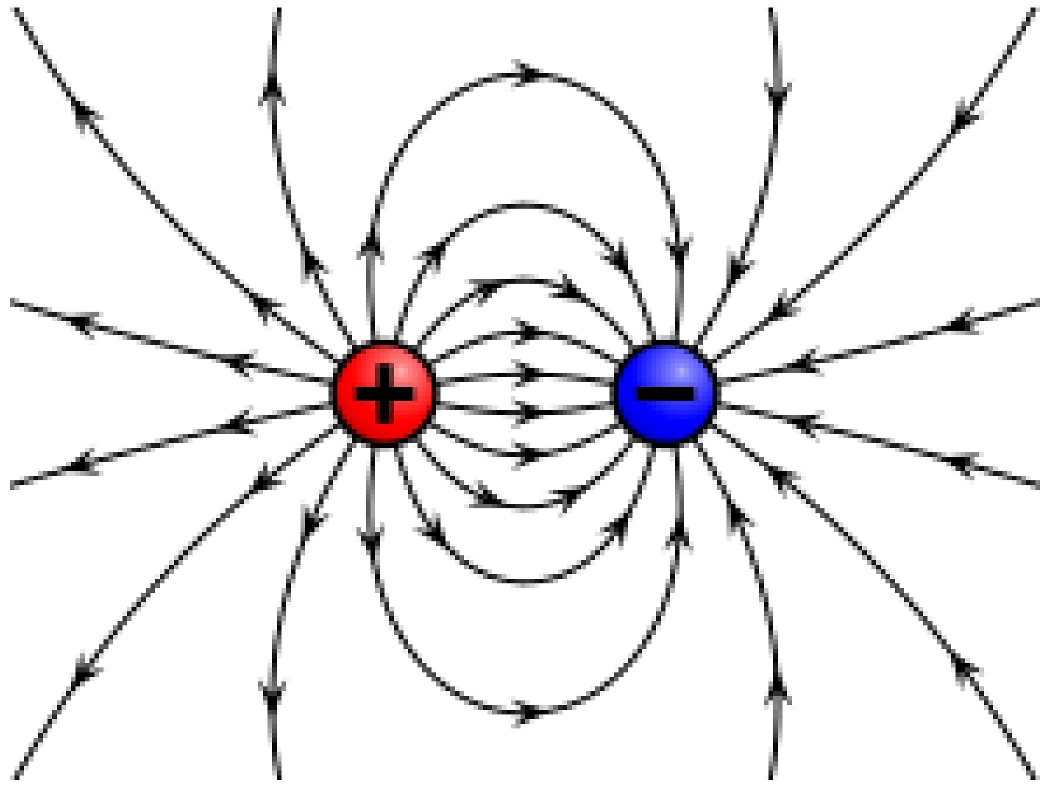
GRAVITATIONAL WAVES

Electromagnetic Waves

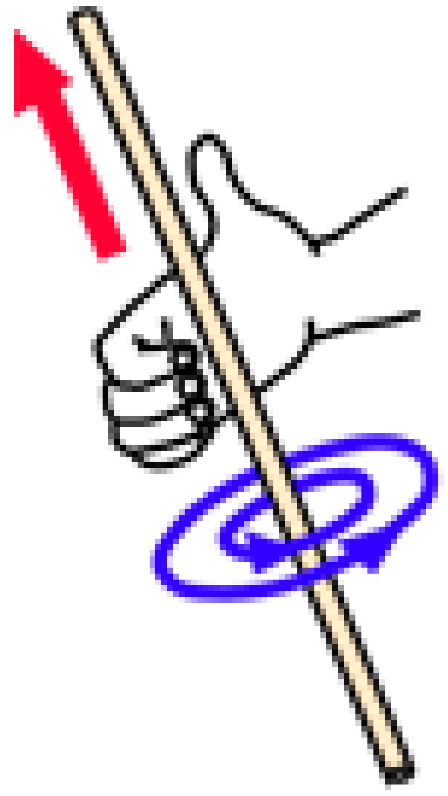
- **Static charged particles** produce **electric fields**



$$\vec{E} = \frac{q}{4\pi\epsilon_0}\frac{\hat{r}}{r^2}$$

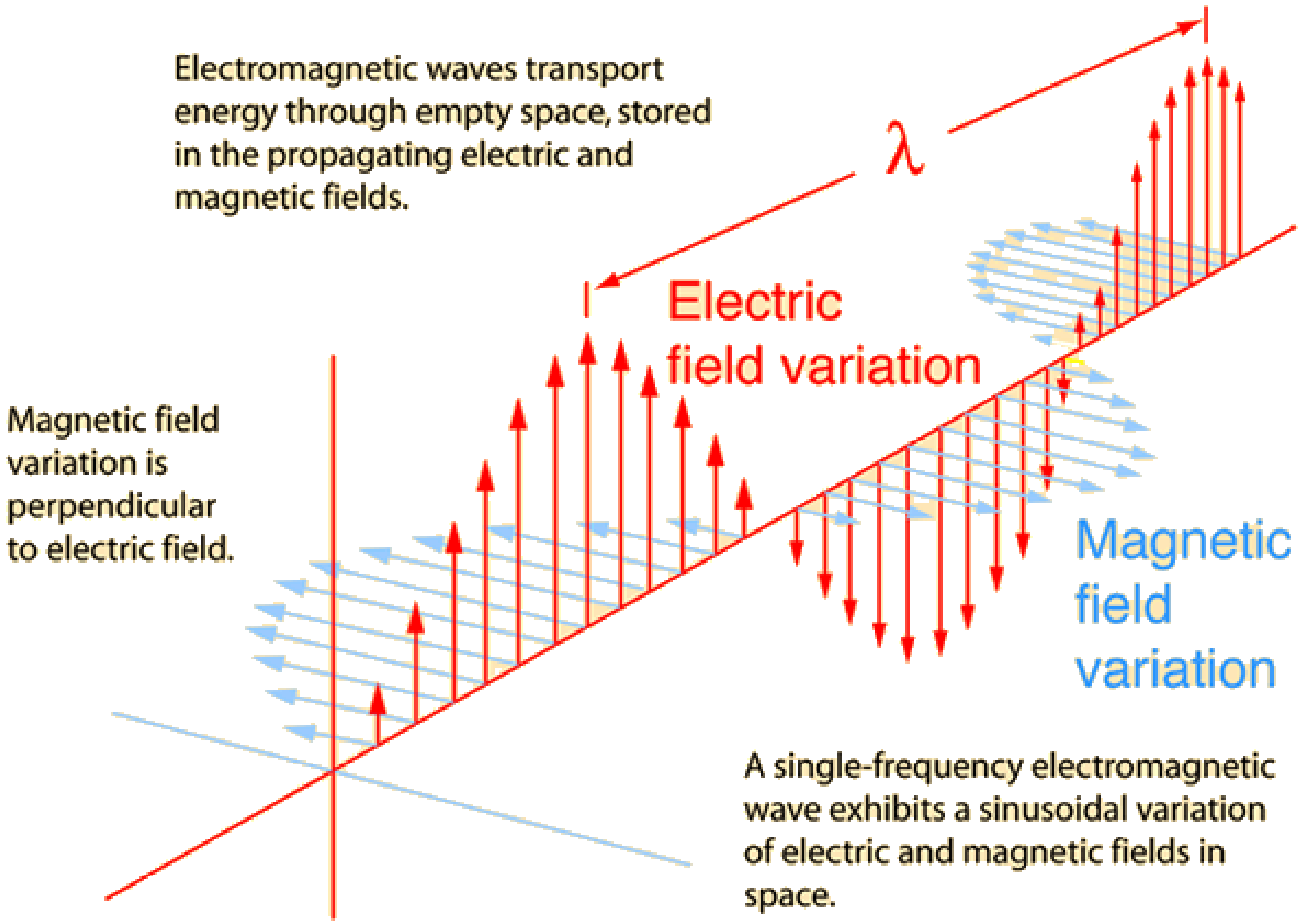
- Charged particles moving with **constant velocity** generate **magnetic fields**

$$\vec{B} = \frac{\mu_0 q}{4\pi}\frac{\vec{v} \times \hat{r}}{r^2}$$



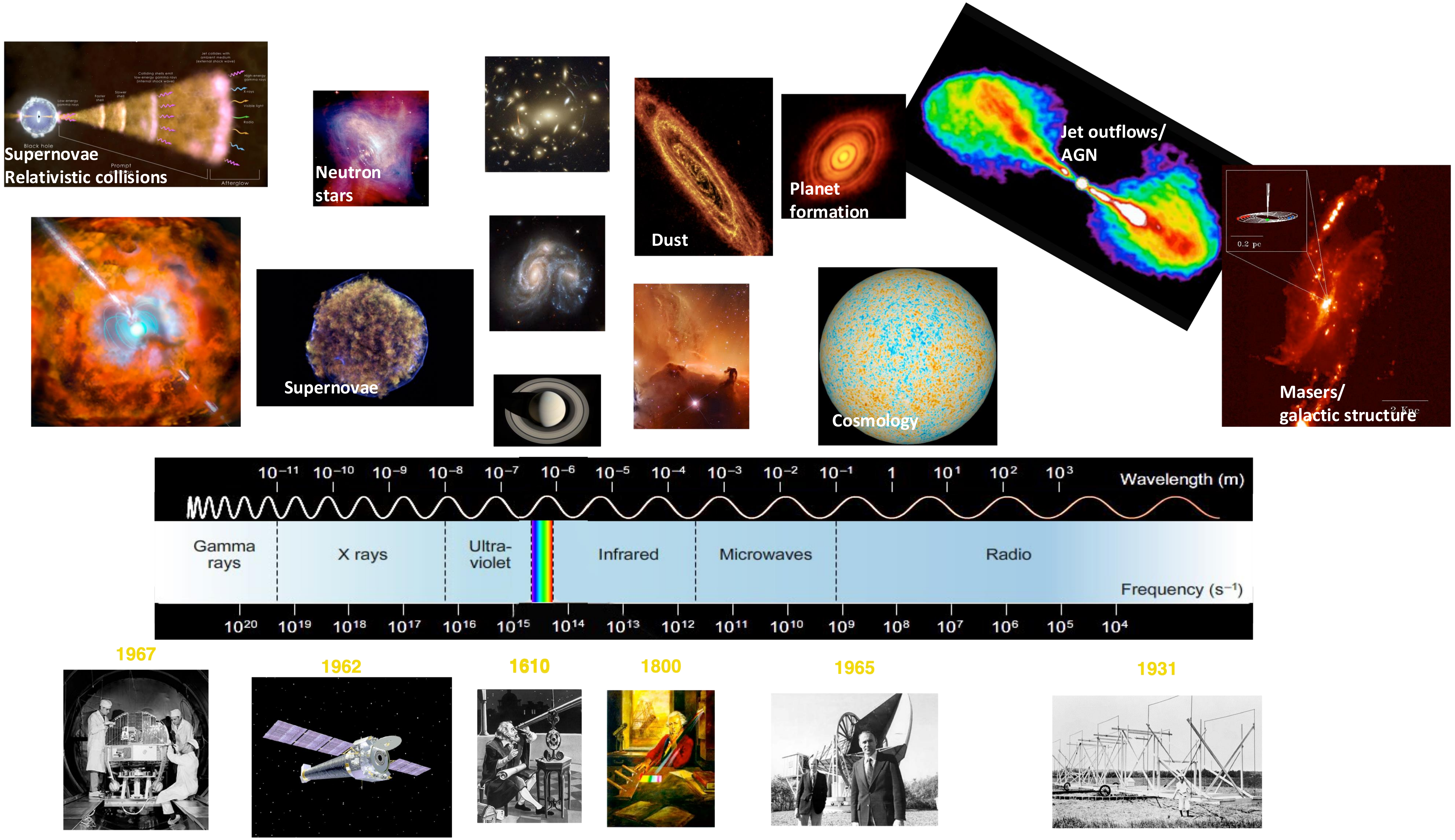
Electromagnetic Waves

- **Accelerating charged particles** generate **electromagnetic waves**.
- Sourced by shaking a charge — **dipolar radiation**.
- Electric and magnetic fields offset by **90 degrees**.

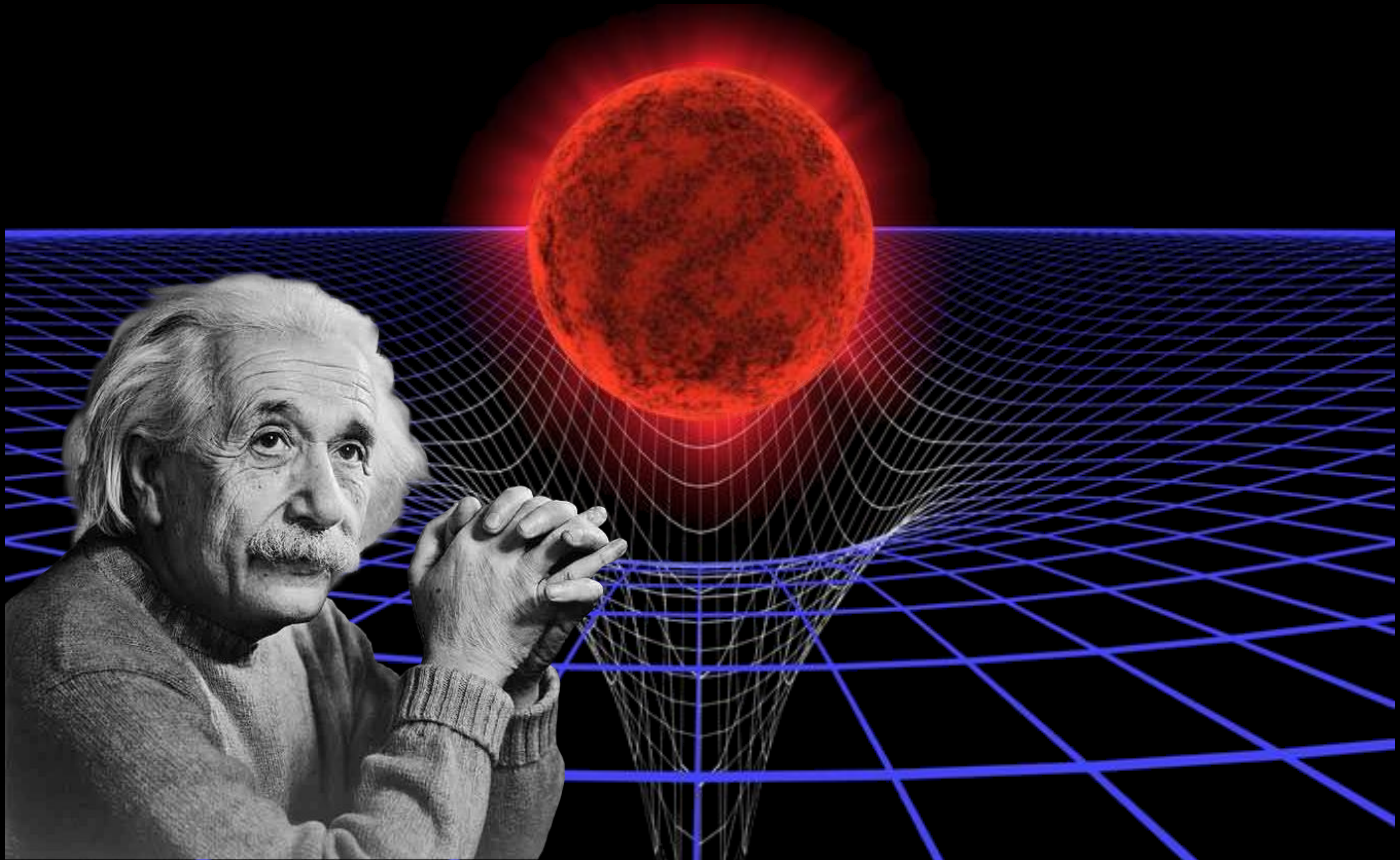


- Lowest order radiation from **dipolar** moment of electric potential.
- **Monopole moment is restricted by conservation of charge.**

