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1) (i) λ^2 is an eigen value of A^2 <u>Show</u>

if $A\vec{x} = \lambda\vec{x}$

then $A \cdot A\vec{x} = A\lambda\vec{x}$

$\Rightarrow A^2\lambda = \lambda \cdot A\vec{x}$

$= \lambda \cdot \lambda\vec{x}$

$= \lambda^2$

$\therefore \lambda^2$ is an eigenvalue of A^2

(ii) <u>show that</u> $\frac{1}{\lambda}$ is an eigenvalue of A^{-1}

if $A\vec{x} = \lambda\vec{x}$

$A^{-1}A\vec{x} = A^{-1}\vec{x} \cdot \lambda$

$I\cancel{x} = A^{-1}\cancel{x} \cdot \lambda$

$I = A^{-1}\lambda$

(iii) Show that $\lambda+1$ is an eigenvalue of $A+I$

$A\vec{y} = \lambda\vec{x}$

$A\vec{x} + I\vec{x} = \lambda\vec{x} + Iv$

$(A+I)\vec{x} = \vec{x}(\lambda+1)$

$$(A+I)\vec{x} = \vec{x}(\lambda + 1)$$

<u>hence true</u>

b) $A = \begin{bmatrix} 0 & 10^4 \\ 0 & 0 \end{bmatrix}$ Find eigenvalues ? and eigenvectors

$$A - \lambda I = \begin{bmatrix} 0 & 10^4 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} -\lambda & 10^4 \\ 0 & -\lambda \end{bmatrix} = 0$$

$$\det(A - \lambda I) = \lambda^2 - 0 = 0$$

$$\cancel{\lambda = 0}$$

say $\lambda = 1$

$$\begin{bmatrix} 0 & 10^4 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \vec{x}$$

$$\begin{bmatrix} 10^4 x_2 \\ 0 \end{bmatrix} = \vec{x}$$

$$\vec{x} = x_2 \begin{bmatrix} 10^4 \\ 0 \end{bmatrix}$$

there is no such λ
and eigenvector