Q4

Thursday, October 29, 2020 8:31 AM

a) $\det(A - I\lambda) = 0$

$$A - I\lambda = \begin{bmatrix} 0 & 1 \\ c & d \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} -\lambda & 1 \\ c & d-\lambda \end{bmatrix}$$

$\det(A - I\lambda) = 0$

$\Rightarrow -\lambda(d-\lambda) - c = 0$

$\Rightarrow -d\lambda + \lambda^2 - c = 0$

$\Rightarrow \lambda^2 - d\lambda - c = 0 \Rightarrow \lambda = 4, \lambda = 7$

are solutions.

$\therefore 16 - 4d - \cancel{c} = 0$

$- \quad 49 - 7d - \cancel{c} = 0$

$16 - 49 - 4d + 7d = 0$

$$\begin{array}{r} 49 \\ -16 \\ \hline 33 \end{array}$$

$3d = 49 - 16$

$3d = 33$

$\boxed{d = 11}$

$16 - 44 = c$

$\boxed{c = -28}$

$$\therefore A = \begin{bmatrix} 0 & 1 \\ -28 & 11 \end{bmatrix}$$

b) Eigen vector for $\lambda = 4$

Say Eigen vector: $\begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$

$\therefore \quad A \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = 4 \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$

$\therefore \quad w_2 = 4w_1$

$-28w_1 + 11w_2 = 4w_2$

$$\begin{array}{r} 44 \\ -28 \\ \hline 16 \end{array}$$

$-28w_1 + 11(4w_1) = 4(4w_2)$

$(-28 + 44)\cancel{w_1} = 16\cancel{w_1}$

✓

$\therefore$ Eigen vectors for $\lambda = 4 = \boxed{\begin{bmatrix} w_1 \\ 4w_1 \end{bmatrix}}$