The Electromagnetic Universe

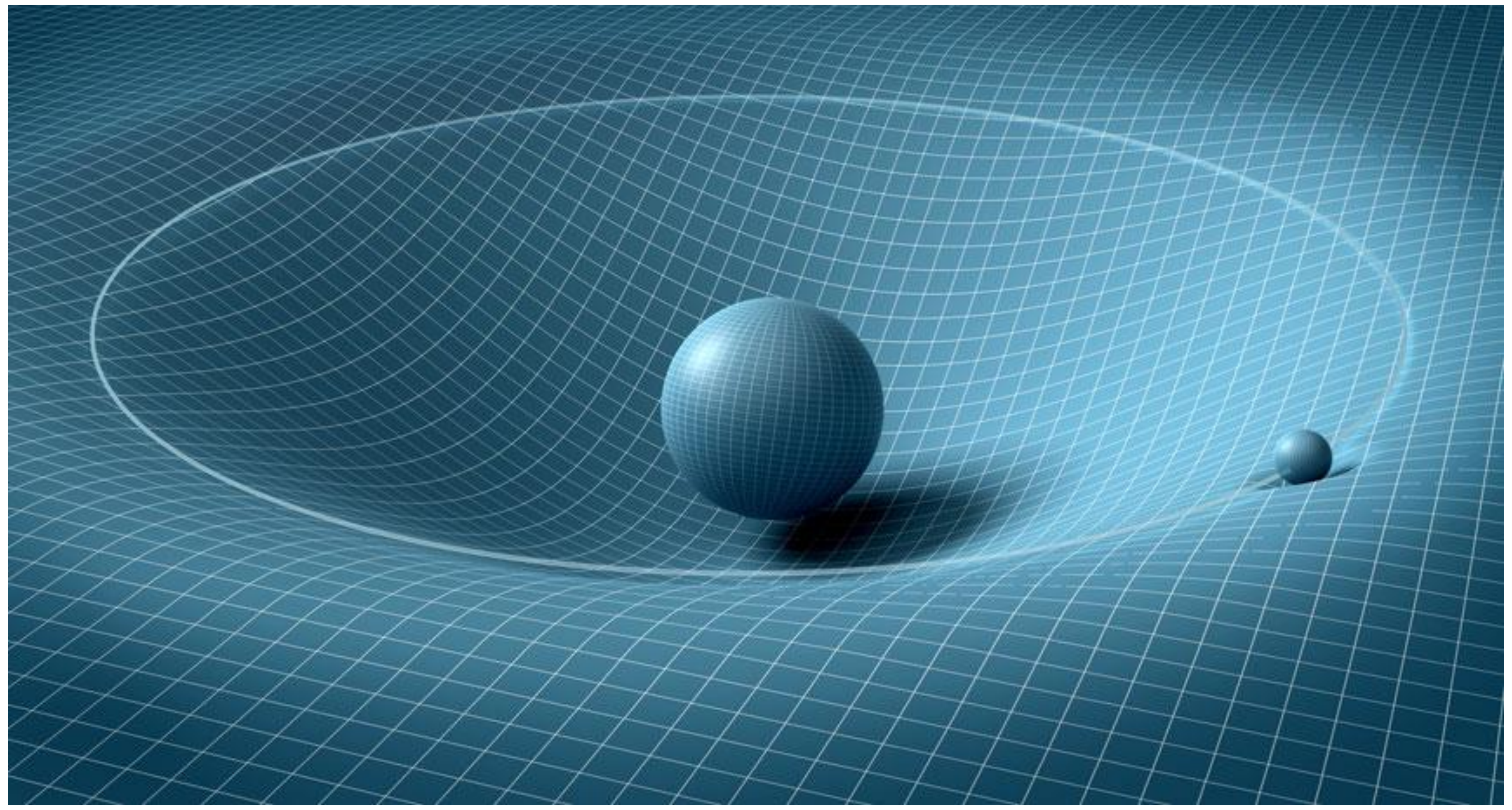


Credit: S. Burke-Spolaor

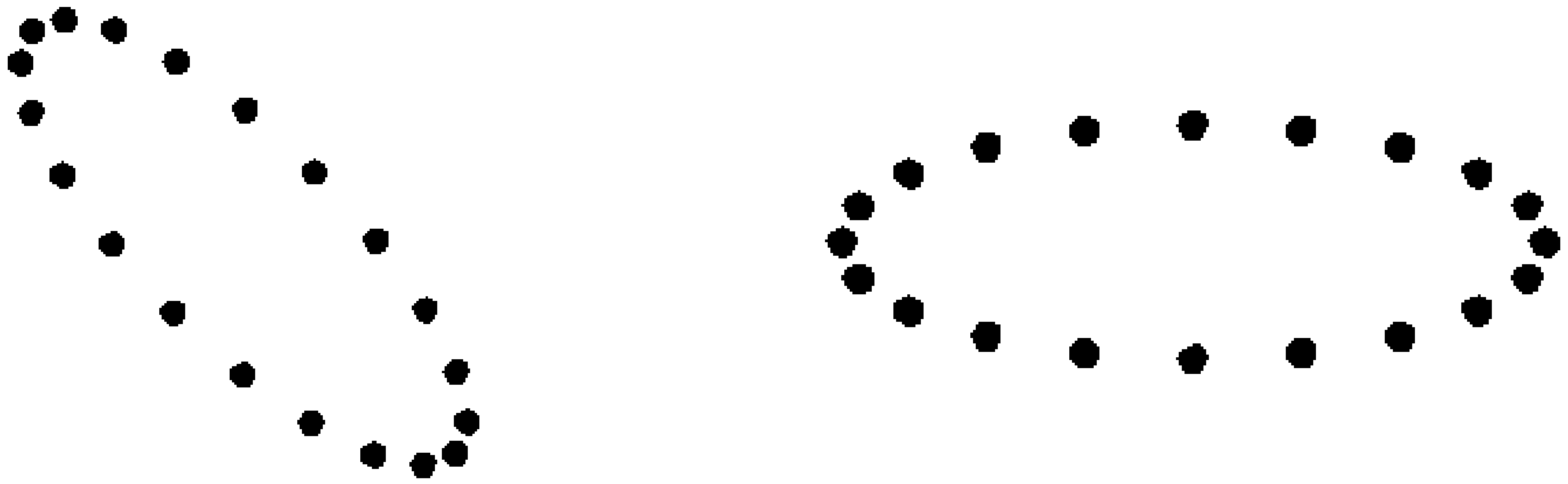


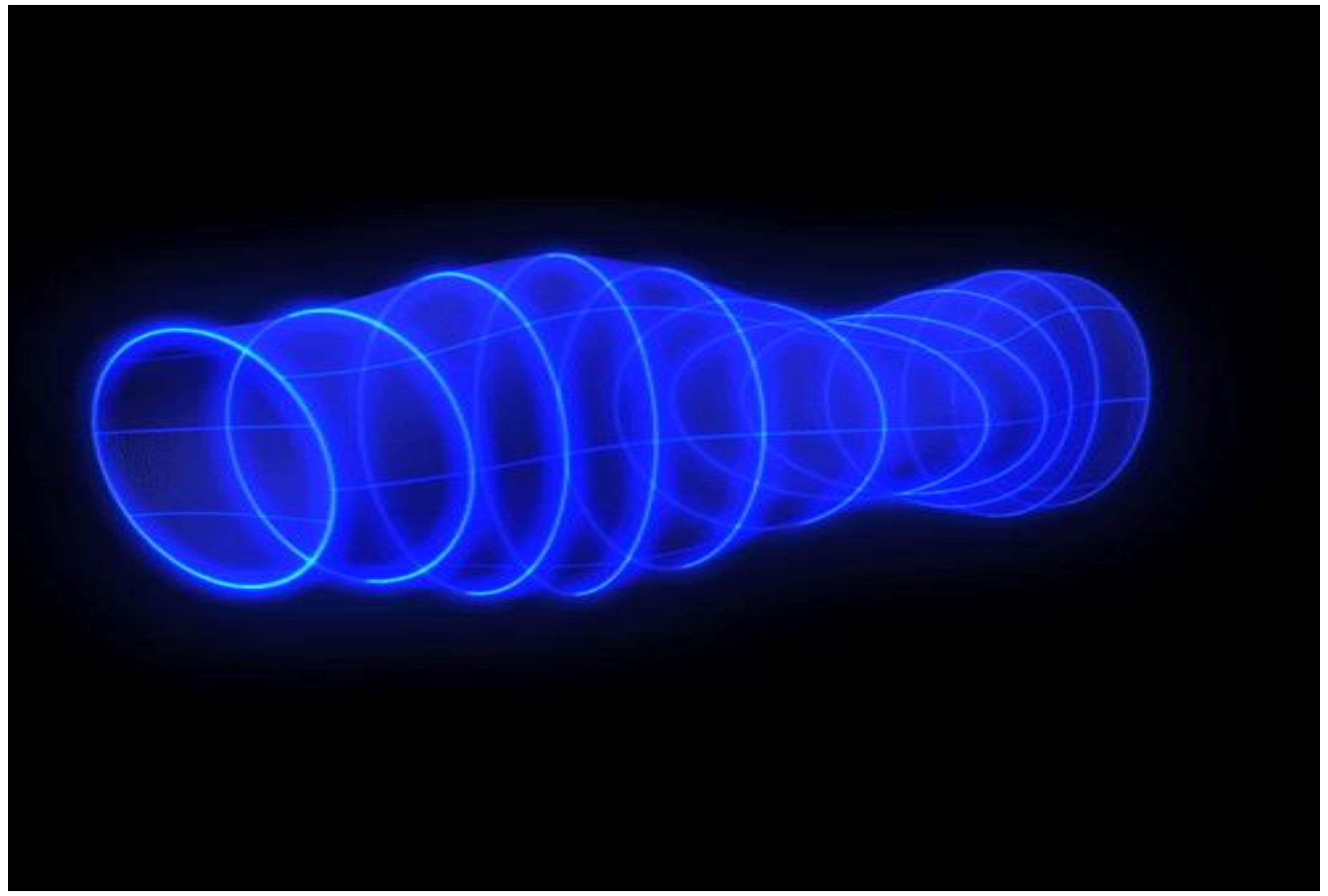
Gravitational Waves

- **General Relativity** — gravity is deformed space-time geometry



- Perturbations to space-time admit wave solutions — **gravitational waves**





Gravitational Waves On The Back of An Envelope

Gravitational waves on the back of an envelope

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(Received 20 June 1983; accepted for publication 25 July 1983)

Using only Newtonian gravity and a little special relativity we calculate most of the important effects of gravitational radiation, with results very close to the predictions of full general relativity theory. Used with care, this approach gives helpful back-of-the-envelope derivations of important equations and estimates, and it can help to teach gravitational wave phenomena to undergraduates and others not expert in general relativity. We use it to derive the following: the quadrupole approximation for the amplitude h of gravitational waves; a simple upper bound on h in terms of the Newtonian gravitational field of the source; the energy flux in the waves, the luminosity of the source (called the "quadrupole formula"), and the radiation reaction in the source; order-of-magnitude estimates for radiation from supernovae and binary star systems; and the rate of change of the orbital period of the binary pulsar system. Where our simple results differ from those of general relativity we quote the relativistic ones as well. We finish with a derivation of the principles of detecting gravitational waves, and we discuss the principal types of detectors under construction and the major limitations on their sensitivity.

Gravitational Waves On The Back of An Envelope

$$\nabla^2 \phi = 4\pi G \rho$$

GR ~ Newtonian Gravity + Special Relativity

$$\phi_N(\mathbf{x}, t) = -G \int \rho(\mathbf{y}, t)\, r^{-1}\, d^3 y, \quad r \equiv |\mathbf{x} - \mathbf{y}|$$

$$\phi_R(\mathbf{x}, t) = -G \int \rho(\mathbf{y}, t - r/c)\, r^{-1}\, d^3 y$$

Put in lightspeed delay

$$\nabla^2 \phi - \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} = 4\pi G \rho$$

Obeys wave equation with speed "c"

Gravitational Waves On The Back of An Envelope

$$h \equiv (\text{time}-\text{dependent part of } \phi_R)/c^2$$

$$r^{-1} = |\mathbf{x}|^{-1} + 2\mathbf{y}\cdot\mathbf{n}|\mathbf{x}|^{-2} + \cdots$$

$$t_0 \equiv t - |\mathbf{x}|/c$$

$$\phi_R = -G|\mathbf{x}|^{-1}\int \rho(\mathbf{y}, t - r/c)\, d^3y + \mathcal{O}(|\mathbf{x}|^{-2})$$

$$\begin{aligned} t - t_0 &= (|\mathbf{x}| - |\mathbf{x}-\mathbf{y}|)/c \\ &= -\mathbf{n}\cdot\mathbf{y}/c + \mathcal{O}(|\mathbf{x}|^{-1}) \end{aligned}$$

$$\phi_R = -G|\mathbf{x}|^{-1}\int \left[\rho(t_0) - c^{-1}\dot\rho(t_0)\,\mathbf{n}\cdot\mathbf{y} + \tfrac{1}{2}c^{-2}\ddot\rho(t_0)(\mathbf{n}\cdot\mathbf{y})^2 + \cdots\right] d^3y + \mathcal{O}(|\mathbf{x}|^{-2})$$

Gravitational Waves On The Back of An Envelope

MONOPOLE

$$\int \rho(t_0)\, d^3y = M = \text{const.}$$

DIPOLE

$$\dot{\rho}(t, \mathbf{y}) + \frac{\partial}{\partial y_j}(\rho V_j) = 0$$

$$\int \dot{\rho} y_i\, d^3y = -\int y_i \frac{\partial}{\partial y_j}(\rho V_j)\, d^3y$$

$$= \int \delta_{ij} \rho V_j\, d^3y$$

$$= \int \rho V_i\, d^3y = P_i = \text{const.}$$

QUADRUPOLE

$$I_{ij}(t) \equiv \int \rho(\mathbf{y}, t)\, y_i y_j\, d^3y$$

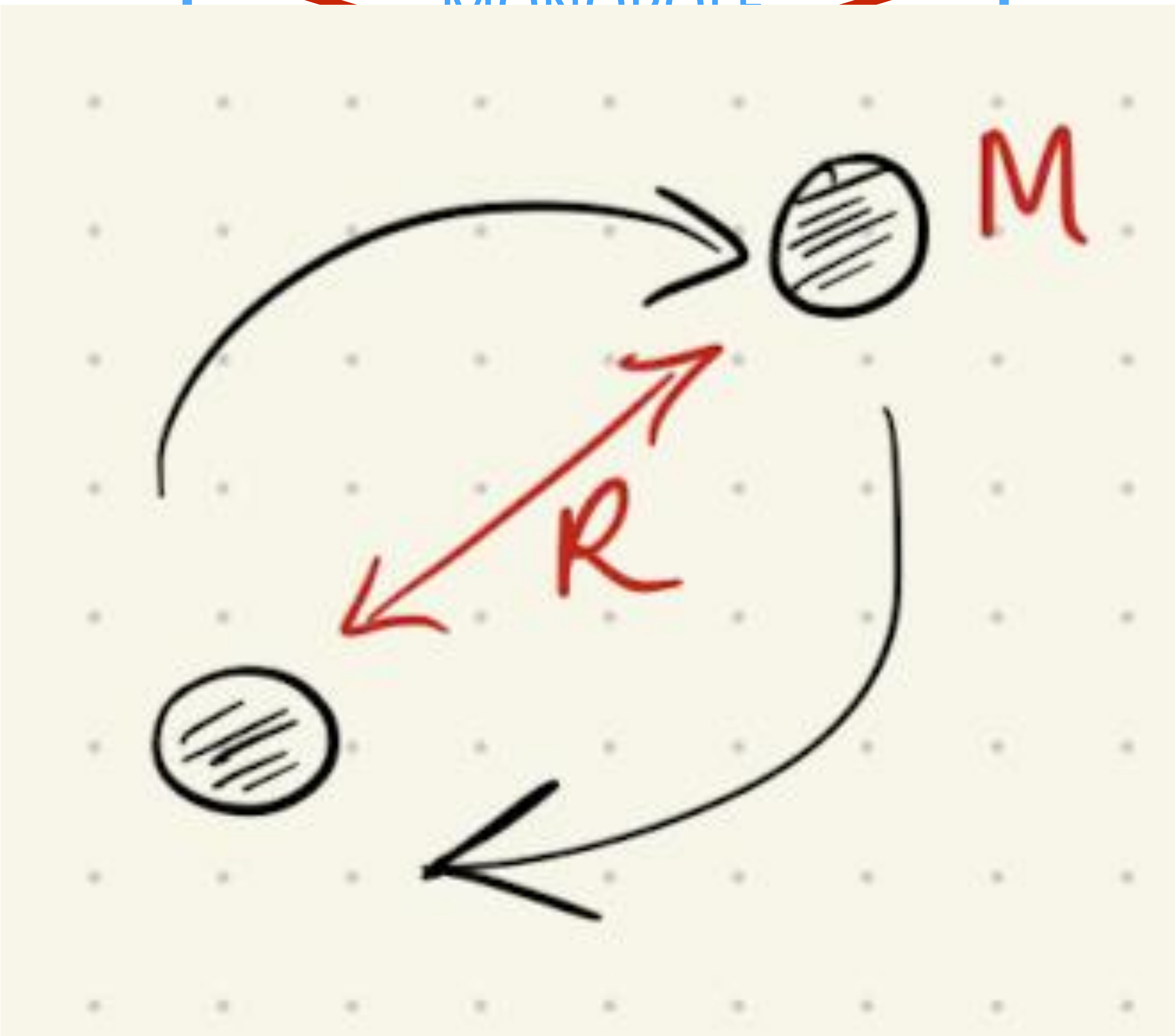
$$\phi_R \simeq -\frac{GM}{|\mathbf{x}|} + \frac{G n_i P_i}{c|\mathbf{x}|} - \frac{G}{2c^2} \frac{\ddot{I}_{ij} n_i n_j}{|\mathbf{x}|},$$

$$h \simeq -\left(\frac{G}{2c^4}\right) \frac{\ddot{I}_{ij} n_i n_j}{|\mathbf{x}|}.$$

$$\int \ddot{\rho}\, y_i y_j\, d^3y = \ddot{I}_{ij}$$

Gravitational Waves On The Back of An Envelope

MONOPOLE



$$= \int \rho V_i \, d^3 y = P_i = \text{const}$$

QUADRUPOLE

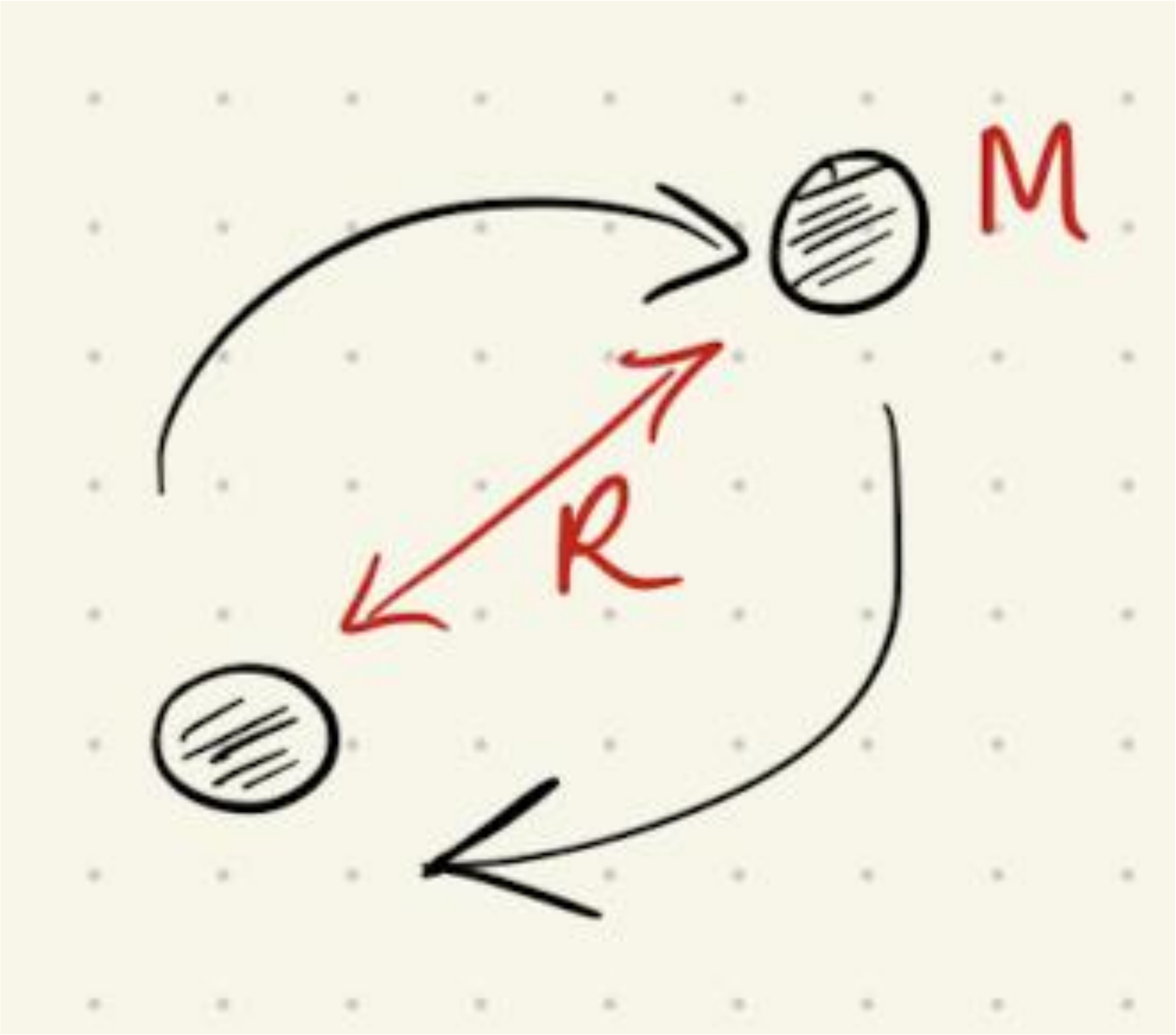
$$I_{ij}(t) \equiv \int \rho(\mathbf{y}, t)\, y_i y_j \, d^3 y$$

$$\phi_R \simeq -\frac{GM}{|\mathbf{x}|} + \frac{G n_i P_i}{c|\mathbf{x}|} - \frac{G}{2c^2}\frac{\ddot{I}_{ij} n_i n_j}{|\mathbf{x}|},$$

$$h \simeq -\left(\frac{G}{2c^4}\right)\frac{\ddot{I}_{ij} n_i n_j}{|\mathbf{x}|}.$$

$$\int \ddot{\rho}\, y_i y_j \, d^3 y = \ddot{I}_{ij}$$

Gravitational Waves On The Back of An Envelope



OOM : $h \propto \frac{1}{r} \cdot \frac{1}{P^2} M R^2$

distance to system

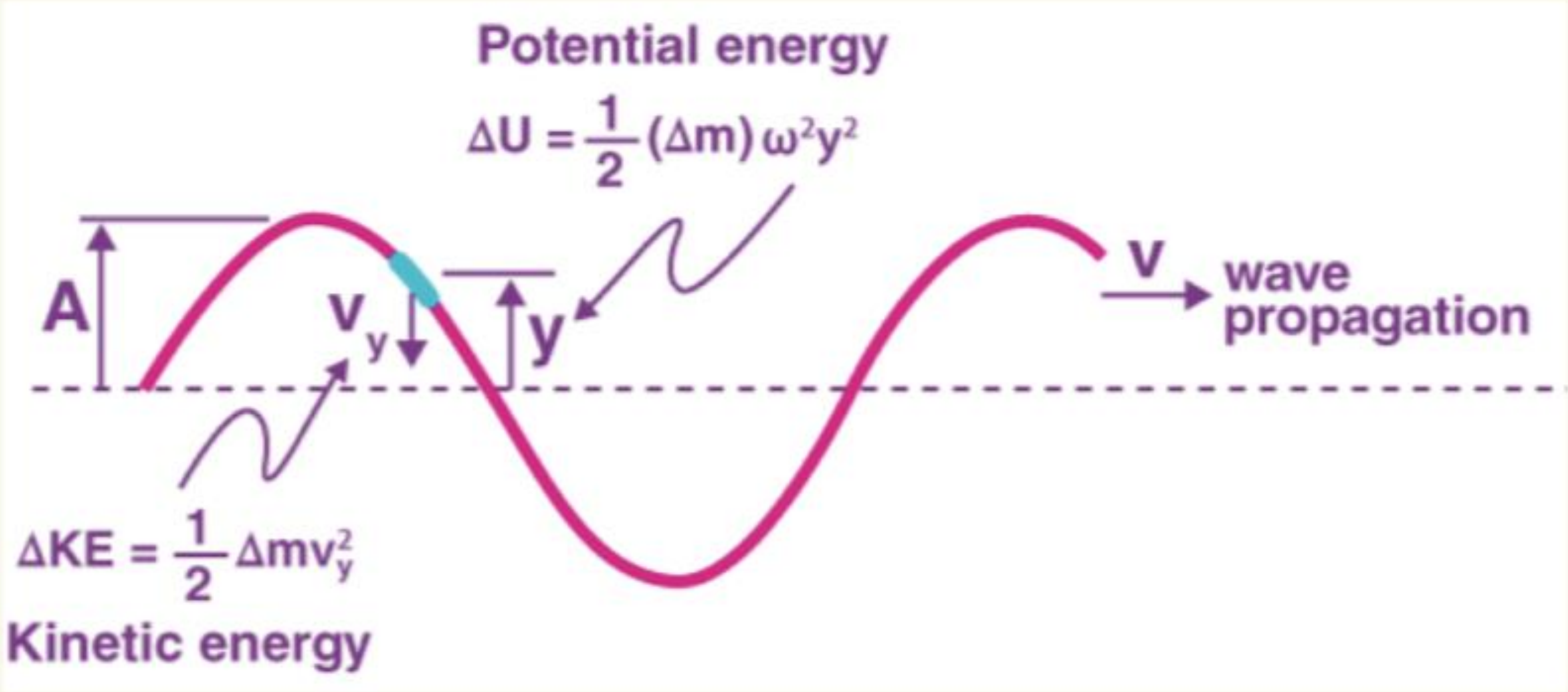
USING K3

$$\propto \frac{M^{5/3} P^{-2/3}}{r}$$

$$\propto \frac{M^{5/3} f_{orb}^{2/3}}{r}$$

$$h \sim 10^{-16} \left(\frac{M}{10^9 M_\odot}\right)^{5/3} \left(\frac{f_{\rm orb}}{10\,{\rm nHz}}\right)^{2/3} \left(\frac{100\,{\rm Mpc}}{r}\right)$$

