# Assignment 10

1. **Fund performance and fees (15 points)**

   Consider a passive mutual fund, an active mutual fund, and a hedge fund. The mutual funds claim to deliver the following gross returns:

$$R_t^{\text{passive fund before fees}} = R_t^{\text{stock index}}$$

$$R_t^{\text{active fund before fees}} = 2.20\% + R_t^{\text{stock index}} + \epsilon_t$$

   The passive mutual fund charges an annual fee of 0.10%. The active mutual fund charges a fee of 1.20% and seeks to beat the same stock market index by about 1% per year after fees. The active mutual fund has a beta of 1 and has a tracking error volatility of $\sqrt{Var(\epsilon_t)} = 3.5\%$. The hedge fund uses the same strategy as the active mutual fund to identify "good" and "bad" stocks, but implements the strategy as a long-short hedge fund, applying 4 times leverage. The risk-free interest rate is 1% and the financing spread is zero (meaning that borrowing and lending rates are equal). Therefore, the hedge fund's return before fees is

$$R_t^{\text{hedge fund before fees}} = 1\% + 4 \times \left(R_t^{\text{active fund before fees}} - R_t^{\text{stock index}}\right)$$

   (a) What is the hedge fund's volatility?

   (b) What is the hedge fund's beta?

   (c) What is the hedge fund's alpha before fees (based on the active mutual fund's alpha estimate)?

   (d) Suppose that an investor has \$40 invested in the active mutual fund and \$60 in cash (measured in thousands, say). What investments in the passive mutual fund, the hedge fund, and cash (i.e., the riskfree asset) would yield the same market exposure, same alpha, same volatility, and same exposure to $\epsilon_t$? As a result, what is the fair management fee for the hedge fund in the sense that it would make the investor indifferent between the two allocations (assume that the hedge fund charges a zero performance fee)?

   (e) If the hedge fund charges a management fee of 2%, what performance fee makes the expected fee the same as above? Ignore high water marks and ignore the fact that returns can be negative, but recall that performance fees are charged as a percentage of the (excess) return after management fees. Specifically, assume the performance fee is a fraction of the hedge fund's outperformance above the risk-free interest rate.

(f) Comment on whether it is clear that hedge funds that charge 2-20 fees (2% management fee and 20% performance fee) are "expensive" relative to typical mutual funds. More broadly, what should determine fees for active management?

**Solution:**

(a) The hedge fund's variance is

$$\begin{aligned} Var\left[R_t^{\text{hedge funds before fees}}\right] &= Var\left[4 \times (R_t^{\text{active fund before fees}} - R_t^{\text{stock index}})\right] \\ &= 16 \times Var\left[2.20\% + R_t^{\text{stock index}} + \epsilon_t - R_t^{\text{stock index}}\right] \\ &= 16 \times Var\left[\epsilon_t\right] = 16 \times (3.5\%)^2 . \end{aligned}$$

Thus, the hedge fund's volatility is $4 \times 3.5\% = 14\%$.

(b) The hedge fund's return is equal to

$$\begin{aligned} R_t^{\text{hedge funds before fees}} &= 1\% + 4 \times \left(R_t^{\text{active fund before fees}} - R_t^{\text{stock index}}\right) \\ &= 1\% + 4 \times \left(2.20\% + R_t^{\text{stock index}} + \epsilon_t - R_t^{\text{stock index}}\right) \\ &= 1\% + 4 \times (2.20\% + \epsilon_t) . \end{aligned}$$

The only risk exposure of the hedge funds is to the tracking error $\epsilon_t$. Thus, the hedge fund's beta is 0.

(c) The hedge fund's alpha before fees is its expected excess return minus its beta times the equity risk premium, i.e.,

$$\begin{aligned} & E\left[R_t^{\text{hedge funds before fees}}\right] - 1\% - \beta \times ERP \\ &= E\left[1\% + 4 \times \left(R_t^{\text{active fund before fees}} - R_t^{\text{stock index}}\right)\right] - 1\% - 0 \times ERP \\ &= E\left[1\% + 4 \times (2.20\% + \epsilon_t)\right] - 1\% \\ &= 4 \times 2.20\% + 4 \times E\left[\epsilon_t\right] = 8.80\%. \end{aligned}$$

(d) The first portfolio, denoted $P1$, provides the investor with the following exposure:

$$\begin{aligned} R_t^{P1 \text{ before fees}} &= 40\$ \times R_t^{\text{active fund before fees}} + 60\$ \times 1\% \\ &= 40\$ \times \left(2.20\% + R_t^{\text{stock index}} + \epsilon_t\right) + 60\$ \times 1\% \\ &= 40\$ \times 2.20\% + 60\$ \times 1\% + 40\$ \times R_t^{\text{stock index}} + 40\$ \times \epsilon_t. \end{aligned}$$

