Investigating Pulsar Glitches Using Baym - Pines Model

Thesis submitted in partial fulfillment of the requirements for the degree of

BACHELOR OF SCIENCE IN PHYSICS

Submitted by

RIDHA FATHIMA M

CB.SC.I5PHY18051

Under the guidance of

Dr Bharat Kishore Sharma
(Asst. Professor)

Department of Sciences
AMRITA VISHWA VIDYAPEETHAM
Ettimadai, Coimbatore (Tamil Nadu), India.
June, 2021

BONAFIDE CERITIFICATE

This is to certify that the thesis entitled **"Investigating Pulsar Glitches Using Baym - Pines Model"** submitted by **Ridha Fathima M, CB.SC.I5PHY18051** in partial fulfillment of the requirements for the award of degree of Bachelor of Science in Physics is a bonafide thesis work carried out by them under my guidance and supervision at Department of Science, Amrita Vishwa Vidyapeetham, Coimbatore and the work has not been submitted elsewhere to award a degree.

Dr Bharat Kishore Sharma
Signature of supervisor

Place: Coimbatore
Date: June 5, 2021

श्रद्धावान् लभते ज्ञानम्

AMRITA
VISHWA VIDYAPEETHAM

DECLARATION

I hereby declare that the matter manifested in this thesis, **"Investigating Pulsar Glitches Using Baym - Pines Model"** is the result of research work carried out by me under supervision and guidance of **Dr Bharat Kishore Sharma** in the Department of Sciences, Amrita Vishwa Vidyapeetham, Coimbatore - 641102, Tamil Nadu, India and to the best of my knowledge this work has not formed the basis for the award of any degree/diploma/associateship/ fellowship or a similar award, to any candidate in any university. In keeping with general practice of reporting scientific observations, due acknowledgements have been made wherever the work described is based on the findings of other investigators.

Ridha fathima. M

Ridha Fathima M
Signature of student

Place: Coimbatore
Date: June 5, 2021

ENDORSEMENT

This is to certify that the thesis entitled **"Investigating Pulsar Glitches Using Baym - Pines Model"** is a bonafide thesis work done by **Ridha Fathima M, CB.SC.I5PHY18051** under the guidance of **Dr Bharat Kishore Sharma**.

Dr Mahadevan S
Signature of Chairman

Place: Coimbatore
Date:

Acknowledgements

The completion of this project was possible through the effort and time of many. I am immensely grateful to the following people.
Dr Bharat Kishore Sharma for the guidance by providing valuable inputs and helping me develop a clear understanding of pulsars and their rotational glitches.
Dr V Sreekanth for coordinating the projects and making this process as smooth as possible.
The Vice Chairperson, Dr. M. Sivakumar, and the Chairperson and Deputy Dean, Dr Mahadevan S. for their support all along.
This project would not have been possible without the knowledge and skills gained through the degree programme. I am thankful to all the faculties who taught us.
Finally, I would like to extend my gratitude to my friends and family for their inspirations and encouragement.
Thank you.

Abstract

Crab pulsar has been an important object of study in astronomy since its discovery in 1968. When sudden speed ups in the pulsar's rotation were observed, it was proposed that quakes occurring in the crust of the star might be the cause of the rotational 'glitches'. In this work, we employ Baym - Pines starquake model to understand the physics behind the glitch phenomenon. In the second part of this thesis, we use Crab pulsar timing data and glitch catalogue from Jodrell Bank Observatory to visualise the model and estimate the energy released during the largest glitch in 2017.

Contents

List of Symbols

$\dot{\nu}$ Frequency derivative

$\dot{P}$ Period derivative

ϵ Oblateness parameter

μ Shear modulus

ν Rotational frequency, $\nu = \frac{\Omega}{2\pi}$

Ω Rotational frequency

ρ Density

σ Stress

τ Characteristic age

P Rotational period

E Energy

G Gravitational constant

I Moment of inertia

M Mass

$M_\odot$ Solar mass

n Braking index

R Radius

V Volume

List of Figures

1 Introduction

Compact objects such as white dwarfs, neutron stars and black holes represent the extremes of physics. Understanding them is crucial to developing a complete picture of the universe. In this project, we focus on a particular class of them called pulsars. Specifically, we study the phenomenon of glitches. This section will delve into the formation and structure of pulsars[1], and the starquake model of glitches.