Gravitational Waves On The Back of An Envelope



POWER IN A WAVE

$$y = A\sin(kx - \omega t)$$

$$\underbrace{\langle P \rangle}_{\text{power averaged over period.}} \propto \omega^2 A^2$$

OOM $\Longrightarrow$ (1) Energy flux quadratic in A ($\therefore \ddot{I}^{ij}$)

(2) $\omega \propto 1/\text{time}$ implies a time derivative

Gravitational Waves On The Back of An Envelope

$$L_{gw} = \frac{1}{5}\left\langle \dddot{I}_{ij}\dddot{I}^{ij}\right\rangle = \frac{128}{5} M^2 R^4 \omega^6$$

$$= \frac{128}{5} 4^{1/3} \cdot \frac{c^5}{G}\left(\frac{\pi G M}{c^3 P}\right)^{10/3}$$

- Consider total (KE + PE) in a binary orbit...

$$E_{NEWTON} = -M^2/4R$$

- Equate $\frac{dE_{NEWTON}}{dt}$ to $-L_{gw}$ ...

$$\frac{dP}{dt} = -\frac{96}{5}\pi 4^{1/3}\left(\frac{2\pi M}{P}\right)^{5/3}$$

$$\frac{df_{orb}}{dt} = \frac{96}{5}\pi 4^{1/3}(2\pi M)^{5/3} f_{orb}^{11/3}$$

negative...
GWs cause
orbit to shrink

Gravitational Waves On The Back of An Envelope

$$r_{\rm ISCO} = 6\frac{GM}{c^2} \longrightarrow \begin{aligned} f_{\rm orb}^{\rm ISCO} &\simeq 2.2\times 10^3\,{\rm Hz}\left(\frac{M_\odot}{M}\right) \\ f_{\rm gw}^{\rm ISCO} &\simeq 4.4\times 10^3\,{\rm Hz}\left(\frac{M_\odot}{M}\right) \end{aligned}$$

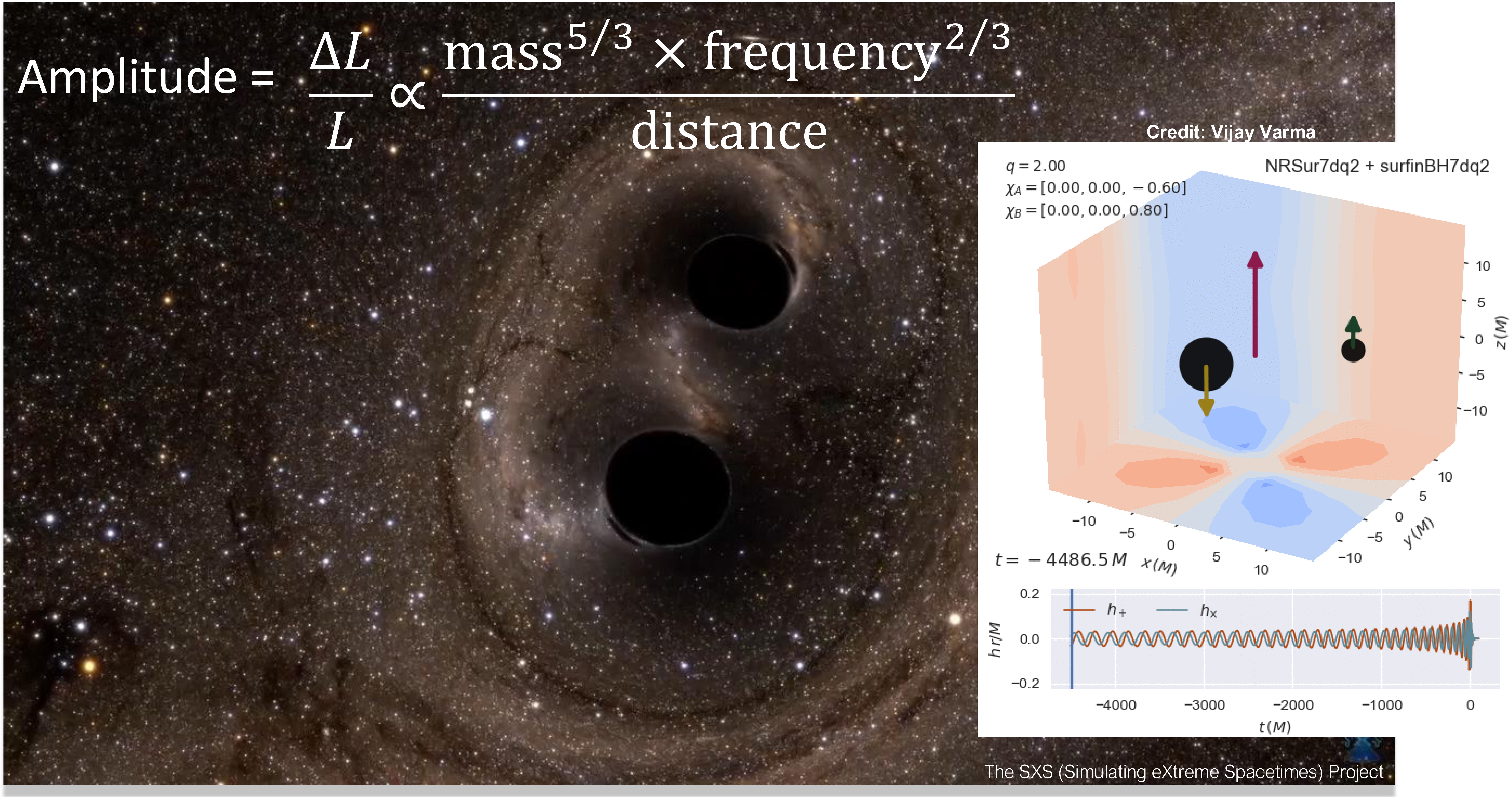
$$t_{\rm coal}(f) \approx 19\,{\rm s}\left(\frac{10\,{\rm Hz}}{f}\right)^{\frac{8}{3}}\left(\frac{30\,M_\odot}{M}\right)^{\frac{5}{3}}$$

$$t_{\rm coal}(f) \approx 7.2\,{\rm hr}\left(\frac{1\,{\rm mHz}}{f}\right)^{\frac{8}{3}}\left(\frac{10^6\,M_\odot}{M}\right)^{\frac{5}{3}}$$

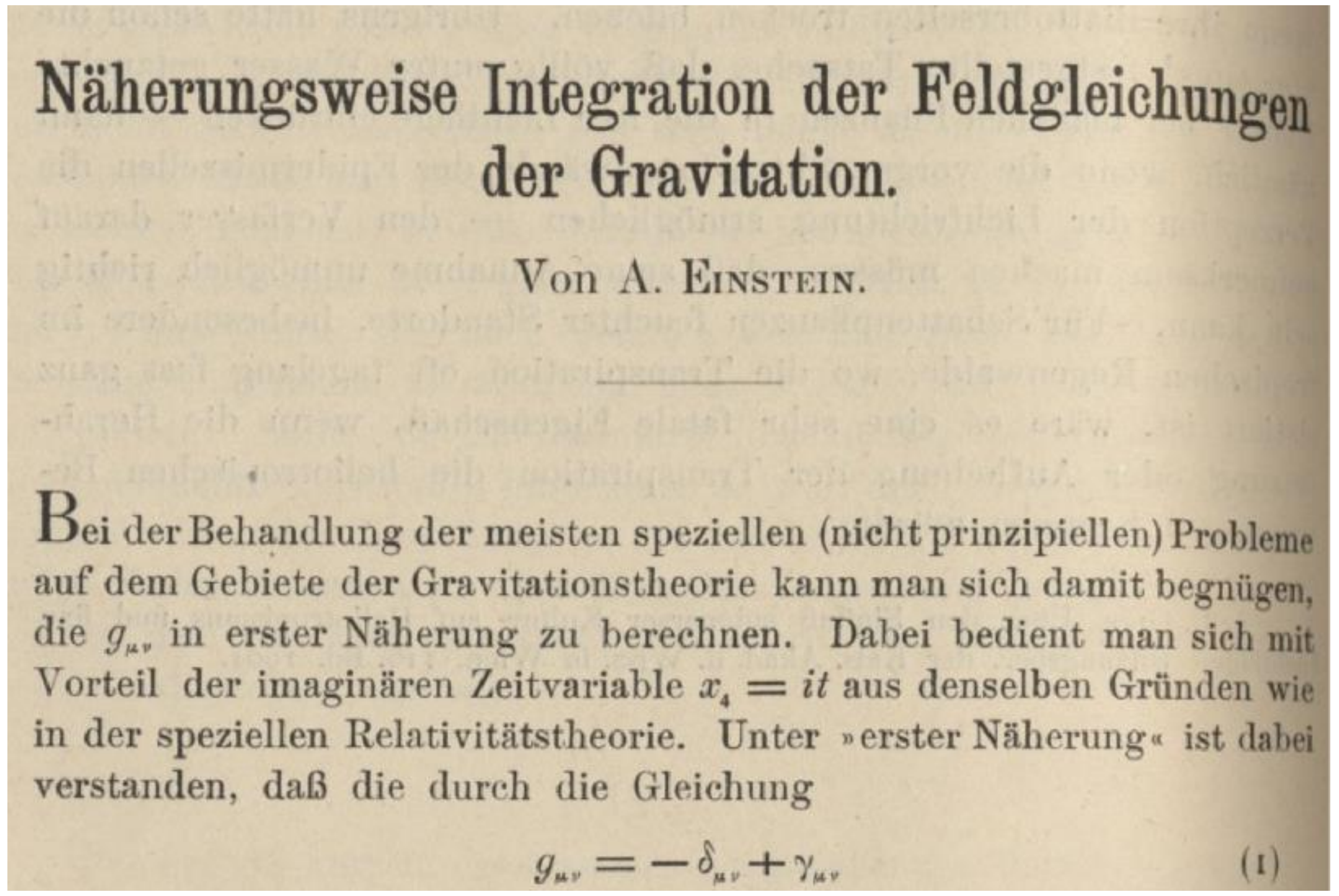
$$t_{\rm coal}(f) \approx 1.8\times 10^5\,{\rm yr}\left(\frac{10\,{\rm nHz}}{f}\right)^{8/3}\left(\frac{10^9 M_\odot}{M}\right)^{5/3}$$

Gravitational Waves

Amplitude = $\frac{\Delta L}{L} \propto \frac{\text{mass}^{5/3} \times \text{frequency}^{2/3}}{\text{distance}}$



A Brief History Of Doubt



Näherungsweise Integration der Feldgleichungen der Gravitation.

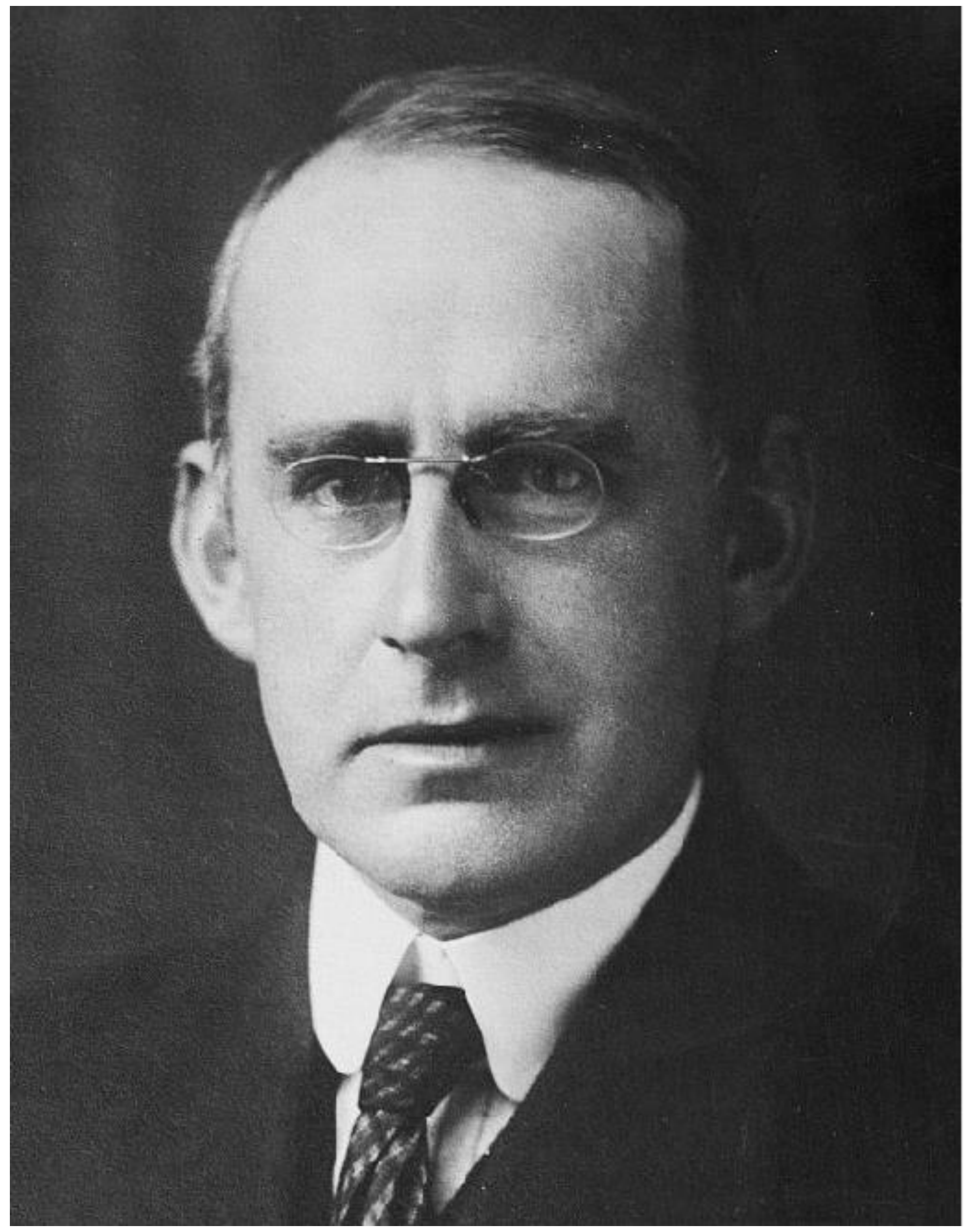
Von A. Einstein.

Bei der Behandlung der meisten speziellen (nicht prinzipiellen) Probleme auf dem Gebiete der Gravitationstheorie kann man sich damit begnügen, die $g_{\mu\nu}$ in erster Näherung zu berechnen. Dabei bedient man sich mit Vorteil der imaginären Zeitvariable $x_4 = it$ aus denselben Gründen wie in der speziellen Relativitätstheorie. Unter »erster Näherung« ist dabei verstanden, daß die durch die Gleichung

$$g_{\mu\nu} = -\delta_{\mu\nu} + \gamma_{\mu\nu} \qquad (1)$$

1916: Einstein first predicts GWs on basis of his field equations

1922: Some wave solutions are coordinate artifacts. Eddington jokes that they propagate "***at the speed of thought***".



A Brief History Of Doubt

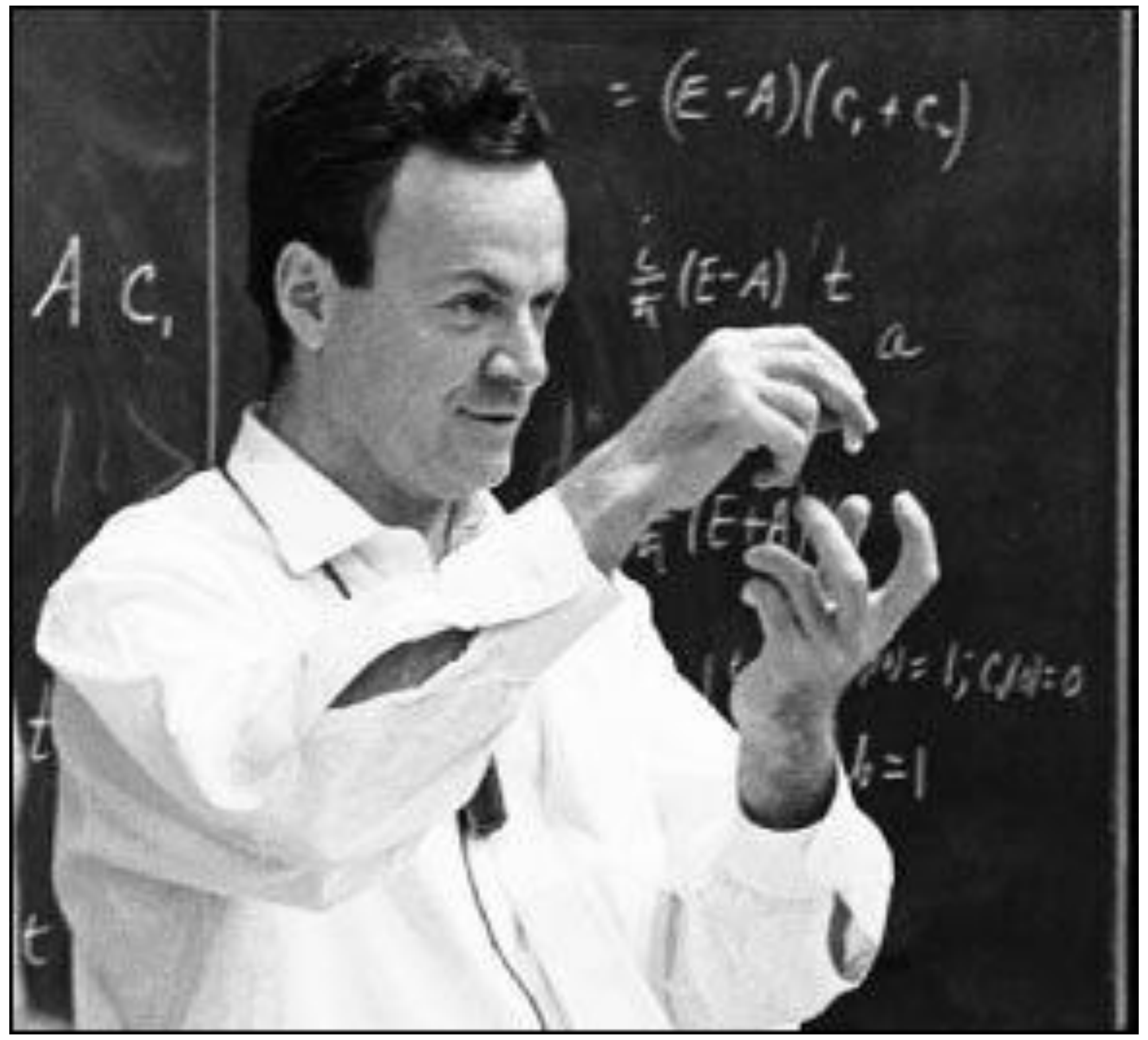
Dear Sir,

We (Mr. Rosen and I) had sent you our manuscript for publication and **had not authorized you to show it to specialists before it is printed**. I see no reason to address the — in any case erroneous — comments of your anonymous expert. On the basis of this incident I prefer to publish the paper elsewhere.

