Q2: Riddhiman Roy: royridd1

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a)

$$\begin{matrix} -2 & 4 \\ -1 & 2 \\ 0 & -1 \\ 1 & 0 \\ 2 & 0 \end{matrix}$$

$$\begin{bmatrix} 1 & -2 \\ 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} A \\ C \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$ This condition must be satisfied for a perfect fit.

b) $\overrightarrow{x_{LS}} = (A^+A)^{-1} \cdot (A^+\vec{b})$

$$A^+ \cdot A = \underset{2\times 5}{\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -2 & -1 & 0 & 1 & 2 \end{bmatrix}} \underset{5\times 2}{\begin{bmatrix} 1 & -2 \\ 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}}$$

$$= \begin{bmatrix} 5 & 0 \\ 0 & 4+1+0+1+4 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 10 \end{bmatrix} \Rightarrow (A^+A)^{-1} = \frac{1}{50}\begin{bmatrix} 10 & 0 \\ 0 & 5 \end{bmatrix} = A^*$$

$$A^+ \cdot \vec{b} = \underset{2\times 5}{\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -2 & -1 & 0 & 1 & 2 \end{bmatrix}} \underset{5\times 1}{\begin{bmatrix} 4 \\ 2 \\ -1 \\ 0 \\ 0 \end{bmatrix}} = \begin{bmatrix} 4+2-1 \\ -8-2+0 \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix} = \vec{b}^*$$

2×1

$$\therefore A^* \cdot \vec{b}^* = \frac{1}{50}\underset{2\times 2}{\begin{bmatrix} 10 & 0 \\ 0 & 5 \end{bmatrix}} \cdot \underset{2\times 1}{\begin{bmatrix} 5 \\ -10 \end{bmatrix}}$$

2×1

$$= \frac{1}{50}\begin{bmatrix} 50 \\ -50 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad \therefore \boxed{C=-1,\ d=+1}$$

c)

$$\begin{matrix} 0 & 0 \\ 0 & 8 \\ 0 & 8 \end{matrix}$$

$$A = \underset{5\times 1}{\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}} \quad A^+ = \underset{1\times 5}{\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix}}$$

0 8

0 20

$\begin{pmatrix} 1 \\ 1 \end{pmatrix}_{5\times 1}$

$\vec{b} = \begin{bmatrix} 0 \\ 8 \\ 8 \\ 20 \end{bmatrix}$

$A^{+} \cdot A = 5$

$A^{+} \cdot b = 0 + 8 + 8 + 20 = 36$

$\therefore \vec{a_{LS}} = \frac{1}{5} \cdot 36 = \boxed{\frac{36}{5}}$