Assignment 2

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Extranef 127

1. **Utility theory (10 points).** Consider an individual whose end of period wealth will be $W = Y + \epsilon$ where Y is his labor income and ϵ is an insurable risk (e.g., car accident). Assume both are jointly normally distributed with $E[Y] = \mu_y$, $E[\epsilon] = -\mu_\epsilon$, $V[Y] = \sigma_Y^2$, $V[\epsilon] = \sigma_\epsilon^2$ and $cor[Y, \epsilon] = \rho$.

   Assume that the investor is risk-averse with negative exponential utility given by:

   $$u(W) = -e^{-aW}, \quad a > 0. \tag{1}$$

   (a) Derive the coefficients of relative risk aversion and absolute risk aversion of this agent

   (b) Compute the maximum insurance premium π the individual would be willing to pay to insure against the risk ϵ. That is, find π such that

   $$E[u(Y + \epsilon)] = E[u(Y - \pi)].$$

   (c) How does π change with $\mu_Y, \sigma_\epsilon, \sigma_Y, \rho, a$. Interpret these findings. Are they reasonable?

2. **Value at Risk, Expected Shortfall, and Expected Utility (10 points).** Consider the following asset return distribution:

Probability	return
.025	-.40
.05	-.20
.10	-.10
.10	0
.10	0.05
.20	0.1
.20	0.15
.20	0.20
.025	0.30

(a) Compute the mean, standard deviation, skewness, kurtosis of this asset return.

(b) Compute the 5% and 1% Value at Risk of an investor who has a $100 million investment in that asset.

(c) Compute the the 5% and 1% Conditional expected shortfall of an investor who has a $100 million investment in that asset.

(d) Suppose an investor with a utility $u(w) = \frac{w^{1-\gamma}}{1-\gamma}$ with coefficient of relative risk-aversion $\gamma = 2$ is considering investing in this asset. The investor has total initial wealth of $W_0 = 100$. She can also invest in a risk-free asset with fixed return r_f. Suppose that the investor has to choose to invest 100% of her wealth either in the risky asset or in the risk-free rate. What is the level of risk-free rate above which she would choose to invest all her wealth in the risk-free asset? How does this cut-off rate change with the risk-aversion coefficient?

(e) Instead suppose she can invest a fraction of her wealth in the risk-free asset. What is the risk-free rate where she would choose to invest nothing in the risk-free asset? Is this rate different (higher or lower) than that identified in the previous question? Why?

3. **Normal distribution for stock returns (10 points).** Using the Python code developed in the first lecture, download daily stock returns over the period December 31^{st}, 1999, to December 31^{st}, 2019 from WRDS for the following 5 companies: Apple, General Electrics, Goldman Sachs, Microsoft, and Procter & Gamble (use the permco's given in Problem Set 1).

   (a) For each stock compute the mean and variance of daily simple returns.

   (b) Plot the empirical density function of stock returns (i.e., an histogram estimate of the underlying density function) and compare to (i) the normal distribution with the same mean and variance as the empirical distribution and (ii) the normal distribution with mean and variance of the 'winsorized' empirical distribution. To compute the 'winsorized' empirical distribution, simply keep only the daily returns with absolute value less than 4%.

   (Effectively you should make one separate graph for each stock with the empirical histogram and two different normal densities, corresponding to the two calibrations of means and variances).

   (*Hint: If* `S` *is a* `panda` *DataFrame, then the method* `S.plot.bin` *might be useful.*)

   (c) For each stock, compute the 95% and 99% Value-at-Risk and Conditional Expected Shortfall from the empirical distribution of returns, and compare to the Value-at-Risk and Conditional Expected Shortfall, that would obtain if the distributions were normal with corresponding means and variances.

   (*Hint: If* `S` *is a* `panda` *DataFrame, then the method* `S.quantile` *might be useful. Also, you might want to import the function* `scipy.stats.norm` *which contains a lot of useful methods linked to the normal distribution.*)

   (d) Based on your computations, does the normal distribution seem to describe appropriately the distribution of daily stock returns?