Respectfully,

P.S. Mr. Rosen, who has left for the Soviet Union, has authorized me to represent him in this matter.

1936: Einstein makes classic GR mistake, and claims GWs don't exist. Seems unfamiliar with peer review…

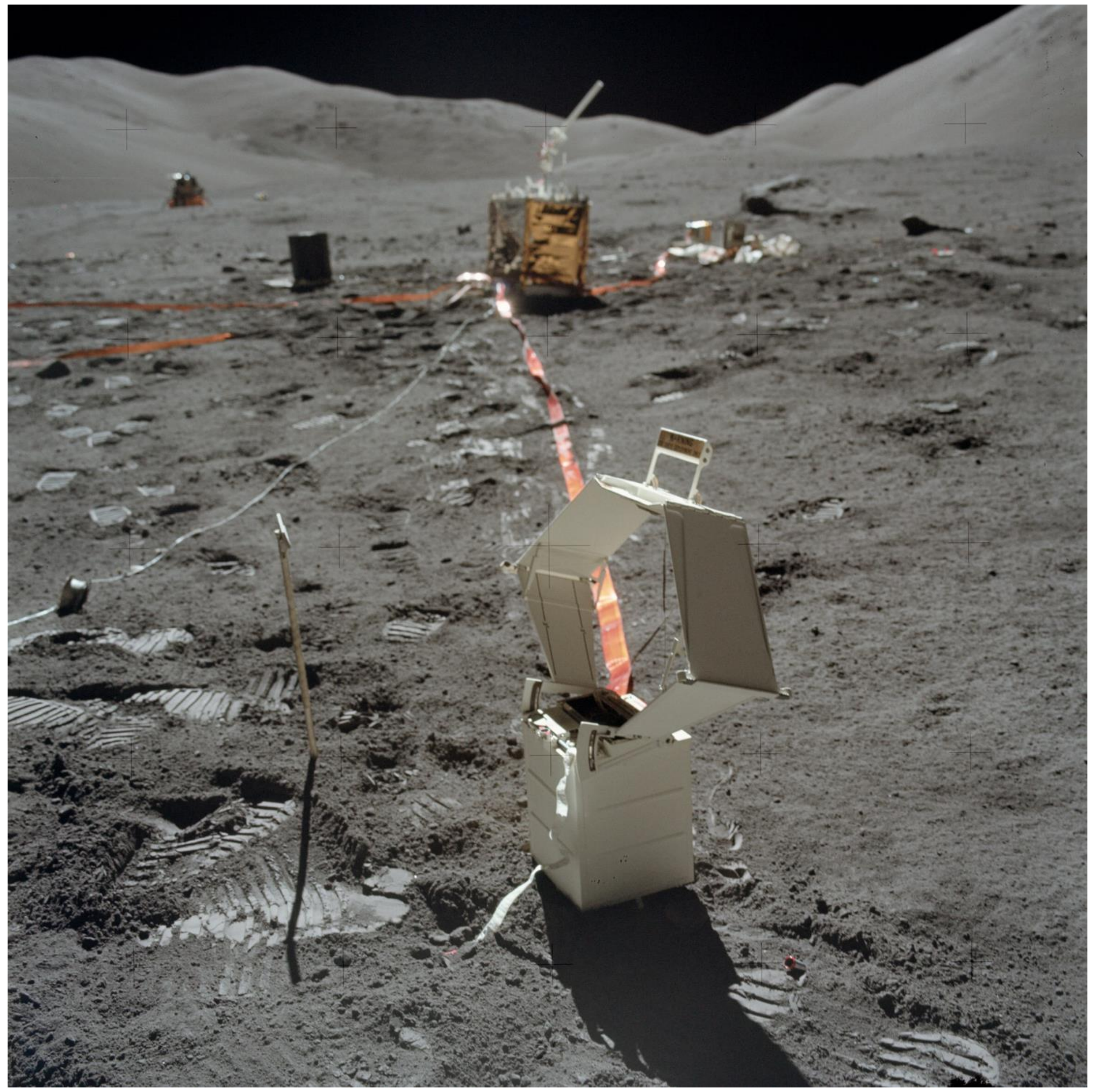


1957: Feynman, Bondi, Pirani conclude that GWs are real because they should transmit energy.

A Brief History Of Doubt

1969: Joe Weber invents experimental GW astronomy, but erroneously claims detection.

1972: GW detector carried to Moon on Apollo 17. Nowhere near sensitive enough…



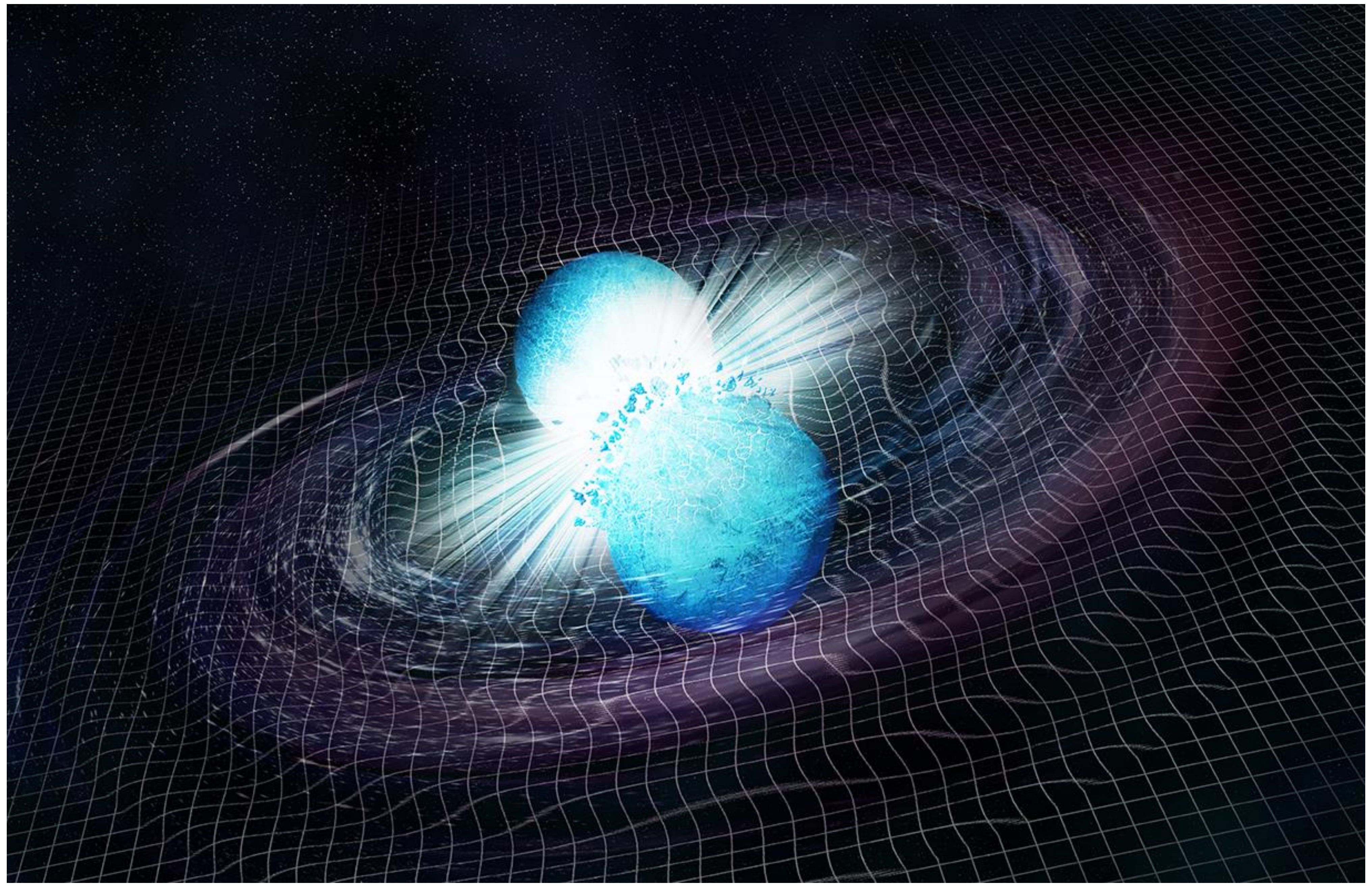
1974: Hulse & Taylor discover a binary pulsar. Orbital decay agrees beautifully with GR prediction due to GW emission. Nobel Prize in ’93.

A Brief History Of Doubt

1984 - 1994: Beginnings of LIGO with Thorne, Drever, Weiss. LIGO is officially funded by NSF.

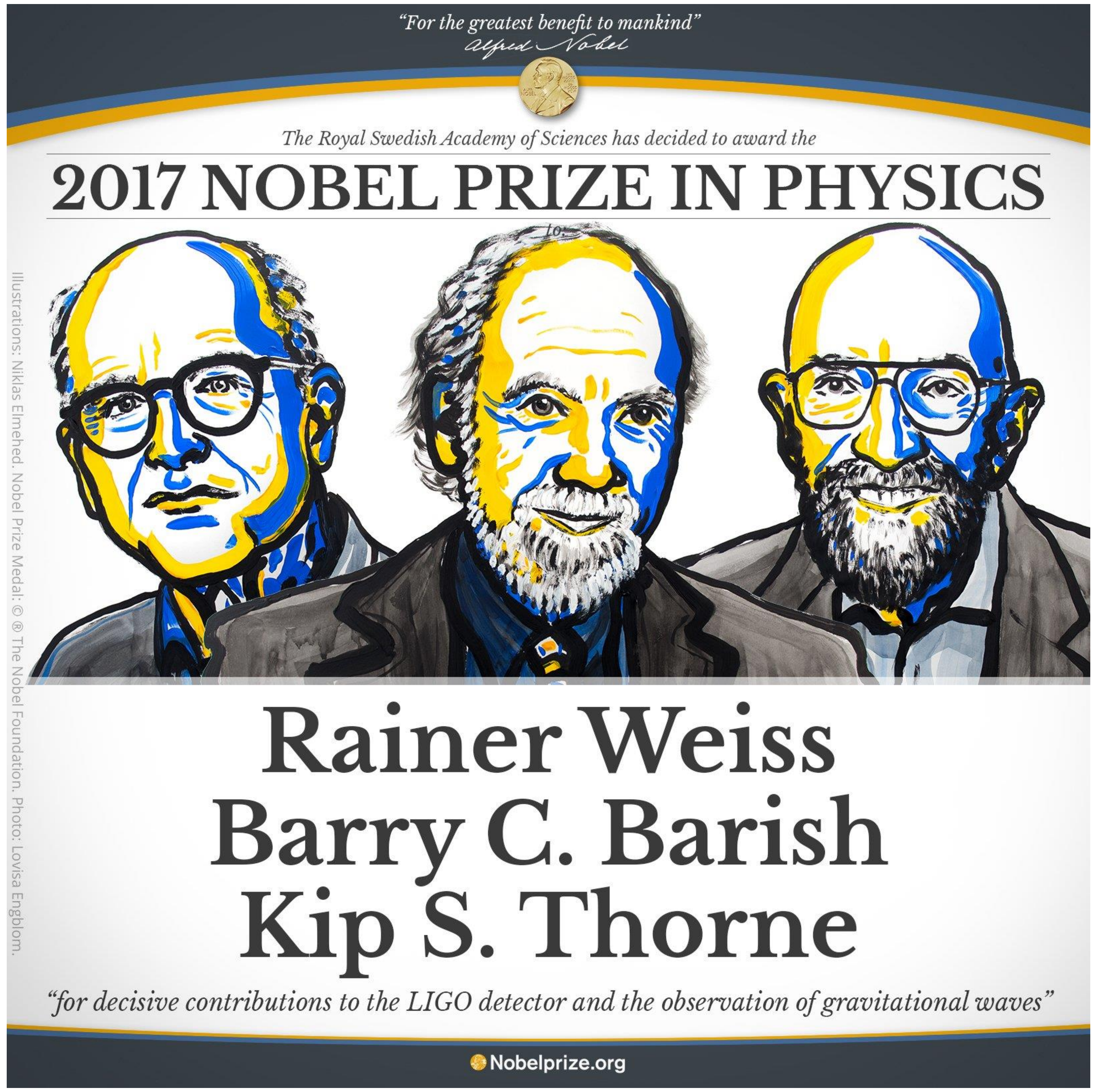
2015: Advanced LIGO switches on. You know how this story goes…

A Brief History Of Doubt



2017: GW170817

2017: Weiss, Barish, Thorne awarded Nobel Prize in Physics



A Brief History Of Doubt

NEWS & EVENTS

NANOGrav Finds Possible 'First Hints' of Low-Frequency Gravitational Wave Background

In data gathered and analyzed over 13 years, the North American Nanohertz Observatory for Gravitational Waves (NANOGrav) has found an intriguing low-frequency signal that may be attributable to gravitational waves.

2020: Pulsar Timing Arrays start to see something…

2023: Pulsar Timing Arrays find evidence for a cosmic GW background, likely from Supermassive Black-hole Binaries

The Washington Post
Democracy Dies in Darkness

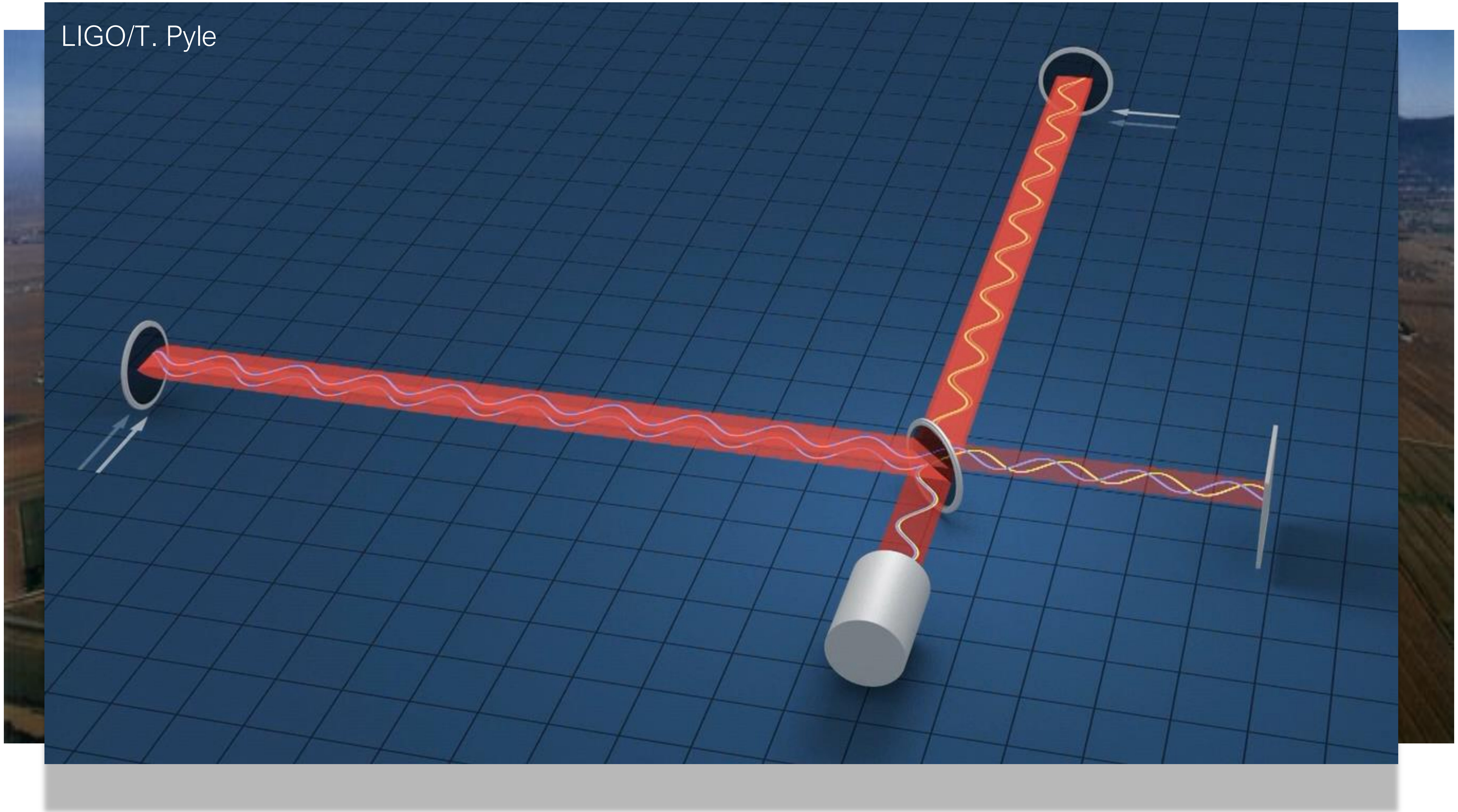
Science Space Animals Health Environment

In a major discovery, scientists say space-time churns like a choppy sea

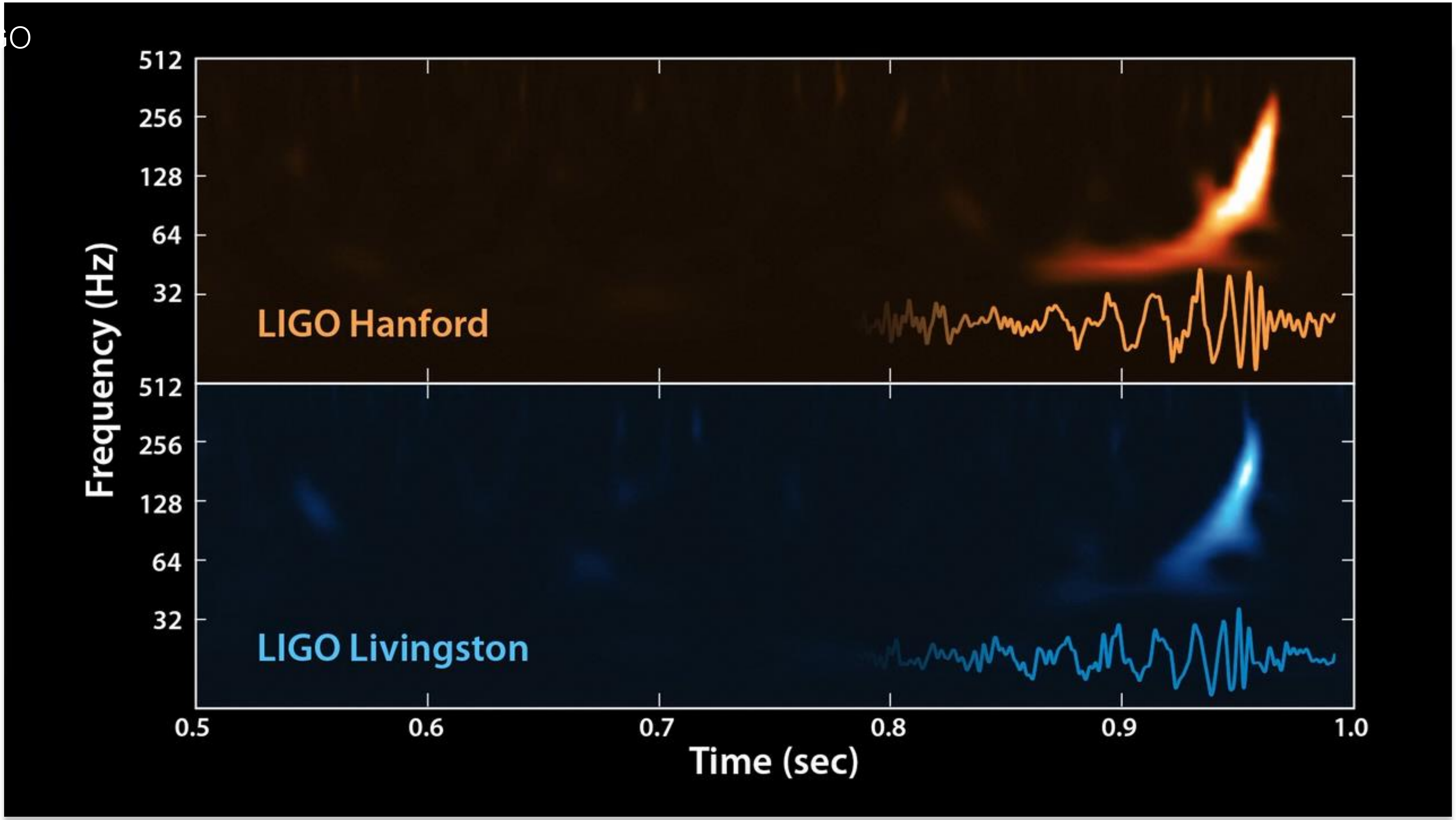
The mind-bending finding suggests that everything around us is constantly being roiled by low-frequency gravitational waves

