross product is not reflexive
    - Cross product is not symmetric
    - Cross product is not antisymmetric
    - Cross product is not linear
    - Cross product is not bilinear
    - Cross product is not multilinear