1.1 Formation of neutron stars

Neutron stars are the end products of stellar evolution of massive stars (>8 $M_\odot$).[2] When such massive stars exhaust their fuel i.e., they cannot produce enough energy to balance the gravitational collapse, the star collapses. For stars like our Sun, the collapse stops when electron degeneracy balances gravity producing a white dwarf as shown in Figure 1. For heavier stars, the collapse continues till a neutron star forms along with a supernova. Further collapse of the neutron star due to gravitational pressure is prevented by neutron degeneracy pressure.

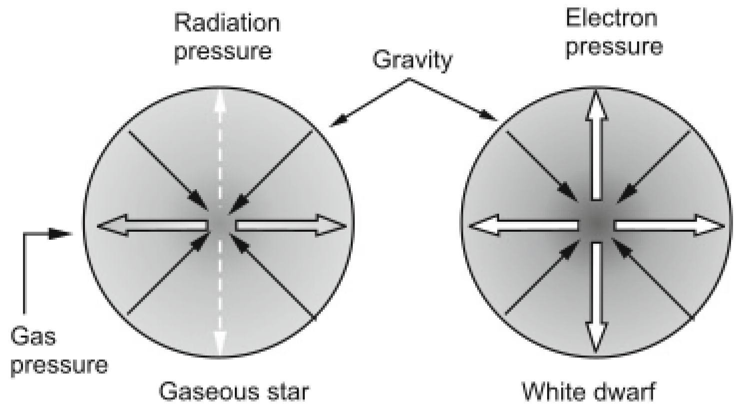


Figure 1: Illustration of hydrostatic equilibrium in gaseous and degenerate matter stars. Credit: *Srinivasan, G., 2014 - Life and Death of the Stars*

1.2 Structure of neutron stars

Though we do not have a complete equation of state for a neutron star, observations put the mass limit between 2-3 $M_\odot$ beyond which the neutron stars will collapse into a black hole. The density of neutron stars is comparable to nuclear densities and continues to increase from the crust to the core. The structure of a neutron star can be separated into 5 regions as shown in Figure 2:

- Atmosphere - A hot plasma of H, He, C and O.
- Outer crust - Rigid lattice of nuclei with relativistic electrons (except close to the surface) and supported by electron degeneracy.

[1]Henceforth, neutron stars and pulsars will be used interchangeably.

[2]Though the mass limit of the progenitor star that leads to formation of a black hole instead of a neutron star is not clear yet, for the purpose of this project we will focus on the formation of neutron stars.

- Inner crust - Beginning at the neutron-drip line ($\rho \sim 4.3 \times 10^{11} gcm^{-3}$), the inner crust is a lattice of heavy nuclei with superfluid neutrons penetrating the lattice. Free electrons contribute to the degeneracy pressure.
- Outer core - It is comprised of superfluid neutrons and traces of superconducting protons. The neutrons contribute to the degeneracy pressure of the region.
- Inner core - The nature of this extremely dense region is still unknown. Theories suggest quark matter can be found here.

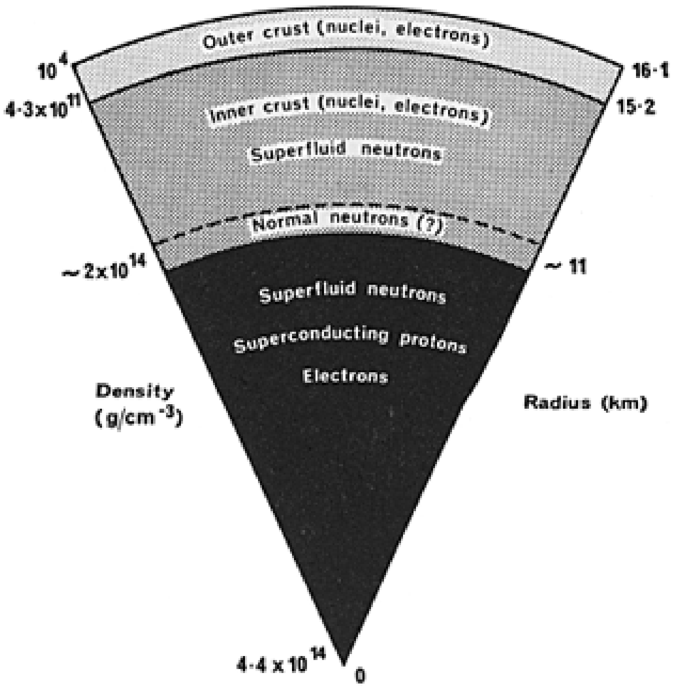


Figure 2: Cross section of a neutron star. Credit: `https://heasarc.gsfc.nasa.gov/docs/objects/binaries/neutron_star_structure.html`