Denote with $X_{passive}$ the amount invested in the passive mutual fund, $X_{HF}$ the amount invested in the hedge fund and $X_{cash}$ the amount invested in cash. The second portfolio, denoted $P2$, provides the investor with the following exposure:

$$\begin{aligned} R_t^{P2 \text{ before fees}} &= X_{passive} R_t^{\text{passive fund before fees}} + X_{HF} R_t^{\text{hedge fund before fees}} + X_{cash} 1\% \\ &= X_{passive} \times R_t^{\text{stock index}} + X_{HF} \times (1\% + 4(2.20\% + \epsilon_t)) + X_{cash} \times 1\% \\ &= 4X_{HF} \times 2.20\% + (X_{HF} + X_{cash}) \times 1\% + X_{passive} \times R_t^{\text{stock index}} + 4X_{HF} \times \epsilon_t. \end{aligned}$$

Comparing terms, we see immediately that $X_{passive} = 40\$$. Also, $4X_{HF} = 40\$$, so $X_{HF} = 10\$$. Finally, $X_{HF} + X_{cash} = 60\$$, so $X_{cash} = 50\$$. The equivalent portfolio invests 40\$ in the passive fund, 10\$ in the hedge fund, and 50\$ in cash. This portfolio has the same market exposure, same alpha before fees, same expected return before fees, same volatility, and same exposure to $\epsilon_t$ .

The fair management fee for the hedge fund, denoted $f_m^{\%}$, is 4.40% in the sense that it would make the investor indifferent between the two allocations since this equates the total fees:

$$\begin{aligned} 40\$ \times 1.20\% &= 40\$ \times 0.10\% + 10\$ \times f_m^{\%} \\ f_m^{\%} &= \frac{40\$ \times (1.20\% - 0.10\%)}{10\$} = 4.40\%. \end{aligned}$$

(e) The hedge fund's expected return in excess of the risk-free rate before fees is 8.80% so the expected excess return net of a management fee of 2% is 6.80%. Therefore, a 35.3% performance fee makes the expected fee the same as above (ignoring high water marks and the fact that returns can be negative).

To see this, denote with $f_p^{\%}$ the percentage performance fee. We have:

$$\begin{aligned}
4.40\% &= f_p^{\%} \times \left(E\left[1\% + 4 \times (2.20\% + \epsilon_t)\right] - f_m^{\%} - 1\%\right) + f_m^{\%} \\
f_p^{\%} &= \frac{4.40\% - f_m^{\%}}{1\% + 4 \times 2.20\% + 4 \times E\left[\epsilon_t\right] - f_m^{\%} - 1\%} \\
f_p^{\%} &= \frac{4.40\% - 2.00\%}{4 \times 2.20\% - 2.00\%} = 35.29\%.
\end{aligned}$$

(f) Based on the example in this exercise, the hedge fund would in fact be cheap relative to the mutual fund if it were to charge 2-and-20 fees. Fees should be judged relative to the alpha that the investment manager delivers.

Hedge fund fees are high, and perhaps too high in some cases, but it is important to make an appropriate comparison when looking at funds with different market exposure, volatility, and other characteristics. As this example shows, some mutual funds may in fact be even more expensive per unit of alpha that they deliver. Another implication is that, if two hedge funds run the same strategy but at different levels of leverage, then the high-leverage fund should charge a larger fee since it should have a correspondingly larger alpha.

2. **Closed-end funds (20 points)**

Suppose a closed-end fund invests in a portfolio of stocks with return $R_t = R_f + \beta(R_{M,t} - R_f) + \epsilon_t$. The fund pays a constant dividend yield $\delta$ of the total NAV to the closed-end fund investors by liquidating a fraction of its portfolio at market value every period. It also pays a management fee (including payments to the managers and annual expenses such as custody fees etc...) a fraction $f$ of the the total NAV every period to the fund manager by liquidating a fraction of its holdings every period. The starting value of the fund is $V_0$. The expected return on the market is $E[R_M] = \mu_M$.

(a) Compute the dynamics of the NAV of the fund.

(b) Assume that the CAPM holds and thus that the discount rate to apply to the fund's cash flows is the CAPM-expected return on the underlying portfolio held by the fund. Compute net present value of the cash-flows paid out to the investor. Compute the net present value of the fees earned by the manager. (hint: note that $E_t[1 + R_{t+1}]/(1 + k) = 1$ where $k$ is the CAPM discount rate.) Do the values for the investor and for the manager depend on the systematic or the idiosyncratic risk of the underlying closed-end fund portfolio? Give some intuition.