June 28, 2023 More than **2 years ago**

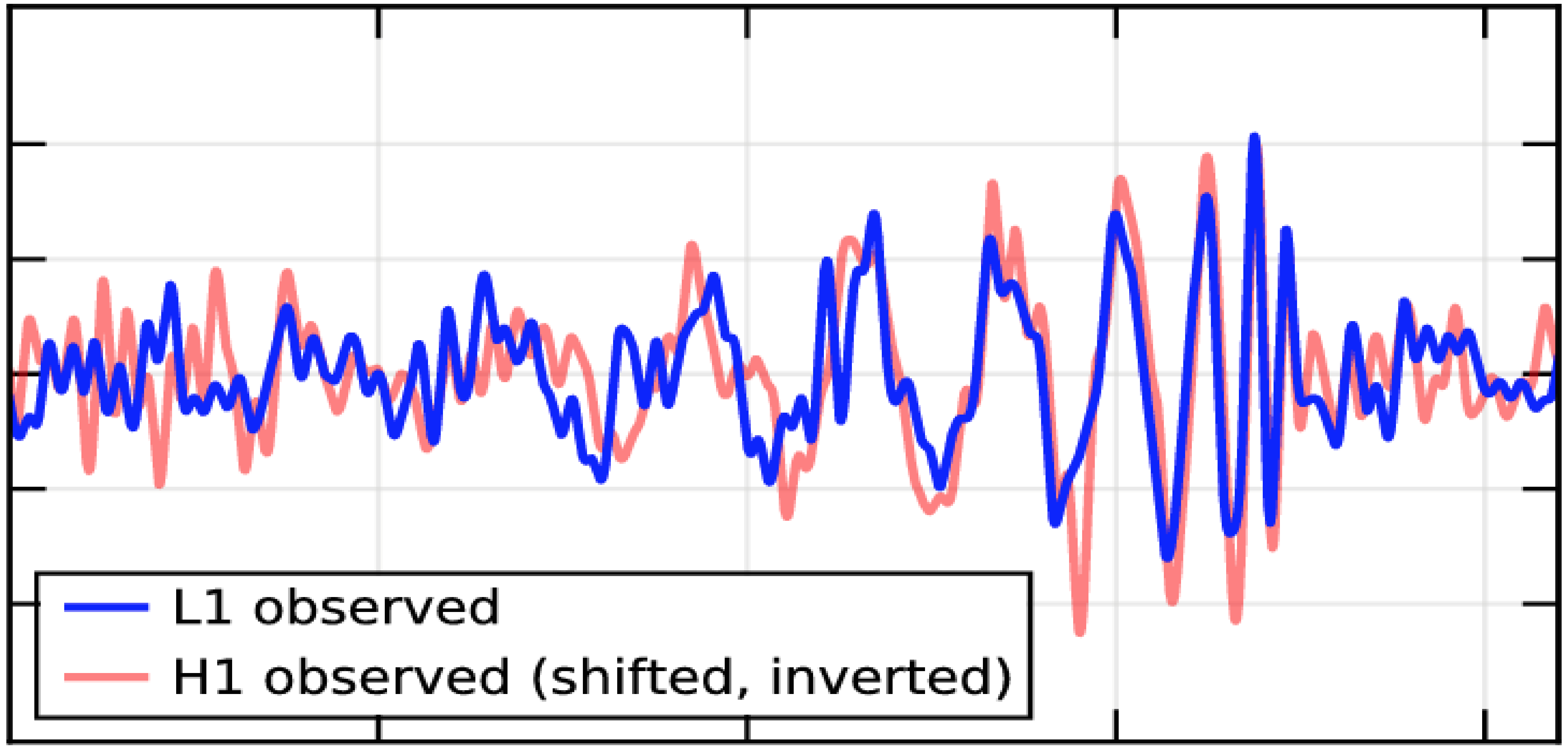
LIGO [~20 Hz to ~1 kHz]



September 14th, 2015

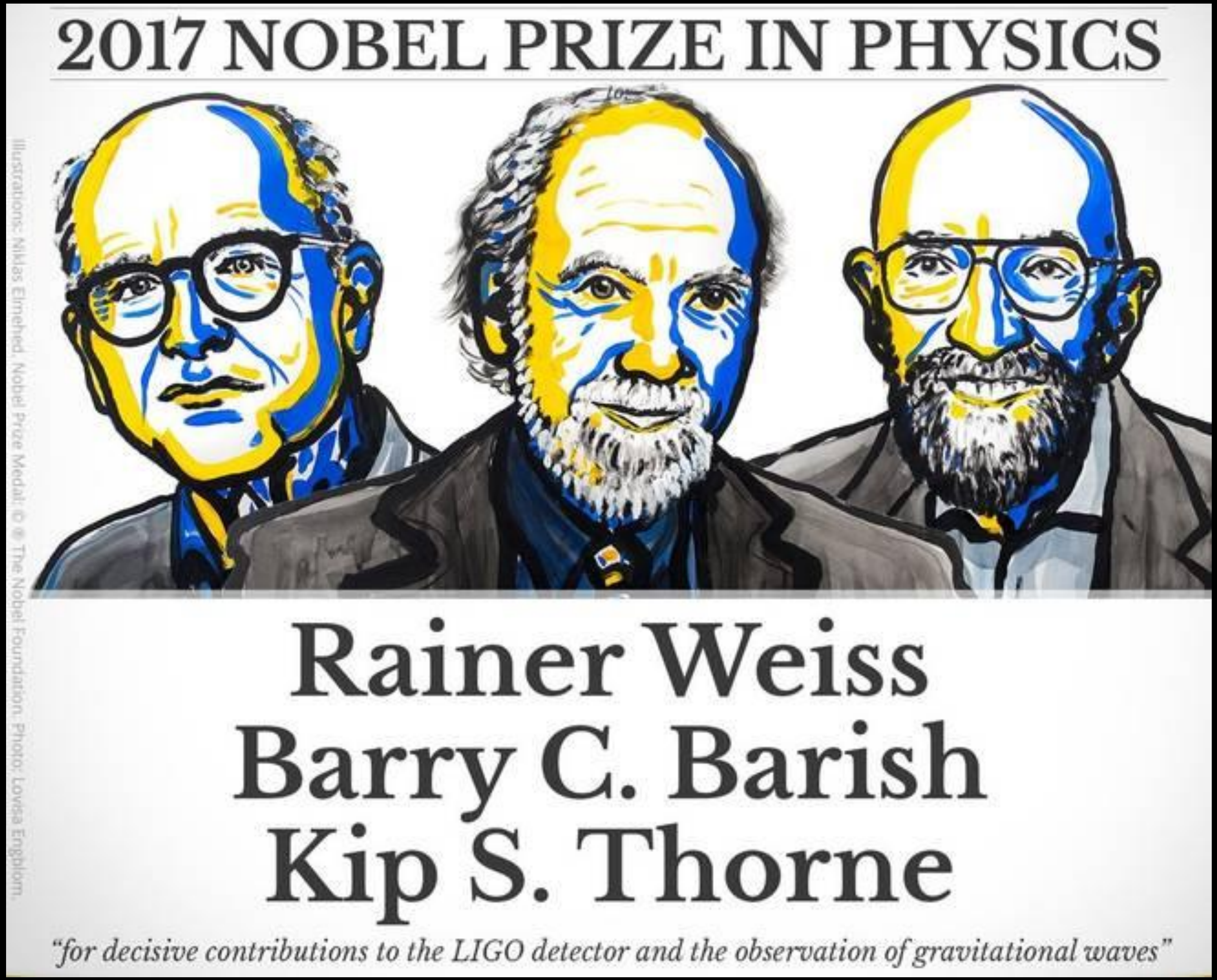


September 14th, 2015

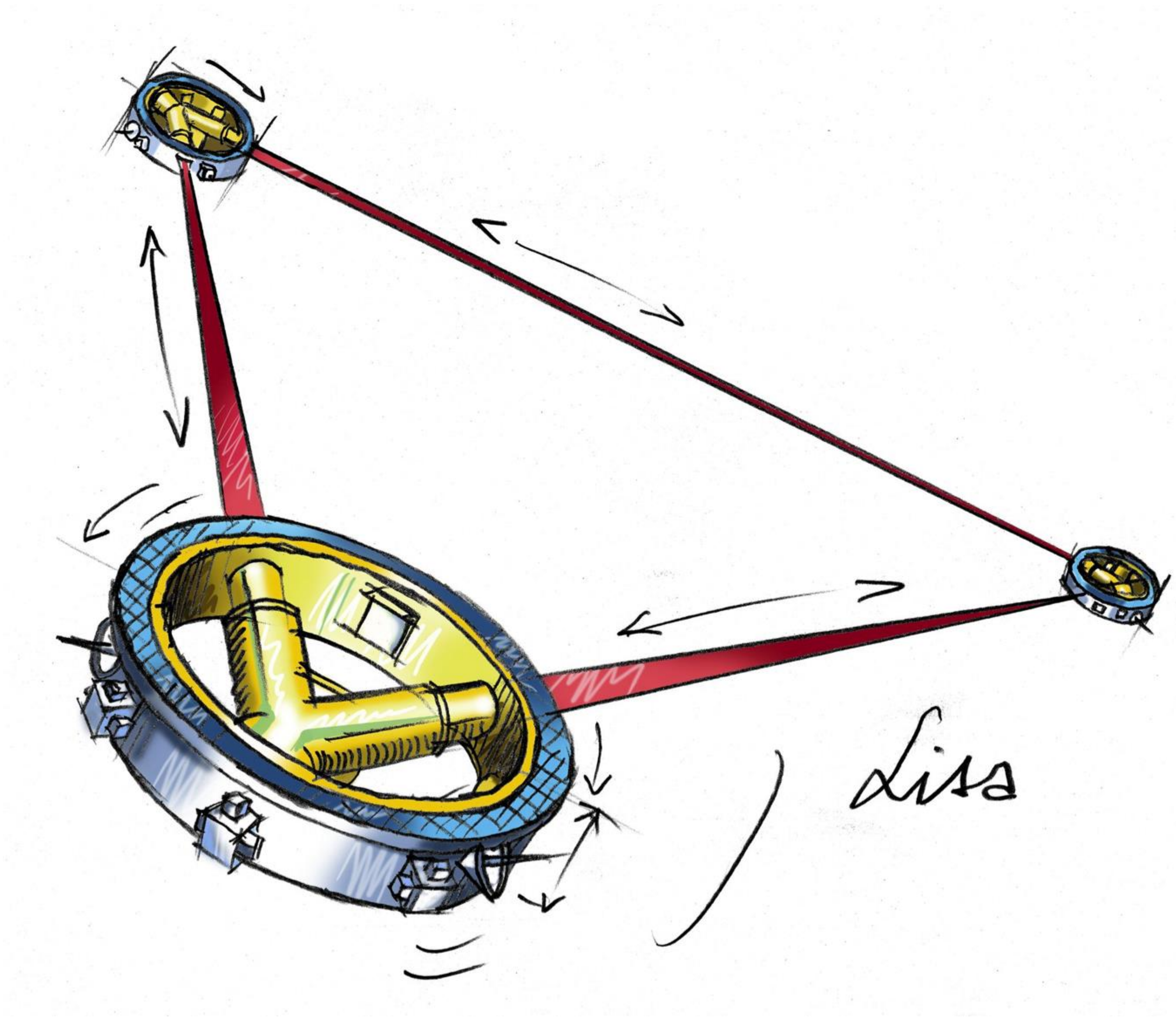


- LIGO still technically in engineering mode.
- Huge BH-BH system detected!
- Signal in band for 200 ms.
- $L \sim 10^{56}\,\mathrm{erg}\,s^{-1}$. Briefly more luminous than entire EM Universe.
- $\sim 3M_\odot$ radiated as GWs!
- $2\times \sim 30M_\odot$ BHs merging.

LIGO

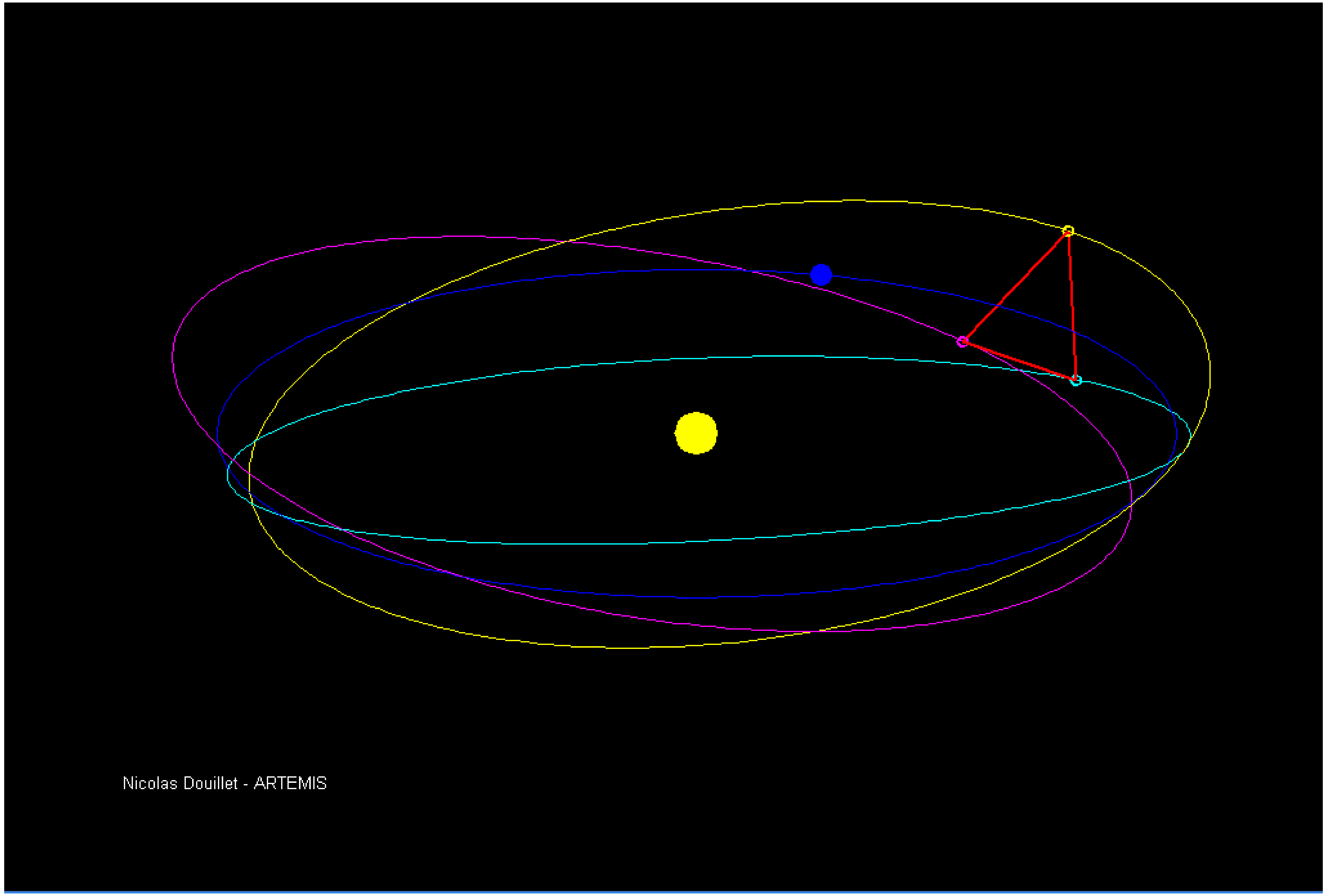

2017 NOBEL PRIZE IN PHYSICS
Illustrations: Niklas Elmehed. Nobel Prize Medal: © ® The Nobel Foundation. Photo: Lovisa Engblom.
Rainer Weiss
Barry C. Barish
Kip S. Thorne
"for decisive contributions to the LIGO detector and the observation of gravitational waves"

LISA [~0.1 - 100 mHz]