1.3 Pulsars

Pulsars are highly magnetised and rapidly rotating neutron stars with emission regions and they appear as a point source every time the emission beam passes our line of sight. Synchrotron radiation from highly energetic electrons in the strong magnetic fields is the source of these radio beams. Their magnetic axis is often displaced from the rotation axis giving rise to the periodic nature of their signal. Only rarely do they coincide and to receive a continuous signal it must be aligned with our line of sight which makes the possibility of such occurrences even smaller. The pulse period, P, can vary from the order of 10 seconds to 1 milli second. Due to the emission, the star continues to lose energy slowly, resulting in a steady spin down that is observed to follow a power law model. After a critical value, the star stops emitting radiation since the rotational energy is not sufficient to power it and the pulsar turns off [Shapiro and Teukolsky, 2005].

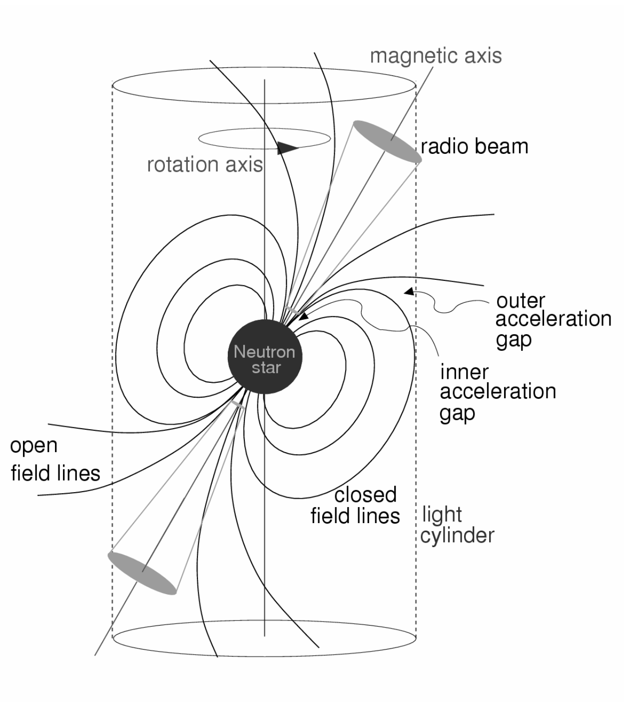


Figure 3: Magnetic dipole model / Lighthouse model of pulsars. Credit: *Lorimer, D. R. and Kramer, M., 2005 - Handbook of pulsar astronomy*

1.3.1 Pulse profile changes

Each pulsar has a characteristic pulse shape depending on its orientation, location and the emission geometry. But the pulse shape and the time interval are not always constant. The variations can occur due to the noise in detector or due to the interstellar medium. But some variations in pulse shape and period are intrinsic .i.e., occur due to changes in the pulsar. These variations are classified into 5 types:

- Drifting - A pulse maybe accompanied by sub-pulses. Recent evidences suggest that multiple emission regions outside the magnetic poles can exist contradicting the lighthouse model. As these regions rotate and pass our line of sight, they produce sub-pulses that add to the main pulse shape appearing as disturbance compared to the average pulse shape.
- Moding - A moding pulsar switches between them two or more states of pulse shapes Janzen [2015].
- Nulling - Some pulsars vanish for certain period and appear again. The emissionless period can be explained as change in the flow of charged particles in the pulsar magnetosphere.
- Timing noises - It is a common phenomenon characterised by random delay in arrival of the pulses due to rotational instabilities.
- Glitches - Sudden increase in rotational frequency of the pulsar causing the pulses arrive early than expected is called glitching. After the observation of speed ups in Crab and Vela pulsars in 1969, Ruderman proposed quakes in the crust as the cause for glitches [Antonopoulou, 2015].

1.4 The starquake model of glitches

In early epoch, star spins the fastest and the crust that solidifies is more oblate. As the star slows down, gravity pulls it in towards less oblate shape causing stress in the rigid crust. The stress builds up till a critical point and releases energy in a quake. Moment of inertia decreases and by conservation of angular momentum, rotational frequency increases. There are 3 possible scenarios:

- Entire oblateness relieved in the quake [Ruderman, 1969].
- Part of stress relieved in quake, remaining causes plastic flow [Smoluchowski, 1970].
- Part of stress relieved in quake, effects of plastic flow comparatively small and next quake depends on the stress relieved during previous one [Baym and Pines, 1971].

In this project, we use the Baym-Pines starquake model to understand the glitching mechanism and the reason for rotational instabilities observed in pulsars.

