

the \mathcal{H}^1 -norm, and \mathcal{H}^1 -convergence of \mathbf{u}_ε to \mathbf{u}_0 follows from the L^2 -convergence of \mathbf{u}_ε to \mathbf{u}_0 .

For the L^2 -convergence of \mathbf{u}_ε to \mathbf{u}_0 , we first show that \mathbf{u}_ε is bounded in $L^2(\Omega)$.

Let $\mathbf{u}_\varepsilon = \mathbf{u}_\varepsilon^1 + \mathbf{u}_\varepsilon^2$ with $\mathbf{u}_\varepsilon^1 \in \mathbf{H}_0^1(\Omega)$ and $\mathbf{u}_\varepsilon^2 \in \mathbf{H}_0^1(\Omega)$ such that

$$\mathbf{u}_\varepsilon^1 = \mathbf{u}_0 + \mathbf{u}_\varepsilon^3, \quad \mathbf{u}_\varepsilon^2 = \mathbf{u}_\varepsilon^4 + \mathbf{u}_\varepsilon^5, \quad \mathbf{u}_\varepsilon^3 \in \mathbf{H}_0^1(\Omega), \quad \mathbf{u}_\varepsilon^4 \in \mathbf{H}_0^1(\Omega), \quad \mathbf{u}_\varepsilon^5 \in \mathbf{H}_0^1(\Omega).$$

Let \mathbf{u}_ε^1 be the solution of the problem

$$-\operatorname{div}(\mathbf{A}_\varepsilon \nabla \mathbf{u}_\varepsilon^1) = \mathbf{f}, \quad \mathbf{u}_\varepsilon^1 = 0 \text{ on } \partial\Omega, \quad \mathbf{u}_\varepsilon^1 \in \mathbf{H}_0^1(\Omega).$$

Let \mathbf{u}_ε^2 be the solution of the problem

$$-\operatorname{div}(\mathbf{A}_\varepsilon \nabla \mathbf{u}_\varepsilon^2) = \mathbf{f}, \quad \mathbf{u}_\varepsilon^2 = 0 \text{ on } \partial\Omega, \quad \mathbf{u}_\varepsilon^2 \in \mathbf{H}_0^1(\Omega).$$

Let \mathbf{u}_ε^3 be the solution of the problem

$$-\operatorname{div}(\mathbf{A}_\varepsilon \nabla \mathbf{u}_\varepsilon^3) = \mathbf{f}, \quad \mathbf{u}_\varepsilon^3 = 0 \text{ on } \partial\Omega, \quad \mathbf{u}_\varepsilon^3 \in \mathbf{H}_0^1(\Omega).$$

Let \mathbf{u}_ε^4 be the solution of the problem

$$-\operatorname{div}(\mathbf{A}_\varepsilon \nabla \mathbf{u}_\varepsilon^4) = \mathbf{f}, \quad \mathbf{u}_\varepsilon^4 = 0 \text{ on } \partial\Omega, \quad \mathbf{u}_\varepsilon^4 \in \mathbf{H}_0^1(\Omega).$$

Let \mathbf{u}_ε^5 be the solution of the problem

$$-\operatorname{div}(\mathbf{A}_\varepsilon \nabla \mathbf{u}_\varepsilon^5) = \mathbf{f}, \quad \mathbf{u}_\varepsilon^5 = 0 \text{ on } \partial\Omega, \quad \mathbf{u}_\varepsilon^5 \in \mathbf{H}_0^1(\Omega).$$

Let \mathbf{u}_ε^6 be the solution of the problem

$$-\operatorname{div}(\mathbf{A}_\varepsilon \nabla \mathbf{u}_\varepsilon^6) = \mathbf{f}, \quad \mathbf{u}_\varepsilon^6 = 0 \text{ on } \partial\Omega, \quad \mathbf{u}_\varepsilon^6 \in \mathbf{H}_0^1(\Omega).$$

Let \mathbf{u}_ε^7 be the solution of the problem

$$-\operatorname{div}(\mathbf{A}_\varepsilon \nabla \mathbf{u}_\varepsilon^7) = \mathbf{f}, \quad \mathbf{u}_\varepsilon^7 = 0 \text{ on } \partial\Omega, \quad \mathbf{u}_\varepsilon^7 \in \mathbf{H}_0^1(\Omega).$$

Let \mathbf{u}_ε^8 be the solution of the problem

$$-\operatorname{div}(\mathbf{A}_\varepsilon \nabla \mathbf{u}_\varepsilon^8) = \mathbf{f}, \quad \mathbf{u}_\varepsilon^8 = 0 \text{ on } \partial\Omega, \quad \mathbf{u}_\varepsilon^8 \in \mathbf{H}_0^1(\Omega).$$

Let \mathbf{u}_ε^9 be the solution of the problem

$$-\operatorname{div}(\mathbf{A}_\varepsilon \nabla \mathbf{u}_\varepsilon^9) = \mathbf{f}, \quad \mathbf{u}_\varepsilon^9 = 0 \text{ on } \partial\Omega, \quad \mathbf{u}_\varepsilon^9 \in \mathbf{H}_0^1(\Omega).$$

Let $\mathbf{u}_\varepsilon^{10}$ be the solution of the problem

$$-\operatorname{div}(\mathbf{A}_\varepsilon \nabla \mathbf{u}_\varepsilon^{10}) = \mathbf{f}, \quad \mathbf{u}_\varepsilon^{10} = 0 \text{ on } \partial\Omega, \quad \mathbf{u}_\varepsilon^{10} \in \mathbf{H}_0^1(\Omega).$$

Let $\mathbf{u}_\varepsilon^{11}$ be the solution of the problem

$$-\operatorname{div}(\mathbf{A}_\varepsilon \nabla \mathbf{u}_\varepsilon^{11}) = \mathbf{f}, \quad \mathbf{u}_\varepsilon^{11} = 0 \text{ on } \partial\Omega, \quad \mathbf{u}_\varepsilon^{11} \in \mathbf{H}_0^1(\Omega).$$