Series Spring Equilibrium Constraint

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1 System

The system is a point mass with a spring connected to ground that moves only vertically w.r.t ground with position x_1, resting length l_1, and spring constant k_1. A second mass is attached vertically with a spring to the first mass and the state $x_2$corresponds to the vertical distance between the two masses. The second spring has resting length l_2, and spring constant k_2

2 Constraint

If the first mass has zero mass, the mass matrix becomes singular without an additional constraint. Try to get a constraint by minimizing potential energy:

$$PE = \frac{1}{2}k_1(x_1 - l_1)^2 + \frac{1}{2}k_2(x_2 - l_2)^2$$

Differentiating this gives:

$$\frac{dPE}{dt} = k_1(x_1 - l_1)\dot{x}_1 + k_2(x_2 - l_2)\dot{x}_2$$

For a constraint matrix form, where the two states are $\dot{x}_1$and $\dot{x}_2$:

$$C = [k_1(x_1 - l_1) \quad k_2(x_2 - l_2)]$$

Which is singular when both springs are at equilibrium and therefore causes problems during integration.