2 Baym - Pines model

The Baym-Pines model is a two component model where the neutron star has a rigid crust and a fluid core. The fluid core is rotating at a rate relatively faster than the crust and hence weakly coupled to the crust. When the crust experiences a quake the coupling time, the time taken for both the crust and core to attain similar rotational frequencies, is several years long and does not significantly help in relieving stress. Thus a large part of the strain energy is dissipated as changes in rotation. In this section we will define oblateness and derive how the frequency changes during a crustquake.

2.1 Oblateness parameter

An oblateness parameter is defined as

$$\epsilon = \frac{I_C - I_{C0}}{I_{C0}} \tag{1}$$

where, I_C is the moment of inertia of the crust and I_{C0} is the moment of inertia of the crust were the star spherical and non-rotating. Thus, the parameter determines the departure of moment of inertia from an ideal star. When the star forms, the crust is the most oblate and has a value ϵ_0. As the star slows down, gravity pulls the crust inward to a less oblate shape and causes strain in the rigid crust. Therefore ϵ_0 reduces to ϵ and the strain energy is given by

$$E_{strain} = B(\epsilon - \epsilon_0)^2 \tag{2}$$

where B is a constant given by

$$B = 0.42 N_A V Z^2 e^2 (\frac{N_A}{2})^{-1/3} \tag{3}$$

Gravitational energy of a rotating spherical star:

$$E_{grav} = E_0 + A_1\epsilon + A_2\epsilon^2 \tag{4}$$

where $E_0 = \frac{M^2 G}{R}$ is the gravitational energy of a stationary star of mass M and radius R. The linear term disappears for a spherical star and let A_2=A=$\frac{-1}{5}E_0$

Rotational energy of the star of moment of inertia I:

$$E_{rot} = \frac{1}{2}I\Omega^2 \tag{5}$$

The total energy of the neutron star:

$$E = E_0 + A\epsilon^2 + \frac{1}{2}I\Omega^2 + B(\epsilon - \epsilon_0)^2 \tag{6}$$

Differentiating with respect to ϵ,

$$\epsilon = \frac{\Omega^2}{4(A+B)}\frac{\partial I}{\partial \epsilon} + \frac{B}{A+B}\epsilon_0 \tag{7}$$

Before the quake, ϵ=ϵ_0 and Ω=Ω_0. Then we get the initial oblateness as

$$\epsilon_0 = \frac{\Omega_0^2}{4A}\frac{\partial I}{\partial \epsilon} \tag{8}$$

2.2 Characterising the starquake

The mean stress is then

$$\sigma = |\frac{1}{V_C}\frac{\partial E_{strain}}{\partial \epsilon}| = |\frac{2B}{V_C}(\epsilon - \epsilon_0)| = \mu(\epsilon_0 - \epsilon) \tag{9}$$

V_C is the volume of crust and $\mu = \frac{2B}{V_C}$ is the rigidity modulus. From equations (3) (4),

$$\sigma = \mu\frac{\Omega_0^2 - \Omega^2}{4(A+B)}\frac{\partial I}{\partial \epsilon} \tag{10}$$

The stress builds up with square of the rotational frequency as the star slows down. When it reaches a critical value σ_c, releasing the stress and the oblateness changes by $\Delta\epsilon$.

$$\Delta\epsilon = \frac{B}{A+B}\Delta\epsilon_0 \tag{11}$$

$\Delta\epsilon_0$ can be imagined as a change in reference point as shown in Figure 4. The stress relieved is

$$\Delta\sigma = \mu\frac{A}{B}\Delta\epsilon \tag{12}$$

From equation (1) it is seen that

$$\Delta\epsilon = \frac{\Delta I_C}{I_{C0}} = -\frac{\Delta\Omega}{\Omega} \tag{13}$$

The change $\Delta\Omega$ corresponds to the sudden increase in frequency that is observed as a glitch.

2.3 Energy released in a starquake

The energy lost during a quake is the change in strain energy due to change in the oblateness parameter. According to conservation energy, the net change in energy is zero. Therefore the energy released during quake is transmitted as increase in rotational frequency and also through gravitational waves. Due to weak coupling of the core, only marginal role is played by plastic flow in relieving the stress.

$$|E_{loss}| = |\Delta E_{strain}| = |2B(\epsilon - \epsilon_0)\Delta\epsilon| = |2A\frac{\sigma_c}{\mu}\Delta\epsilon| \tag{14}$$

The change in rotational energy is

$$|\Delta E_{rot}| = |\frac{\Omega^2}{2}\frac{\partial I}{\partial \epsilon}| \tag{15}$$

Dividing equation (14) by (15)

$$|\frac{E_{loss}}{\Delta E_{rot}}| = |\frac{\sigma_c}{\mu\epsilon}| \tag{16}$$

The resultant increase in rotational frequency is given by

$$\Omega = \Omega_0(1 - \frac{\sigma_c}{\mu\epsilon_0}) \tag{17}$$

When ϵ_0 reaches a value $\frac{\sigma_c}{\mu}$, no further quakes will occur.