(c) The observed discount on the Tri-Continental Corporation closed-end fund over a 26-year period was 14.4%. The average annual manager's fee was 0.44% of the NAV, and the dividend yield 2.27%. Based on these numbers compute the average closed-end fund discount implied by your formula above.

(d) Do you think management fees are a good explanation for the closed-end fund discount puzzle?

**Solution:**

(a) Let $V_t$ denote the NAV of the fund at time $t$ *net of all payouts*. From time $t$ to $t+1$ the NAV *net of all payouts* will rise or fall at the gross return $1 + R_t$, and fall by the amount of the payouts. The dynamics of the NAV of the fund are therefore given by

$$\begin{aligned} V_{t+1} &= \overbrace{V_t(1+R_{t+1})}^{\text{NAV gross of payouts}} - \overbrace{(\delta+f)V_t(1+R_{t+1})}^{\text{total payouts}} \\ &= \underbrace{V_t(1+R_{t+1})(1-\delta-f)}_{\text{NAV net of payouts}}. \end{aligned}$$

(b) Note first that

$$\begin{aligned} V_{t+n} &= V_{t+n-1}(1-\delta-f)(1+R_{t+n}) \\ &= V_{t+n-2}(1-\delta-f)^2(1+R_{t+n-1})(1+R_{t+n}) \\ &= \ldots \\ &= V_t(1-\delta-f)^n(1+R_{t+1}) \times \cdots \times (1+R_{t+n}) \\ &= V_t(1-\delta-f)^n \prod_{j=1}^{n}(1+R_{t+j}). \end{aligned}$$

Using the fact that the returns of the underlying stock portfolio are i.i.d., it follows that

$$\mathrm{E}_t[V_{t+n}] = V_t(1-\delta-f)^n(1+k)^n,$$

where $k = \mathrm{E}[R_t] = R_f + \beta(\mu_M - R_f)$ is the CAPM-expected return on the portfolio held by the fund.

In period $t+n$, the investor receives the dividend

$$\overbrace{V_{t+n-1}(1+R_{t+n})}^{\text{NAV gross of payouts}}\,\delta = \frac{V_{t+n}\delta}{1-\delta-f}.$$

The time-$t$ expected discounted value of the payouts to investors, $P_t^I$, is then

$$\begin{aligned}
P_t^I &= \mathrm{E}_t\left[\sum_{n=1}^{\infty}\frac{V_{t+n}\delta}{(1-\delta-f)(1+k)^n}\right] \\
&= \frac{\delta}{1-\delta-f}\sum_{n=1}^{\infty}\frac{\mathrm{E}_t[V_{t+n}]}{(1+k)^n} \\
&= \frac{\delta}{1-\delta-f}\sum_{n=1}^{\infty}\frac{V_t(1-\delta-f)^n(1+k)^n}{(1+k)^n} \\
&= \delta V_t\sum_{n=0}^{\infty}(1-\delta-f)^n \\
&= \delta V_t\frac{1}{1-(1-\delta-f)} \\
&= V_t\frac{\delta}{\delta+f}.
\end{aligned}$$

By the same logic, the net present value of the fees earned by the manager is

$$\begin{aligned}
P_t^F &= \mathrm{E}_t\left[\sum_{n=1}^{\infty}\frac{V_{t+n}f}{(1-\delta-f)(1+k)^n}\right] \\
&= \frac{f}{1-\delta-f}\sum_{n=1}^{\infty}\frac{\mathrm{E}_t[V_{t+n}]}{(1+k)^n} \\
&= V_t\frac{f}{\delta+f}.
\end{aligned}$$

Neither the values for the investor nor for the manager depend on the systematic or the idiosyncratic risk of the underlying closed-end fund portfolio. The reason is that the fund grows at a fair expected return. Since the investor and manager discount expected future cash flows at a discount rate that is consistent with the fair expected return, the two effects cancel out.

(c) The discount to the NAV is simply,

$$discount = \frac{V_t - V_t \frac{\delta}{\delta+f}}{V_t} = \frac{f}{\delta + f}.$$

Based on these numbers,

$$discount = \frac{0.44\%}{2.27\% + 0.44\%} = 16.24\%.$$

(d) It is likely that many phenomena contribute to explaining the discount associated with closed-end funds. Thus, we should not expect such a simple model to explain perfectly the closed-end fund puzzle. Yet, it is interesting to note that the formula generates a quantitatively realistic value for the discount. We conclude that management fees likely play an important role in explainining the closed-end fund discount puzzle.

Finally, even though the model can explain why closed-end funds tend to trade at a discount after their IPO, it does not explain the important issue of why investors buy such funds in the IPO.