Assignment 10

1. **Fund performance and fees (15 points)**

   Consider a passive mutual fund, an active mutual fund, and a hedge fund. The mutual funds claim to deliver the following gross returns:

$$R_t^{\text{passive fund before fees}} = R_t^{\text{stock index}}$$

$$R_t^{\text{active fund before fees}} = 2.20\% + R_t^{\text{stock index}} + \epsilon_t$$

   The passive mutual fund charges an annual fee of 0.10%. The active mutual fund charges a fee of 1.20% and seeks to beat the same stock market index by about 1% per year after fees. The active mutual fund has a beta of 1 and has a tracking error volatility of $\sqrt{Var(\epsilon_t)} = 3.5\%$. The hedge fund uses the same strategy as the active mutual fund to identify "good" and "bad" stocks, but implements the strategy as a long-short hedge fund, applying 4 times leverage. The risk-free interest rate is 1% and the financing spread is zero (meaning that borrowing and lending rates are equal). Therefore, the hedge fund's return before fees is

$$R_t^{\text{hedge fund before fees}} = 1\% + 4 \times \left(R_t^{\text{active fund before fees}} - R_t^{\text{stock index}}\right)$$

   (a) What is the hedge fund's volatility?

   (b) What is the hedge fund's beta?

   (c) What is the hedge fund's alpha before fees (based on the active mutual fund's alpha estimate)?

   (d) Suppose that an investor has \$40 invested in the active mutual fund and \$60 in cash (measured in thousands, say). What investments in the passive mutual fund, the hedge fund, and cash (i.e., the riskfree asset) would yield the same market exposure, same alpha, same volatility, and same exposure to ϵ_t? As a result, what is the fair management fee for the hedge fund in the sense that it would make the investor indifferent between the two allocations (assume that the hedge fund charges a zero performance fee)?

   (e) If the hedge fund charges a management fee of 2%, what performance fee makes the expected fee the same as above? Ignore high water marks and ignore the

fact that returns can be negative, but recall that performance fees are charged as a percentage of the (excess) return after management fees. Specifically, assume the performance fee is a fraction of the hedge fund's outperformance above the risk-free interest rate.

(f) Comment on whether it is clear that hedge funds that charge 2-20 fees (2% management fee and 20% performance fee) are "expensive" relative to typical mutual funds. More broadly, what should determine fees for active management?

2. **Closed-end funds (20 points)**

Suppose a closed-end fund invests in a portfolio of stocks with return $R_t = R_f + \beta(R_{M,t} - R_f) + \epsilon_t$. The fund pays a constant dividend yield δ of the total NAV to the closed-end fund investors by liquidating a fraction of its portfolio at market value every period. It also pays a management fee (including payments to the managers and annual expenses such as custody fees etc...) a fraction f of the the total NAV every period to the fund manager by liquidating a fraction of its holdings every period. The starting value of the fund is V_0. The expected return on the market is $E[R_M] = \mu_M$.

(a) Compute the dynamics of the NAV of the fund.

(b) Assume that the CAPM holds and thus that the discount rate to apply to the fund's cash flows is the CAPM-expected return on the underlying portfolio held by the fund. Compute net present value of the cash-flows paid out to the investor. Compute the net present value of the fees earned by the manager. (hint: note that $E_t[1 + R_{t+1}]/(1 + k) = 1$ where k is the CAPM discount rate.) Do the values for the investor and for the manager depend on the systematic or the idiosyncratic risk of the underlying closed-end fund portfolio? Give some intuition.

(c) The observed discount on the Tri-Continental Corporation closed-end fund over a 26-year period was 14.4%. The average annual manager's fee was 0.44% of the NAV, and the dividend yield 2.27%. Based on these numbers compute the average closed-end fund discount implied by your formula above.

(d) Do you think management fees are a good explanation for the closed-end fund discount puzzle?