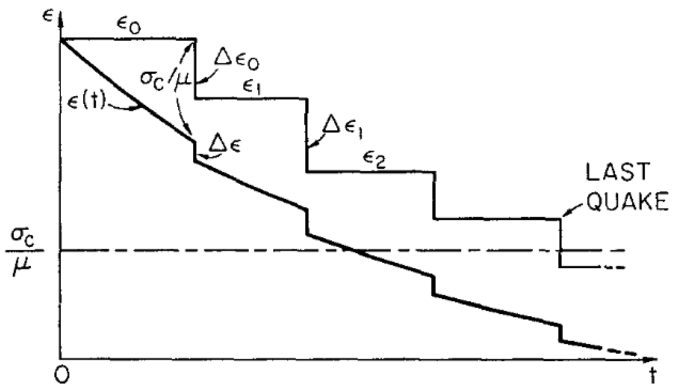


Figure 4: Representation of shift in reference point during quakes. Credit: *Baym G. and Pines, D., 1971 - Neutron starquakes and pulsar speedup*

2.4 Frequency of starquakes

Following a quake the next quake will occur when the stress builds up and reaches the critical value again. Therefore, the time between quakes is given by

$$t_q = |\frac{\Delta\sigma}{\dot{\sigma}}| \tag{18}$$

The rate of build-up of stress is

$$\dot{\sigma} = \frac{\mu}{2A}\frac{\partial I}{\partial \epsilon}\frac{\Omega^2}{T} \tag{19}$$

where T = -$\frac{\Omega}{\dot{\Omega}}$ is the rate at which the pulsar slows down due to loss of rotational energy.

$$t_q = \frac{A}{B}|\frac{\Delta\epsilon}{2\epsilon}|T \tag{20}$$

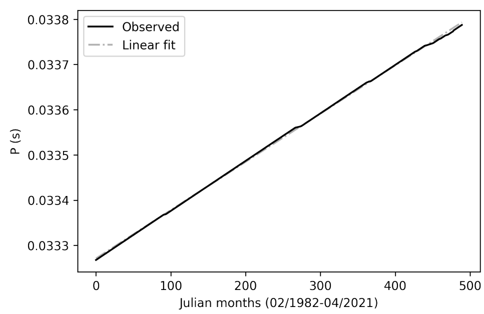
3 Crab Pulsar

Pulsars were first observed in 1967 by Jocelyn Bell Burnell proving the existence of neutron stars theorised much earlier. The discovery of Crab pulsar in the Crab nebula in 1968 confirmed that neutron stars are formed in supernova. Jodrell Bank Observatory maintains a open archive of timing data[3] of the pulsar from 1982 to present day and a catalogue of 500+ glitch events[4] at the time of writing. In the beginning of the section, we make use of the JPL Crab timing data to calculate the period, braking index and characteristic age of the pulsar. Later in the section we will look at the changes in oblateness parameter during glitch events and the large glitches recorded so far.

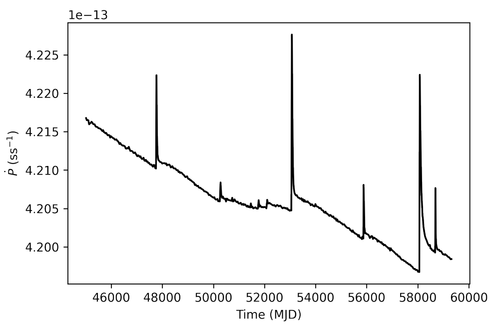
Figure 5: Combined X-ray and optical image of Crab nebula. Credit: `https://hubblesite.org/contents/news-releases/2002/news-2002-24.html`

3.1 Characteristics

3.1.1 Period and period derivative



(a)



(b)

Figure 6: Period (P) and Period derivative ($\dot{P}$) vs Time (MJD)

The period is the time interval between two signals which gives the rotation time of the pulsar. Crab pulsar has a rotation period of ~ 33 milliseconds. The period derivative is

[3] `http://www.jb.man.ac.uk/research/pulsar/crab.html`
[4] `http://www.jb.man.ac.uk/pulsar/glitches/gTable.html`

the rate at which the period changes. As seen in Figure 6, the period continues to increase, meaning the pulsar is slowing down steadily, while the period derivative decreases. The decrease of $\dot{P}$ is clearly non-linear and shows that the increase in period is not linear. The peaks in $\dot{P}$ corresponds to glitches.

