Q5

Thursday, October 29, 2020 8:31 AM

a) $3\vec{v} + 6\vec{w} + 9\vec{z} = \vec{b}$

$\vec{b} = A\vec{x}$ $\quad \vec{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$A = \begin{bmatrix} 3 & 0 & 0 \\ 3 & 3 & 0 \\ 3 & 3 & 3 \end{bmatrix}$

$A\vec{x} = \begin{bmatrix} 3 & 0 & 0 \\ 3 & 3 & 0 \\ 3 & 3 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 3+6 \\ 3+6+9 \end{bmatrix}$

$3\vec{v} + 6\vec{w} + 9\vec{z} = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix} + \begin{bmatrix} 0 \\ 6 \\ 6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 9 \end{bmatrix} = \begin{bmatrix} 3 \\ 9 \\ 18 \end{bmatrix}$

$\therefore A = \begin{bmatrix} 3 & 0 & 0 \\ 3 & 3 & 0 \\ 3 & 3 & 3 \end{bmatrix}$ $\vec{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$\boxed{A\vec{x} = \vec{b}}$

b) $x_1 = 1$

$\therefore \vec{v} \perp x_2\vec{w} + x_3\vec{z} = 0$

$5(1 + 4x_2 + 7x_3) = 0 \quad = 5 + 20x_2 + 35x_3 = 0$

$4(2 + 5x_2 + 8x_3) = 0 \quad = 8 + 20x_2 + 32x_3 = 0$

$-3 \qquad +3x_3 = 0$

$\boxed{x_3 = +1}$

$1 + 4(x_2) + 7 = 0$

$c = -4x_2$

$\boxed{x_2 = -2}$

$x_2 = -2$

$x_1 = 1$

$x_2 = -2$

$x_3 = 1$

c) $\vec{v} \times \vec{\omega}$: $\begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix}$ $= (12-15)i - (6-12)j + (5-8)k$

$= -3\vec{i} + 6\vec{j} - 3k$

$= \begin{bmatrix} -3 \\ 6 \\ -3 \end{bmatrix} : \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}$

$\pi: -x + 2y - z = D$

since $(0,0,0)$ is on plane, $D = 0$

$\therefore$ $-x + 2y - z = 0 : \pi$

$\begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 7 \\ 6 \\ 9 \end{bmatrix} = 0$

$-7 + 16 - 9 = 0$ ✓