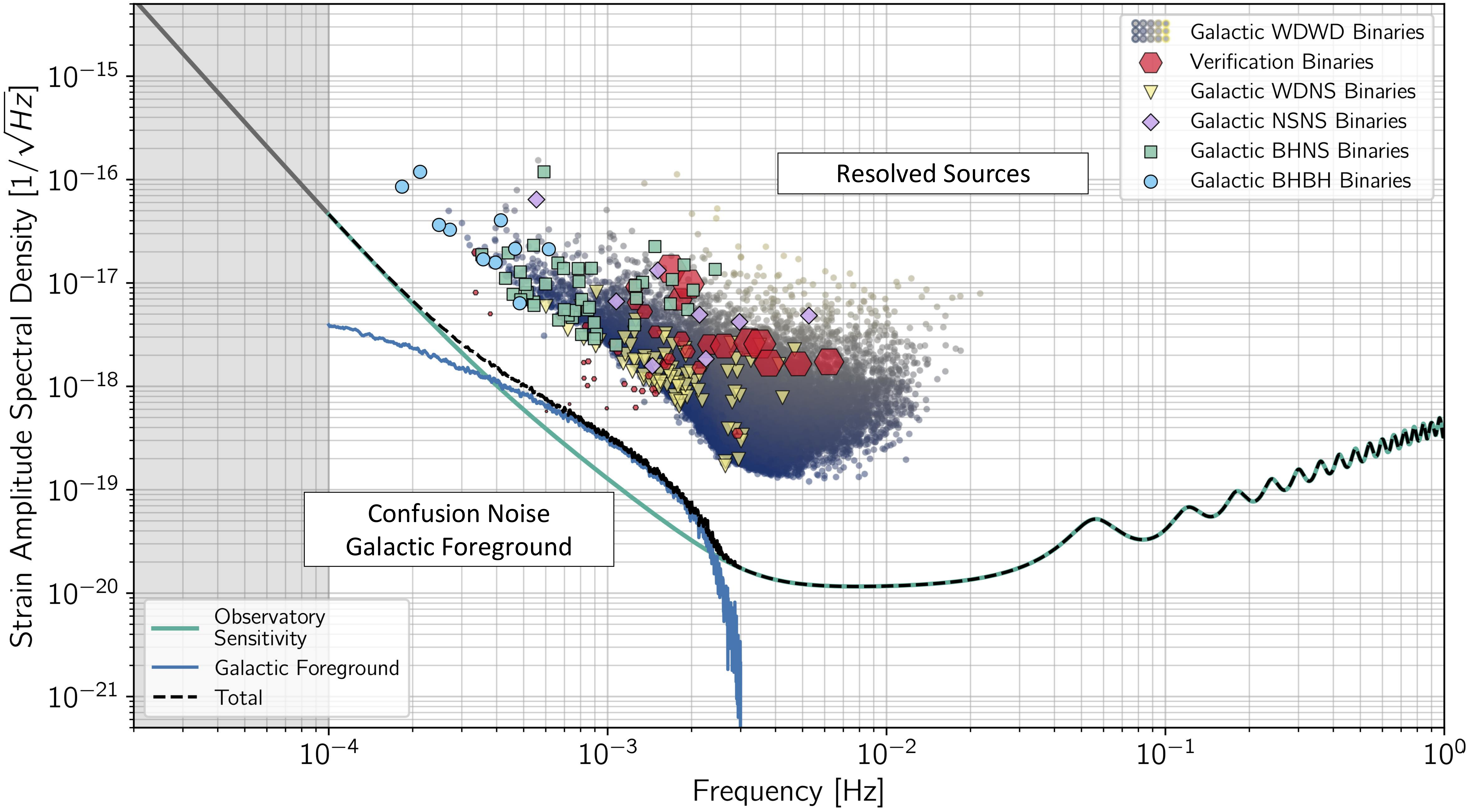


ESA-C. Vijoux

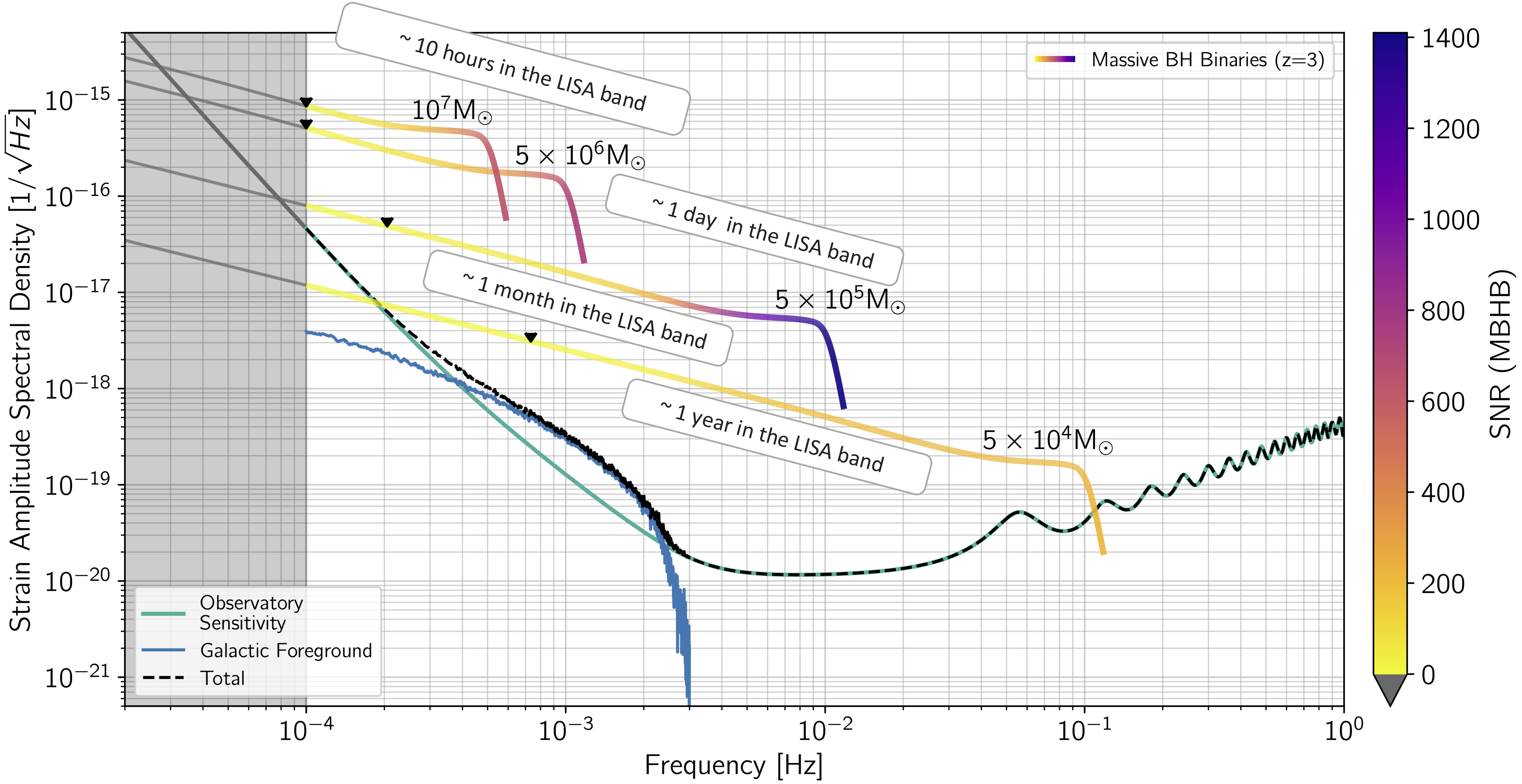
LISA



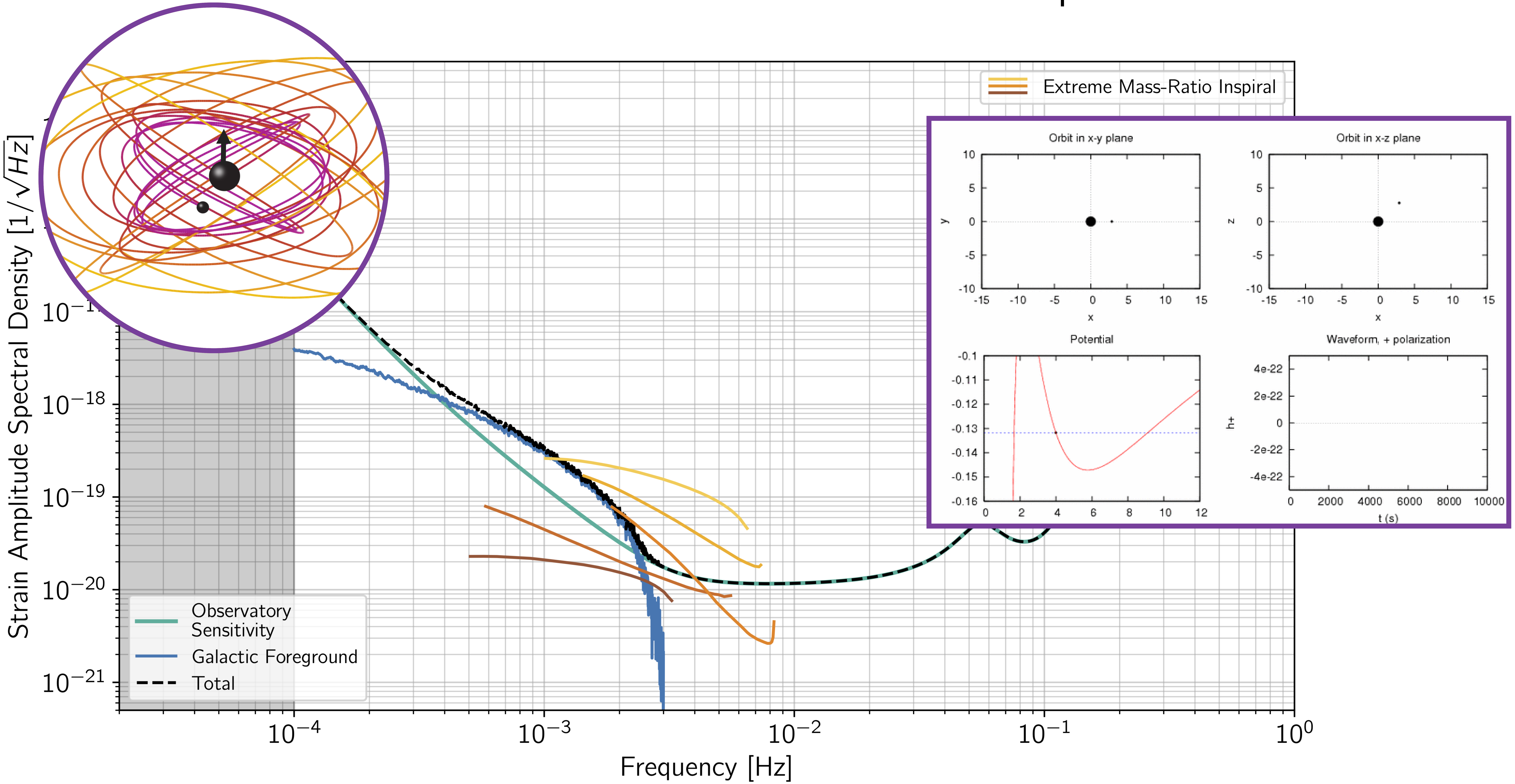
Galactic binaries



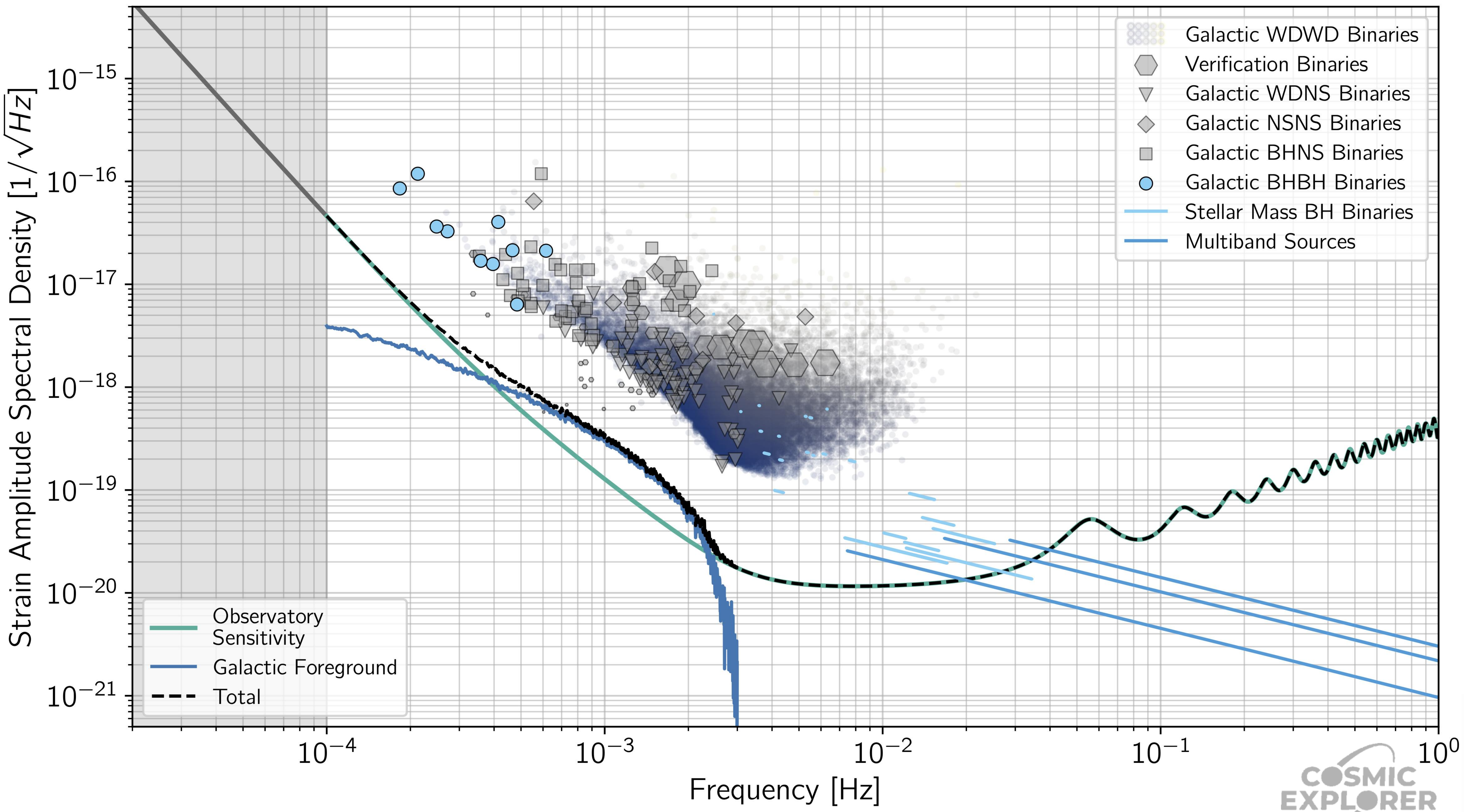
Massive black holes



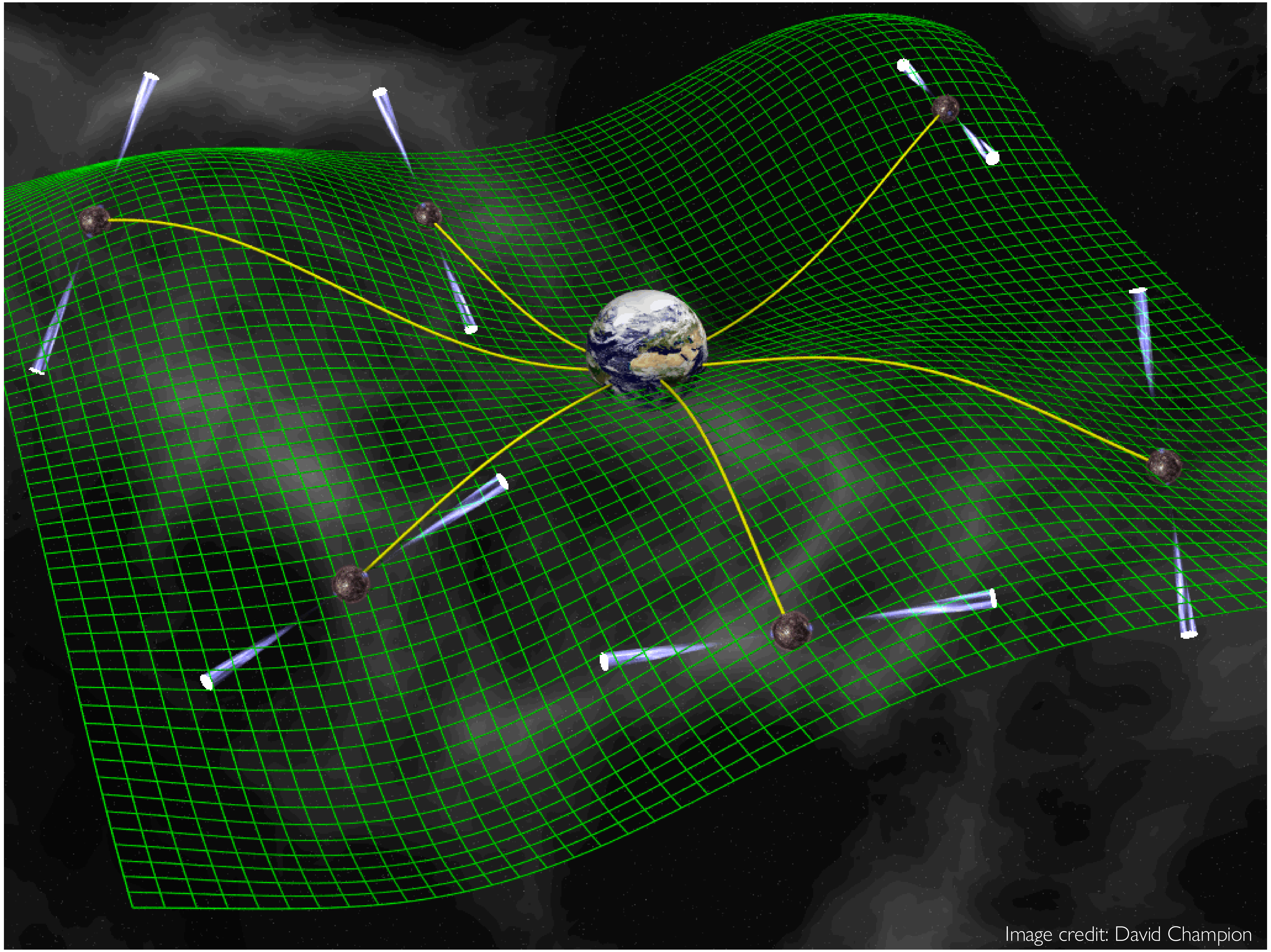
Extreme and intermediate mass ratio inspirals



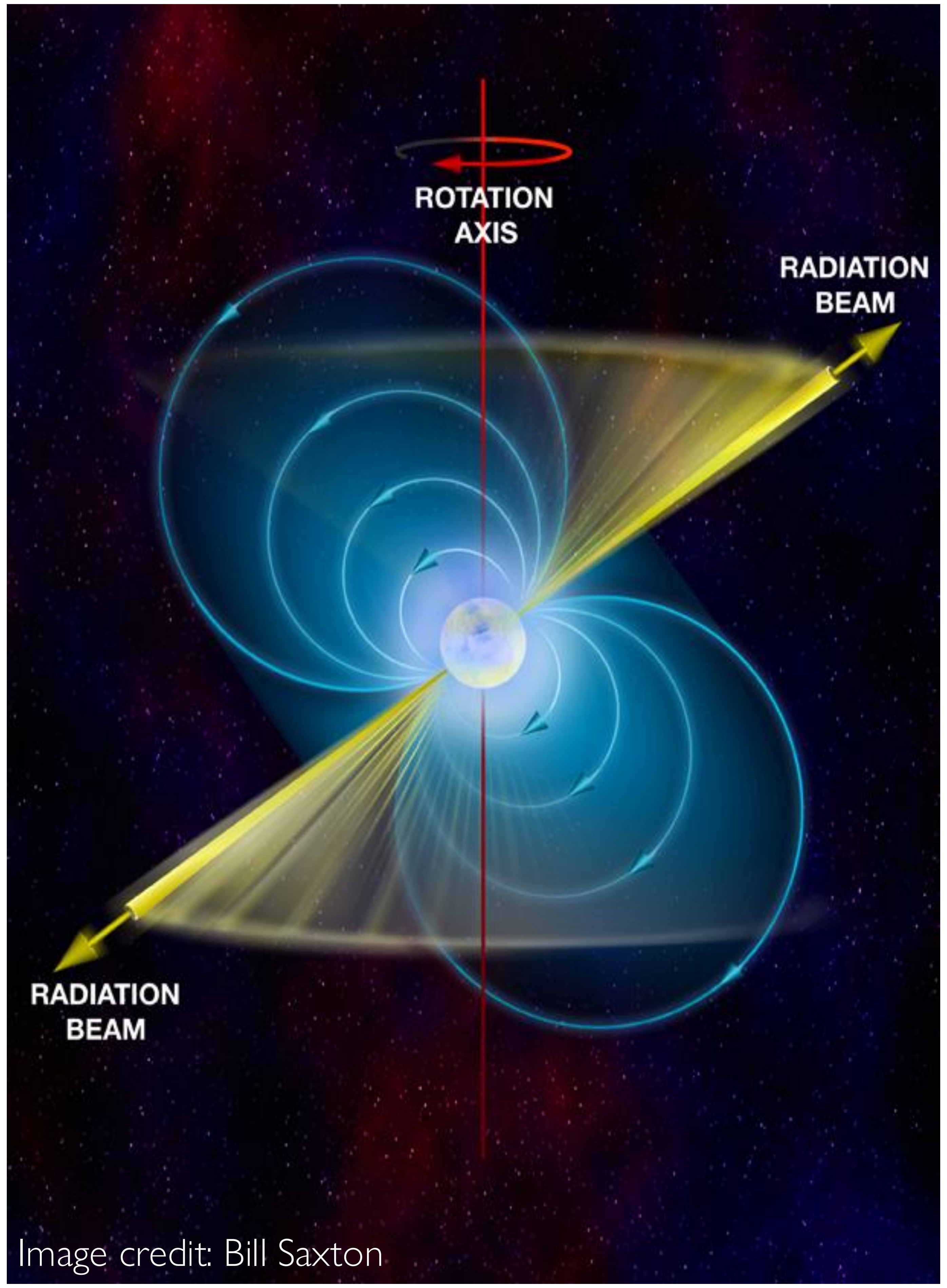
Stellar mass black holes

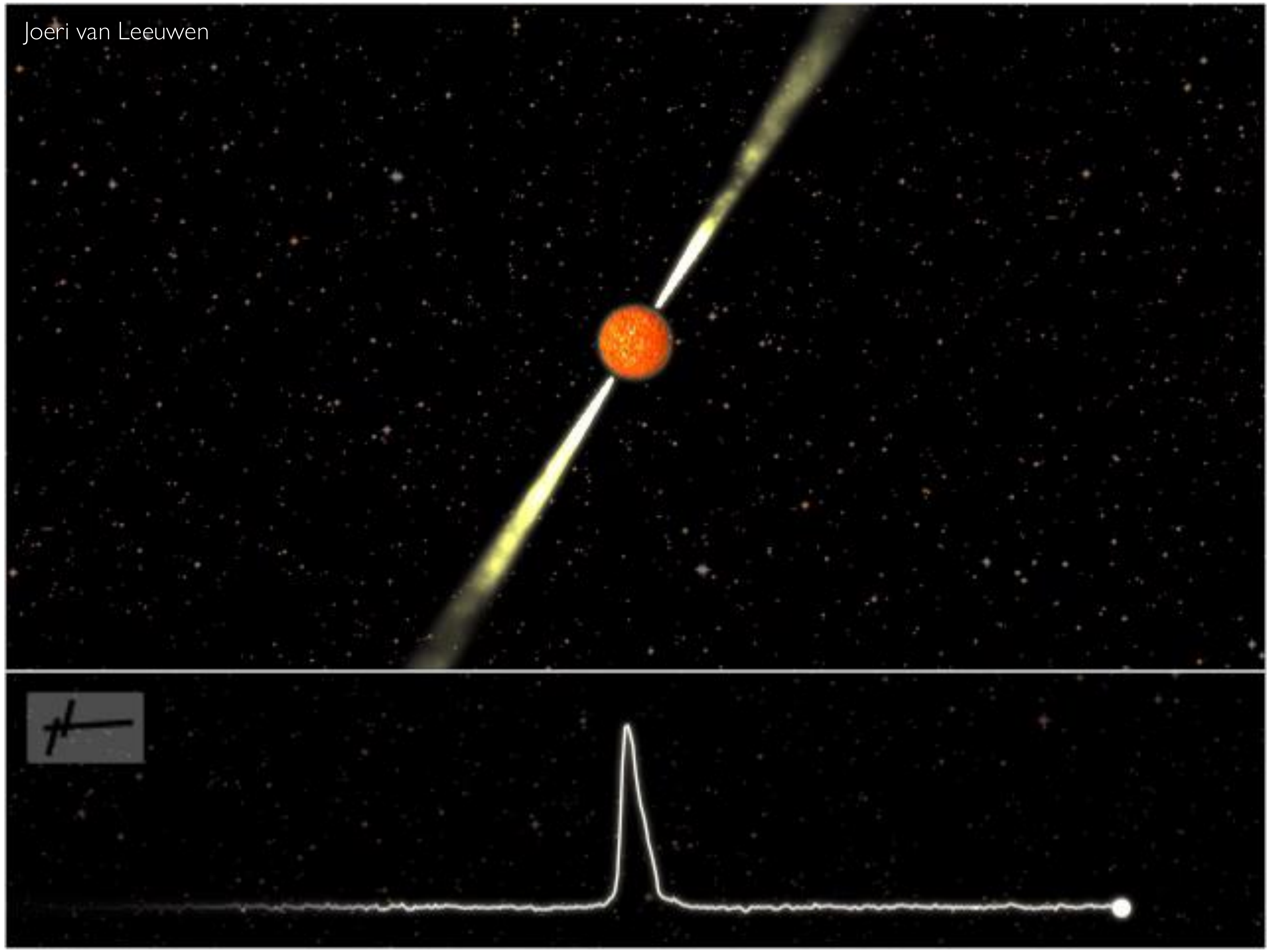


Pulsar Timing Arrays [~1 - 100 nHz]