3.1.2 Braking index

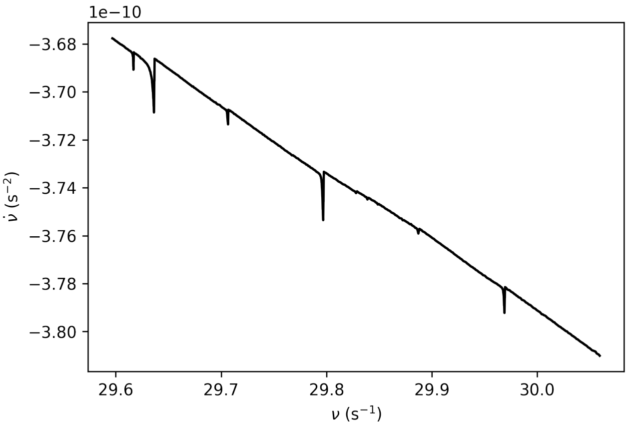


Figure 7: Rotational frequency (ν) vs Frequency derivative ($\dot{\nu}$)

The deceleration of Crab pulsar follows a power law model [Lyne et al., 1993].

$$\dot{\nu} = -k\nu^n \tag{21}$$

where k is a constant and n is the braking index. Using `lmfit` package to fit the data, the braking index for the pulsar is calculated to be 2.240 $\pm$ 0.254.

3.1.3 Characteristic age

Using the braking index, period and period derivative, we can calculate the approximate age of the pulsar.

$$\tau = \frac{P}{(n-1)\dot{P}}[1 - (\frac{P_0}{P})^{(n-1)}] \tag{22}$$

Assuming initial spin to be much faster, $P >> P_0$, and choosing a braking index of 3 according to the magnetic dipole model, τ becomes

$$\tau = \frac{P}{2\dot{P}} \tag{23}$$

The mean characteristic age we obtain is $\sim$ 1263 years which is close to the actual value of 967 years based on historical records. The deviation arises from the approximations made above and the assumption that the tilt of magnetic axis with respect to the rotation axis of pulsar and the strength of the magnetic field is constant.

3.2 Glitches

From equation (13)

$$|\Delta\epsilon| = |\frac{\Delta\nu}{\nu}| \tag{24}$$

Using MJD values of the events and their corresponding $\frac{\Delta\nu}{\nu}$ values from the catalogue data, we arrive at the step function[5] similar to Figure 4. The height of the step is proportional to

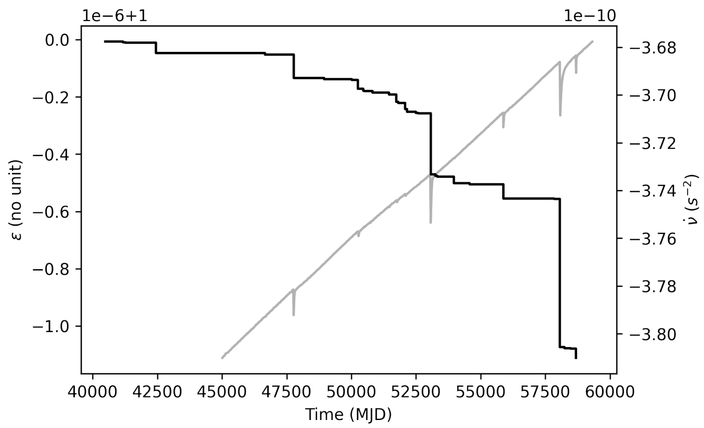
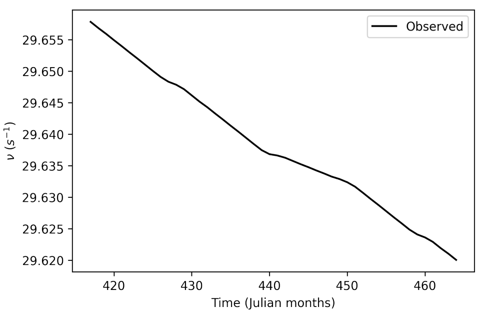


Figure 8: Step function of oblateness parameter (ϵ, in black) superimposed with timing data ($\dot{\nu}$ in grey) vs Time (MJD)

