Q3

Thursday, October 29, 2020 8:31 AM

a) $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^1$

$$T\left(\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}\right) = v_1 - v_2$$

P_1: $T(c\vec{v}) = c \cdot T\vec{v}$

LHS

$$c\vec{v} = \begin{bmatrix} cv_1 \\ cv_2 \end{bmatrix}$$

$$T(c\vec{v}) = cv_1 - cv_2$$

RHS

$$c \cdot T\vec{v} = c(v_1 - v_2)$$
$$= cv_1 - v_2$$

P_1 Pass

P_2: $T(\vec{v} + \vec{w}) = T(\vec{v}) + T(\vec{w})$

LHS

$$T\left(\begin{bmatrix} v_1 + w_1 \\ v_2 + w_2 \end{bmatrix}\right) = (v_1 + w_1) - (v_2 + w_2)$$
$$= v_1 + w_1 - v_2 - w_2$$

RHS

$$T(\vec{v}) + T(\vec{w}) = (v_1 - v_2) + (w_1 - w_2)$$
$$= v_1 - v_2 + w_1 - w_2$$
$$= v_1 + w_2 - v_2 - w_2$$

$\therefore P_2$ pass

linear transform

b)

$$T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = 1$$

$$T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = -1 \qquad \boxed{M_T = [1, -1]}$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \cdot [1, -1] = v_1 - v_2 \quad \checkmark$$

c) $T(\vec{0}) = \vec{0}$

c) $T(\vec{0}) = \vec{0}$

$$\vec{w} = -\vec{v}$$

$$T(\vec{v} - \vec{v}) = T(\vec{v}) + T(-\vec{v})$$

$$= T(\vec{v}) - T(\vec{v}) = 0$$

QED