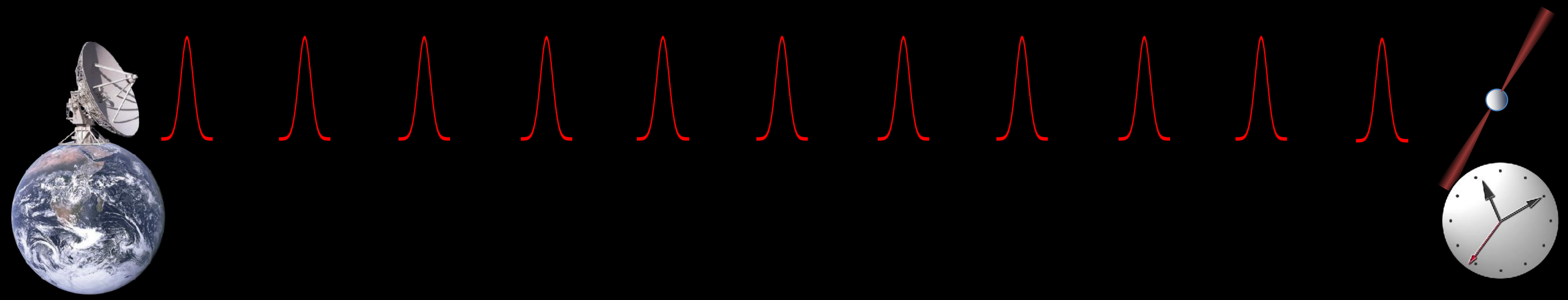


Pulsars


ROTATION AXIS
RADIATION BEAM
RADIATION BEAM
Image credit: Bill Saxton


Joeri van Leeuwen

Sophisticated timing model for each pulsar includes **spin period**, **spindown rate**, **astrometric effects**, **interstellar-medium dispersion**, etc.

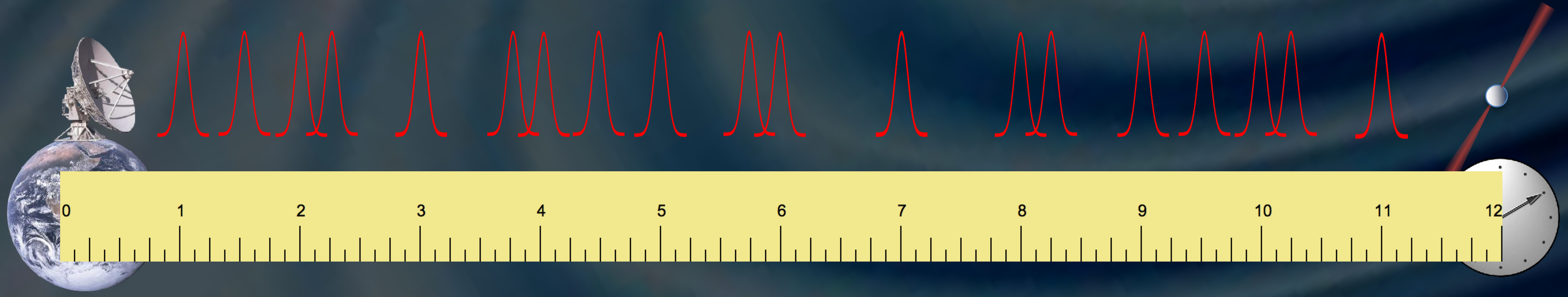


Gravitational Waves

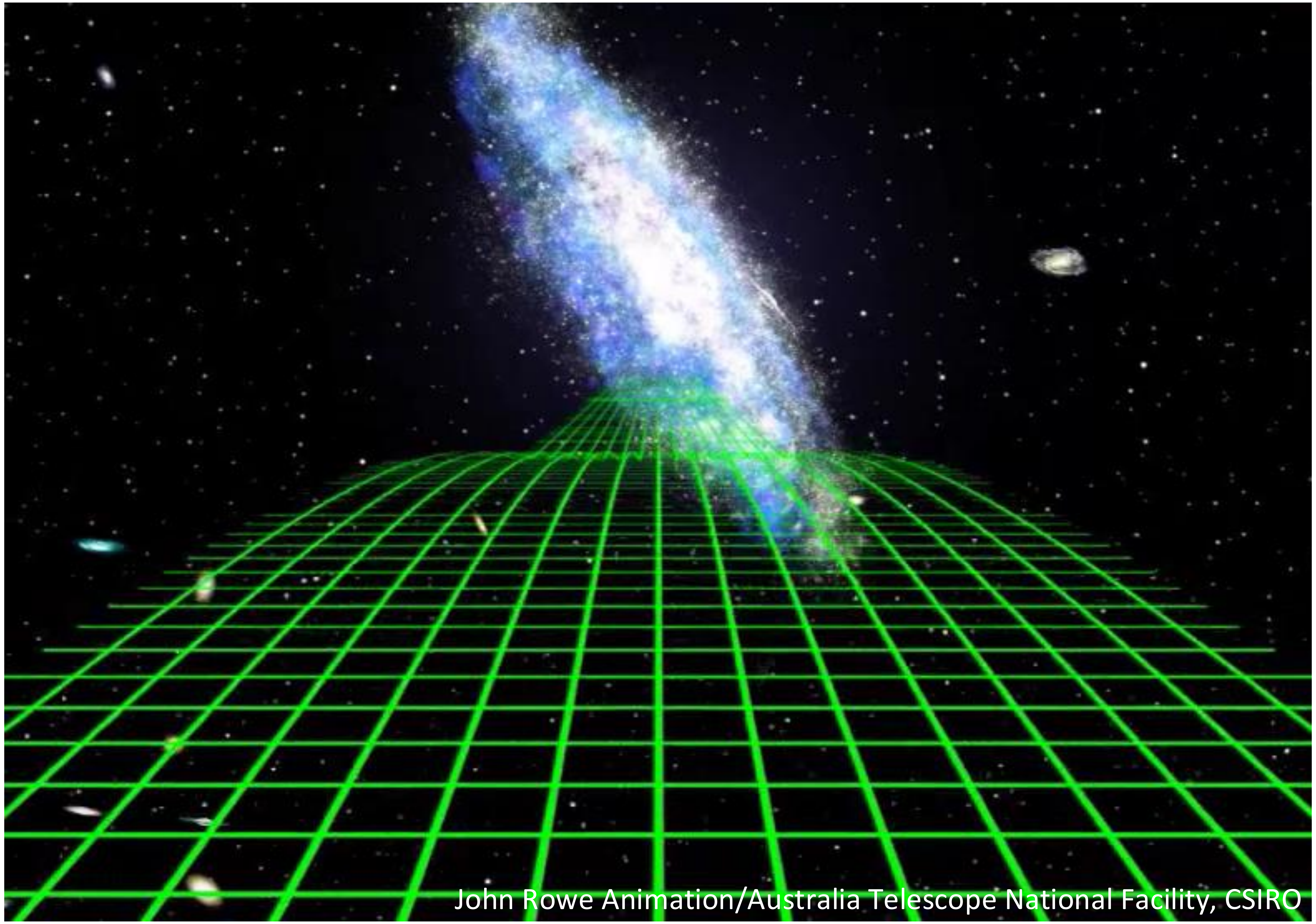
change the proper separation

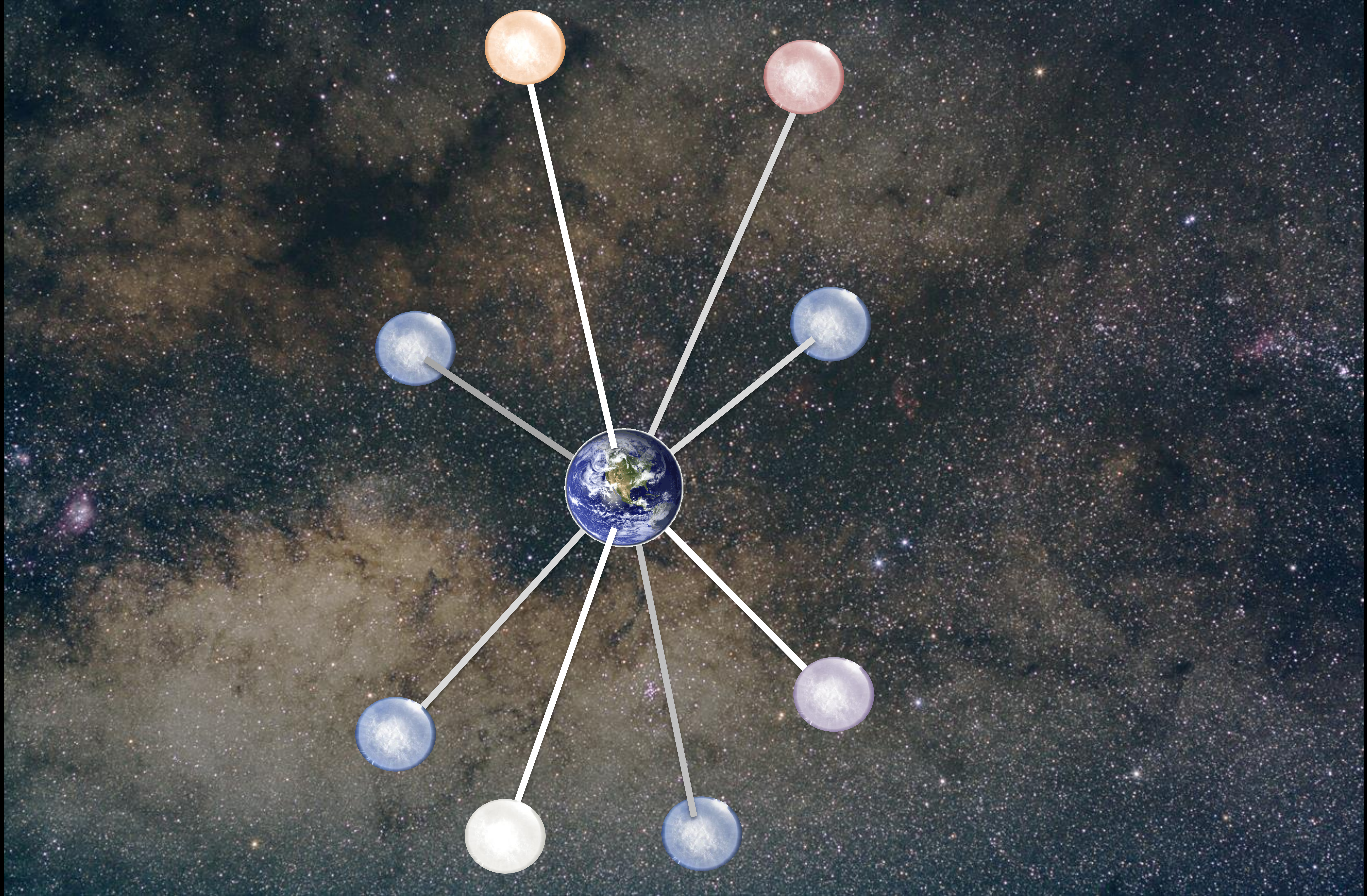
between Earth and pulsar.

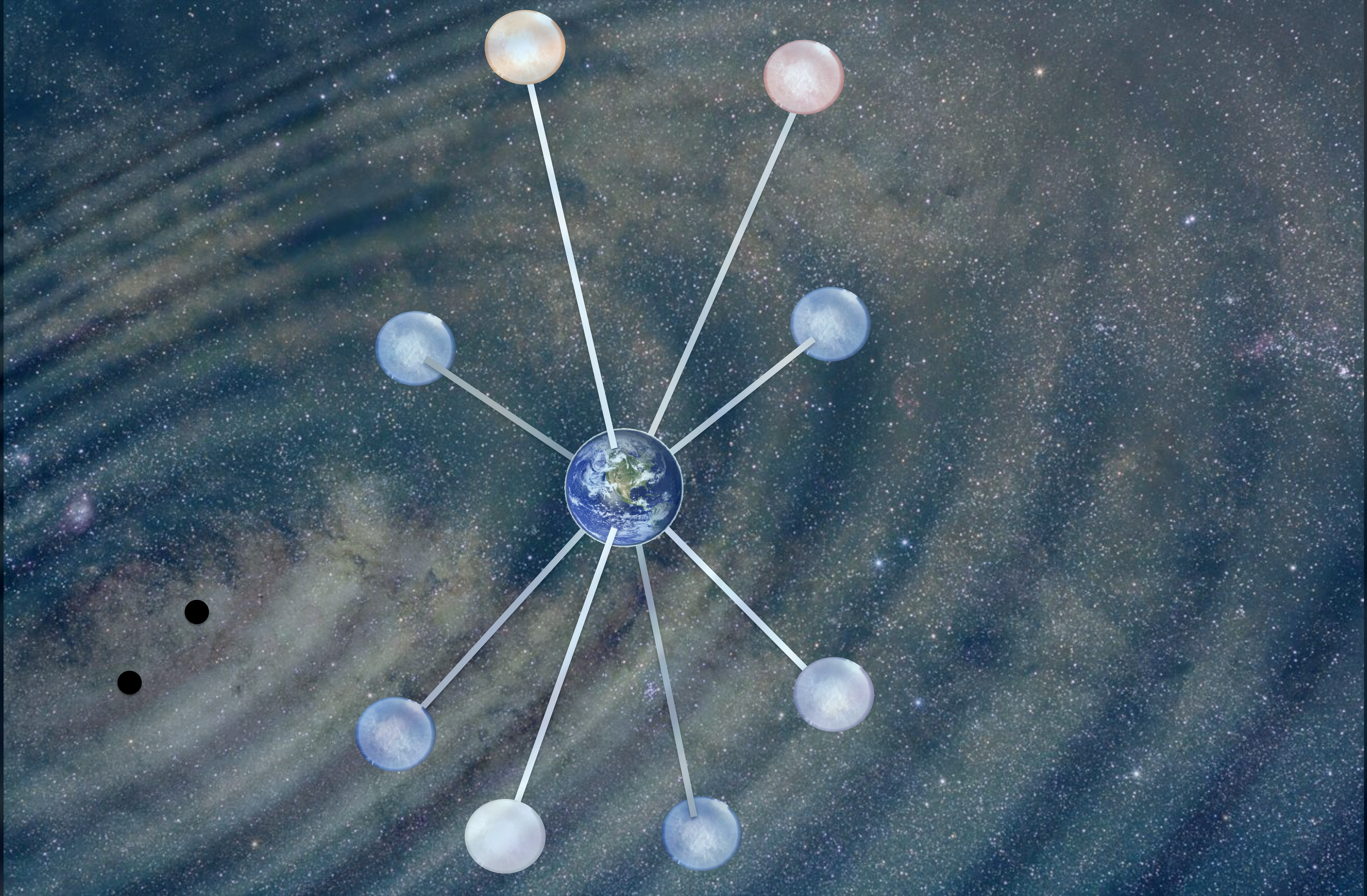
Pulses arrive *ahead* of or *behind* schedule



