Q6: Riddhiman Roy: royridd1

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a) $A = \begin{bmatrix} 2 & c & c \\ c & c & c \\ 8 & 7 & c \end{bmatrix}$

$$\begin{bmatrix} 2 & c & c \\ c & c & c \\ 8 & 7 & c \end{bmatrix}$$

$$\xrightarrow[R_1 = R_1 - R_2]{} \begin{bmatrix} 2-c & 0 & 0 \\ c & c & c \\ 8 & 7 & c \end{bmatrix}$$

$$\xrightarrow[R_2 = R_2 + R_1]{} \begin{bmatrix} 2-c & 0 & 0 \\ 2 & c & c \\ 8 & 7 & c \end{bmatrix}$$

$$\xrightarrow[R_2 = R_3 - R_2]{} \begin{bmatrix} 2-c & 0 & 0 \\ 6 & 7-c & 0 \\ 8 & 7 & c \end{bmatrix}$$

$$\xrightarrow[R_3 = R_3 - R_2]{} \begin{bmatrix} 2-c & 0 & 0 \\ 6 & 7-c & 0 \\ 2 & c & c \end{bmatrix}$$

$$\xrightarrow[R_1 = R_2 - 3R_3]{} \begin{bmatrix} 2-c & 0 & 0 \\ 0 & 7-4c & -3c \\ 2 & c & c \end{bmatrix}$$

$$\xrightarrow[R_3 = R_3 - R_1]{} \begin{bmatrix} 2-c & 0 & 0 \\ 0 & 7-4c & -3c \\ c & c & c \end{bmatrix}$$

if A is invertible

then

$$\begin{bmatrix} 2-c & 0 & 0 \\ 0 & 7-4c & -3c \\ c & c & c \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

if $c = 1$

then $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -3 \\ 1 & 1 & 1 \end{bmatrix} \neq I$

if $c = 0$

then $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 7 & -3 \\ 0 & 0 & 0 \end{bmatrix} \neq I$

if $c = -1$

then $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 11 & 3 \\ -1 & -1 & -1 \end{bmatrix} \neq I$

$\boxed{\text{then } c = -1, 0, 1}$

b) $\left[\begin{array}{cccc|cccc} 1 & -a & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -b & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -c & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$

$$1 \quad 0 \quad 0 \quad 0 \quad 1 \; \Big| \; 0 \quad 0 \quad 0 \quad 1$$

$$\xRightarrow[R_3 = c \cdot R_4 + R_3]{} \left[\begin{array}{cccc|cccc} 1 & -a & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -b & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\xRightarrow[R_2 = bR_3 + R_2]{} \left[\begin{array}{cccc|cccc} 1 & -a & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & b & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\xRightarrow[R_1 = aR_2 + R_1]{} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & a & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & b & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\boxed{\therefore A^{-1} = \begin{bmatrix} 1 & a & 0 & 0 \\ 0 & 1 & b & 0 \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$