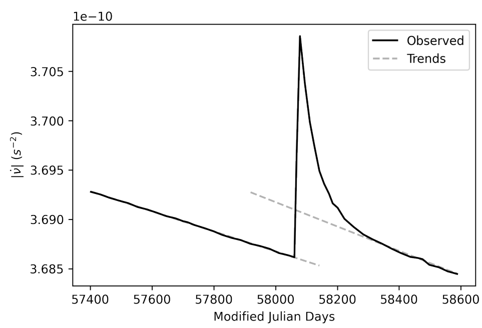
the oblateness relieved. The step function obtained is dependent on the number of events that were detected. Since the telescope observations are spaced by 30 days, changes within that time might go unrecorded. 30 glitches have been detected at Jodrell Bank Observatory at the time of writing this thesis.[6]

3.3 The largest glitch

The largest step on the right in Figure 8 corresponds to the glitch that occurred in 2017 having $\Delta\nu/\nu = 516.37 \pm 0.10 \times 10^{-9}$ and $\Delta\dot{\nu}/\dot{\nu} = 6.969 \pm 0.021 \times 10^{-7}$ [Shaw et al., 2018]. The ν and $\dot{\nu}$ changes during the glitch is shown in Figure 9.



(a) Frequency (ν) vs Time



(b) Frequency derivative ($\dot{\nu}$) vs Time

Figure 9: The 2017 glitch

Using equations (13), (14) and (17), the energy released during a quake is

$$E_{quake} = 2I_0\Omega^2\Delta\epsilon \tag{25}$$

[5]The python program is attached in Appendix ii

[6]The catalogue of Crab glitches is attached in Appendix i

where we have applied an upper bound of $\Omega_0 = 2\Omega$. Taking $I_0 = 10^{46}$ gcm^2 based on current mass and radius estimates, the energy released int 2017 glitch is estimated to be in the order of 10^{42} ergs. Baym and Pines estimated an energy release of 10^{40} ergs for the 1969 glitch in the Crab pulsar. Scargle and Harlan [1970] noted the changes in the wisps near the centre of Crab nebula and predicted an energy release of $10^{41} - 10^{44}$ ergs. They also mention that 10^{41} ergs is an underestimate and that internal energy release might not be the only factor causing the observed changes in the local wisps of the nebula.

4 Conclusion

The Baym - Pines model provides a neat understanding of the mechanics of a pulsar. In its formulation, it ignores magnetic field and this can be justified by the fact that pulsars of varying magnetic field strength including the magnetars follow similar power law trend and other properties as shown in Fuentes et al. [2017]. If magnetic field of the star is also affected by the glitch, there should be an accompanying emission which has not been observed yet. The model was able to successfully characterise the 1969 Crab glitch. It also suggested that the glitch mechanism is slowing down the spin down of pulsars and this has been verified [Downs, 1981]. The main failure of the model is that it suggests predictability of pulsars that was never detected [Wang et al., 2001]. The model can not explain the behaviour of Vela pulsar. Despite the failures, it still remains as a foundation for other advanced models such as vortex pinning. An extension of this work can take vortex pinning into account, wherein increasing stress in the crust causes increase in frictional force leading to coupling between the crust and the core.

Bibliography

D. Antonopoulou. *Thesis: Rotational glitches in radio pulsars and magnetars*. 2015.

G. Baym and D. Pines. Neutron starquakes and pulsar speedup. *Annals of Physics*, 1971.

G. Downs. Jpl pulsar timing observations - i the vela pulsar. *The Astrophysical Journal*, 1981.

J. R. Fuentes, C. M. Espinoza, A. Reisenegger, B. Shaw, B. W. Stappers, and A. G. Lyne. The glitch activity of neutron stars. *Astronomy Astrophysics*, 2017.

D. Janzen. *Bachelor thesis : Methods for identifying pulsar modes at radio frequencies*. 2015.

A. G. Lyne, R. S. Pritchard, and F. G. Smith. 23 years of crab pulsar rotational history. *Monthly Notices of the Royal Astronomical Society*, 1993.

M. Ruderman. Neutron starquakes and pulsar periods. *Nature*, 1969.

J. Scargle and E. Harlan. *Astrophys. J. Letters 159*, 1970.

S. Shapiro and S. Teukolsky. *Black holes, white dwarfs and neutron stars*. 2005.

B. Shaw, A. G. Lyne, B. W. Stappers, P. Weltevrede, C. G. Bassa, and A. Y. Lien. The largest glitch observed in the crab pulsar. *Monthly Notices of the Royal Astronomical Society*, 2018.