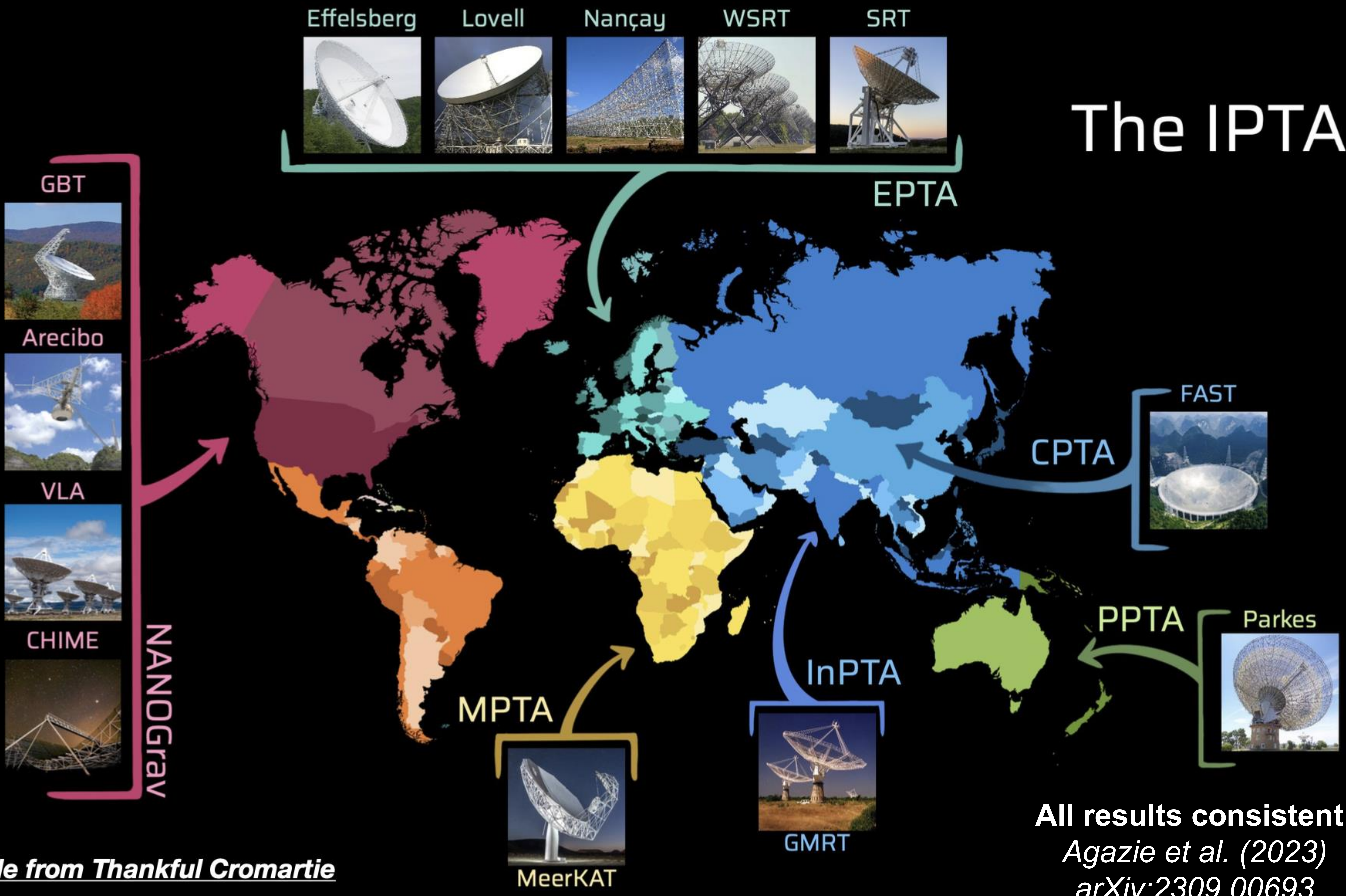
Pulsar Timing Arrays







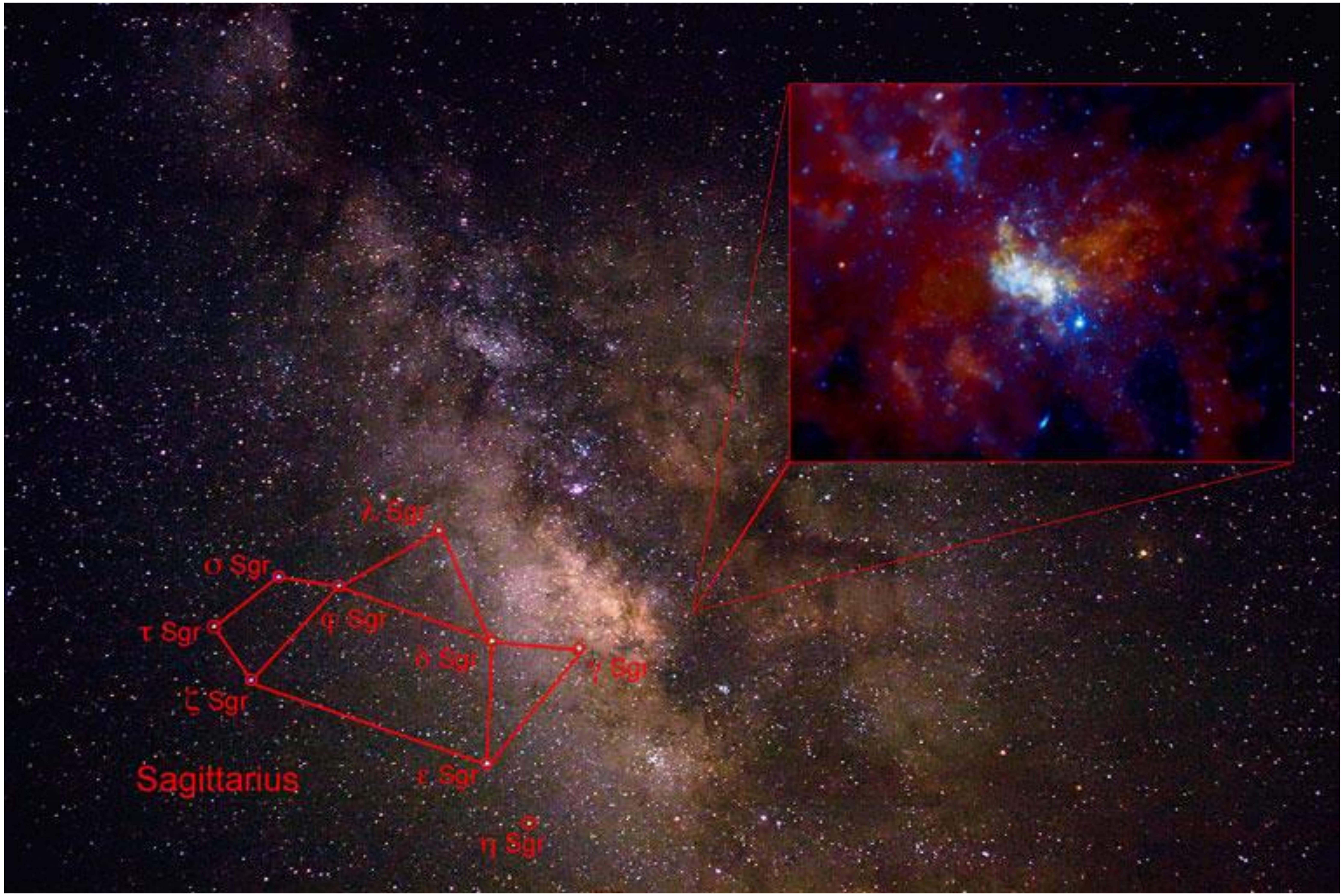
The IPTA



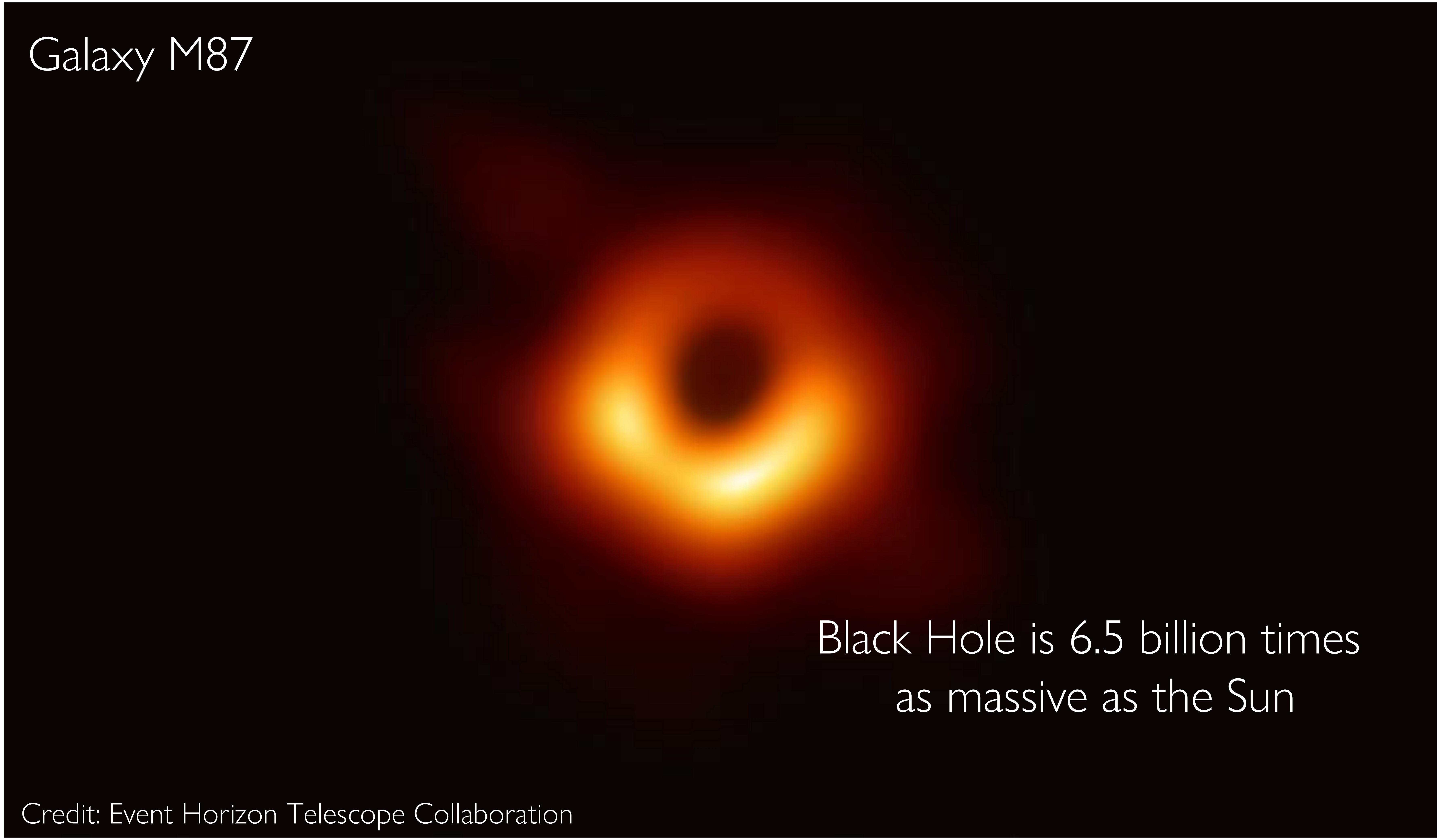
Slide from Thankful Cromartie

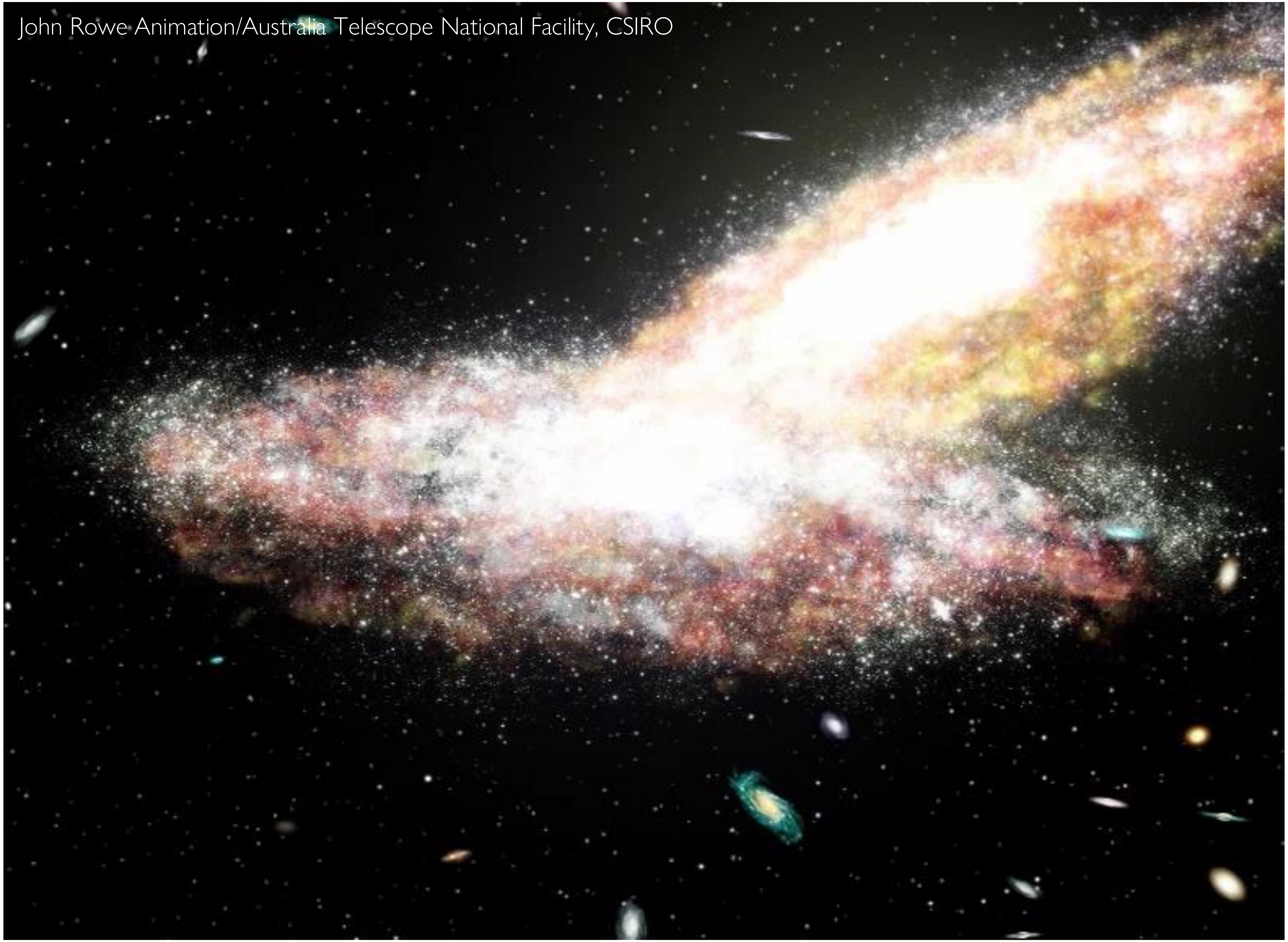
All results consistent!
Agazie et al. (2023)
arXiv:2309.00693

supermassive **Black Holes**



supermassive **Black Holes**

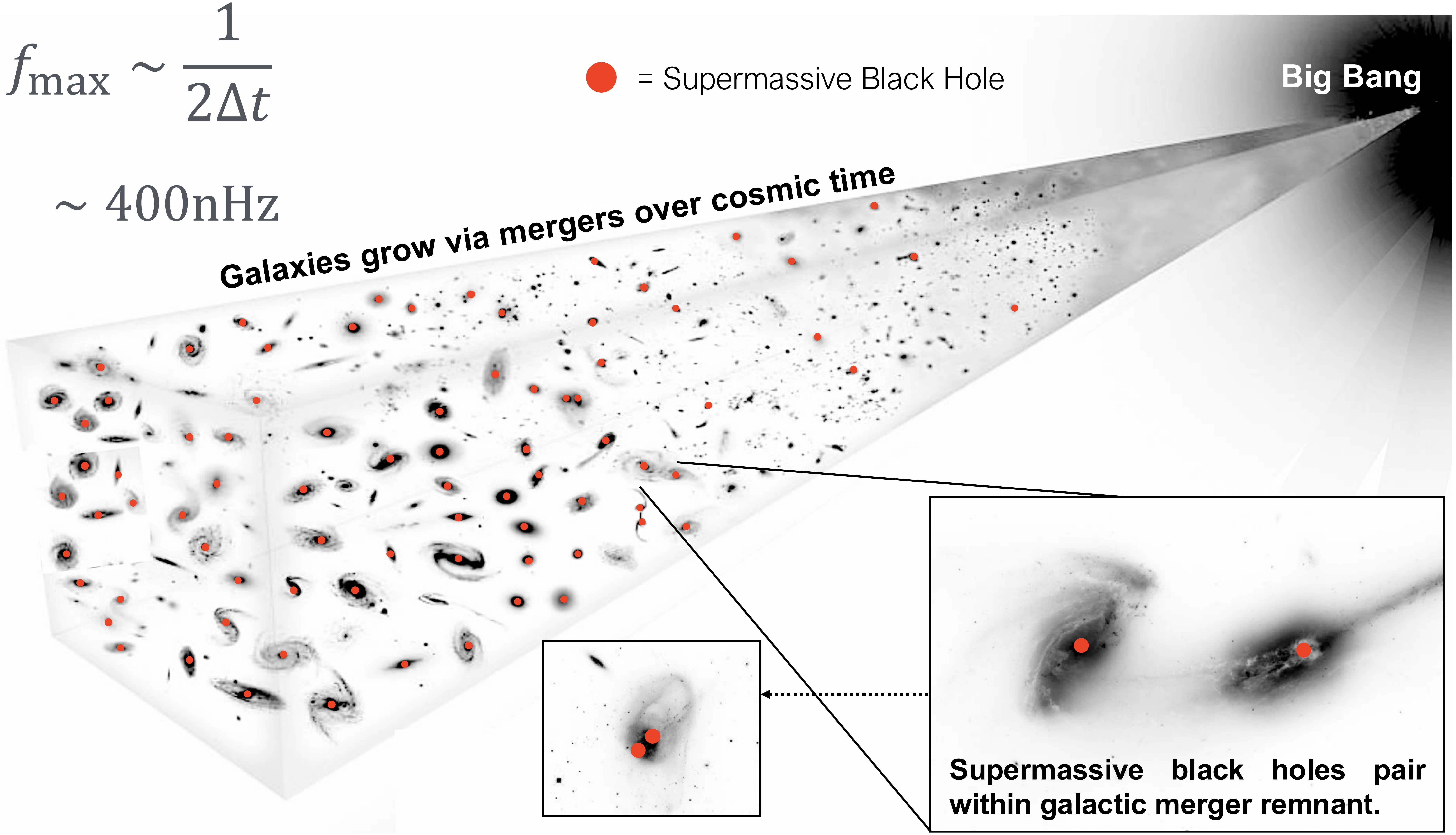



John Rowe Animation/Australia Telescope National Facility, CSIRO

Supermassive Binary Black Holes

$$f_{\rm min} = \frac{1}{T_{\rm obs}} \qquad f_{\rm max} \sim \frac{1}{2\Delta t}$$

$$\sim 2\mathrm{nHz} \qquad \sim 400\mathrm{nHz}$$



NANOGrav: The North American Nanohertz Observatory for Gravitational waves

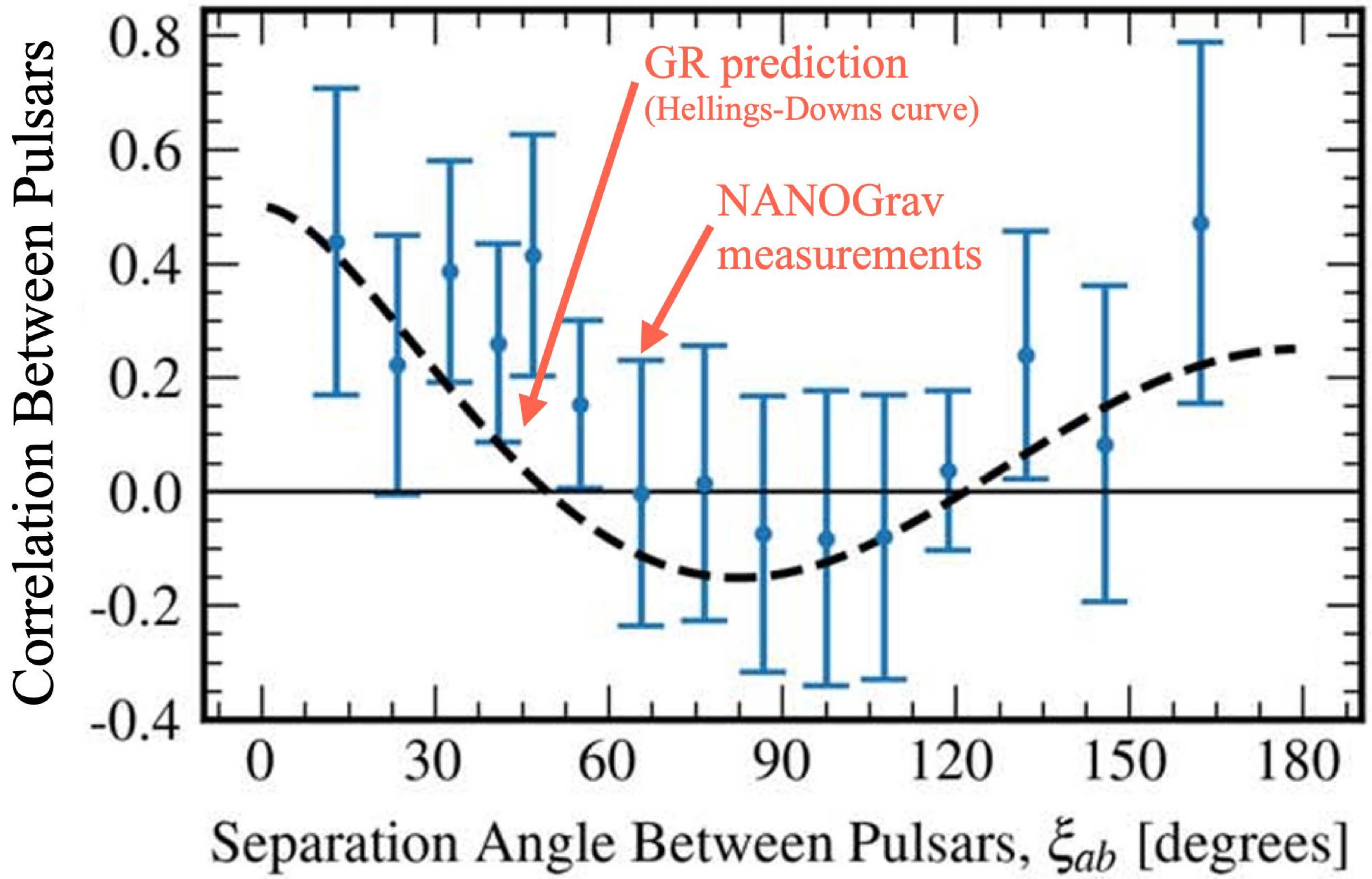
Credit: Pat Meyers

> 180 members
> 80 institutions (several outside of North America)
Radio-astronomers, GW scientists, BH & NS astrophysicists

Formally created in 2007, but data goes back to 2004
Founding member of the International Pulsar Timing Array
Funded by NSF as a Physics Frontier Center (>$30M since 2015)

Evidence for a Gravitational Wave Background

UTC 00:00, June 29 2023



Agazie et al. (2023), ApJL, Vol. 951, Issue 1

NSF Announcement, June 29 2023

Attended by Kip Thorne. Jocelyn Bell-Burnell joined virtually.

The Universe Is A Symphony