

a) (i) $b_3 = x_3$
 $x_2 + x_3 = b_2$
 $x_1 + x_2 + x_3 = b_1$

$$\therefore \vec{b} = \begin{bmatrix} x_1 + x_2 + x_3 \\ x_2 + x_3 \\ x_3 \end{bmatrix}$$

(ii) $b_3 = 0$
 $x_2 + x_3 = b_2$
 $x_1 + x_2 + x_3 = b_1$

$$\vec{b} = \begin{bmatrix} x_1 + x_2 + x_3 \\ x_2 + x_3 \\ 0 \end{bmatrix}$$

(iii) $b_3 = x_3 = b_2$
 $b_1 = x_1 + x_2 + x_3$

$$\vec{b} = \begin{bmatrix} x_1 + x_2 + x_3 \\ x_3 \\ x_3 \end{bmatrix}$$

b) $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 5 \\ 0 & 4 & 0 \end{bmatrix}$ LU decomp

$$R_2 = R_2 - 2R_1 \quad \begin{bmatrix} 1 & 1 & 1 \\ 2-2 & 4-2 & 5-2 \\ 0 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 4 & 0 \end{bmatrix} \quad E_1 = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_3 = R_3 - 2R_2 \quad \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & -6 \end{bmatrix} \quad E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & -6 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

c) $E_1 = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$E_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

$$1: E_1^{-1} \cdot E_2^{-1}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix} = L$$

$$\text{check: } \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 4 & 5 \\ 0 & 4 & 0 \end{pmatrix} \quad \checkmark \checkmark$$