R. Smoluchowski. Frequency of pulsar starquakes. *Physical review letters*, 1970.

N. Wang, X.-J. Wu, R. N. Manchester, J. Zhang, A. G. Lyne, and A. Yusup. A large glitch in the crab pulsar. *Chinese Journal of Astronomy and Astrophysics*, 2001.

Appendix

i. Catalogue of Crab Glitches

The full catalog is available at `http://www.jb.man.ac.uk/pulsar/glitches/gTable.html`
Note: Modified Julian Date (MJD) is the Julian Date - 240000.5

No.	MJD(days)	+/-(days)	$\Delta\nu/\nu$ ($\times 10^{-9}$)	+/- ($\times 10^{-9}$)	$\Delta\dot{\nu}/\dot{\nu}$ ($\times 10^{-3}$)	+/- ($\times 10^{-9}$)
1	40491.80	0.03	7.2	0.4	0.44	0.04
2	41161.98	0.04	1.9	0.1	0.17	0.01
3	41250.32	0.01	2.1	0.1	0.11	0.01
4	42447.26	0.04	35.7	0.3	1.6	0.1
5	46663.69	0.03	6	1	0.5	0.1
6	47767.50	0.003	81.0	0.4	3.4	0.1
7	48945.60	0.1	4.2	0.2	0.32	0.03
8	50020.04	0.02	2.1	0.1	0.2	0.01
9	50260.03	0.004	31.9	0.1	1.73	0.03
10	50458.94	0.03	6.1	0.4	1.1	0.1
11	50489.7	0.2	0.8	0.3	-0.2	0.1
12	50812.59	0.01	6.2	0.2	0.62	0.04
13	51452.02	0.01	6.8	0.2	0.7	0.1
14	51740.656	0.002	25.1	0.3	2.9	0.1
15	51804.75	0.02	3.5	0.1	0.53	0.03
16	52084.072	0.001	22.6	0.1	2.07	0.03
17	52146.758	0.0003	8.9	0.1	0.57	0.01
18	52498.257	0.002	3.4	0.1	0.70	0.02
19	52587.20	0.01	1.7	0.1	0.5	0.1
20	53067.078	0.0002	214	1	6.2	0.2
21	53254.109	0.002	4.9	0.1	0.2	0.1
22	53331.17	0.01	2.8	0.2	0.7	0.1
23	53970.190	0.0003	21.8	0.2	3.1	0.1
24	54580.38	0.01	4.7	0.1	0.2	0.1
25	55875.5	0.1	49.2	0.3	X	X
26	57839.92	0.06	2.14	0.11	0.27	0.03
27	58064.555	0.003	516.37	0.10	6.969	0.021
28	58237.357	0.005	4.08	0.22	0.46	0.11
29	58470.939	0.006	2.36	0.04	0.36	0.05
30	58687.565	0.004	31.7	1.2	0.341	0.033

ii. Python programs

```
import pandas as pd
import numpy as np
import astropy
import matplotlib.pyplot as plt

from lmfit.models import PowerLawModel

def powfit(x,y):

    pl_model = PowerLawModel(prefix='pl_')
    pl_params = pl_model.make_params()
    pl_params['pl_amplitude'].set(value=1.0)
    pl_params['pl_exponent'].set(value=-2.5)
    pl_fit = pl_model.fit(y, pl_params, x=x)
    a = pl_fit.params['pl_exponent'].value
    return a,pl_fit

from astropy.io import ascii
data = ascii.read('crab') #Timing

freq = data['nu_Hz']
time = np.arange(len(freq))
period =  1/freq
fdot = data['nudot_10-15sec-2']*10**-15
pdot = -(period**2)*fdot

# Calculating braking index and characteristic age

n,vdotdot = powfit(freq,fdot)
print('Braking index:',n)

age = period/(2*pdot)
age = age/(60*60*24*365.25)
print('Characteristic age:',np.mean(age))

# Producing the step function

from astropy.io import ascii
glitches = ascii.read('glitches') #Catalogue

e = 1 #initial oblateness is chosen to be 1
E = []
t = glitches['MJD']
de = glitches['dF/F']*10**(-9)

for i in de:
    e = e-i
```

```
    E.append(e)

fig, ax1 = plt.subplots()
ax2 = ax1.twinx()

a = ax1.step(t, E, where='post', color='black', label=r'$\epsilon$')
b = ax2.plot(time, fdot, color='black', alpha=0.3, label=r'$\dot{\nu}$')

ax1.set_xlabel('Time (MJD)')
ax1.set_ylabel(r'$\epsilon$ (no unit)')
ax2.set_ylabel(r'$\dot{\nu}$ ($s^{-2}$)'
```