Assignment 8

1. **The CAPM (20 points)**

   You are given the following information about possible investments:

Asset	Expected Return	Std. Dev.	Correlation with Market	Beta
A	3.2%	15%	0.4	0.3
B	?	25%	0.6	?
C	?	35%	???	1.4
T-bills	2%	0%	0	0
Market port.	?	?	?	?

   (a) What is the volatility of the market portfolio, the beta of asset B, and the correlation of asset C with the market portfolio?

   Solution:
   From

$$\beta_A = 0.3 \tag{1}$$

   one gets

$$\sigma_M = \frac{\sigma_A corr(R_A, R_M)}{\beta_a} = \frac{0.15 \cdot 0.4}{0.3} = 0.2. \tag{2}$$

   One can now calculate

$$\beta_B = corr(R_B, R_M)\frac{\sigma_B}{\sigma_M} = 0.6 \cdot \frac{0.25}{0.2} = 0.75 \tag{3}$$

   and

$$corr(R_C, R_M) = \beta_C \frac{\sigma_M}{\sigma_C} = 1.4 \cdot \frac{0.2}{0.35} = 0.8. \tag{4}$$

   (b) Assume all assets are priced correctly according to the CAPM. What are the expected returns of the market portfolio, asset B, and asset C?

   Solution:
   The CAPM specifies

$$E(R_A) = R_{T-bill} + \beta_A(E(R_M) - R_{T-bill}), \tag{5}$$

implying

$$E(R_M) = R_{T-bill} + \frac{E(R_M) - R_{T-bill}}{\beta_A} = 3\%. \tag{6}$$

One now gets

$$E(R_B) = R_{T-bill} + \beta_B(E(R_M) - R_{T-bill}) = 5\% \tag{7}$$

and

$$E(R_C) = R_{T-bill} + \beta_C(E(R_M) - R_{T-bill}) = 7.6\%. \tag{8}$$

To summarize:

Asset	Expected Return	Std. Dev.	Correlation with Market	Beta
A	3.2%	15%	0.4	0.3
B	5%	25%	0.6	0.75
C	7.6%	35%	0.8	1.4
T-bills	2%	0%	0	0
Market port.	6%	20%	1	1

(c) In addition to the assets described above, you are told that the expected return on an additional asset D with beta of 1.2 is 8% and its standard deviation is 28%. Is this additional asset over- or undervalued relative to the existing set of assets? What is its systematic risk, its residual risk and its alpha relative to the CAPM and Information ratio.

Solution:

The CAPM implies the following return for asset D:

$$E_{CAPM}(R_D) = R_{T-bill} + \beta_D(E(R_M) - R_{T-bill}) = 6.8\%. \tag{9}$$

Since the actual expected return is 8%, the stock has an alpha of 1.2% and is therefore undervalued. The systematic risk is given by

$$\beta_D \cdot \sigma_M = 24\%. \tag{10}$$

The idiosyncratic risk is given by

$$\sigma_{\varepsilon_D} := \sqrt{0.28^2 - 0.24^4} = 14.4\bar{2}\%. \tag{11}$$

(d) Derive the maximum Sharpe ratio portfolio and give its composition, it, expected return, volatility, and sharpe ratio (assume you can invest in all assets, A,B,C, Tbills, Market, D).

hint: consider the optimal portfolio of a mean-variance investor with a risk-aversion of 1.

Solution:
The Sharpe ratio of the market is

$$SR := \frac{0.06 - 0.02}{0.2} = 0.2. \tag{12}$$

Since assets A,B,C and T-Bills are valued according to the CAPM, they have an information ratio of zero. Asset D has an information ratio of

$$IR := \frac{0.012}{0.1442} = 0.2. \tag{13}$$

The maximum Sharpe ratio is thus given by

$$SR_{max} = \sqrt{SR^2 + IR^2} = 0.216617. \tag{14}$$

2. **The momentum factor (40 points).** In this exercise we will test the CAPM using portfolio sorted based on beta.

(a) As in Problem sets 6 an 7, download data on all stocks traded on NYSE. This time, let the sample start in January, 1970 and let it end in December, 2019. Also, keep the stock that have been traded only during a limited part of the sample period. Finally, also download the HML, SMB and Momentum factors from Kenneth French's website.

(b) For each stock and each month t, calculate the average return of the stock between month t-12 and t-2. Then, in each month, sort the stocks into 10 deciles based on that average return. Compute the value-weighted returns on those portfolios.

Compute the returns on a zero-cost portfolio that goes long in the group with the highest past returns and short in the group with the lowest past returns. Compute the alpha of this strategy with respect to the market as well as the SMB and HML factors. What do you observe?

(c) Sort the stocks in each month into two groups based on the previous month's market capitalization (such that the stocks in one group are larger than in the other group). Repeat question b) for the two groups. When computing the alphas for the long-short strategies, also control for the momentum factor from Kenneth French's website. What do you observe?